Chapter 2
Optimal Energy Management for Smart Homes

2.1 System Model

Consider a smart home owning a mix of DERs such as renewable generators, storage devices, and energy loads. It may purchase power from the grid under the real-time pricing program. We assume a discrete-time model with time period indexed by t, where the length of each time period could be 15 min.

We first describe the mathematical models for renewable energy generation, energy storage, and electricity market we use in this chapter for the energy supply part as shown in Fig. 2.1. We then present the control objective, which is to minimize the long-term time-average expected electricity cost. In the next two sections, we consider two different types of energy demand, i.e., the inelastic energy demand and the elastic one, and further analyze the electricity cost minimization problem.

2.1.1 Renewable Energy Generation

Let $S(t)$ denote the amount of renewable energy generated in slot t and we assume that this energy is first stored in battery before it can be used in the next time slot. A controller is to regulate the portion $\gamma(t)$ of the generated energy stored into battery for each slot t in order to prevent battery overflow. The other portion is spilled. Hence, we have

$$0 \leq \gamma(t) \leq 1. \tag{2.1}$$

Moreover, there is a maximum value S_{max} for $S(t)$, that is,

$$0 \leq S(t) \leq S_{max}. \tag{2.2}$$

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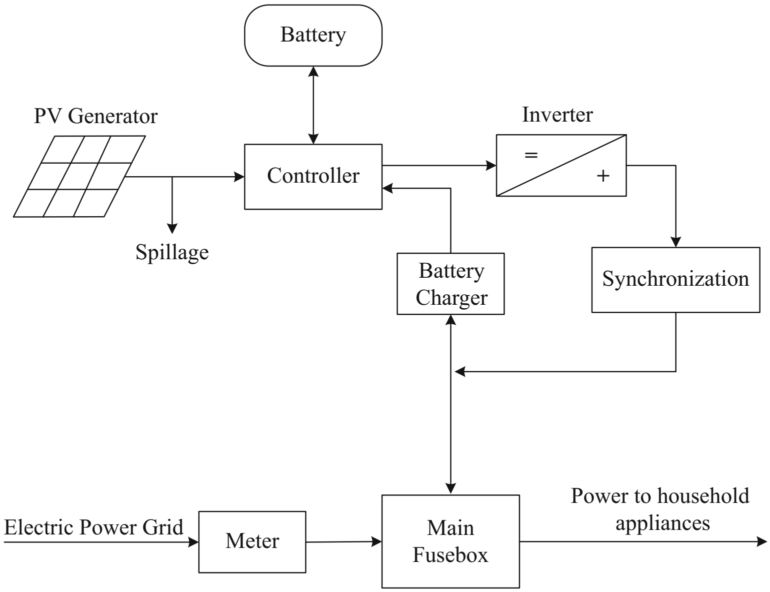

Fig. 2.1 A PV-utility grid system with battery storage for a smart home in smart grids

2.1.2 Energy Storage

In practice, the battery is not ideal and has the following physical properties as analyzed in [1, 2]. First, the battery lifetime depends on both the number of times it undergoes charging/discharging and the depth of discharge during its operation. This relationship is illustrated as battery lifetime chart [3]. Second, there is an energy conversion loss during the charging or discharging process, which is measured as charging/discharging efficiency. Finally, as the time goes, the battery will leak some energy stored in it. To simplify our analysis, we assume a battery model without any inefficiency in charging or discharging. As the time slot we choose is typically small (e.g., 5 min period), energy leakage over time can be neglected. However, more complicated battery model can be easily incorporated into our model without significant impact on our analysis.

We assume that in each time slot t, energy amount $G_b(t)$ can be drawn from the traditional power grid (or simply power grid) to recharge the battery in order to utilize the time-diversity of electricity price. The intuition is that if we recharge the battery when electricity price is low, the overall electricity cost may be reduced with proper design.

