

# Preface

This book concerns the foundations of Quantum Gravity, in particular from a conceptual point of view. It provides a self-contained introduction to this topic, resting on particular features of the accepted Paradigms of Physics: Newtonian Physics, Special Relativity (SR), Quantum Mechanics (QM), Quantum Field Theory (QFT) and General Relativity (GR). In approaching Quantum Gravity, many conceptual issues turn out to be related to notions of time. This occurs because notions of time are substantially different across these Paradigms. A first example in which this occurs is QM versus GR. Isham and Kuchař formalized the study of such discrepancies between notions of time. They did so by giving a conceptual classification of the many time-related reasons why a wide range of attempted approaches to Quantum Gravity fail to be satisfactory, in two seminal Reviews in the early 1990's [483, 586]. The current book's titular 'Problem of Time' refers to this conceptual classification. This is a multi-faceted collection of very interesting problems which turn out to be heavily interlinked. Quite a few of these problems were first glimpsed in the pioneering works of Wheeler and DeWitt in the 1960's [237, 897, 899] on the geometrodynamical formulation of GR.

The Problem of Time is, in greater generality, a consequence of the mismatch between Background Dependent and Background Independent [12, 363] Paradigms of Physics. Newtonian Physics, SR, QM, and QFT are all Background Dependent, whereas GR is Background Independent and many approaches to Quantum Gravity expect this to be Background Independent as well. So, whereas there has been quite widespread belief among theoretical physicists that the Problem of Time is a quantum matter, this is a misconception since clearly also Classical Physics can exhibit mismatches between Background Dependent and Background Independent Paradigms. Once this is taken into account, models exhibiting classical versions of the Problem of Time turn out to provide substantial conceptual insight into the harder quantum versions of the Problem of Time.

It is thus clear that further explanation of what this book (and [483, 586]) takes the Problem of Time to consist of is best done after the following.

- A) Presenting the standard Paradigms of Physics and explaining how notions of time differ across these.
- B) Outlining what each of Quantum Gravity and Background Independence are.

N.B. that A)—in Chaps. 1 to 8’s account of notions of time and of space and of the diversity of physical laws across accepted Paradigms in Physics—serves as a *preamble*. It is not to be mistaken for *introduction of the material which the rest of the book greatly expands upon*, which is, rather, Chaps. 9, 10 and 12 on the Problem of Time and Background Independence issues which underlie this. Chapters 1 to 8 enter, rather, into assembling check lists to test foundational and Quantum Gravitational candidate times against, to see if these merit to be called timefunctions, and toward building up toward plausible Quantum Gravitational laws in Chap. 11. While these may be somewhat unexpected and indirect uses of Chaps. 1 to 8’s material, this is the intended use of these Chapters in writing this book. By way of explanation, this book’s main topic happens to benefit from a preliminary presentation of the types of law and notions of time and space that each of the established theories has. This is prudent given that this book’s main topic is a systematic analysis of a wide range of more speculative foundational and Quantum Gravitational programs in which *only subsets* of standard theories’ laws and temporal and spatial notions are kept.

## From the Accepted Paradigms of Physics to Quantum Gravity

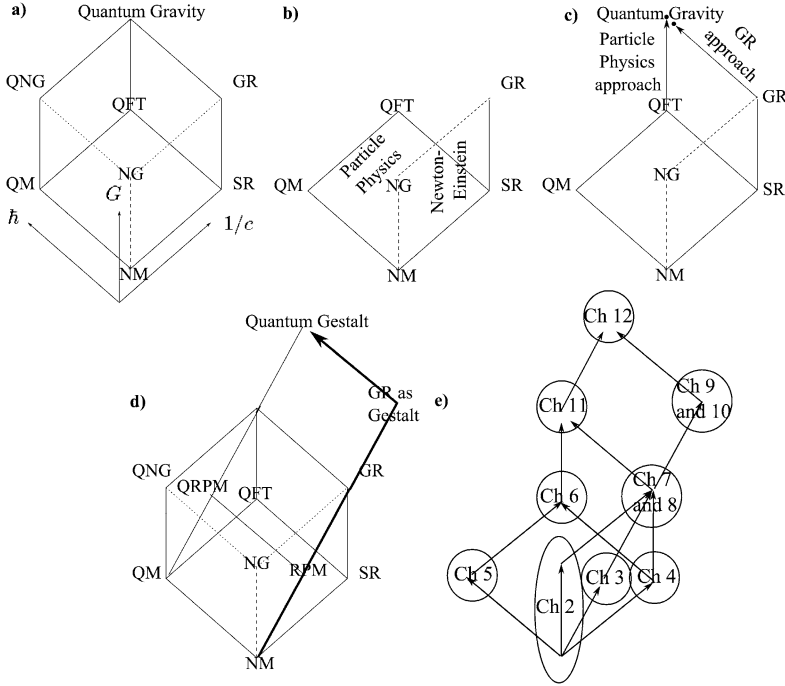
This book thus begins by considering time and clock concepts, alongside supporting notions of space, length-measuring devices, spacetime and frames. Chapter 1 gives a largely theory-free conceptual outline of these, intended for a very wide and diverse multidisciplinary audience.

