

Preface to the Third Edition

Grid generation codes represent an indispensable tool for solving field problems in applied mathematics, mechanics, physics, and other areas of practical applications. Despite the considerable success achieved in grid generation technologies, development of more efficient and sophisticated algorithms and computer codes for generating grids still remains an important problem. Serious difficulties arise in grid generation in domains with complicated boundary geometries, specifically, with discretely defined boundary segments and in the case when grids have to be adapted to solution singularities, such as boundary and interior layers, shocks, detonation waves, combustion fronts, high-speed jets, and phase transition zones. A promising tool to deal with the numerical problems having such singularities is adaptive grid generation technology. With increasing complexity of the physical problems, there is an increased need for more reliable, robust, and fully automated grid generation codes which enable one to generate suitable meshes in a uniform “black box” mode, without human interaction. The development of such grid systems is a challenging problem in computational physics and applied mathematics.

Grid technology still remains a rapidly advancing field of computational and applied mathematics. New achievements are being added by the creation of more sophisticated techniques, modification of the available methods, and implementation of more subtle tools as well as the results of the classical theories of differential equations, calculus of variations, and Riemannian geometry in the formulation of grid models and analysis of grid properties. Therefore, there is a clear need of students, researchers, and practitioners in the field of applied mathematics and industry for the creation of new books and/or updated editions of the available books which will complement the existing ones, providing a description of current developments relating to grid methods, grid codes, and their applications to the solving of actual problems.

This third edition of the monograph “Grid Generation Methods” is significantly expanded with new material that discusses recent advances in grid generation technology. It includes a description of updated grid generation methods, which were partly presented in the former monograph of the author, as well as new adaptive approaches for structured and unstructured grids and numerical algorithms

for their generation. Special attention is paid to a review of those promising approaches and methods which have been developed recently and/or have not been sufficiently covered in other monographs. In particular, this book includes an application for generating grids for immersed boundary methods. It also describes a stretching method adjusted to the numerical solution of singularly perturbed equations having large-scale solution variations, e.g., those modeling high-Reynolds-number flows. A number of functionals related to conformality, orthogonality, energy, and alignment are described. This book includes differential and variational techniques for generating uniform, conformal, and harmonic coordinate transformations on hypersurfaces for the development of a comprehensive approach to the construction of both fixed and adaptive grids in the interior and on the boundary of domains in a unified manner. The monograph is also concerned with the description of all essential grid quality measures such as skewness, curvature, torsion, angle and length values, and conformality. It gives a more detailed and practice-oriented description of control metrics for providing the generation of adaptive, field-aligned, and balanced numerical grids by means of the numerical solution of inverted Beltrami and diffusion equations in the control metrics. Some numerical algorithms are described for generating surface and domain grids. One more new feature of this book is the implementation of adaptive grid technology to the numerical solution of problems in mechanics, physics, fluids, plasmas, and nanotechnologies. Emphasis is placed on mathematical formulations, explanations, and examples of various aspects of grid generation and their applications.

This book will introduce a reader to structured and unstructured grid methods, as well as automated technologies for the generation of adaptive grids for the numerical solution of applied problems with complicated domain segments and complicated solution structures. These technologies are based on advanced algebraic, elliptic, variational, Delaunay, advancing-front, and quad-octree methods, as well as on the methods of finite differences and volumes. The technologies are indispensable for the numerical solution of differential equations, modeling various complex physical processes in energetics, ecology, industry, as well as the medical sphere. Furthermore, this book includes chapters devoted to the implementation of comprehensive grid methods into numerical codes and to the application of the codes to the numerical solution of a range of mechanical, fluid, and plasma-related problems. The new and fast-developing computational tools discussed throughout this book enable a detailed analysis of real-world problems that simply lie beyond the reach of traditional methods.