The state of charge (SOC) level $B(t)$ in the battery evolves according to the following equation:

$$B(t+1) = B(t) - D(t) + \gamma(t)S(t) + G_b(t), \tag{2.3}$$

where $D(t)$ is the amount of energy that is discharged from battery to supply demand in slot t. Obviously, we should have the following "energy-availability" and finite capacity constraint for each time slot t:

$$D(t) \leq B(t) \leq B_{max}, \tag{2.4}$$

where B_{max} is the battery capacity. There is a maximum discharge rate D_{max} of the battery for one time slot, i.e.,

$$0 \leq D(t) \leq D_{max}. \tag{2.5}$$

The energy amount that can be drawn from the electric power grid to recharge battery for one time slot is also bounded by G_b^{max}, i.e.,

$$0 \leq G_b(t) \leq G_b^{max}. \tag{2.6}$$

2.1.3 Electricity Market

As shown in [4], electricity price in the real-time electricity market has both time-diversity and location-diversity. In this paper, as the proof of concept, we concentrate on one single residential customer (one household), who is subject to a time-varying electricity price. Assume that the time-varying electricity price, $C(t)$, is sent to the customer's smart meter by the utility company at the beginning of each slot t. The cost of using renewable energy generated by the customer itself is assumed to be zero. Denote $G_l(t)$ as the power drawn from the electric power grid to directly supply the energy demand in slot t. Since the total electricity drawn from the electric power grid is $G_b(t) + G_l(t)$, the electricity cost for each time slot t is $(G_b(t) + G_l(t))C(t)$. In our analysis, the unit electricity price $C(t)$ does not depend on the total amount of energy drawn from the power grid. However, if the unit electricity price depends on the total power consumed, such as inclining block rate in [5], it can be still integrated easily into our model as in [1].

2.1.4 Control Objective

In this chapter, we are interested in long-term electricity cost. Hence, our objective here is to minimize the long-term time-average expected electricity cost as described below:

$$P = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C(t)(G_l(t) + G_b(t))\}, \tag{2.7}$$

where the expectation is w.r.t. possibly randomized control actions as well as the distribution of electricity price $C(t)$.

2.2 Inelastic Energy Demand

In a smart home, some energy demands are inelastic, such as lighting, TV watching, as well as computers. For this kind of energy demands, the energy requests must be met exactly at the time t when needed. Based on the models presented in the previous section, we come up with a schematic of power management for inelastic energy demands depicted in Fig. 2.2, where both the power flow and the information flow are shown. Due to the information and communication infrastructure deployed in smart grids, a residential house would form a home area network, which enables communication between components for information gathering and dissemination.

We assume the inelastic energy demand generated in time slot t is $A_{ine}(t)$. For each time slot t, we have

$$G_l(t) + D(t) = A_{ine}(t). \tag{2.8}$$

Thus, our problem can be formulated as the following stochastic optimization, called **Problem One**:

$$\min_{D(t),G_b(t),G_l(t),\gamma(t)} P_1 = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C(t)(G_l(t) + G_b(t))\} \tag{2.9}$$

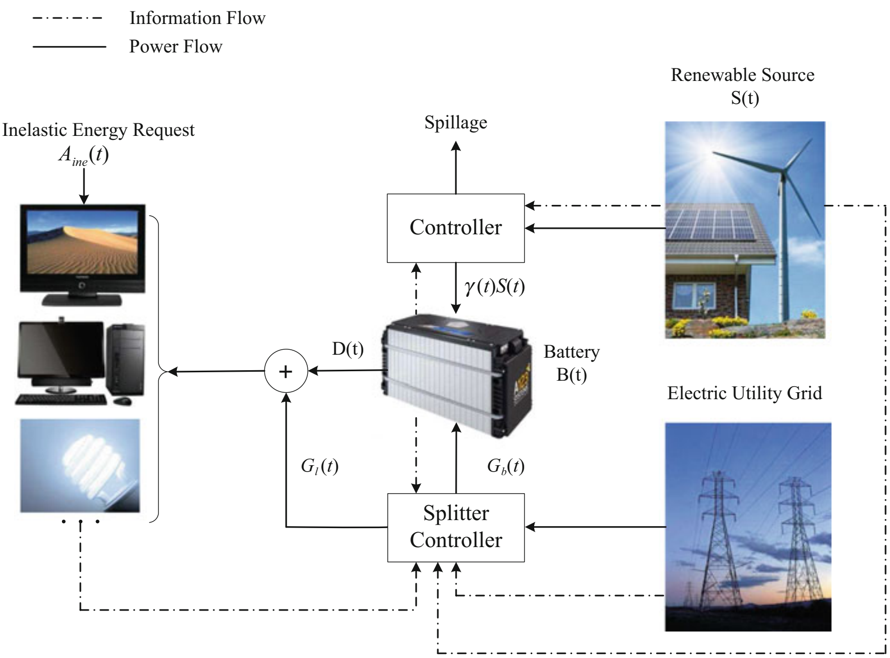

Fig. 2.2 A schematic of power management with inelastic energy demands in a smart home