Each of the Newtonian Paradigm, SR, QM, QFT and GR are then covered in turn, in Chaps. 2 to 7. This treatment includes in outline how these Paradigm Shifts affect time, clock, space, length-measuring, spacetime and frame concepts.

This theory by theory development has the further complication of not being a linear venture: these Paradigms of Physics fan out from Newtonian Mechanics as indicated in Fig. 1.a). Three distinct theoretical developments each bring in one of the three known fundamental constants of Nature:<sup>1</sup> Newton’s gravitational constant  $G$ , the reciprocal of the speed of light  $c$ , and Planck’s constant  $\hbar$ , as follows.

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<sup>1</sup>Values of—and uncertainties in—these fundamental constants are as follows [661]. The speed of light in vacuo  $c$  is *defined* to be exactly  $299,792,458 \text{ m s}^{-1}$  due to the metre itself being defined in terms of  $c$  (see Chap. 1.13). Planck’s constant  $\hbar = 1.054571800(13) \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$ . Newton’s gravitational constant  $G = 6.67408(31) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ . The error analysis for the Planck units is very straightforward: since  $G$  is by far the least accurately known of the fundamental units, the error in this swamps the others.



**Fig. 1** **a)** Planckian cube of fundamental physical theories. Here, NM stands for Newtonian Mechanics and (Q)NG stands for (Quantum) Newtonian Gravity. **b)** indicates the ‘Newton–Einstein’ (alias ‘classical’, in the sense of ‘non-quantum’) plane, and the ‘Particle Physics’ (i.e. ‘non-gravitational’) plane. **c)** Gordian cube: here it is perceived that different routes along the edges to the ‘final Quantum Gravity vertex’ may not commute (in the algebraic sense, upon viewing its edges as maps). **d)** Cutting the Gordian cube? [Or ‘thinking outside of the box’ that is **a)**?] Here, (Q)RPM stands for (Quantum) Relational Particle Mechanics; see the next Subsection for an outline of what (Quantum) Gestalt means. **e)** Indicates the corresponding coverage by Chapters in Part I

- 1)  $G$  is significant when gravitational force is non-negligible in comparison to whichever forces dominate the physics.  $G$  was originally formulated in Newtonian Gravity, which lies within Newton’s Paradigm of Physics, whereas 2) and 3) each additionally represent introducing a new Paradigm.
- 2)  $c$  is non-negligibly finite in SR [736]; this is relevant to objects whose velocities  $v$  are non-negligible compared to  $c$ .
- 3)  $\hbar$  is significant in Quantum Mechanics (QM) [599], due to certain quantities coming in minimum-sized packets. For instance, angular momentum comes in  $\hbar$  (or  $\hbar/2$ ) sized packets. This is relevant in situations involving quantities comparable in size to the corresponding minimum packets.

Pairwise incorporations of these constants are furthermore as follows (Fig. 1.a).

- 4) Relativistic QFT [712] involves  $c$  and  $\hbar$  together, corresponding to the *Compton wavelength*

$$l_C = \hbar / m c \quad (1)$$

length-scale for a ‘particle’ of mass  $m$ .

- 5) GR [874]—in the sense of a Relativistic Theory of Gravitation—considers  $c$  and  $G$  together, corresponding to strongly gravitating fast-moving objects e.g. confined to around the scale given by the *Schwarzschild radius*,

$$r_{\text{Schw}} = G M / 2 c^2. \quad (2)$$

Each of the ‘Particle Physics’ and ‘Newton–Einstein’ planes indicated in Fig. 1.b) are self-consistent two-step Paradigm Shifts.

