

Contents

1	Orthogonal Designs	1
1.1	Hurwitz-Radon families	2
2	Non-existence Results	7
2.1	Weighing Matrices	7
2.2	Odd Order	8
2.3	Algebraic Problem	13
2.4	Orthogonal Designs' Algebraic Problem	13
2.5	Geramita-Verner Theorem Consequences	15
3	Algebraic Theory of Orthogonal Designs	19
3.1	Generalities on Quadratic and Bilinear Forms	19
3.2	The Matrix Formulation	21
3.3	Mapping Between Bilinear Spaces	22
3.4	New Spaces From Old	23
3.5	Bilinear Spaces Classification Theorems	24
3.6	Classification of Quadratic Forms Over \mathbb{Q}	25
3.7	The Similarities of a Bilinear Space	30
3.8	Linear Subspaces of $Sim(V)$	31
3.9	Relations Between Rational Families in the Same Order	36
3.10	Clifford Algebras	37
3.11	Similarity Representations	38
3.12	Some Facts About Positive Definite Forms Over \mathbb{Q}	40
3.13	Reduction to Powers of 2	43
3.14	Orders 4 and 8	46
3.14.1	Order 4	46
3.14.2	Order 8	48
3.15	Order 16	51
3.15.1	Case 1: 9-member rational families.	53
3.15.2	Case 2: 7-member rational families.	53
3.15.3	Case 3: 8-member rational families.	54

3.16	Order 32	56
3.17	Solution of the Algebraic Problem	57
3.18	Combining Algebra with Combinatorics	59
3.18.1	Alert	61
4	Orthogonal Designs Constructed via Plug-in Matrices	63
4.1	Introduction	63
4.2	Some Orthogonal Designs Exist	63
4.3	Some Basic Matrix Results	68
4.3.1	Supplementary Difference Sets, their Incidence Matrices and their Uses as Suitable Matrices.....	74
4.4	Existence of Weighing Matrices	76
4.5	Constructions for $W(h, h)$ and $W(h, h - 1)$	82
4.6	Using Circulants–Goethals–Seidel Array and Kharaghani Array	89
4.7	Constraints on construction using circulant matrices	95
4.8	Eades’ Technique for Constructing Orthogonal Designs	96
4.9	Some Arrays for Eight Circulants.....	107
4.10	Amicable Sets and Kharaghani Arrays	110
4.11	Construction using 8 Disjoint Matrices	111
4.11.1	Hadamard Matrices.....	115
4.12	Baumert–Hall Arrays.....	117
4.13	Plotkin Arrays	124
4.13.1	Kharaghani’s Plotkin arrays	126
4.14	More Specific Constructions using Circulant Matrices	126
4.15	Generalized Goethals–Seidel Arrays	129
4.15.1	Some Infinite Families of Orthogonal Designs	133
4.15.2	Limitations	134
4.16	Balanced Weighing Matrices	134
4.16.1	Necessary Conditions for the Existence of Balanced Weighing Matrices.....	135
4.16.2	Construction Method for Balanced Weighing Designs .	136
4.16.3	Regular Balanced Weighing Matrices	139
4.16.4	Application of the Frobenius Group Determinant Theorem to Balanced Weighing Matrices	141
4.16.5	Balanced Weighing Matrices with $v \leq 25$	143
4.16.6	No Circulant Balanced Weighing Matrices $BW(v, v - 1)$ Based on $(v, v - 1, v - 2)$ Configurations .	144
4.17	Negacyclic Matrices.....	148
4.17.1	Constructions	152
4.17.2	Applications	153
4.17.3	Combinatorial Applications	154

