A Sustainable Bi-objective Approach for the Minimum Latency Problem

Nancy A. Arellano-Arriaga[1,2](✉), Ada M. Álvarez-Socarrás[1], and Iris A. Martínez-Salazar[1]

[1] Facultad de Ingeniería Mecánica y Eléctrica, Universidad Autónoma de Nuevo León, San Nicolás de los Garza, Nuevo León, Mexico
{nancy.arellanoarg,iris.martinezsalaz}@uanl.edu.mx, nancy.arellano@uma.es, ada.alvarezs@uanl.mx

[2] Departamento de Economía Aplicada (Matemáticas), Universidad de Málaga, El Ejido, Málaga, Spain

Abstract. Nowadays, sustainability is a major factor to consider in the decision-making process. Specifically, for companies trying to stay competitive and having some advantage in the market it is a vital issue. In this study, we introduce a multi objective problem which aims to minimize distance and latency of a route with enough capacity to serve a set of clients. We assume that a vehicle leaves an established depot, visits all clients and returns to the depot before the end of the workday. With this bi-objective problem, we aim to improve the sustainability of the company by improving their economic and environmental contribution, through the minimization of the traveled distance of the vehicle along with the improvement of their social service by the minimization of the total waiting time of the customers. We call this problem Minimum Latency-Distance Problem (MLDP) and in this paper, we introduce a mathematical formulation which describes it.

Keywords: Combinatorial optimization · Mathematical formulation · Multiple objective programming · Multi objective optimization · Multi objective problem · Latency · Distance · Multi objective routing problem

1 Introduction

Humanity developed the notion of sustainability by the awareness of the damage caused by living in big cities, the waste of natural resources among other environmental issues [1–4]. Sustainability is nowadays a global concept which rests on three important pillars: the environmental, the economic and the social. The first pillar refers to climate protection alongside the protection of the natural resources and the diversity of Earth's flora and fauna [5,6]. The second pillar refers to business and industries by focusing in regulating them with an agreement of responsibility towards the environment and the community. This

E. Alba et al. (Eds.): Smart-CT 2017, LNCS 10268, pp. 11–19, 2017.
DOI: 10.1007/978-3-319-59513-9_2

agreement deals with the reduction of the waste and the environmental pollution the companies generate as well as to improve the contribution of all social improvements created by the companies [7–9]. The last pillar of sustainability refers to the equality among humanity by referring to the importance of all human beings, their integrity, and in general, the satisfaction of their needs within ecological constraints [10,11]. Summarizing, sustainability attempts to protect the environment and the human beings integrity meanwhile respecting the structure and the resources of the Earth.

It is known that in general, deterministic relationships between the sustainability pillars are inadequate and lead to make several trade-offs [12,13]. Dobson [14,15] states that environmental and social sustainability are not compatible, this means that to gain in one pillar we have to lose in another, and concludes that deterministic decisions are only served in an incomplete way. Therefore, it is important to introduce new approaches to already known problems which attempt to aim several objectives focusing on the improvement of all pillars at the same time, targeting to the generation of more sustainable conditions for everyone. Multi-objective optimization specializes in taking decisions based on several objectives [16] and to choose a suitable trade-off for all involved stakeholders in the decision making.

In this paper, we present a bi-objective problem which arises in the context of logistic activities of distribution-and-service companies which specialize in customer service by attending requests of product delivery and/or maintenance services. We propose a bi-objective approach which attempts to obtain a trade-off that benefits all involved pillars of sustainability: environmental, economic and social, by simultaneously minimizing distance and latency of a route under certain assumptions.

The proposed problem is a direct application of two well-known routing problems: the minimum latency problem [17–19], and the traveling salesman problem [20]. The first one focuses on minimizing the total waiting time of a set of clients in a route, regardless the traveled distance of the vehicle. Meanwhile, the second problem deals with minimizing the total traveled distance of the vehicle, regardless the waiting time of the clients. By combining both objectives, we propose a client-centered approach which considers equally important the company resources and the quality of the service, or the social resources, in the decision making. The contribution to the environmental pillar of sustainability, goes along the minimization of the traveled distance of the vehicle by assuming the correspondence of less moving distance, fewer emissions the vehicle generates.

The main contribution of this paper is the development of a mixed integer formulation to represent this bi-objective problem. In Sect. 2, the description of the proposed problem is presented as well as its mathematical formulation. In Sect. 3, the computational experimentation is presented. And lastly, the conclusions are shown in Sect. 4.

