Chapter 2
Friedmann Cosmology with Changing Dark Energy

The energy-momentum tensor (1.6) can be written in the form (1.5) of a perfect fluid,

$$T_{\alpha\beta} = -p_{tot}\, g_{\alpha\beta} + (\rho_{tot} + p_{tot}) u_\alpha u_\beta \ , \tag{2.1}$$

whose density, ρ_{tot}, and pressure, p_{tot}, are defined as

$$\rho_{tot} = \rho + \rho_{vac}, \qquad p_{tot} = p + p_{vac} = p - \rho_{vac} \ ; \tag{2.2}$$

the last expression here is implied by the DE equation of state (1.4).

We study the Friedmann cosmology using the Robertson–Walker metric

$$ds^2 = dt^2 - a^2(t)\left[\left(1 - kr^2\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right] \ , \tag{2.3}$$

with the dimensionless scale factor $a(t)$ being the only unkown, and $k = 0,\ -1,\ 1$ for the flat, open and closed universe, respectively; we use the system of units with $c = G = 1$.

For the expressions (2.1) and (2.3) Einstein's equations are known to reduce to the Friedmann equations, which we write as:

$$3\left(\frac{\dot a}{a}\right)^2 = 8\pi\rho_{tot} - \frac{k}{a^2}; \qquad \dot\rho_{tot} = -(\rho_{tot} + p_{tot})\, 3\,\frac{\dot a}{a} \tag{2.4}$$

(the dot always denotes the derivative in time). The second of this equations is one of the four conditions (1.7) in co-moving coordinates ($u_0 = 1,\ u_1 = u_2 = u_3 = 0$) with $\beta = 0$; it expresses energy conservation in a co-moving volume (see below). The other three conditions (1.7) require that ρ_{vac} is independent of all spatial coordinates, which is also clear from the assumptions made.

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Equation (2.4) can be combined to yield the expression for the acceleration,

$$3\,\frac{\ddot{a}}{a} = -4\pi\left(\rho_{tot} + 3p_{tot}\right)\,. \tag{2.5}$$

It shows that the expansion accelerates, decelerates, or proceeds uniformly depending on the sign of the 'effective gravitating density' $\rho_{tot} + 3p_{tot}$ (negative, positive or zero, respectively). If only DE is present, i.e., $\rho_{tot} = \rho_{vac},\ p_{tot} = p_{vac} = -\rho_{vac}$, then

$$\rho_{tot} + 3p_{tot} = -2\rho_{vac} < 0\,,$$

and expansion accelerates; thus heavy vacuum gravity is repulsive. In the opposite case of pure matter, $\rho_{vac} = 0$, the sign depends on its equation of state; usually matter is attractive, leading to deceleration.

It is convenient to introduce the co-moving volume as

$$V(t) = a^3(t), \qquad a(t) = V^{1/3}(t)\,, \tag{2.6}$$

and rewrite the Friedmann equations (2.4) in terms of it. Using $V(t)$ in the second equation as an independent variable instead of time, we obtain:

$$\frac{1}{3}\left(\frac{\dot{V}}{V}\right)^2 = 8\pi\rho_{tot} - \frac{k}{V^{2/3}}; \qquad \frac{d\left(\rho_{tot}V\right)}{dV} = -\,p_{tot}\,. \tag{2.7}$$

The first of these equations determines the time dependence of the volume, and hence of the scale factor and all other parameters. Indeed, as soon as the total density is known as a function of the volume, $\rho_{tot} = \rho_{tot}(V)$, the dependence of the latter on time is obtained by direct integration, namely:

$$t - t_0 = \int_{V_0}^{V(t)} \frac{dV}{V\sqrt{3\left[8\pi\rho_{tot}(V) - k\,V^{-2/3}\right]}}, \qquad V_0 = V(t_0)\,. \tag{2.8}$$

Thus everything reduces to the second of Eq. (2.7), which describes conservation of total energy, $\rho_{tot}V$, in the co-moving volume by implying

$$d\left(\rho_{tot}V\right) + p_{tot}\,dV = 0\,.$$

This single equation, however, contains three unknown functions, $\rho,\ \rho_{tot}$ and p:

$$\frac{d\left[(\rho_{vac} + \rho)V\right]}{dV} = -\,p_{tot} = -(p + p_{vac}) = -(p - \rho_{vac})\,. \tag{2.9}$$

The equation of state (EOS) of a single–phase matter relates its pressure and density, reducing thus the number of unknowns to two, ρ and ρ_{vac} Of course, it is impossible to determine both of them simultaneously form the single Eq. (2.9). The situation is even worse when there are $N > 1$ matter components, each with its own EOS; then all the $N + 1$ densities are unknown, with just the same single equation for all of them (see chapter III A).

Clearly, what is missing yet is the law of interaction between DE and matter, which would provide the second equation needed to determine the expansion completely. Its physical derivation, especially from the first principles, is an outstanding problem of physics and cosmology, and a great challenge to the physics theory. Since we currently do not know how to derive this equation, the only way to understand possible features of the universe seems to rely on certain plausible models (and hope that at least some of them are not very far from reality!).

In what follows we model the interaction of heavy vacuum with matter, and study cosmological solutions that stem form these models; some of the solutions exhibit remarkable properties. We will use the energy conservation Eq. (2.9) in the form more convenient for our purposes:

$$\frac{d\left[(\rho_{vac} + \rho)\right]}{dV} = -\frac{(\rho + p)}{V} \,. \tag{2.10}$$

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Silbergleit, A.S.; Chernin, A.

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