subject to

$$B(t+1) = B(t) - D(t) + \gamma(t)S(t) + G_b(t),$$
$$D(t) \le B(t) \le B_{max},$$
$$G_l(t) + D(t) = A_{ine}(t),$$
$$0 \le D(t) \le D_{max},$$
$$0 \le G_l(t) \le G_l^{max}, 0 \le G_b(t) \le G_b^{max},$$
$$0 \le \gamma(t) \le 1.$$

Define the optimal objective value of the optimization problem above as P_1^*. In the following, we apply the Lyapunov optimization framework [6, 7] to find an approximate solution, which attains an analytical performance guarantee within $O(1/V)$ of the optimal objective value, where V is a tunable control parameter related to the battery capacity.

The problem above is challenging mainly because of the time-coupling property brought by the battery constraint (2.4). To be specific, in our problem, the current control action may impact the future control actions in the sense that a current action may overuse the battery and leave insufficient energy for future use, or the current action may leave less available capacity and the future generated renewable energy cannot be utilized efficiently. Previous methods to handle this time-coupling problem are usually based on dynamic programming, which suffers from the "curse of dimensionality" problem [8] and requires detailed knowledge of statistics of $C(t)$, $S(t)$ and $A_{ine}(t)$ in our problem. However, in reality, the statistics of $C(t)$, $S(t)$ and $A_{ine}(t)$ may be unknown or difficult to obtain, and we need to design an optimal control algorithm under uncertainty. We use the recently developed Lyapunov optimization framework [7] and find a modified Lyapunov function to develop our algorithm. A salient feature of our algorithm is that it does not need any future knowledge of the system states and can be implemented in real-time.

In the next subsection, instead of solving the above stochastic optimization problem exactly, we study a relaxed problem, whose solution is easy to characterize based on the framework of Lyapunov optimization [6, 7].

2.2.1 Relaxed Problem

We define the time-average expected value of utilized renewable energy, charging and discharging rate under any feasible control policy of **Problem One**, respectively, as follows.

$$\overline{\gamma S} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{\gamma(t)S(t)\},$$

$$\overline{G_b} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{G_b(t)\},$$

$$\overline{D} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{D(t)\}.$$

Since the battery SOC level evolves according to (2.3), summing over all $t \in \{0, 1, 2, \ldots, T-1\}$ and taking expectation on both sides, we have

$$\mathbb{E}\{B(T)\} - B_0 = \sum_{t=0}^{T-1} \mathbb{E}\{\gamma(t)S(t) + G_b(t) - D(t)\},$$

where $B(0) = B_0$ is the initial battery SOC level. As $0 \le B(t) \le B_{max}$ for any time slot t, dividing both sides with T and taking $T \to \infty$, we have $\overline{D} = \overline{\gamma S} + \overline{G_b}$. Hence, we obtain the following relaxed problem, called **Problem Two**:

$$\min_{D(t),G_b(t),G_l(t),\gamma(t)} P_1 = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C(t)(G_l(t) + G_b(t))\} \tag{2.10}$$

subject to

$$\overline{D} = \overline{\gamma S} + \overline{G_b},$$

$$G_l(t) + D(t) = A_{ine}(t),$$

$$0 \le D(t) \le D_{max},$$

$$0 \le G_l(t) \le G_l^{max}, 0 \le G_b(t) \le G_b^{max},$$

$$0 \le \gamma(t) \le 1.$$

Denote the optimal objective value of **Problem Two** as $P_{1,rel}^*$. From the discussion above, we observe that any feasible solution to **Problem One** is also a feasible solution to **Problem Two**, i.e., **Problem Two** is less constrained than **Problem One**. Therefore, $P_{1,rel}^* \le P_1^*$.