The major area of attention of this book is grid-mapping techniques. In addition, however, the author has also included an elementary introduction to basic unstructured approaches to mesh generation. A more detailed description of unstructured mesh techniques and corresponding aspects related to parallel processing, mesh quality enhancement, and mesh modification and optimization can be found in the books of the leading experts on these technologies: *Computational Grids: Adaptation and Solution Strategies* by G.F. Carey (1997), *Delaunay Triangulation and Meshing* by P.-L. George and H. Borouchak (1998), *Mesh Generation Application to Finite Elements* by P.J. Frey and P.-L. George (2008),

and *Finite Element Mesh Generation* by D.S.H. Lo (2015). These books, though, do not give a detailed introduction to advanced mapping approaches developed in recent years. Thus, the current monograph and these books complement each other, presenting a comprehensive description of all the popular grid generation approaches. As grid generation methodology is well on its way to becoming a formal subject in university curricula, the books mentioned and the current book taken together will provide materials fully sufficient to support a one-year university course related to structured and unstructured mesh technologies.

Since grid technology has a widespread application across nearly all field problems, this new edition of the monograph will be of significant interest to a broad range of readers: teachers, students, researchers, and practitioners in applied mathematics, mechanics, physics, and other areas of application. In addition, it could be used as a textbook for advanced undergraduates or for first-year postgraduate students or as a tutorial for mathematicians, engineers, and scientists who are engaged in the computation of equations in multidimensional domains with complicated boundary geometries.

Chapter 1 of this book provides a general introduction to the subject of grids. It gives an outline of structured, unstructured, hybrid, overlapping, and composite grids. The chapter delineates some of the basic classes of methods, in particular manual or semiautomatic methods, mapping methods, and unstructured methods. The chapter also includes a description of various types of grid topology and touches on certain issues of comprehensive grid codes.

Chapter 2 deals with several mathematical relations that are necessary only for the generation of grids by means of the mapping approach and which are connected with and derived from the metric tensors of coordinate transformations. As an example of an application of these relations, the chapter presents a technique aimed at obtaining conservation-law equations in new fixed or time-dependent coordinates. In the procedures described, the deduction of the expressions for the transformed equations is based only on the formula for the differentiation of the Jacobian of the coordinate transformations.

Very important issues of grid generation, connected with a description of grid quality measures in forms suitable for formulating grid techniques and efficiently analyzing the necessary mesh properties, are discussed in Chap. 3. The definitions of the grid quality measures are based on the metric tensors and on the relations between the metric elements considered in Chap. 2. Special attention is paid to the invariants of the metric tensors, which are the basic elements for the definition of many important grid quality measures. Clear algebraic and geometric interpretations of the invariants are presented.

Chapter 4 describes a stretching method based on the application of special nonuniform stretching coordinates in the regions of large variation of the solution. The use of stretching coordinates is extremely effective for the numerical solution of problems with boundary and interior layers. The chapter acquaints the reader with various types of singularity arising in solutions to equations with a small parameter affecting the higher derivatives. The solutions of these equations undergo large variations in very small boundary and interior zones, called boundary or

interior layers, respectively. The chapter gives a detailed description of the qualitative properties of solutions in such layers. Besides the well-known exponential layers, three types of power layer common to bisingular problems having complementary singularities arising from reduced equations are described. Such equations are widespread in applications, for example, in viscous gas dynamics. The specification of the stretching functions is given for each type of basic singularity. The functions are defined in such a way that the singularities are automatically smoothed with respect to the new stretching coordinates. The chapter gives the description of a procedure to generate intermediate coordinate transformations with the stretching functions. The transformations are suitable for smoothing both exponential and power layers. The grids derived with such coordinate transformations are often themselves well adapted to the expected physical features. Therefore, they make it easier to provide dynamic adaptation by taking part of the adaptive burden on themselves.

The simplest and fastest technique of grid generation is the algebraic method of transfinite interpolation described in Chap. 5. Of central importance in transfinite interpolation are the blending functions which provide the matching of the grid lines at the boundary and interior surfaces. Examples of various types of blending functions are reviewed, in particular the functions defined through the basic stretching coordinate transformations for singular layers. These transformations are dependent on a small parameter so that the resulting grid automatically adjusts to the respective physical parameter, e.g., viscosity, Reynolds number, or shell thickness, in practical applications. This chapter also gives a description of a procedure for generating triangular, tetrahedral, or prismatic grids through the method of transfinite interpolation. The chapter ends with a concise presentation of drag and sweeping meshing methods.