- 6) Considering  $\hbar$  and  $G$  together gives ‘Quantum Newtonian Gravity’; this is however much less relevant (Ex VI.0). A characteristic lengthscale here would be

$$l_g := \hbar^2 / 2 G M m^2. \quad (3)$$

- 7) Finally, ‘Quantum Gravity’ [75, 154, 194, 237–239, 385, 471, 474, 475, 483, 485, 552, 586, 746, 845] is often held to be ‘the’ triple combination at the last vertex of Fig. 1.a)’s ‘cube’ of theories. The three fundamental constants combine here to form the *Planck units*:

$$l_{\text{Pl}} = \sqrt{\hbar G / c^3} \simeq 1.616228(38) \times 10^{-35} \text{ m}, \quad (4)$$

$$t_{\text{Pl}} = \sqrt{\hbar G / c^5} \simeq 5.39116(13) \times 10^{-44} \text{ s}, \quad (5)$$

$$m_{\text{Pl}} = \sqrt{\hbar c / G} \simeq 2.176470(51) \times 10^{-8} \text{ kg}. \quad (6)$$

The first two of these are very small compared to ‘ordinary physical quantities’. [Compare  $l_{\text{Pl}}$  with the atomic  $\simeq 10^{-10}$  m and nuclear  $\simeq 10^{-15}$  m lengthscales, and with the maximum precision of displacement detection in existing gravitational wave detectors corresponding to displacements of  $\simeq 10^{-18}$  m. Compare also the ratio of  $t_{\text{Pl}}$  to the timescales of observational Physics with the maximally accurate clock precision of currently around 1 part in  $10^{16}$ , as per Chap. 1.] On the other hand,  $m_{\text{Pl}}$  is very large upon considering its interpretation as a single ‘fundamental particle’ mass: compare e.g. the proton mass  $1.672621898(21) \times 10^{-27}$  kg. By SR’s  $E = mc^2$ ,  $m_{\text{Pl}}$  corresponds to an energy scale  $E_{\text{Pl}} = 1.220910(29) \times 10^{19}$  GeV. N.B. this is much larger than the  $10^2$  to  $10^4$  GeV range of the most powerful particle accelerator to date: CERN’s Large Hadron Collider (LHC). Moreover, as detailed in Chap. 11, the Planck regime is expected to feature in some parts of Black Hole Physics and Early-Universe Cosmology. In particular, this book covers the Quantum Cosmology arena and simpler model arenas that exhibit features of this.

## Differing Roles of Time and Space Throughout the Paradigms of Physics

Space and especially time are moreover not consistently conceived of throughout ‘the Planckian cube of theories’. Due to this, consideration of which units can be

built out of fundamental constants may not suffice as a conceptual framework within which to reconcile the Paradigm Shifts of Physics. Indeed, this book expounds that GR involves qualitatively distinct concepts of time, space, spacetime and frame from those used in Particle Physics.

On these grounds, this book contends that conceiving of ‘Quantum Gravity’ solely in terms of ‘the Planckian cube’ is a misleading simplification. This conceptual disparity points instead to different paths around this ‘cube’ not commuting (Fig. 1.c). By this disparity and the subsequent notorious difficulty with its resolution, it might be more apt to name the cube not ‘Planckian’ but ‘Gordian’: after a notorious knot of the ancient world that was presented to Alexander the Great. He is supposed to have dealt with this knot by ‘thinking out of the box’. Accounts differ, however, as to whether this involved cutting it or removing it from the wooden pole it was mounted upon. Indeed, suggestions for approaching ‘Quantum Gravity’ differ amongst themselves as well.

A further interplay is that the Theoretical Physics literature often pays little attention to the properties entailed in calling an entity a time or a clock. This is unfortunate, because a number of such purported time quantities do not stand up against a suitable list of temporal properties. We emphasize this point in this book firstly by attributing mathematical properties to ‘timefunctions’, and operational character to ‘clocks’, in contradistinction to physical, philosophical and conceptual discussion of aspects of time.<sup>2</sup> Secondly, we refer to *candidate* times, timefunctions and clocks until enough properties of these have been established. There is moreover not a unique list of properties to check against, since different physical theories involve different lists of properties, as the ‘Gordian cube’ in Fig. 1.c foreshadows.

