

## Chapter 2

# The Logic of Guilt, Innocence and Legal Discourse

Andreas Kapsner

**Abstract** Even though the notions of guilt and innocence are two of the most important ones for any legal system, there are surprisingly deep and complicated confusions that entangle them. There is much clarifying work to be done here, part of which is to investigate the logical principles governing these concepts. In this paper, I build on Larry Laudan's analysis in his book *Truth, Error and Criminal Law*. Though I have great respect for the conceptual lines he draws, I will show that the logical relationships of these concepts is richer than Laudan makes out. In order to do so, I will use one of the oldest aids to logical reasoning, the square of oppositions. The square captures these logical relations well, albeit only with some idealizing assumptions in place. I will go on to argue that these idealizations are in harmony with a useful way to model legal deliberation in constructive logic.

**Keywords** Legal reasoning · Legal discourse · Guilt · Innocence · Constructive logic · Larry Laudan

## 2.1 Introduction

It is clear that the notions of guilt and innocence are two of the most important ones for any legal system. It might even seem that we could not claim to understand what the legal system is about at all if there should be any principled difficulty in understanding these concepts. But in fact it turns out that there are surprisingly deep and complicated confusions that entangle these two notions. There is much clarifying work to be done here, part of which is to investigate the logical principles governing these concepts.

In this essay, I will concentrate on the conceptualization of guilt and innocence in the jurisdiction of the so-called common law countries, especially the United States, where criminal trials are judged by lay juries. There are several reasons for this, the most important of which is the following: My paper is heavily influenced by, and in

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part a commentary on, Larry Laudan's book *Truth, Error, and Criminal Law*, which focuses on the legal system of the U.S. [10].

As most readers of this volume will know, Laudan is one of the leading philosophers of science of our time, and the great joy in reading his work on legal matters is that he asks precisely the questions a philosopher would ask.<sup>1</sup> These are at times very different from the questions legal scholars are accustomed to ask and answer, and it is very illuminating to read an exposition of their thinking in a language that is much closer to home.<sup>2</sup> Though the book is altogether informal, by natural extension this language includes the notions of philosophical logic, and I hope to further push Laudan's project along by illustrating and extending his discussion of guilt and innocence with the aid of logical tools.

Laudan's main point about the usage of "guilt" and "innocence" by legal scholars and practitioners is that it is ambiguous: A factual reading (What happened?) is often conflated with an evidential reading (What does the evidence indicate happened?). His proposal is to disambiguate these readings by introducing new terminology that allows unequivocal reference to the different concepts at play.

I will show that the logical relationships of these concepts is richer than Laudan makes out. In order to do so, I will use one of the oldest aids to logical reasoning, the square of oppositions. The square captures these logical relations well, albeit only with some idealizing assumptions in place. I will go on to argue that these idealizations are in harmony with a useful way to model legal deliberation in constructive logic.

## 2.2 "Not Guilty" and "Innocent"

It would be unfair to say that it took a philosopher like Laudan to point out that there is widespread confusion about the legal meaning of the terms "guilt" and "innocence". Instead, this is something that legal scholars and practitioners themselves are quite aware of. Some of them, however, seem to believe that this kind of confusion is only prevalent *outside* of the legal profession.

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<sup>1</sup>It is with pleasure that I follow the suggestion of an anonymous referee to mention Susan Haack's recent [7] as another example of an eminent philosopher taking up legal matters. In contrast to both Laudan's book and the present paper, she also addresses questions of proof outside of criminal trials, i.e. in private law. An interesting discussion of the difference between the notions of proof in a criminal trial and proof in a civil case is [17], which supplies a quite different analysis of the nature of proofs of guilt or innocence from the present paper.

<sup>2</sup>In turn, it is highly interesting to see the reaction of legal scholars to the book. Here is one example: "Enter Laudan, the distinguished epistemologist, training his sophisticated analytical apparatus on the legal system. [...] The result is an insightful, trenchant, exhilarating, bracing, disturbing exploration of the legal system that leaves no doubt that factual accuracy could be pursued considerably more ruthlessly, and that Laudan thinks it is a reproach to the legal system that it does not do so." [1], p. 199. It is intriguing, and maybe somewhat worrying as well, that the author of that review is not inclined to agree with Laudan on the last point.

Even if true, this would of course not mean that these confusions should be left unattended by legal professionals, especially in a legal system that relies on lay people (the jury members) to decide its most important questions about particular instances of guilt and innocence. In a very interesting piece, lawyer Hugh Duvall describes the point at which he usually addresses the problem in his closing argument, thereby relating the “most important piece of information I pass to the jury before it begins its deliberations.”<sup>3</sup>

It is the point at which I educate jury members to the distinction between their ability to return a verdict of “not guilty” and their inability to return a verdict of “innocent.”

[...]

Even if you are very sure the defendant is guilty, but the state has not proven it “beyond a reasonable doubt,” then it is your sworn duty to return a verdict of “not guilty.” [...] I attempt to hammer home, relentlessly, that the jury’s job has very little to do with the concept of “innocence.” Its job is [...] one of analyzing what evidence the state has presented and determining whether it is enough to satisfy the jury that there is no reason to doubt the defendant’s guilt. It is the jury’s job to sniff out the reasonable possibility that the defendant may not be guilty.