Chapter 6 is concerned with grid generation techniques based on the numerical solution of systems of partial differential equations. Generation of grids from these systems of equations is largely based on the numerical solution of elliptic, hyperbolic, and parabolic equations for the coordinates of grid lines which are specified on the boundary segments. The elliptic and parabolic systems reviewed in the chapter provide grid generation within blocks with specified boundary point distributions. These systems are also used to smooth algebraic, hyperbolic, and unstructured grids. A very important role is currently played in grid codes by a system of Poisson equations defined as a sum of Laplace equations and control functions. This system was originally considered by Godunov and Prokopov and further generalized, developed, and implemented for practical applications by Thompson, Thames, Mastin, and others. The chapter describes the properties of the Poisson system and specifies expressions for the control functions required to construct nearly orthogonal coordinates at the boundaries. Hyperbolic systems are useful when an outer boundary is free of specification. The control of the grid spacing in the hyperbolic method is largely performed through the specification of volume distribution functions. Special hyperbolic and elliptic systems are presented for generating orthogonal and nearly orthogonal coordinate lines, in particular those proposed by Ryskin and Leal. The chapter also reviews some parabolic and

high-order systems for the generation of structured grids and describes adaptive mesh generation for steady and unsteady simulations.

Chapter 7 reviews the development of variational methods applied to grid generation. Variational grid generation relies on functionals related to grid quality: smoothness, orthogonality, regularity, aspect ratio, adaptivity, etc. By the minimization of a combination of these functionals, a user can define a compromise grid with the desired properties. The chapter discusses a variational approach for generating harmonic maps through the minimization of energy functionals, which was suggested by Dvinsky. Several versions of the functionals from which harmonic maps can be derived are identified.

Methods developed for the generation of grids on curves and surfaces are discussed in Chap. 8. The chapter describes the development and application of hyperbolic, elliptic, and variational techniques for the generation of grids on parametrically defined curves and surfaces. The differential approaches based on the Beltrami equations were proposed by Warsi and Thomas, while the variational methods rely on functionals of surface grid quality measures.

Chapter 9 is devoted to the implementation of inverted Beltrami equations with respect to control metrics for the generation of multidimensional adaptive grids via a mapping approach. The approach is based on mapping from a simple structured or an unstructured grid in the logical domain to a curvilinear grid with the desired properties in the physical domain. The control metrics provide efficient and simply defined conditions for various types of grid adaptation, particularly grid clustering according to given function values and/or gradients, grid alignment with given vector fields, and combinations thereof. The corresponding formulas of control metrics providing these grid properties are demonstrated. Using this approach, both adaptive and fixed grids can be generated in a unified manner, in arbitrary domains.

Numerical algorithms for generating grids by mapping approaches based on the solution of inverted Beltrami and diffusion equations are presented in Chap. 10. Furthermore, the chapter includes numerical methods (finite differences, finite elements, and spectral elements) for finding grid nodes via the solution of the inverted Beltrami equations. Basic approaches in parallelizing the mesh generating process are outlined in Chap. 10.

Chapter 11 describes techniques aimed at controlling grid properties with special control metrics in inverted Beltrami and diffusion equations. The control metrics provide efficient and straightforwardly defined conditions for various types of grid adaptation, particularly grid clustering according to given function values and/or gradients, grid alignment with given vector fields, and combinations thereof.