## Background Independence in Mechanics, GR and Quantum Gravity

Following Einstein, a second perspective on the nature of—and motivation for—GR is as a freeing from absolute or background structures. From this perspective, GR is more than just a Relativistic Theory of Gravitation. Such perspectives have subsequently been dubbed Background Independence [12, 363, 483, 485, 552, 752]; contrast with how Background Dependent absolute structures pervade all six of the other non-final vertices of the cube. This book considers GR as embodying both of these perspectives at once, phrasing this in the shorthand that GR is a ‘*gestalt*’ of a Relativistic Theory of Gravitation and of Background Independence.

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<sup>2</sup>This book prioritizes conceptual matters concerning time over giving detailed specifics of accurate timekeeping [82, 783]. It does contain some comments on sidereal and ephemeris astronomical times, atomic clocks, and SR and GR implications for timekeeping. Some of the theoretical concepts outlined in this book, moreover, may eventually become relevant to timekeeping: space clocks, extending Earth or Solar System based reference systems to galactic and cosmological scales, and clocks in physically extreme regimes.

Some programs in Physics confine themselves to Background Dependent theories. Such approaches work for Quantum Theory within the Newtonian and Minkowskian Paradigms, while amounting to dismissing features of GR that are inconvenient in these Paradigms. This is to be contrasted with seeking new Paradigms that reconcile GR and Quantum Theory by having features of mixed quantum and GR origin! If GR's second meaning—Background Independence—is retained in passing to the quantum level, this book terms such an approach '*Quantum Gestalt*'. This book ceases to use the name 'Quantum Gravity' in this context due to it implicitly giving complete priority to the gravitational perspective on GR's identity over the Background Independent perspective. *Quantum Gestalt is a proposed family of Paradigms for Physics that encompass a number of theories and programs which implement Background Independence.* Since some kind of Background Independence is widely considered in approaches to 'Quantum Gravity', the Quantum Gestalt family includes a number of well-known examples, such as Geometrodynamics [483, 581, 899], Loop Quantum Gravity [752, 845], the Canonical Approach to Supergravity [232], and M-Theory [136, 719]. In this way, this book covers how Background Independence—and consequently the intriguing and difficult Problem of Time—are manifested in a wide range of well-known current approaches to 'Quantum Gravity'.

Both by resting upon Background Independence and by inherent interest in temporal matters, the Problem of Time is also a topic of interest more widely in Foundations of Physics and Philosophy of Physics, as well as in Theoretical Physics generally and in Background Independent Quantum Gravity in particular. None the less, this book mostly concerns Theoretical Physics rather than Philosophy of Physics.<sup>3</sup>

While attempting to combine QM and GR is an *example* of Background Dependence versus Independence clash, it is a *very hard* example, so it very much helps to first point out that there are other simpler examples. In particular, I) Classical Physics already exhibits such mismatches: there is also a Classical Problem of Time, which is more straightforward to resolve. II) Finite models (Minisuperspace GR and Mechanics models) already exhibit many of the Problem of Time's mismatches as well. III) Some otherwise simplified models (Midisuperspace, or, even more simply, 'slightly inhomogeneous'—perturbative—semiclassical such) exhibit all of them. I), II) and III) turn to be very insightful topics to consider prior to the Problem of Time between QM and full GR. In this vein, we outline the Classical Problem of Time in Chaps. 9 and 10 prior to its quantum counterpart in Chap. 12, and we always start by considering the above kinds of model arenas.

Returning to the Gordian cube, one can view 'freeing from absolute structures' or Background Independence as a *fourth* departure (see Chap. 3) from Newtonian Mechanics which can be adopted independently from Relativity, Gravitation and the Quantum, and their considerations of fundamental units. A simple opening here turns out to involve a Mechanics that satisfies relational criteria which arose from Leibniz's and Mach's criticisms of the Newtonian Paradigm. While no concrete

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<sup>3</sup>For the philosophically-minded reader, most of what few such matters are mentioned in this book involve Leibniz, Mach or Broad.

such Mechanics was available in their day, Barbour and Bertotti's [105] Relational Particle Mechanics is a satisfactory such (with or without Newtonian Gravitation: Fig. 1.d). Relational Particle Mechanics is based on the following Background Independence principles.

- 1) *Temporal Relationalism* is that there is no meaningful time for the Universe as a whole. We shall see that this is implemented by actions which are, firstly, free of extraneous time-like quantities, and, secondly, Manifestly Reparametrization Invariant, by which there is no physically meaningful role for 'label times' either.