The idea that Duvall nicely summarizes and that, according to Laudan, is “taught in every law school in the world”,<sup>4</sup> is thus the following: Even though “guilty”/“guilt” and “innocent”/“innocence” are understood to be antonyms in vernacular English, one should not commit the error of thinking that a verdict of “not guilty” was the same as a proclamation of the innocence of the defendant. Indeed, a common law jury will never decide on the innocence of the defendant. As Duvall rightly notes, the only possible verdicts are “guilty” or “not guilty”. His claim is that these notions are to be understood as evidentially constrained ones. “Guilty” is the appropriate verdict if the evidence produced by the prosecution suffices to meet a given standard of proof, normally “beyond reasonable doubt”. “Not guilty” is the appropriate verdict if the evidence fails to meet this strict standard. Innocence, on the other hand, is not an evidentially constrained concept. A person is innocent of a crime if and only if she did not commit that crime, simple as that.<sup>5</sup>

Although the conceptual distinction is exactly on the right track, there are some problems with Duvall’s claim that it is captured already in the legal use of the terms, or even that it can possibly be so captured without the introduction of further vocabulary.

Standing in the way of the latter possibility are seemingly mundane problems of easy expressibility: while the notion of “not guilty” has a natural contradictory, “guilty”, there is a lack of a suitable or commonly used expression that is contradictory to “innocent”. On the other hand, “innocent” is readily turned into a noun,

<sup>3</sup>See <http://www.defendingoregon.com/innocent-v-guilty/>, last accessed on July 5th 2017.

<sup>4</sup>Reference [10], p. 99.

<sup>5</sup>A note on pronouns: When they refer to unspecified persons (which, as it happens, is only the case when I talk about defendants), these will be male in odd numbered sections and female in even numbered sections.

“innocence”, whereas “not guilty” does not allow such a transition.<sup>6</sup> These may seem like accidental and trivial problems, but when one tries to regiment the use of such terms, it is important to give people the linguistic resources to express all they want using the right concepts. Otherwise, the distinction will quickly be disregarded.

And indeed, one finds that common usage does not really respect the distinction between “not guilty” and “innocent” in the way Duvall wants to have it understood. Especially in press reports, one often reads “The defendant was pleading ‘innocent’” or “The verdict was ‘innocent’”. One interesting explanation for this Duvall offers is that the journalists are well aware of their equivocation, but are willing to pay the price of inaccuracy for the gain of greater safety (ironically enough, greater *legal* safety): In writing “not guilty” one incurs the risk that somewhere along the way from draft to printing press, the word “not” is lost. Thus one risks great danger to the reputation of the defendant and, in form of legal charges, to the news company one works for. The meaning of “innocent”, on the other hand, will survive many typos and virtually never turn into its opposite; thus, the above noted lack of a lexicalized contradictory comes as a virtue here.

This, so Duvall claims, is how a distinction that is clearly understood by the legal profession gets corrupted by the accidental needs of the publishing industry and ends up in widespread confusion on the side of the laymen. And indeed, the point about the press can be corroborated. The Associated Press Style Book, for example, advises: “Use *innocent*, rather than *not guilty*, in describing a defendant’s plea or a jury’s verdict, to guard against the word *not* being dropped inadvertently.”<sup>7</sup>

## 2.3 New Vocabulary

Duvall correctly locates the most important source of confusion: To mistake assertions about the strength of evidence for a crime with assertions about the crime itself. However, Laudan’s book brings in a lot of evidence that the distinction is not quite as clearly marked and adhered to in legal language as Duvall takes it to be.<sup>8</sup> Laudan takes this to warrant the introduction of some clarifying terminology. His solution is to use “guilt” and “innocence” as the natural antonyms that they are and

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<sup>6</sup>There is “acquittal” but that carries a factual commitment to the actual outcome of a trial that I think neither is nor should be part of the meaning of “guilty”, and should therefore not be part of the meaning of a candidate antonym either.

<sup>7</sup>Reference [6], p. 122.

<sup>8</sup>Apart from many uses of the terms by members of the legal profession that showcase the same ambiguity that common use does, the distinction is also used in other, quite different senses. Sometimes is used to mark a difference in culpability of the defendant. Here, the idea is that, even though someone may not be “innocent” because he in fact has done what the prosecution claims, he may indeed be “not guilty”, because he did not act voluntarily (cases of insanity or hypnosis might fall under this category) or in self-defense. It seems clear that it would be better to have different terminology to mark this distinction, but I will for the most part ignore this usage and the cases that give rise to it in this paper.

to disambiguate the two ways of understanding them by giving them labels, namely “material” and “probatory”.<sup>9</sup>

*Material innocence* is given if the defendant simply did not commit the crime. Likewise, *material guilt* is given if the defendant did commit the crime. *Probatory innocence*, on the other hand, is given if at the trial there is not enough evidence to meet the relevant standard of proof to lead to a conviction. *Probatory guilt*, finally, is given if there is enough inculpatory evidence to lead to a conviction. It is the probatory notions, not the material ones, that the verdict of a jury should be expressed in. The normal verdict “guilty” corresponds to “probatorily guilty”, “not guilty” corresponds to “probatorily innocent”. “Materially innocent” is what legal scholars, on Duvall’s account, call simply “innocent”, while “materially guilty” has no simple correlate in the legal vernacular.

As these concepts are at the core of this paper, I sum them up in a table for further reference:

Laudan	Traditional	Explication
Materially innocent	Innocent	Did not commit crime
Materially guilty		Did commit crime
Probatorily innocent	Not guilty (verdict)	ev. below standard of proof
Probatorily guilty	Guilty (verdict)	ev. above standard of proof

Maybe surprisingly, at least one legal scholar, Ronald Allen, applauds this call to linguistic reform by an outsider. He writes:

[Laudan] makes a powerfully useful distinction between material guilt (did the guy actually do the crime) and probatory guilt (does the evidence satisfactorily prove that the guy actually did the crime) and shows, convincingly, how much present confusion rests upon the failure of legal scholars to have previously made and understood the significance of this distinction [1], p.199).