2 Minimum Latency-Distance Problem

Let us consider a company in charge of attending a set of customers which demands a service, for example delivery or a maintenance issue; this company has an agent or a vehicle which is responsible of attending all the requests. This agent searches for a route that leaves from a known depot, visits all clients and returns to the depot, minimizing the travel distance of the vehicle, as well as the total clients' waiting time, or total latency of the route. All service times he may take to fullfill the request in each client and all travel times among clients are known, this is to say, every service and travel time are known before hand.

This bi-objective problem will be referenced from this moment on as the *Minimum Latency-Distance Problem* (MLDP). Due to the differences between the objectives involved in the MLDP and the fact that service and travel times are non-zero in real applications, in literature, it is known that both, distance and latency objectives are not calculated with the same metric [18,21]. Therefore, we assume several asseverations to formulate a model to describe the MLDP:

- For simplicity, we assume that traveling time is linearly proportional to traveled distance. Hence, the larger the traveled distance, larger the traveling time required to reach the client. This is a major assumption and in terms of applications, this is not always a true statement. We will assume all clients are reachable in a constant amount of time and the vehicle in charge of visit them will travel at the same speed all the time.
- One route is enough to serve all clients. Hence, this vehicle has infinite capacity.
- We assume the proportionality of the environmental savings to the traveled-distance savings. This is to say, less traveled distance, less emissions are generated by the vehicle.
- The environmental savings are not responsibility of the company in charge of providing the service to the clients. Environmental savings are a direct consequence of the minimization of the traveled distance, as previously stated.

Please note that this bi-objective approach has not been studied in the operations research area, and therefore it is a base model to consider the non-linearity of the objectives as future research. Previous studies [22], showed that it is not convenient to use a formulation designed for dealing with the objective of minimizing distance to handle an objective of minimizing the total latency of a route. Therefore, for the modelling of the MLDP, we take advantage of a model developed for the objective of latency. The issues related to the distance objective are later incorporated. Reported in literature, there are several formulations for the minimum latency problem. Some examples are the formulation presented in Méndez Díaz et al. [23], Gouveia and Voß [24], Picard and Queyranne [25], the implementation of Picard's formulation by Sarubbi et al. [26], and lately, Angel-Bello et al. [21]. However, [21] outbid them all by proposing two mathematical models that showed a better performance than all existing formulations. Therefore, our formulation takes as reference one of those models, called by Angel-Bello et al. [21] as "Model A".

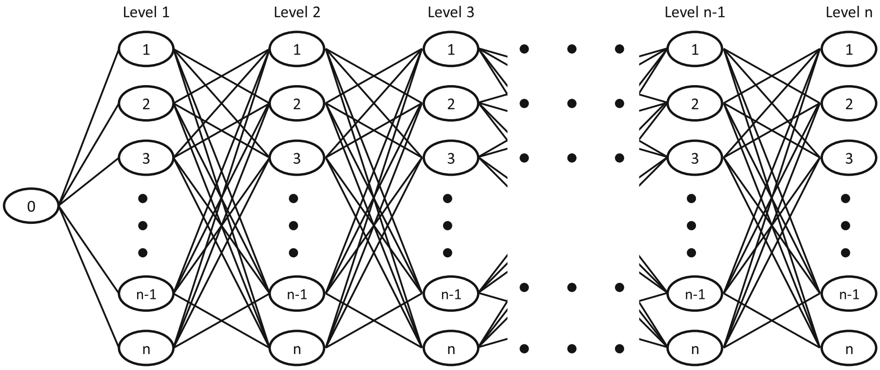

Fig. 1. Multi-level network used to formulate the MLDP.

Consider a directed and complete graph $G = (V, A)$. Let A be the arc set and $V = \{\{0\} \cup I\}$ the vertex set, where 0 is defined as the depot and the subset $I = \{1, 2, \ldots, n\}$ are the clients meant to be visited. Each client $i \in I$ has a service time s_i associated and each arc $(i, j) \in A$ has associated a travel time from client i to j, defined as t_{ij}. The connections among the depot and all the clients are also considered. For the formulation, a cost matrix $C = c_{ij}$ is built, considering:

$$c_{ij} = \begin{cases} t_{0j}, & \text{for } i = 0; j = 1, 2, \ldots, n; \\ s_i + t_{ij}, & \text{for } i = 1, 2, \ldots, n; j = 0, 1, \ldots, n; j \neq i. \end{cases}$$

This formulation is based on a multi-level network (see Fig. 1) with $n + 1$ levels inspired by one previously proposed by Picard and Queyranne in 1978, which was designed for a time-dependent travelling salesman problem [25].

