Preface

Implementations of optimisation methods often rely on the assumption that the underlying problem is posed in the Euclidean space. In many applications however, the optimisation control is a function over some Hilbert space rather than being a number or a vector in $\mathbb{R}^n$. Consideration of this point is important since the choice of the underlying Hilbert space determines the notion of angles and distances, induced by its associated inner product. Since such quantitative concepts are typically used by algorithms, for example to compute a search direction or to decide whether convergence criteria have been meet, it seems logical that the Euclidean assumption might lead to problems in the performance of these algorithms in cases where the control is not Euclidean.

One example of this is in the case of gradient-based methods, that is methods that use the gradient of an objective functional. It is a consequence of the Riesz representation theorem that the representation of the derivative of a functional depends on the underlying inner product. Methods which do not respect the inherent or natural control space and its associated inner product become dependent on the underlying discretisation of the domain – with detrimental consequences for the algorithm's performance. The exploration and analysis of this phenomenon, that we term mesh dependence, in theory and numerical simulations represent the core of this work.

Scenarios whose controls are functions lying in some Hilbert space regularly arise when solving optimisation problems for a physical system which is subject to constraints in terms of partial differential equations (PDEs). A classical example of such an optimisation problem is: what is the optimal shape of an aeroplane in terms of maximum fuel efficiency or lift-to-drag ratio under the influence of air flow around the body? It seems natural to represent the shape by a continuous function. Applications of PDE-constrained optimisation appear in a wide range of other fields of industry as well as in medical research and in economics.

This work provides a discussion and analysis which quantifies the performance loss induced by assuming the Euclidean inner product for gradient-based methods for PDE-constrained optimisation problems with continuous controls. The authors also hope to promote best practice for the development and usage of optimisation tools in the context of problems posed in Hilbert spaces as a consequence on their analysis.

The intended audience of this book mainly consists of graduate students and researchers in computational science, mathematics, engineering and physics with an interest in PDE-constrained optimisation and their real-world applications. The application oriented character of the book may also make it of interest to users of optimisation tools and practitioners that work with PDE-constrained problems. Developers of optimisation tools or users interested in best practice for mesh-independent methods are a further potential audience.

The reader is provided with a gentle introduction into the field of PDE-constrained optimisation in Chap. 1. This includes an introduction to solving PDEs using the finite element method, to adjoints used for the effective computation of derivatives and an overview of some popular gradient-based optimisation methods. In addition, this chapter introduces the relationship between primal and dual space, which underpins the analysis in the subsequent chapter. In Chap. 2 the impact of mesh-dependence is treated theoretically for a simple optimisation problem, as well as numerically for a slightly more advanced generic PDE-constrained optimisation problem. In Chap. 3, a case study considering the optimisation of tidal turbine array layouts in the Pentland Firth, Scotland, demonstrates the relevance of the analysis from the preceding chapter. This further represents a valuable example of a real-world application of optimisation methods used in the context of PDE-constrained problems.

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