It seems clearly worthwhile, then, to further investigate these reframed concepts.

## 2.4 Logical Relationships

In particular, I am interested in the logical relationships holding between the material and the probatory notions. On this topic, Laudan notes:

There is a salient asymmetry between the two pairs of distinctions. It consists in the fact that a) while a finding of [probatory guilt] sustains (fallibly) the assertion of [material guilt] (that is, the legal system justifiably assumes that someone proved to be guilty is genuinely

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<sup>9</sup>Though Laudan does not discuss the above problems with expressibility, his solution meets them admirably.

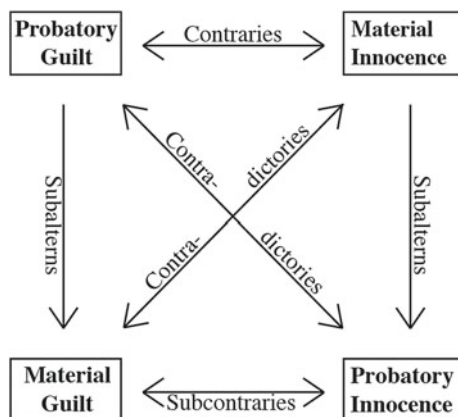
guilty), b) a finding of [probatory innocence] (a “not-guilty” verdict) warrants no inference about [material innocence] ([10], p. 96).<sup>10</sup>

I take it to be admissible to interpret his talk of “sustaining an assertion” as a form of logical entailment (ignore the “fallibly” for a moment, I will come back to it).

Now, although the difference that Laudan points out might indeed be salient, the alleged asymmetry turns into quite a pleasing *symmetry* once we arrange and inspect the items adequately.

Indeed, the four concepts can be neatly mapped to the four corners of a *square of oppositions*, one of the oldest items in the logician’s toolbox. Used traditionally to analyze modal and quantified statements,<sup>11</sup> it has recently regained scholarly attention and been applied in a variety of new areas.<sup>12</sup>

When we apply it to the topic at hand, Laudan’s four new notions of guilt and innocence, we get the following:



THE SQUARE OF GUILT AND INNOCENCE

The traditional explanation of the relations in this diagram were the following:

- Two propositions are *contradictory* iff (if and only if) they cannot both be true and they cannot both be false.
- Two propositions are *contraries* iff they cannot both be true but can both be false.
- Two propositions are *subcontraries* iff they cannot both be false but can both be true.
- A proposition is a *subaltern* of another iff it must be true if its superaltern is true, and the superaltern must be false if the subaltern is false.

<sup>10</sup>Notation altered. Laudan abbreviates his new notions with subscripted p’s and g’s, as in “guilt<sub>p</sub>”. This economy maybe pays off in a book length endeavor such as his, but I find the spelled out terms easier to read.

<sup>11</sup>See [13] for a history of the square as well as an account of certain problems involving non-denoting terms, which we can safely ignore, as they will not be relevant to the following application.

<sup>12</sup>See, for example, [3].

Given common (but not uncontroversial) assumptions about entailment, one can reformulate the last statement by saying that the supernaltern (one of the upper corners) entails the subaltern (the corner straight below it).

Now, on to motivate the relations the square suggests: The diagonals deserve the least amount of explanation. We take it for granted that either the defendant committed the crime, or that she did not commit the crime.<sup>13</sup> Thus, material guilt and material innocence form a pair of contradictories. More debatable, we here assume that in all trials, the relevant standard of proof has either been reached or not. Thus, we take probatory guilt and probatory innocence to be contradictories as well, even though, admittedly, it is (at the very least) often difficult to *know* whether the standard of proof has been reached.<sup>14</sup>

That probatory guilt and material innocence are contraries means that we assume that against someone who genuinely did not commit a crime, there will not be enough evidence to meet the standard of proof. One could say that this standard is set as high as it is, in the US for example “beyond reasonable doubt”, in an attempt to assure the contrariness of the two notions.

On the other hand, because of the high standard of proof it may well happen that someone who actually committed a crime will be acquitted, because her guilt could not be proven beyond all reasonable doubt. Thus, material guilt and probatory innocence might well hold at the same time, which is entailed by the subcontrary relation between them.

Lastly, looking at the vertical sides of the square, we see that probatory guilt entails material guilt, just as Laudan has pointed out. Also, as Laudan has noted, there is *no* implication from probatory innocence to material innocence. Probatory innocence just means that the evidence presented is not enough to put to rest all reasonable doubt. However, there is an implication in the other direction, for the same reason that justifies the contrariety arrow at the top of the square: If the defendant did not commit the crime, it will not be possible to present enough evidence to meet the standard of proof.

Now, it should be clear that there is some idealization involved in proclaiming these logical relations. Maybe a bit less clear is that these relations depend heavily on some core values that have shaped Western legal systems. I will first turn to these and then return to the question of idealization.

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<sup>13</sup>As noted above, I put aside cases of self-defense, insanity etc. Putting them back into the picture would complicate things considerably (but not impossibly). The trouble is not just terminological, but is rooted in different standards of proof and distributions of the burden of proof attached to such cases. I will go on to show that these matters have a profound influence on the logical relationships between the concepts.

<sup>14</sup>I will briefly come back to the standard of proof below; for an in depth exploration, see Chap. 3 of Laudan's book.