In this multi-level network level 0 represents the depot, while levels $1, 2, \ldots, n$ are n copies of the set of clients. Note that an MLDP solution is a permutation of the n clients with node 0 in the first position of the permutation. Consequently, any solution can be represented on this multilevel network having a single node active in each level, with no repetitions between levels. These facts entail to define the decision variables x_i^k and y_{ij}^k as

$$x_i^k = \begin{cases} 1, & \text{If node } i \text{ is selected on level } k \text{ on the network,} \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{ij}^k = \begin{cases} 1, & \text{If node } j \text{ on level } k+1 \text{ follows node } i \text{ on level } k, \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, MLDP is then formulated as:

$$min\ F_1 = n\sum_{i=1}^{n} c_{0i}x_i^1 + \sum_{k=1}^{n-1}\sum_{i=1}^{n}\sum_{j=1}^{n}(n-k)c_{ij}y_{ij}^k \tag{1}$$

$$min\ F_2 = \sum_{i=1}^{n} c_{0i}x_i^1 + \sum_{k=1}^{n-1}\sum_{i=1}^{n}\sum_{j=1}^{n}c_{ij}y_{ij}^k + \sum_{i=1}^{n} c_{i0}x_i^n \tag{2}$$

s.a

$$\sum_{k=1}^{n} x_i^k = 1,\ i = 1, 2, \ldots, n \tag{3}$$

$$\sum_{i=1}^{n} x_i^k = 1,\ k = 1, 2, \ldots, n \tag{4}$$

$$\sum_{\substack{j=1\\ j\neq i}}^{n} y_{ij}^k = x_i^k,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, n-1 \tag{5}$$

$$\sum_{\substack{j=1\\ j\neq i}}^{n} y_{ji}^k = x_i^{k+1},\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, n-1 \tag{6}$$

$$x_i^k \in \{0, 1\},\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, n \tag{7}$$

$$y_{ij}^k \geq 0,\ i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n, j \neq i,\ k = 1, 2, \ldots, n \tag{8}$$

The minimization of the total waiting time of the clients is defined by Eq. (1). This Equation is a mathematical representation of the social interest in this proposed problem. In Eq. (2) lays the minimization of the total traveled distance and therefore, the minimization of the cost this route represents to the company. By assuming that minimizing traveled distance is proportional to minimizing emissions, the minimization of the emissions generated by the vehicle goes along this equation and therefore, this Equation is the mathematical representation of the economic and environmental interests in this proposed problem. Note that in Eq. (1) the arc denoting the return of the vehicle to the depot is not considered, meanwhile this returning arc has to be considered on Eq. (2). The reason of this is we do not consider the returning of the vehicle to the depot as client's waiting time.

Equation (3), guarantees that each node occupies a single position in any feasible solution. Equation (4), guarantees that each position is occupied by no more than one node in any feasible solution. Equation (5), ensures that only one arc leaves from position k, exactly from the node taking that position and Eq. (6) imposes that only one arc at a time can arrive to position $k+1$, exactly to the node occupying that position. The nature of Eqs. (3) to (6), lays in the fact that all possible solutions for the MLDP can be represented on the multi-level network previously defined.

Lastly, Eqs. (7) and (8) correspond to the nature of the variables. Note that variables y_{ij}^k are previously defined as binary but they are handled as continuous variables in the model. This is achieved by Eqs. (5) and (6), which guarantee that only one arc leaves from position k and arrives exactly to the node occupying the position $k+1$. These pair of constraints assures y_{ij}^k takes a value of one or a value of zero. This formulation consists of n^2 binary variables, $n^3 - 2n^2 + n$ real variables and $2n^2$ constraints.

3 Computational Experimentation

Because both single-objective problems involved are NP-hard [27,28], it is known before hand that to obtain a true Pareto front, it could take an exponential-crescent time according to the size of the instance. The study of the complexity for this proposed problem is left for future research. To test the mathematical formulation, we implemented the *Augmecon2* procedure, proposed by Mavrotas et al. [29].

The Augmecon2 procedure, in general, fixes one objective and attempts to solve the multi-objective formulation as a single objective one, considering all the original constraints plus the rest of the objectives as constraints as well. In our particular implementation of Augmecon2, we defined the distance objective as the main objective the one to pursue. The algorithm requires the definition of a step, in our particular implementation this step is defined as the difference between the optimal solution of the mono-objective latency problem and the optimal distance solution, evaluated on the function of latency, divided by 10 as it was proposed by Mavrotas et al. [29].

