Preface

> "A teacher can never truly teach unless he is still learning himself. A lamp can never light another lamp unless it continues to burn its own flame. The teacher who has come to the end of this subject, who has no living traffic with his knowledge but merely repeats his lessons to his students, can only load their minds; he cannot quicken them."
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> Rabindranath Tagore
> Nobel Prize Winner for Literature (1913)

The origin of wavelet analysis can be traced to the classic theory of harmonic analysis and the seminal contributions of Joseph Fourier, Alfred Haar, and Paul Levy. Since the appearance of the pioneering work of Morlet and Grossman in the 1980s wavelet methodology has been introduced to the literature as a regular alternative for analyzing irregular situations where the data/signal contains scaling properties, discontinuities, sharp spikes, etc. These contributions were followed by the introduction of the general idea of multiresolution analysis by Mallat and Meyer and the notion of orthogonal wavelet bases by Daubechies in the late 1980s. Thus, we can say that the development and advancement of the theory of wavelets came through the efforts of mathematicians with a variety of backgrounds and specialties, and of engineers and scientists with an eye for better solutions and models in their applications. Nowadays, there is no doubt that the introduction of wavelet transform was one of the most important events in mathematics over the past few decades. They have fascinated the scientific, engineering, and mathematical community with their versatile applicability and are now considered as a nucleus of shared aspirations and ideas. The application areas for wavelets have been growing for the last two decades at a very rapid rate. They have been applied in a number of fields including signal processing, image processing, sampling theory, turbulence, approximation theory, geophysics, astrophysics, quantum mechanics, computer graphics, statistics, economics and finance, quality control, differential and integral equations, numerical analysis, neuroscience, medicine, neural networks, chemistry, nano-technology, and even in political time series. A consequence of this interest is the appearance of several books, journals, and a large volume of research articles on this subject. Currently, there are many books in the market, with more being written everyday, which treat the subject of wavelets from a wide range of perspectives and with several areas of a large spectrum of possible applications. Workers in the field judge some of these "excellent." So, why bother to publish an additional one?

The answer lies in the fact that there seems to be no textbook that provides a systematic introduction to the subject of wavelet transforms. While teaching courses on integral transforms and wavelet transforms, the authors have had difficulty choosing textbooks to accompany lectures on wavelet transforms at the senior undergraduate and/or graduate levels. Many hours of study convinced us that there is a need for lecture notes on wavelet transforms for mathematicians, scientists and engineers that provide both a systematic exposition of the basic ideas and results of wavelet analysis. The selection, arrangement, and presentation of the material in these lecture notes have carefully been made based on our past and present teaching, research, and professional experience. In particular, drafts of these lecture notes have been used by us for regular teaching courses in wavelet transforms and their applications at the University of Texas–Pan American, USA, and the University of Kashmir, India. These notes differ from many textbooks with similar titles due to major emphasis placed on numerous topics and systematic development of the underlying theory before presenting applications and the inclusion of many new and modern topics such as fractional Fourier transforms, windowed canonical transforms, fractional wavelet transforms, fast wavelet transforms, spline wavelets, Daubechies wavelets, harmonic wavelets, and nonuniform wavelets. Therefore, our primary goal is to show how different types of wavelets can be constructed, illustrate why they provide us with a particularly powerful tool in mathematical analysis, and indicate how they can be used in applications. Our secondary goal is to develop required analytical knowledge and skills on the part of the reader, rather than focus on the importance of more abstract formulation with full mathematical rigor. Indeed, our major emphasis is to provide an accessible working knowledge of the analytical and computational methods with proofs required in pure and applied mathematics, physics, and engineering.

This monograph is written from the ground level and up. The presentation is as simple as possible, but to paraphrase Einstein “it should not be simpler.” We have attempted to make the monograph as self-contained as possible. Mathematics, science, and engineering students need to gain a sound knowledge of mathematical and computational skills by the systematic development of underlying theory with varied applications and provision of carefully selected fully worked-out examples combined with their extensions and refinements through addition of a large set of a wide variety of exercises at the end of each chapter. Numerous standard and challenging worked-out examples and exercises are included so that they stimulate research interest among senior undergraduates and graduate students. Another special feature of this book is to include sufficient modern topics which are vital prerequisites for subsequent advanced courses and research in mathematical, physical, and engineering sciences.