## 2.5 Norms, Values and Legal Principles

The norms and values that are embedded in this square are indeed amongst the most central in our legal reasoning. They boil down to this: It is more harmful to society to convict innocent citizens than to let guilty criminals go free.

Laudan has collected a very nice succession of quotes that show how different thinkers have tried to quantify this difference in harm. The options range from *twice as harmful* to *a thousand times as harmful*,<sup>15</sup> hardly a range that indicates even a rough consensus. Indeed, there is even legal and philosophical scope for questioning this imbalance *in principle*<sup>16</sup>; nonetheless, few would dispute that it is deeply engrained in our traditional understanding of justice, no matter that we cannot flesh it out with exact numbers.

There are many ways in which the legal systems try to avoid punishing the innocent, even if that means that guilty criminals will go free. Among the most important ones are the following principles: The distribution of the *burden of proof*, the *presumption of innocence* and the notion of *proof beyond reasonable doubt*. If we squint at them a bit, it is possible to see the latter two principles rolled up into the first one, the distribution of the burden of proof. The point of taking this somewhat simplifying view is that we will be able to do with one parameter in the logical model of legal litigation later on.

The *burden of proof* determines who has to prove what. To safeguard against false convictions, the burden of proof is on the prosecution in criminal cases.<sup>17</sup> From the outset the prosecution has to deliver proof of the defendant's guilt, not the defendant of his innocence. Both his testimony and his plea (we assume he pleads "Not guilty") will stand as long as what he says is not disproven.

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<sup>15</sup> 'Tis much more Prudence to acquit two Persons, tho' actually guilty, than to pass Sentence of Condemnation on one that is virtuous and innocent. Voltaire (1749).

It is better that five guilty persons should escape punishment than one person should die. Matthew Hale (1678).

It is better that ten guilty persons escape [punishment] than that one innocent suffer. William Blackstone (eighteenth century).

I should, indeed, prefer twenty guilty men to escape death through mercy, than one innocent to be condemned unjustly. John Fortesque (1471).

It is better a hundred guilty persons should escape than one innocent person should suffer. Benjamin Franklin (1785).

It is better . . . to acquit a thousand guilty persons than to put a single innocent man to death. Moses Maimonides (twelfth century) [10], p. 63.

<sup>16</sup> Laudan himself has some doubts, and legal scholar Daniel Epps has just published a thought provoking all out attack on the idea in the Harvard Law Review ([5]). He argues that it is a remnant of a time in which capital punishment was the only possible outcome of a trial that found defendants guilty of major crimes, and that it has outlived its social utility in a time when most verdicts are of a more reversible nature.

<sup>17</sup> This distribution of the burden of proof is as basic as a legal principle can get. Nonetheless, the British government recently gave out an information brochure that managed to get this basic point exactly backwards. See <http://www.independent.co.uk/news/uk/home-news/ministry-of-justice-gets-law-terribly-wrong-in-its-guide-to-courts-10016086.html>, last accessed on July 5th 2017.



However, the burden of proof does not necessarily have to stay on the prosecution. The course of a trial might shift the burden of proof away from them and onto the defense. For example, if the defendant pleads not guilty on grounds of insanity, then, depending on the country and state, he might have to bear the burden of proof for this claim.<sup>18</sup> It is the task of the judge to keep track of, decide on, and inform the jury about the burden of proof.

The *presumption of innocence* assures that at the beginning of the trial, the default assumption is that the defendant is innocent. Only if there is enough evidence to ensure his guilt should this presumption be given up. It is actually a matter of scholarly debate whether this is an independent principle or just a corollary of the burden of proof being on the prosecution. As Laudan writes, it seems to follow almost automatically, just as the presumption of innocence seems to entail the distribution of the burden of proof:

If the state bears the full burden of proof, then, of course, one might say, the defendant is presumed innocent. Contrariwise, if the defendant is genuinely presumed innocent, then it naturally follows that the state must defeat that presumption by proving his guilt ([10], p.90).

Laudan's subsequent discussion shows that things are in fact not that simple,<sup>19</sup> but we will leave it at that.

The last item is the high *standard of proof* that is set at criminal trials. It can be seen as an elaboration on what exactly the burden of the burden of proof amounts to. It is not enough that the evidence makes it slightly more likely that the defendant is guilty than not. In the US, the evidence needs to prove this guilt "beyond all reasonable doubt". Though it is not completely clear what this means in practice, it is clear that it is meant to be an extremely high standard of proof.

There have been many attempts to somehow quantify this imbalance, for example in terms of probabilities, but the practical feasibility of this is evidently questionable. Not only is it hard in practice to know how exactly a given piece of evidence influences the probability of material guilt and innocence. It also seems that in order to set the right threshold, we should first have to determine how exactly the optimal ratio between false acquittal and false convictions should look; not a straightforward task, as hinted at above. How to determine the standard of proof, how to express it and how to interpret the expressions that have been given in the past is one of the major

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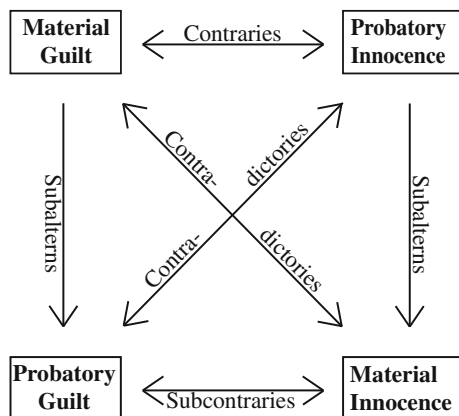
<sup>18</sup>In many states of the U.S., this shift of the burden of proof was legislated after a famous assassination attempt on president Reagan in 1982. The attacker's plea of *not guilty on grounds of insanity* was successful, not because he could prove his insanity, but because the prosecution could not prove his sanity. After this unpopular outcome, many states changed their laws so that a man or a woman in similar circumstances would be convicted.