We tested our implementation with the same group of instances used on the work of Angel-Bello et al. [21]. These instances are called *GTRP* and were obtained by randomly generated points with real coordinates from a uniform distribution between 0 and 100. The travel times defined in these instances were taken as Euclidean distances and rounded down to integers. Service times s_i were considered as zero (GTRP-S_0) and non-zero, defined in between the interval $[0, (t_{max} - t_{min})/2]$ (GTRP-S_1). In these intervals, $t_{max} = max\{t_{ij}\}$ and $t_{min} = min\{t_{ij}\}$. We maintained the original classification of the instances by the type of service times they included (GTRP-S_0 and GTRP-S_1). For each type, each set of instances has several sizes: 10, 15, 20, 25, 30, 35, 40, 60, 80 and 100 clients, each size with 25 instances. All selected instances were solved in a Xenon ®Intel ®CPU E3-1245 v3 @ 3.40 GHz, with 16.0 GB of RAM and the MILP solver selected was ILOG CPLEX C++ Concert Technology.

The selected limit time on each instance was 10,800 s, three hours, CPU Times taken by this algorithm can be seen on Table 1. Note that for both service times we could only reach the true Pareto Front for instances of 25 clients, for larger instances we reached the limit time before finding the exact frontier. In Fig. 2, an example of a 25-client instance exact Frontier is depicted. Note that to select a single solution in a Pareto front, it is advisable to review all existing methodologies and select the most appropriate [30].

Table 1. Average CPU Times obtained using Augmecon2 procedure.

Instance	Size	CPU Time (s)
GTRP-S_0	10	15.340
	15	113.678
	20	1274.710
	25	4263.970
GTRP-S_1	10	18.610
	15	122.830
	20	1587.842
	25	10195.500

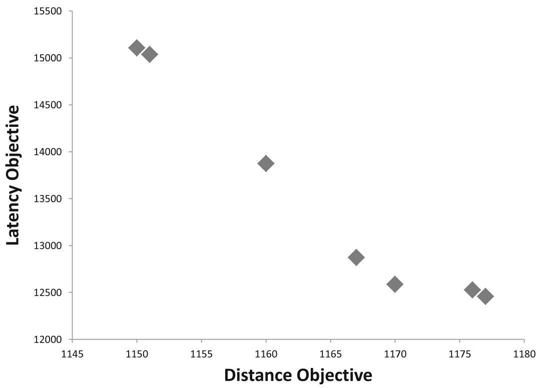

Fig. 2. Points of the exact Pareto front of a GTRP-S_0 instance of size 25, found with the described implementation of the Augmecon2 procedure.

4 Conclusions

In this paper, we introduced a bi-objective problem that seeks to deal with sustainability issues within a routing problem. We propose to minimize the traveled distance and the total latency of a route, to minimize the environmental impact, to improve the economy of a business and to improve the social interaction among the company and the clients, by the integration of the client in the decision-making process. This approach is useful, not only to reduce the cost that the traveling represents to the company, but to improve customer service by minimizing the total waiting time of all the clients. By bringing together the economic and social aspects of this problem, the company can obtain bigger benefit in general.

We presented a mathematical formulation for the MLDP problem and solved it to obtain exact Pareto fronts. We were able to solve instances up to 25 clients with both service times, zero and non-zero, before reaching limit time.

In future research, it is important to consider all involved times stochastic. It would be advisable to introduce a better way to consider the emissions this vehicle generates and to redefine the assumptions made about the linearity between traveled time and traveled distance. The introduction of several vehicles to visit all clients, as well as the introduction of capacity in the vehicles are other issues to add.

Acknowledgements. The first author would like to thank CONACYT, the Mexican National Council for Science and Technology, which supports her studies. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