If time is not primary, moreover, we need to study whatever other entities that are still regarded as primary. One approach to this begins by considering configurations and configuration spaces  $\mathcal{Q}$ . This book is consequently also a sizeable resource on such 'spaces of shapes' [301, 539] (Appendices G–I and N).

- 2) *Configurational Relationalism* involves taking into account that a continuous group of transformations  $\mathfrak{g}$  acting on the system's configuration space  $\mathcal{Q}$  is physically irrelevant. For Mechanics, these transformations are usually translations and rotations of space, though in general Configurational Relationalism also covers physically irrelevant internal transformations, as occur in the most common types of Gauge Theory. Configurational Relationalism can be resolved, at least in principle, by *Best Matching*, which is bringing two configurations into minimum incongruence with each other by application of  $\mathfrak{g}$ 's group action.

Relational Particle Mechanics furthermore points to a theory of Quantum Background Independence, which, from the perspective of Quantum Gestalt, is the complement of 'Quantum Gravity' interpreted literally.

The significance of Temporal and Configurational Relationalism significantly increases upon realizing that GR itself can be recast in terms of these principles. These give two precise senses in which GR is 'Machian'. Mach's work is widely of foundational interest; for instance, some of Mach's concepts played a role in Einstein's search for GR. Moreover, the two precise senses alluded to—Mach's Time Principle and Mach's Space Principle—do not coincide with how Einstein interpreted a partly different set of Mach's ideas; also his historical route to GR ended up making at best indirect use of Machian themes. As Wheeler argued [660, 899], however, there are many routes to the same theory of GR. Some of these arrive at a dynamical formulation of GR: a theory of evolving spatial geometry: Geometrodynamics [899]. It then turns out that a more specific formulation of GR as Geometrodynamics is Machian after all ([62, 109] and Chap. 9). Finally, GR in Machian Geometrodynamics form can furthermore even be rederived from Temporal and Configurational Relationalism first principles ([62, 109] and Part II).

The proposal then is to 'cut the Gordian cube' by taking the following path (Fig. 1.d) along the 'space of fundamental theories'. a) Relational Particle Mechanics, b) the same with Newtonian Gravity, c) GR in Machian Geometrodynamics form (with SR recovered as a limiting case), d) Quantum Gestalt.

In this book, we let the physical theories themselves determine which notions of time are appropriate; see e.g. [483, 519–521, 581, 584, 586, 589, 899] for earlier

such physical and conceptual approaches. With Quantum Gravity and its Quantum Gestalt subset being an unfinished subject with disputed foundations, this book considers conceptual notions to take precedence; only then is one to ask what Mathematics is required for the suitable concepts to be modelled well. This is as opposed to picking a theory for its mathematical tractability at the expense of whether it models suitable physical concepts. The Appendices provide supporting basic Pure Mathematics, Geometry and its application to configuration spaces and the Principles of Dynamics. They also provide theorems for full GR's configuration spaces and partial differential equation theorems, and various other levels of structure for Classical and Quantum Physics.

This approach is commensurate with how Physics has quite often required the development of new Mathematics that is suitable for its concepts: Calculus, Linear Algebra, Analysis. . . . There are moreover many tractable types of Mathematics that, however, as far as we know, Nature makes no use of. So it may be tenuous to let oneself be guided by solvability in a scientific subfield that has no experimental or observational input. This is not to be confused with how internal consistency *is* a valid guiding principle for theories. The point is that internal consistency is not a guarantee that the Universe will be as imagined, since this criterion is not by itself a guarantor of uniqueness.

## The Problem of Time

We have now reached a position in which we can comment on a useful introductory breakdown of what the Problem of Time consists of. It has nine facets—closely following Isham and Kuchař [483, 586]—resulting from nine corresponding aspects of Background Independence (identified in subsequent work).

*Aspect 1)* Temporal Relationalism leads to the notorious *Frozen Formalism Problem: Facet 1)*. At the quantum level, this is the presence of an apparently frozen quantum wave equation—the Wheeler–DeWitt equation—where one would expect an equation which is dependent on (some notion of) time. This quantum Frozen Formalism Problem is very well known, but is unfortunately often confused with the entire multi-faceted Problem of Time.