<sup>19</sup>Mark the name of the principle, for a start. If Duvall had been right and innocence had (obviously to legal professionals) nothing to do with the job of the jury, then why should the jurors presume something that is patently unrelated to their task? Of course, this may once again only be an allowance to accidental features, this time the above noted lack of a lexicalized noun form of "not guilty". The "presumption of not-guiltiness" might well have been too clumsy for the legislators, but to suppose that they willingly introduced an intense confusion into a clear conceptual outlay just because they could not find a proper way to express themselves seems a bit too drastic. Unlike journalists, legal professionals should have been unwilling to pay such a high price.

areas of puzzlement in legal epistemology. A good start on these matters is, again, Laudan's book; another logically informed discussion is [4]. For the rest of the paper, I will suppose that these problems are somehow surmountable and return to my claim that these values and principles are reflected in the square.

Without these principles, the square would indeed look quite different. Imagine a society, maybe one scared by terrorist attacks, that thinks it is more important not to let guilty criminals go free than to avoid convicting innocent people. In the legal system such a society might set up, a defendant would have to bear the burden of proof and have to prove, maybe beyond reasonable doubt, his innocence. If unsuccessful, he would be convicted on the basis of a presumption of guilt.

Not surprisingly, the notions in such a legal system would also form a logical square, only this time with the lower corners on top:



THE SQUARE OF TERROR

Now, probatory guilt and material innocence could well exist together, when the innocent defendant is unable to reach the high standard of proof for his innocence. Likewise, it would be impossible to be materially guilty and probatorily innocent, that is, to be a criminal and escape punishment.

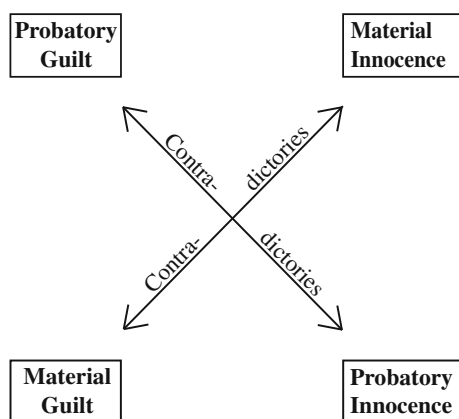
Of course, this square would likewise be subject to the idealizing assumptions I mentioned above, and which I will now discuss in more detail.

## 2.6 Idealizations

Let us then turn back to our original square and address the idealizations that are necessary to justify the logical relations in it. For example, the square invites us to draw an inference from the material innocence of a defendant to her probatory innocence. If this really would be such a sure conclusion, then why would we need skilled defense attorneys in the first place?

In truth, without a fair bit of idealization, there are no logical implications at all between the four corners, except for the diagonals. Indeed, if we were to drop the assumption that it will always be possible to tell whether the standard of proof has been met, even the contradictory arrows between probatory guilt and probatory innocence might become suspect. Note, though, that it isn't clear at all what we should say about the statement "The defendant is guilty" in a case in which we can't tell whether the standard of proof has been met. Is it neither true nor false? Might it be both?

Even if we leave that complication aside (as I will for the remainder of this paper), we will not be able to sustain the horizontal and vertical lines. It is clear that there can be situations where there is enough evidence against a materially innocent defendant to leave the jury no choice but to declare him probatorily guilty. Somewhat surprisingly, one can check that this case alone already gives a counterexample to all relations denoted by the horizontal and vertical arrows.



THE BARE SQUARE OF REALISM

However, the original arrows still hold up "fallibly", as Laudan says. The idealizing assumption, one that we might have to take back in practice, is for the most part<sup>20</sup> about the *quality* of the evidence.<sup>21</sup>

We can sustain the relations of the square if we suppose that the evidence we gather will be veridical. If this sounds too naive, maybe we can be a bit more sophisticated

<sup>20</sup> Among other things, we need to assume that the members of the jury understand the instructions (not as trivial an assumption as one might have thought, as the issues in this paper testify) and are able to draw the right conclusions from the evidence. Also, even purely veridical evidence can point in the wrong direction, if it is of the circumstantial sort and not counterbalanced by evidence for the innocence of the defendant.

<sup>21</sup> If we idealize away restrictions of quantity of evidence as well, then we only end up with *two* corners: If we assume not only that all evidence is veridical, but that we can collect all such veridical evidence pertinent to the case, then material guilt and probatory guilt will come to the same thing, as will material and probatory innocence. This "line of guilt and innocence" might be a blisteringly naive conception, but it is both an idealization and an ideal:

in our idealizing assumption and suppose that misleading pieces of evidence will often cancel each other out by contradicting each other, such that in most cases the veridical evidence will bear more weight at the end of the trial.