1. Daly, H.E.: Toward a Steady State Economy. Freeman, San Francisco (1973)
2. Daly, H.E., Cobb, J.B.: For the Common Good. Beacon Press, Boston (1989)
3. Daly, H.E.: Allocation, distribution and scale: towards an economics which is efficient, just and sustainable. Ecol. Econ. **6**(3), 185–193 (1992)
4. Wackernagel, M., Rees, W.: Our Ecological Footprint: Reducing Human Impact on the Earth. The New Catalyst. Bioregional Series. New Society Publishers, Gabriola Island (1998)
5. Goodland, R.: The concept of environmental sustainability. Ann. Rev. Ecol. Syst. **26**, 1–24 (1995)
6. Esty, D.C., Levy, M., Srebotnjak, T., De Sherbinin, A.: Environmental Sustainability Index: Benchmarking National Environmental Stewardship, pp. 47–60. Yale Center for Environmental Law & Policy, New Haven (2005)
7. Dangelico, R.M., Pujari, D.: Mainstreaming green product innovation: why and how companies integrate environmental sustainability. J. Bus. Eth. **95**(3), 471–486 (2010)
8. Baral, N., Pokharel, M.P.: How sustainability is reflected in the S&P 500 companies strategic documents. Organ. Environ. (April 24, 2016). doi:10.1177/1086026616645381
9. Schaltegger, S., Hansen, E.G., Ldeke-Freund, F.: Business models for sustainability: origins, present research, and future avenues. Organ. Environ. **29**(1), 3–10 (2016)
10. Polèse, M., Stren, R.: The Social Sustainability of Cities: Diversity and the Management of Change. University of Toronto Press, Toronto (2000)
11. Littig, B., Griessler, E.: Social sustainability: a catchword between political pragmatism and social theory. Int. J. Sustain. Dev. **8**(1), 65–79 (2005)
12. Goodland, R.: Sustainability: Human, Social, Economic and Environmental. Encyclopedia of Global Environmental Change. Wiley, London (2002)
13. Costanza, R., Graumlich, L., Steffen, W., Crumley, C., Dearing, J., Hibbard, K., Leemans, R., Redman, C., Schimel, D.: Sustainability or collapse: what can we learn from integrating the history of humans and the rest of nature? Ambio **36**(7), 522–527 (2007)
14. Dobson, A.: Justice and the Environment: Conceptions of Environmental Sustainability and Theories of Distributive Justice. Clarendon Press, Oxford (1998)
15. Dobson, A.: Fairness and Futurity: Essays on Environmental Sustainability and Social Justice. Oxford University Press, Oxford (1999)
16. Ehrgott, M.: Multicriteria Optimization. Springer, Heidelberg (2005)

17. Lucena, A.: Time-dependent traveling salesman problem-the deliveryman case. Networks **20**(6), 753–763 (1990)
18. Blum, A., Chalasani, P., Coppersmith, D., Pulleyblank, B., Raghavan, P., Sudan, M.: The minimum latency problem. In: Proceedings of the Twenty-Sixth Annual ACM Symposium on Theory of Computing (STOC 1994), pp. 163–171. ACM, New York (1994)
19. Chaudhuri, K., Godfrey, B., Rao, S., Talwar, K.: Paths, trees, and minimum latency tours. In: Proceedings of the 44th Annual IEEE Symposium on Foundations of Computer Science, pp. 36–45. IEEE (2003)
20. Laporte, G.: The traveling salesman problem: an overview of exact and approximate algorithms. Eur. J. Oper. Res. **59**(2), 231–247 (1992)
21. Angel Bello, F., Álvarez, A., García, I.: Two improved formulations for the minimum latency problem. Appl. Math. Model. **37**(4), 2257–2266 (2013)
22. Angel-Bello, F., Martinez-Salazar, I., Alvarez, A.: Minimizing waiting times in a route design problem with multiple use of a single vehicle. In: INOC (2013)
23. Méndez-Díaz, I., Zabala, P., Lucena, A.: A new formulation for the traveling deliveryman problem. Discrete Appl. Math. **156**(17), 3223–3237 (2008)
24. Gouveia, L., Voß, S.: A classification of formulations for the (time-dependent) traveling salesman problem. Eur. J. Oper. Res. **83**(1), 69–82 (1995)
25. Picard, J.C., Queyranne, M.: The time-dependent traveling salesman problem and its application to the tardiness problem in one-machine scheduling. Oper. Res. **26**(1), 86–110 (1978)
26. Sarubbi, J., Luna, H., Miranda, G.: Minimum latency problem as a shortest path problem with side constraints (2008)
27. Afrati, F., Cosmadakis, S., Papadimitriou, C.H., Papageorgiou, G., Papakostantinou, N.: The complexity of the travelling repairman problem. RAIRO Theor. Inf. Appl. **20**(1), 79–87 (1986)
28. Papadimitriou, C.M.: Computational Complexity. Addison-Wesley, Massachusetts (1994)
29. Mavrotas, G., Florios, K.: An improved version of the augmented -constraint method (augmecon2) for finding the exact pareto set in multi-objective integer programming problems. Appl. Math. Comput. **219**(18), 9652–9669 (2013)
30. Ferreira, J.C., Fonseca, C.M., Gaspar-Cunha, A.: Methodology to select solutions from the pareto-optimal set: a comparative study. In: Proceedings of the 9th Annual Conference on Genetic and Evolutionary Computation (GECCO 2007), pp. 789–796. ACM, New York (2007)

http://www.springer.com/978-3-319-59512-2

Smart Cities
Second International Conference, Smart-CT 2017,
Málaga, Spain, June 14-16, 2017, Proceedings
Alba, E.; Chicano, F.; Luque, G. (Eds.)
2017, X, 165 p. 53 illus., Softcover
ISBN: 978-3-319-59512-2