*Aspect 2)* Configurational Relationalism leads to—in the case of GR, for which  $\mathfrak{g} = \text{Diff}(\Sigma)$ : the spatial diffeomorphisms—the *Thin Sandwich Problem*, which is *Facet 2)*. [The Thin Sandwich is a particular GR specialization of the previously mentioned notion of Best Matching.]

Each of Temporal and Configurational Relationalism moreover provides constraint equations. In the case of GR, these are the well-known Hamiltonian and momentum constraints respectively. Indeed, the above-mentioned Wheeler–DeWitt equation is the quantum Hamiltonian constraint, whereas the Thin Sandwich Problem is a particular approach to solving the momentum constraint at the classical level.



It is next natural to ask whether one has found all of the constraints: algebraic *Constraint Closure* is *Aspect 3*). This is approached by introducing a suitable brackets structure and systematically applying the Dirac Algorithm. If the answer is in the negative, one has a *Constraint Closure Problem: Facet 3*).

The objects which brackets-commute with all the constraints—or with specific subalgebraic structures thereof—are of subsequent interest. These objects—*observables* or *beables*—are useful objects due to their physical content, whereby *Aspect 4*) is *Assignment of Beables*. If obtaining a sufficient set of these to do Physics is in practice blocked—a common occurrence in Gravitational Theory—then one has a *Problem of Beables: Facet 4*).

Since GR is also a theory with a meaningful and nontrivial notion of spacetime, it has more Background Independence aspects than Relational Particle Mechanics does. Indeed, the Einstein field equations of GR determine the form of GR spacetime, as opposed to SR Physics unfolding on a fixed background spacetime. From a dynamical perspective, GR's geometrodynamical evolution *forms spacetime itself*, rather than being a theory of the evolution of other fields *on* spacetime or *on* a sequence of fixed background spatial geometries. Regardless of whether spacetime is primary or emergent, there is now also need for the following.

*Aspect 5*) is *Spacetime Relationalism*, whereby the *diffeomorphisms* of spacetime itself,  $\text{Diff}(\mathfrak{M})$ , are physically redundant transformations. Whereas this is straightforwardly implemented in the classical spacetime formulation of GR, it becomes harder to implement at the quantum level. For instance, it feeds into the *Measure Problem* of Path Integral Approaches to Quantum Gravity, so *Facet 5*) is indeed nontrivial.

Foliations of spacetime play major roles, both in dynamical and canonical formulations, and as a means of modelling the different possible fleets of observers within approaches in which spacetime is primary. Background Independence Physics is moreover to possess *Foliation Independence* : *Aspect 6*). If this cannot be established, or fails, then a *Foliation Dependence Problem* is encountered: *Facet 6*).

Starting with less structure than spacetime—assuming just one or both of spatial structure or discreteness—is particularly motivated by Quantum Theory [899]. Moreover, in such approaches the spacetime concept is to hold in suitable limiting regimes: *Spacetime Constructability*—*Aspect 7*)—is required. If this is false, or remains unproven, then we have a *Spacetime Construction Problem: Facet 7*).

Finally, *Aspect 8*) is *Global Validity* and *Aspect 9*) is *No Unexplained Multiplicities*. These apply to all the other aspects, facets and strategies toward resolving these; contentions with these are termed, respectively, *Global Problems of Time: Facet 8*) and *Multiple Choice Problems of Time: Facet 9*).

All in all, the Problem of Time is a multi-faceted subset of the reasons why forming 'Quantum Gravity' Paradigms is difficult and ambiguous; Further reasons are purely technical, or a mixture of both.

The classical versions of Background Independence and the Problem of Time are more straightforward, so this book presents these before their quantum counterparts. Similarly, this book makes use of the simpler Relational Particle Mechanics and Minisuperspace model arenas prior to passing to more complicated cases.

Diffeomorphisms are, moreover, crucial [483] as regards a number of Problem of Time facets, and require inhomogeneous GR models so as to feature nontrivially. Balancing this requirement, enough simplicity for calculations, and cosmological applications, this book's third choice of model arena is Slightly Inhomogeneous Cosmology: a type of perturbative Midisuperspace model. This furthermore permits investigating whether galaxies and cosmic microwave background hot-spots could have originated from quantum cosmological fluctuations [419]. Finally, this choice of model arenas amounts to concentrating on Quantum Cosmology rather than Black Hole models.