Even though this weaker assumption can sustain the relations in the square, the first version has the virtue that it opens up the possibility of an elegant logical model of the trial proceedings. The rest of the paper will aim to outline a framework that can be used to reason about the probative notions and the dialectical situation in trials, applying logical ideas I've developed earlier.<sup>22</sup>

## 2.7 Constructive Models for Legal Deliberation

The core of the following framework will belong to the family of constructive logics.<sup>23</sup> The best known constructive logic, of course, is intuitionistic logic, but it is not the only one that has been proposed; I have argued elsewhere at length that intuitionistic logic is not the most useful constructive logic once we leave the realm of mathematics behind and venture into more empirical areas, such as legal epistemology. What follows will instead be based on David Nelson's logic  $N_3$ .<sup>24</sup>

An immediate clarification seems appropriate: The square reports relations between evidential situations and actual facts (that is, between probatory and material notions). One might think that a constructive logic might be unsuitable in such a setting, due to the philosophical heritage of constructive logic. Constructive logicians have traditionally often been inspired by anti-realistic ideas, roughly that there is no realm of facts independent from our evidential situation. Though Laudan has found some legal scholars who seem to be endorsing such a view about criminal cases, it is clear that most would reject this. The facts of criminal deeds are not taken to be in some way ontologically dependent on the ability of a court to come to know of them.

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(Footnote 21 continued)

It leaves out real-world complications, but it can serve as an epistemic ideal that we should set our legal institutions up to strive for. The square of guilt and innocence, in contrast, cannot, on its own, be said to embody such an ideal: It is quite easy to set up a legal institution that satisfies all the relations shown in the square: One that allows no evidence at all to be considered by the judge or jury and that leaves the burden of proof on the prosecution. It seems clear that this institution isn't even aiming at what it should. To make it a viable ideal, we should also stipulate that the state should make a serious attempt to find facts pertinent to the case and bring them to the attention of the court, and that this effort should not be (overly) thwarted.

<sup>22</sup>See Ref. [9].

<sup>23</sup>One referee remarked that the application of a constructive logic to the problem at hand is a rather obvious one. I was glad to read that, hoping that "obvious" implies "plausible". That said, I have not seen the application of a constructive logic to legal epistemology in print before, let alone the system I will propose below. Intuitionistic logic is discussed in [14]; however, the topic is not evidence but rather the nature of legal rules.

<sup>24</sup>See Ref. [11, 12, 15] for more on this logic.

Having such an anti-realist standpoint is, however, not a prerequisite for using a constructive logic. One might well be a logical pluralist who employs different logics at the material and at the probatory level (see [2] for one account of how this might play out; there are other possible ways as well, and I wish to make no commitment to one of those ways in particular).

One of the issues between the traditional realist / anti-realist debate we do not have to enter into when we take such a pluralist point of view is the question of truth. Truth need not be epistemically constrained, and those who like to take a naively realist view on truth are welcome to do so at the material level. I will, however, pay attention to the surrogate notion anti-realists like to give for truth, *correct assertibility*, without trying to use it to substitute truth. Material truth about a crime can co-exist peacefully with correct assertibility at trial, warranted by evidence. As we've seen, however, what determines the outcome of the trial is not so much material truth but rather whether the jury judges the assertions of the defense or those of the prosecution to be correct.

## 2.8 Nelson Logic

To get started on the logic, then, let me first introduce  $N_3$  by way of Kripke models, which are rather similar to the Kripke models for intuitionistic logic. A model for  $N_3$  will be a structure,  $[W, \leq, v]$ , where  $W$  is a non-empty set of information states,<sup>25</sup> and  $\leq$  is a binary relation on those states which is reflexive, transitive and anti-symmetric, that is, a partial order. This is an accessibility relation on the information states that has a strong temporal flavor: One stage comes literally *after* the other. Intuitively, the stages track the progress of the trial and collect the evidence that has been presented so far.

The valuation function  $v$  assigns a truth value, 1 or 0, to each atomic statement  $p$  at each stage. We give both of the values 1 and 0 a substantive reading: 1 stands for “verified by the evidence”, 0 for “falsified by the evidence”.<sup>26</sup>

We allow  $v$  to be a *partial* function, so that statements might not receive either value at a given stage. This reflects the fact that at a stage of investigation, a statement might be neither verified nor falsified by the evidence presented so far.

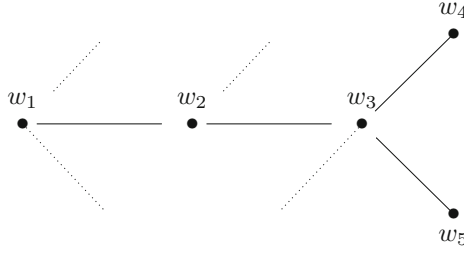
The idealization I discussed above comes in when we impose the *heredity constraint* on our models. As all evidence is veridical, we will never have to go back and revise our account of what has been verified and what has been falsified by the evidence. Formally, this comes to:

For all  $p$  and all worlds  $w$  and  $w'$ , if  $w \leq w'$  and  $w \Vdash_1 p$ , then  $w' \Vdash_1 p$ , and  
for all  $p$  and all worlds  $w$  and  $w'$ , if  $w \leq w'$  and  $w \Vdash_0 p$ , then  $w' \Vdash_0 p$ .

<sup>25</sup>Or “worlds”, for those who are more accustomed to such talk.

<sup>26</sup>This is in contrast to the semantics of intuitionistic logic, in which one of the values marks the constructive notion, mathematical proof, and the other the mere *absence* of that notion.

Here is a diagram and a valuation that suits the requirements of  $N_3$ :



	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$
$p$	-	-	1	1	1
$q$	-	0	0	0	0
$r$	1	1	1	1	1
$s$	-	-	-	1	0

Both 1 and 0 project forward, because both of them record a constructive achievement (verification and falsification respectively) that is taken to be permanent. There is a third option, here represented by “-”: a gap, a mere absence of either verification or falsification. At the outset of the trial, none of the statements relating to the crime will receive a constructive value, i.e., all of them will be gaps.