Now it is time to give some indications on the contents of the monograph. The book has 5 chapters, which are described briefly here to show how the monograph’s main ideas are developed. Wavelet transforms can be considered as a modern supplement to classical Fourier transforms, and for this reason we give a more detailed presentation of Fourier transforms in Chapter 1. We start with the motivation of Fourier series and Fourier transforms in $L^1(\mathbb{R})$ and $L^2(\mathbb{R})$ followed by their basic properties. Several important results including the approximate identity theorem, general Parseval's relation, and Plancherel theorem are discussed in some detail. Discrete Fourier transform, fast Fourier transform, and fractional Fourier transform are also discussed briefly for the purpose of comparing them with the continuous, discrete, and fractional wavelet transforms. Applications of the fractional Fourier transform in solving generalized nonhomogeneous differential equations including the generalized wave and heat equations are also given. Special attention is also given to the Heisenberg's uncertainty principle.

Chapter 2 is devoted to a fairly detailed examination of the joint time-frequency analysis of signals. The main goal here is to set the foundation for the development of continuous and discrete wavelet transforms. We begin with the time-frequency localization of signals which leads us to the windowed Fourier transform. This is followed by the Gabor transform and its basic properties, including the inversion formula. Special attention is also given to the Zak transform and its basic properties. Based on the relationship between the Fourier transform and linear canonical transform, a hybrid windowed transform, namely the windowed linear canonical transform, has been introduced. Its basic properties and several results including the orthogonality relation and inversion formula are also discussed.

The heart of the wavelet theory is covered in Chapters 3 and 4 in a comprehensive approach. We start Chapter 3 with the introduction of wavelets and wavelet transforms with examples. The basic ideas and properties of wavelet transforms are discussed with special attention given to the use of different wavelets for resolution and synthesis of signals. This is followed by the discrete version of wavelet transform and the construction of orthonormal dyadic wavelet basis. Special attention is given to fairly exact mathematical treatment of the fractional wavelet transform, and several important results including Parseval's formula and inversion theorem are proved. Chapter 4 contains an exposition of the general notion of a multiresolution analysis together with several examples. Special attention is given to properties of scaling functions and orthonormal wavelet bases. This is followed by a method of constructing orthonormal bases of wavelets from an MRA. In the end, the fast wavelet transform is briefly discussed.

Chapter 5 is devoted to several generalizations and extensions of orthonormal wavelet bases in $L^2(\mathbb{R})$. To construct wavelets with greater degrees of smoothness and having compact support, we construct wavelets that are smooth and piecewise polynomials, usually known as spline wavelets. The well-known Franklin and Battle-Lemarié wavelets are the special cases of these wavelets. This is followed by Daubechies algorithm for the construction of compactly supported wavelets. Then, we discuss another intersecting class of orthonormal wavelets called harmonic wavelets. Finally, we present a novel and simple procedure for the construction of nonuniform wavelets associated with nonuniform MRA. In this nonstandard setting, the associated translation set is no longer a discrete subgroup of $\mathbb{R}$ but a spectrum associated with a certain one-dimensional spectral pair, and the associated dilation is an even positive integer related to the given spectral pair.

In preparing the monograph, the authors have been encouraged by and have benefited from the helpful comments and criticisms of a number of faculty and postdoctoral and doctoral students of several universities in the USA and India. The authors express their grateful thanks to these individuals for their interest in the book. The editor from Birkhäuser-Springer, Benjamin Levitt, deserves a special vote of thanks for his cooperation and for the exemplary patience he displayed. It goes without saying, however, that all responsibility for errors, imperfections and residual or outright mistakes, is shared jointly by both of us. However, we hope that these are both few and obvious and will cause minimum confusion.

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http://www.springer.com/978-3-319-59432-3

Lecture Notes on Wavelet Transforms
Debnath, L.; Shah, F.
2017, XII, 220 p. 32 illus., 3 illus. in color., Softcover
ISBN: 978-3-319-59432-3
A product of Birkhäuser Basel