Isham and Kuchař's reviews [483, 586] on the Problem of Time are of a very high standard. The current book, however, further advances the subject in the following ways. This book's first advance is that we have enough room to trace the Problem of Time Facets back to more basic and well-known temporal concepts which reside within the Newtonian Paradigm, SR, GR, QM and QFT, by presenting the latter in prequel chapters. This book's second advance is that we present many improvements in the conceptualization of Problem of Time facets and linking them to underlying notions of Background Independence. These conceptual advances point to renaming a number of facets and aspects so as to more truly reflect their content. Consult Fig. 12.3 at the end of Part I to keep track of the evolution and end-product form of these names for the multiple parts of the Problem of Time. This book's third advance is due to those Reviews now being over 20 years old, and containing almost no mention of Supergravity, String and M-Theory, Loop Quantum Gravity, or other modern approaches to Quantum Gravity. In this way, the current book updates and expands on the range of theories considered. In particular, Canonical Supergravity turns out to be a valuable counter-example to the suggestion that passing from one GR-like theory to another leaves the Problem of Time largely unchanged. Apologies are offered as regards not covering all Quantum Gravity programs. This is beyond what can be covered in a single book. Besides, our intent is to focus on time and notions of Background Independence underlying this rather than on diversity of Quantum Gravity programs per se.<sup>4</sup>

## Concerning This Book's Three Parts, Appendices and Epilogues

Part I is a 'first track' introduction to the above-mentioned topics that is widely accessible, including for Freshers new to a graduate school or PhD program, and for advanced undergraduates. This outlines each Fundamental Theory of Physics, alongside explaining the notion of time used in each. Conceptual outlines are subsequently provided, both of Quantum Gravity and of the nine aspects of Background Independence with the ensuing nine facets of the Problem of Time as piecemeal

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<sup>4</sup>Readers new to Quantum Gravity may complement this book with Kiefer's [552] wide overview, in addition to introductory literature more specific to their intended research program, some of which is outlined in Chap. 11 and in Exercise Sets V and VI.

entities. Finally, Part I outlines a number of different strategies that have been suggested to deal with these facets.<sup>5</sup> Part I and its supporting (unstarred) Mathematical Appendices additionally contain Exercises for students to actively expand their horizons. The more challenging Exercises are marked with †, with those marked †† being considerably more challenging.

Parts II (classical) and III (quantum) concern the rather more advanced—‘second track’—material necessary for a more full and up to date an account of the Problem of Time. These are supported by the starred Mathematical Appendices. Parts II and III reflect that *the Devil is in the detail*. For, as Isham and Kuchař argued, the Problem of Time facets turn out to be heavily interlinked, and none of the strategies proposed to date work when examined in sufficient detail. N.B. that this heavy interlinking takes the form that if one resolves a facet piecemeal, and then attempts to extend this resolution to resolve a second facet, then this extension has a strong tendency to spoil the resolution of the first facet. Because of this, little overall progress has arisen from treating Problem of Time facets piecemeal. This is why it is very important to list the facets together in explaining what the Problem of Time is. In particular, studying just one facet—most commonly the Frozen Formalism Problem—misses out not only the other facets but also how these interfere with each other, which is a very major part of the subject. One reason for this interference is that the facets share temporal and Background Independence roots; they have a *common origin* due to the Background Dependent and Background Independent Paradigms not fitting together. This strongly suggests that they should ultimately be approached together rather than piecemeal. Moreover, another reason for this interference is that modelling each facets’ concepts brings in its own distinct type of mathematics. Indeed, this book’s fourth advance is to go much further than previous authors in identifying the mathematics that modelling each facet requires.

This book’s fifth advance is to *temporarily* present each facet’s concepts and consequent mathematical modelling by itself, in full awareness that these approaches will subsequently need to be combined, and that the lion’s share of the difficulty is in the latter. This temporary presentation of the individual facets is of pedagogical value: Part I’s account, by steering clear of facet interferences, is rather probably simpler to understand than Kuchař and Isham’s reviews, so it serves as an overall introduction to this subject area. This and this book’s first advance combine to make Part I a useful introduction to read prior to Kuchař and Isham’s reviews as well as Parts II and III.