Now, the logical connectives can be defined on this structure (we will stay at the propositional level). There are some possible choices here, but the connectives are not my main concern in this paper. Therefore, I simply list the clauses of  $N_3$  and refer the interested reader to [9] for detailed discussion of the connectives:

$w \Vdash_1 A \wedge B$  iff  $w \Vdash_1 A$  and  $w \Vdash_1 B$

$w \Vdash_0 A \wedge B$  iff  $w \Vdash_0 A$  or  $w \Vdash_0 B$

$w \Vdash_1 A \vee B$  iff  $w \Vdash_1 A$  or  $w \Vdash_1 B$

$w \Vdash_0 A \vee B$  iff  $w \Vdash_0 A$  and  $w \Vdash_0 B$

$w \Vdash_1 \neg A$  iff  $w \Vdash_0 A$

$w \Vdash_0 \neg A$  iff  $w \Vdash_1 A$

$w \Vdash_1 A \supset B$  iff for all  $x \geq w$ ,  $x \nVdash_1 A$  or  $x \Vdash_1 B$

$w \Vdash_0 A \supset B$  iff  $w \Vdash_1 A$  and  $w \Vdash_0 B$

Instead of investigating the plausibility of those clauses here, I want to focus on the definition of logical consequence that is appropriate for our application. In  $N_3$ , consequence is defined as follows:

$\Gamma \models A$  iff in every model and every  $w \in W$ , if  $w \Vdash_1 B$  for every  $B \in \Gamma$ , then  $w \Vdash_1 A$ .

To wit, we want to go from verified statements to other verified statements. That makes sense in many situations; here, however, we have a slightly more complex situation. It is the consequence relation that will be reformulated to capture the burden of proof.

## 2.9 Modeling the Burden of Proof

The jury needs to decide which assertions and claims have been correctly made: Those by the defense or those by the prosecution. But the burden of proof dictates that these assertions have to be judged differently. The party bearing the burden of proof is indeed only making correct assertions when what it says is verified. The other party, on the other hand, is making correct claims *as long as what it says is not falsified*. That means that we are interested in two different sets of statements: The ones that are verified and receive a 1 by the valuation function, and those that are at least not falsified and thus do not receive value 0. Of course, the former is a subset of the latter.

To understand this difference is essential not only to decide which assertions have been made correctly. Also, to know what follows from a set of correct claims, one will have to take into account whether it was made under the burden of proof or not. Suppose, then, that we want to know which statements can correctly be asserted given the current distribution of the burden of proof and the correct assertions that have been made in the course of the trial. These earlier correct assertions will fall into two categories: Those made under the burden of proof, and those made not bearing the burden of proof.

I'll write  $\Gamma$  for the set of statements correctly asserted under the burden of proof, and  $\Delta$  for the set of statements correctly asserted while not under the burden of proof. As the burden of proof can shift from the prosecution to the defense (and back) in the course of a trial, I will not identify the set of correct assertions by the prosecution with  $\Gamma$  and the set of correct assertions made by the defense with  $\Delta$ . We will instead think of the burden of proof attached to single assertions: Either an assertion was made bearing the burden of proof, or not, and depending on that alone it will be sorted into  $\Gamma$  or  $\Delta$ .

We will also have two classes of consequences we'll be interested in, depending on whether the next statement we wish to make will be made under the burden of proof or not. Thus, logical consequence will relate the ordered pair of sets of statements  $\Gamma$  and  $\Delta$  to other pairs of sets of statements,  $\Phi$  and  $\Psi$ . If the burden of proof will lie on the consequence, then  $\Phi$  will contain a single statement and  $\Psi$  will be the empty set, and the other way around if the burden of proof will not lie on the consequence. By "the consequence" I simply mean the statement that is in the singleton  $\Phi$  or  $\Psi$ . I will simply omit any of  $\Gamma$ ,  $\Delta$ ,  $\Phi$  and  $\Psi$  in case they are empty.

Now, I redefine logical consequence<sup>27</sup>:

$\Gamma \mid \Delta \models_{hyb} \Phi \mid \Psi$  iff in every model and every  $w \in W$ , if  $w \Vdash_1 B$  for every  $B \in \Gamma$  and if  $w \nVdash_0 C$  for every  $C \in \Delta$ , then  $w \Vdash_1 D$  for every  $D \in \Phi$  and  $w \nVdash_0 E$  for every  $E \in \Psi$ .

The idea is that  $\Gamma$  collects all that we know is verified, because correctly uttered under the burden of proof, and  $\Delta$  all that we know is at least not falsified, because uttered correctly by speakers not bearing the burden of proof. Whether or not the burden of proof is on the conclusion will decide whether we're interested in conclusions before or behind the bar. So, for example, the consequence  $A$  of an inference  $\Gamma \mid \Delta \models_{hyb} A \mid$  will be such that we can utter it correctly if the burden of proof is on us, given that  $\Gamma$  and  $\Delta$  collect verified and unfalsified statements, respectively.<sup>28</sup>

If, on the other hand, we know our assertion will not be made under the burden of proof, we'll be interested in the inferences of the form  $\Gamma \mid \Delta \models_{hyb} \mid A$ .

Again, for more information about the sorts of inferences this setup allows, I refer the interested reader to [9]. My main point in this section was to outline how Laudan's distinction between the material and the probatory level can motivate a constructive logic, and how it is possible to give a notion of logical consequence that allows space for the burden of proof to decide the correctness of assertion in adversarial situations like criminal trials.

