Preface

Duality is a beautiful concept that pervades all natural phenomena. In classical mechanics, each potential energy principle is linked with a unique complementary energy principle via the traditional Legendre transformation. The popular Hellinger–Reissner principle is actually a special saddle Lagrangian duality theory in convex analysis and optimization, which lays a foundation for hybrid/mixed finite element methods in computational mechanics and primal-dual interior point methods in mathematical programming. However, this one-to-one duality is broken in nonconvex systems due to a so-called duality gap produced by the modern Fenchel–Moreau transformation. In finite elasticity, the existence of a pure stress-based complementary energy principle was a well-known open problem, existing for several decades. In global optimization and computer science, many nonconvex problems are considered as NP-hard (Non-deterministic Polynomial-time hard) due to the lack of global optimality criteria. Unfortunately, this well-known difficulty is not fully recognized in computational mechanics due to the significant gap between engineering mechanics and global optimization. Indeed, engineers and scientists are mistakenly attempting to use traditional finite element methods and commercial software for solving nonconvex mechanics problems.

Canonical duality theory is a breakthrough methodological theory that can be used not only for modeling complex systems within a unified framework, but also for solving a large class of challenging problems in multidisciplinary fields of engineering, mathematics, and sciences. The concept of canonical (i.e. one-to-one) duality is from the traditional Chinese Yin-Yang philosophy. Niels Bohr realized its value in quantum mechanics. Based on this philosophy, a unified canonical duality framework in mathematical physics was first proposed in the work by Gao and Strang in 1989. This framework reveals an intrinsic duality in nonconvex systems and lays a foundation for the canonical duality theory. The canonical duality theory was developed originally from nonconvex mechanics (1989–2000) and then generalized to global optimization (2000–2010). This theory is composed mainly of (1) a canonical dual transformation, which can be used to formulate perfect dual problems without duality gap; (2) a complementary-dual principle, which solved the open problem in finite elasticity and provides a unified analytical solution form for general nonconvex/nonsmooth/discrete problems; (3) a triality theory, which can be used to identify both global and local optimality conditions and to develop powerful algorithms for solving challenging problems in complex systems. During the past 10 years, the canonical duality theory has been applied successfully for solving a wide class of real-world problems in chaotic dynamics, computational biology, filter design, information technology, logistics and transportation, machine learning, network communication, nonlinear partial differential equations (PDEs) in finite deformation theory, operations research, post-bifurcation, phase transitions in solid mechanics, and materials science, as well as modeling of complex systems, etc.

The original motivation of this book was a colloquium talk presented by David Yang Gao at UC Berkeley in 2013. This volume provides a comprehensive review of the canonical duality theory, its methodology, and algorithms for solving challenging problems in complex systems with applications in nonconvex analysis, variational inequalities, large deformation problems, global optimization, and computational mechanics. It is the authors' hope that by reading this book, the readers should be able to see the beauty and unity of the canonical duality theory and its potential applications in multidisciplinary fields.

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