This book’s sixth advance is in demonstrating that if the mathematics needed to model each facet is taken far enough, then resolutions of different facets can be

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<sup>5</sup>Recommended books to read beforehand or alongside this one are Lanczos [598] or Goldstein [371] for Principles of Dynamics formulations of Mechanics, Landau–Lifshitz [599] and Isham [487] for QM, Rindler [736] for SR and a brief introduction to GR, and Chaps. 1 to 4 of Peskin and Schroeder [712] for a brief introduction to QFT. Wald [874] and the entirety of Peskin and Schroeder are good books for *second* studies of GR and QFT respectively, and the first two chapters of Dirac [250] or the first four of Henneaux and Teitelboim [446] constitute a second course in specifically-constrained Principles of Dynamics.

combined after all. One caveat here is that almost all of Parts II and III concentrate on *a local* resolution of the Problem of Time, i.e. on joint treatment of all the facets bar the Global one and the Multiple Choice one. Within this restriction, we take each of few-particle Relational Particle Mechanics, Minisuperspace, Slightly Inhomogeneous Cosmology, and full GR as far as we can. Some issues considered are additionally only taken as far as the Semiclassical Quantum Cosmology regime. Within these limitations and considering the classical case first, the fourth, fifth and sixth advances can none the less be made.

Indeed, Parts II and III show *how far* the mathematics needed to model each local facet needs to be taken before it can accommodate considering multiple facets. For instance, it has long been known that replacing Euler–Lagrange actions by Jacobi actions implements Temporal Relationalism. In comparison, this book shows that to *maintain* Temporal Relationalism upon considering further facets, one needs to follow the Jacobi action up by reformulating the entirety of the Principles of Dynamics, spacetime foliation kinematics, Canonical Quantization and the Path Integral Approach to Quantum Theory. It is also very satisfying to see that some already-known key objects such as configurations, momenta, actions and constraint equations remain unchanged in the process. However, this amounts to around two orders of magnitude more work than simply replacing the Euler–Lagrange action by the Jacobi action. In this way, this book contends that the previous five decades of attempts at concurrently resolving multiple Problem of Time facets failed to get round facet interference by seldom, if ever, coupling such a level of thoroughness to mathematics which carefully fits each facet’s conceptual basis.

The main ‘A Local Resolution of the Problem of Time’ program that this book concentrates on building up as a Combined Semiclassical, Histories and Timeless Approach. This is a Machian extension of earlier work by Halliwell, shown to hold beyond the Minisuperspace arena he explicitly investigated. The book also reviews other conceptually interesting strategies that have been suggested over the years toward attempting to resolve the Problem of Time: attempting a Klein–Gordon interpretation for Quantum GR, hidden time, matter time, and a further variety of Timeless,, Path Integral and Histories Approaches.

These results are subsequently tempered with brief accounts of Global Problems and Multiple-Choice Problems in Epilogues at the end of each of Parts II and III. Finally, further Epilogues are provided to outline Background Independence and the Problem of Time at deeper levels of mathematical structure than the usually assumed metric and differentiable structure level: the topological manifold level, topological space level, at the level of sets, and alternatives thereto. Whereas in the context of GR, Background Independence is almost always meant at the metric and differentiable manifold (diffeomorphism) levels of mathematical structure, there is no conceptual reason to stop at this level. These Epilogues represent ‘third track’ material, extending one’s mathematical framework to include either of these would be considerably harder than the book’s local resolution; these lie on, or somewhat beyond, the frontier of current research. We do however point to suitable mathematics to model the Global and Multiple-Choice Problems: stratified manifolds, sheaves, deformed cohomology, Topos Theory. . . . Some of these topics are supported by the double-starred Mathematical Appendices.

Since Part II and III's material is more advanced, these contain not Exercises but Research Projects. Many of the largest such—program rather than paper sized—are gathered in Part III's Conclusion and in the Epilogues. The website <https://conceptsofshape.space/problem-of-time-book/> will be periodically updated to keep track of which of these projects have received significant progress. The Author can be contacted at [Dr.E.Anderson.Maths.Physics@protonmail.com](mailto:Dr.E.Anderson.Maths.Physics@protonmail.com).

Let us end with two recommendations for readers who are interested in pursuing Quantum Gravity and are relatively new to this field. Firstly, take a wide overview.<sup>4</sup> Only by comparing different thoughts about conceptually similar questions—and how answers to these pan out under diverse assumptions—can one get a feel for whether the specifics of a particular Quantum Gravity program are likely to have lasting significance in humanity's understanding of Nature. Secondly, think for yourself.

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