## 2.10 Future Work

The logical setup above is, of course, only of interest as long as we reason about the probatory level.<sup>29</sup> As it is based on the same idealizing assumption as the square of oppositions, however, we can use the relations in the square as a set of bridge principles between these levels. Given the idealizing assumption, if the information

<sup>27</sup>I use, as in [9],  $\models_{hyb}$ , for “hybrid consequence”, to denote this consequence relation.

<sup>28</sup>One may wonder whether the information recorded in  $\Delta$ , which is essentially information about unfalsified statements, could be relevant at all if the conclusion has to be verified. Indeed, it does make a difference, as the following example will show. Let  $A \vee B$  be the only element of  $\Gamma$ , and  $\neg B$  the only element of  $\Delta$ . Then we will find that  $\Gamma \mid \Delta \models_{hyb} A \mid$ , while  $\Gamma \mid \nmodels_{hyb} A \mid$ . Intuitively speaking, the burden of proof-bearing speaker who correctly asserted  $A \vee B$  had to have either a verification of  $A$  or a verification of  $B$ . The latter case is excluded by the fact that someone else, not bearing the burden of proof, was able to utter  $\neg B$ .

Another matter worth mentioning is that the semantics, as it stands, allows for  $A \wedge \neg A$  to be unfalsified. The rationale for that is just analogous to the reasoning that leads intuitionists to reject the validity of  $A \vee \neg A$ . To prove a disjunction, on the constructive view, you need to be able to prove one of the disjuncts. Likewise, on the view proposed here, to falsify a conjunction, you need to be able to falsify (at least) one of the conjuncts. Of course, to lead a party to contradict itself is a powerful move in legal discourse, no matter whether that party has to bear the burden of proof at the moment or not. That means that we should have a means of recognizing self-contradictory statements as defective in the logical system. Chap. 9 of [9] contains a mechanism for doing just that.

<sup>29</sup>The logic that is suitable for the material level might well be classical logic, or whichever logic one deems suitable to deal with facts about criminal events.



recorded at a stage of trial is enough to prove guilt beyond a reasonable doubt, then we can infer material guilt. Under the same assumption, the material innocence of the defendant will rule out all information stages that prove guilt beyond any reasonable doubt in the model of the possible ways the trial might unfold.

We saw that giving up the idealization meant the loss of the vertical and horizontal arrows of the square. In other words, the material and the probatory level will get dissociated, or at least, their connection will only be a fallible one, as Laudan had originally put it.

But even at the probatory level itself we will have to implement changes; evidence can not only be misleading, but also be discredited and outweighed by later counter evidence. What has been taken to be probatorily established at one stage of the trial might have become dubious at a later stage.

This will mean that the heredity constraint will have to be weakened in some way, or that some other way will have to be implemented to turn the logic above into a non-monotonic logic. Though there have been interesting attempts to base a non-monotonic logic on Nelson's  $N_3$ <sup>30</sup> that could be modified along the lines above, I think it is fair to say that a suitable system that can be adapted for the purpose at hand remains on the agenda for future work.

A very different set of questions concerns what interdisciplinary use could be made of the analysis. I believe that especially the material of the first part of this essay might be adapted for instructive uses. One could use the square of oppositions to teach people with no background in formal logic about the connections between guilt and innocence, e.g. at a law school. It would of course be highly beneficial if the jurors would understand the relations reported in the square, but in all its elegance I think it is too complicated to be of any use in pre-trial jury instructions.

In any case, I hope to have shown that Laudan's regimentation of the vocabulary allows new and fruitful routes of investigation that use both formal and informal methods.

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## References

1. Allen, R. (2010). Laudan, Stein, and the limits of theorizing about juridical proof. *Law and Philosophy*, 29(2), 195–230.
2. Beall, J., & Restall, G. (2006). *Logical pluralism*. Oxford: Oxford University Press.
3. Bèziau, J. Y., & Jacquette, D. (Eds.). (2012). *Around and beyond the square of opposition*. Berlin: Springer.

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<sup>30</sup>See Ref. [8, 16].

4. Clermont, K. (2013). *Standards of decision in law*. Carolina Academic Press.
5. Epps, D. (2015). The consequences of error in criminal justice. *Harvard Law Review*, 128(4), 1065–1151.
6. Goldstein, N. (Ed.). (2002). *Stylebook*. The Associated Press.
7. Haack, S. (2014). *Evidence matters. science, proof and truth in the law*. Cambridge: Cambridge University Press.
8. Jaspars, J. (1995). Partial up and down logic. *Notre Dame Journal of Formal Logic*, 36(1), 134–157.
9. Kapsner, A. (2014). *Logics and falsifications*. Cham: Springer.
10. Laudan, L. (2006). *Truth, error and criminal law*. Cambridge: Cambridge University Press.
11. Nelson, D. (1949). Constructible falsity. *Journal of Symbolic logic*, 14(1), 16–26.
12. Odintsov, S. (2008). *Constructive negations and paraconsistency*. Berlin: Springer.
13. Parsons, T. (2014). The traditional square of opposition. In by E. D. Zalta (Ed.), *The stanford encyclopedia of philosophy (Spring 2014 Edition)*. <http://plato.stanford.edu/archives/spr2014/entries/square/>.
14. Philipps, L. (1964). Rechtliche regelung und formale logik. *Archiv für Rechts- und Sozialphilosophie*, 1964, 317–329.
15. Wansing, H. (1993). *The logic of information structures*. In Lecture notes in AI 681. Berlin: Springer.
16. Wansing, H. (1998). *Displaying modal logic*. Dordrecht: Kluwer Academic Publishers.
17. Wexler, S. (2010). Two forms of legal proof and the so-called blue bus problem. *International Zeitschrift*, 6, 3.

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