5	Amicable Orthogonal Designs	155
5.1	Introduction	155
5.2	Definitions and Elementary Observations	157
5.2.1	n Odd	158
5.2.2	$n = 2b$, b Odd	160
5.3	More on Variables in an Amicable Orthogonal Design	162
5.4	The Number of Variables	164
5.5	The Algebraic Theory of Amicable Orthogonal Designs	168
5.6	The Combinatorial Theory of Amicable Orthogonal Designs	171
5.6.1	Cases $a = 2, 3$ or 4	175
5.7	Construction of Amicable Orthogonal Designs	178
5.8	Construction Methods	182
5.9	Specific Orders 2^n	183
5.9.1	Amicable OD of order 2	183
5.9.2	Amicable Orthogonal Designs of Order 8	184
5.10	Amicable Hadamard Matrices	194
5.11	Amicable Hadamard Matrices and Cores	202
5.12	Strong Amicable Designs	205
5.13	Structure of Amicable Weighing Matrices	206
5.14	Generalizations	207
5.15	Repeat and Product Design Families	211
6	Product Designs and Repeat Designs (Gastineau-Hills)	213
6.1	Generalizing Amicable Orthogonal Designs	213
6.1.1	Product Designs	214
6.1.2	Constructing Product Designs	215
6.2	Constructing Orthogonal Designs from Product Designs	218
6.2.1	Applications	221
6.3	Using Families of Matrices – Repeat Designs	221
6.3.1	Construction and Replication of Repeat Designs	224
6.3.2	Construction of Orthogonal Designs	225
6.4	Gastineau-Hills on Product Designs and Repeat Designs	227
6.5	Gastineau-Hills Systems of Orthogonal Designs	232
6.6	Clifford-Gastineau-Hills Algebras	236
6.7	Decomposition	238
6.8	Clifford-Gastineau-Hills (CGH) Quasi Clifford Algebras	242
6.9	The Order Number Theorem	246
6.10	Periodicity	253
6.11	Orders of Repeat Designs	256
6.12	Orders of Product Designs and Amicable Sets	261
7	Techniques	267
7.1	Using Cyclotomy	267
7.2	Sequences with Zero-autocorrelation Function	275
7.2.1	Other sequences with zero auto-correlation function	282

7.3	Current Results for Non-Periodic Golay Pairs	284
7.4	Recent Results for Periodic Golay Pairs	285
7.5	Using complementary sequences to form Baumert-Hall arrays	285
7.6	Construction using complementary sequences	291
7.7	6-Turyn-type Sequences	294
8	Robinson's Theorem	295
9	Hadamard Matrices and Asymptotic Orthogonal Designs .	305
9.1	Existence of Hadamard Matrices	305
9.2	The Existence of Hadamard Matrices	306
9.3	Asymptotic Existence Results for Orthogonal Designs	309
9.4	n -Tuples	314
9.4.1	Description of the Construction Algorithm.....	316
9.4.2	Implementing the Algorithm	318
9.4.3	n -Tuples in Powers of 2 With No Zeros.....	319
9.5	Enough Powers of Two: Asymptotic Existence.....	321
9.5.1	The Asymptotic Hadamard Existence Theorem	323
9.5.2	Ghaderpour and Kharaghani's Uber Asymptotic Results.....	323
9.6	The Asymptotic Existence of Amicable Orthogonal Designs ..	329
9.7	de Launey's Theorem	332
10	Complex, Quaternion and Non Square Orthogonal Designs	335
10.1	Introduction	335
10.2	Complex orthogonal designs	336
10.3	Amicable orthogonal designs of quaternions	337
10.4	Construction techniques	340
10.4.1	Amicable orthogonal designs	341
10.5	Amicable orthogonal design of quaternions.....	342
10.6	Combined Quaternion Orthogonal Designs from Amicable Designs	348
10.7	Le Tran's Complex Orthogonal Designs of Order Eight	352
10.8	Research Problem	355
A	Orthogonal Designs in Order 12, 24, 48 and $3 \cdot q$	357
A.1	Number of possible n -tuples	357
A.2	Some Theorems	358
A.3	Order 12	358
A.4	Order 24	360
A.5	Order 48	366
B	Orthogonal Designs in Order 20, 40 and 80	369
B.1	Some Theorems	369
B.2	Order 20	369
B.3	Order 40	370
B.4	Order 80	375

C	Orthogonal Designs in Order 28 and 56	379
C.1	Some Theorems	379
C.2	Order 28	379
C.3	Order 56	385
C.4	Further Research	385
D	Orthogonal Designs in Order 36 and 72	389
D.1	Some theorems	389
D.2	Order 36	389
D.3	Order 72	390
E	Orthogonal Designs in order 44	395
E.1	Some theorems	395
E.2	Order 44	395
F	Orthogonal Designs in Powers of 2	403
F.1	Some Theorems	403
F.2	Orthogonal Designs in Order 16	404
F.3	Order 32	409
F.4	Order 64	415
G	Some Complementary Sequences	417
H	Product Designs	425
	References	429
	Index	441

Orthogonal Designs

Hadamard Matrices, Quadratic Forms and Algebras

Seberry, J.

2017, XXIII, 453 p. 10 illus., Hardcover

ISBN: 978-3-319-59031-8