The subject of unstructured grid generation is discussed in Chap. 12. Unstructured grids may be composed of cells of arbitrary shape, but they are generally composed of triangles and tetrahedrons. Tetrahedral grid methods described in the chapter include Delaunay procedures and the advancing-front method. The Delaunay approach connects neighboring points (of some previously defined set of nodes in the region) to form tetrahedral cells in such a way that the sphere through the vertices of any tetrahedron does not contain any other points. In the advancing-front method, the grid is generated by building cells one at a time,

marching from the boundary into the volume by successively connecting new points to points on the front until all the unmeshed space is filled with grid cells. The chapter also outlines the quod-octree approaches.

Chapter 13 is devoted to the implementation of adaptive grid techniques to provide solutions to applied problems. The equations for the evolving grid are incorporated into a single implicit time step in which the grid and the physical solution evolve together. The chapter discusses numerical solutions of three-dimensional diffusion equations with boundary and interior layers on adaptive grids with node clustering in the zones of large solution variations and also touches some applications of adaptive grid technologies to solutions of two-dimensional gas dynamics problems. The application of adaptive grids to calculations of some magnetically confined plasma problems and a tsunami wave run-up on a coastal region is described as well in Chap. 13. This chapter describes the results of numerical modeling of temperature gauged in burning solid fuel via an inserted thermocouple. Subsurface thermocouple sensors are very important technical devices employed to gauge heat fluxes in complicated heat-stressed frameworks in various heat-diffusion mechanisms and in burning solid fuels. Incidental problems related to the accuracy of the sensors' temperature data may appear. The primary source of inaccuracy is the difference in thermal properties of the materials of the thermocouple and of the surrounding medium. High gradients of temperature in heated materials lead, as a rule, to an increased heat transfer from the surface, since the thermal conductivity of metallic thermocouples is higher than that of the surrounding substance. Additional problems may appear due to a variation in the distances to the surface of heat transfer caused by the pyrolysis of the substance. The chapter presents an adaptive grid technology to investigate these problems. Chapter 13 also demonstrates the use of adaptive grids for the numerical simulations of nanopore formations in metals. It also includes a new grid generation approach aimed at solving problems by immersed boundary methods. This approach to grid generation does not involve the initial triangulation of the boundary of a domain, but a global numerical grid is first constructed in a larger domain with mesh refinement near the specified boundary points. Next, boundary and interior cells of the domain under consideration are selected, which comprise its numerical grid. As a result, a grid with thin cells near the boundary is obtained. The thinner these cells, the better the approximation of the domain. The global grid is generated by means of inverted diffusion equations for a spherical metric tensor. This approach is also suitable for domains with boundaries specified by an implicit analytic function.

Acknowledgement. The author is greatly thankful to G.S. Liseikin who prepared the text of the manuscript in L^AT_EX code. Thanks go as well to G. Lukas for correcting the author's English.

The author is very conscious of helpful suggestions in geometry, algebra, and numerical techniques made by his colleagues, Profs. Borisov, Churkin, Glasser, Kuzminov, Sharafutdinov, and Shvedov.

The author is also greatly obligated to the researchers who responded to his requests and sent files of their papers, namely T.J. Baker, D.A. Field, E. Ivanov, P. Knupp, G. Liao, M.S. Shephard, N.P. Weatherill, and P.P. Zegeling, and to those who prepared illustrations, namely B.S. Azarenok, A.A. Charakhch'yan, V.A. Garanja, A.H. Glasser, S.A. Ivanenko, A.M. Kharitonchick, A.V. Kofanov, I.N. Kudryavtseva, A.S. Lebedev, Yu.V. Likhanova, G.S. Liseikin, D.V. Patrakhin, A.D. Rychkov, and I.A. Vaseva.

Work on this book was supported in part by RFBR through Grant No. 16-01-00455 and by the Institute of Computational Technologies SB RAS and Novosibirsk State University.

This book was published with the permission of the Academic Councils of the Institute of Computational Technologies of the Siberian Branch of the Russian Academy of Sciences and Novosibirsk State University.

Novosibirsk, Russia
September 2016

Vladimir D. Liseikin

Grid Generation Methods

Liseikin, V.D.

2017, XX, 530 p. 151 illus., 14 illus. in color., Hardcover

ISBN: 978-3-319-57845-3