It is easy to find the optimal solution to **Problem Two** due to the removal of dependence between battery SOC levels across time slots. As given by the following lemma, the optimal solution to **Problem Two** can be obtained by a randomized, stationary control policy that only chooses $D(t)$, $G_l(t)$, $G_b(t)$ and $\gamma(t)$ every slot purely as a (possibly randomized) function of $C(t)$, $S(t)$ and $A_{ine}(t)$. That means the control policy is independent of battery SOC level. This fact is stated as below:

Lemma 2.1 *If the* $\{A_{ine}(t), C(t), S(t)\}$ *are i.i.d. over slots, then there exists a stationary and randomized policy that takes control decisions* $\hat{D}_{ine}(t)$*,* $\hat{G}_{l,ine}(t)$*,* $\hat{G}_{b,ine}(t)$ *and* $\hat{\gamma}_{ine}(t)$ *every slot t purely as a function (possibly randomized) of current system states* $\{A_{ine}(t), C(t), S(t)\}$ *while satisfying the constraints above and providing the following guarantees:*

$$\mathbb{E}\{\hat{D}_{ine}(t)\} = \mathbb{E}\{\hat{\gamma}_{ine}S(t)\} + \mathbb{E}\{\hat{G}_{b,ine}(t)\}, \tag{2.11}$$

$$\mathbb{E}\{C(t)(\hat{G}_{l,ine}(t) + \hat{G}_{b,ine}(t))\} = P^*_{1,rel}, \tag{2.12}$$

where the expectations are w.r.t. the stationary distribution of $\{A_{ine}(t), C(t), S(t)\}$ *and randomized control decisions.*

The proof is similar to that in [1, 9] and follows directly from the framework of Lyapunov optimization in [6, 7], which is omitted here for brevity.

To derive such a policy, we need to know the statistical distribution of all combinations of $\{A_{ine}(t), C(t), S(t)\}$, which suffers from the "curse of dimensionality" problem [8] if being solved by dynamic programming. Moreover, this control policy may not be a feasible solution to **Problem One**. Instead, we use the existence of such a policy to help us design our control policy that meets all constraints of **Problem One** and derive the performance results for our algorithm.

2.2.2 *Our Proposed Algorithm*

Before presenting our algorithm, we define another variable $X_{ine}(t)$ as a shifted version of battery SOC level $B(t)$ for each time slot t as follows:

$$X_{ine}(t) = B(t) - V_{ine}C_{max} - D_{max}, \tag{2.13}$$

where V_{ine} is a control parameter to be specified later. $X_{ine}(t)$ is used to ensure that the constraint (2.4) of battery SOC level is satisfied in our algorithm. The intuition behind $X_{ine}(t)$ is to construct the algorithm based on a quadratic Lyapunov function, but carefully perturb the weights used for decision making, so as to push the SOC level in battery towards certain nonzero values to avoid underflow. According to (2.3) of $B(t)$, we have the same update equation for $X_{ine}(t)$,

$$X_{ine}(t+1) = X_{ine}(t) - D(t) + \gamma(t)S(t) + G_b(t). \tag{2.14}$$

In the latter part of this paper, we will prove that through our algorithm, $X_{ine}(t)$ is bounded in some range so that the constraint (2.4) on $B(t)$ is always satisfied for each slot t.

The proposed algorithm for inelastic energy demand is shown in Algorithm 1. Note that the algorithm only uses the current system states $X_{ine}(t)$, $C(t)$, $S(t)$ and $A_{ine}(t)$, and does not require any knowledge of the statistics of renewable energy generation, electricity price, and energy demand arrival process.

foreach *Time slot t* **do**

1 Measure the system states $X_{ine}(t)$, $C(t)$, $S(t)$ and $A_{ine}(t)$;

2 Choose control decisions $D^*_{ine}(t)$, $G^*_{l,ine}(t)$, $G^*_{b,ine}(t)$ and $\gamma^*_{ine}(t)$ as the solution to the following optimization problem, called **Problem Three**:

min $[V_{ine}C(t) + X(t)]G_b(t) + [X(t)S(t)]\gamma(t) + [V_{ine}C(t)]G_l(t) - X(t)D(t)$

s.t.

$$D(t) + G_l(t) = A_{ine}(t),$$

$$0 \leq G_l(t) \leq G_l^{max}, 0 \leq G_b(t) \leq G_b^{max},$$

$$0 \leq D(t) \leq D_{max},$$

$$0 \leq \gamma(t) \leq 1;$$

3 $X(t+1) = X(t) - D^*_{ine}(t) + \gamma^*_{ine}(t)S(t) + G^*_{b,ine}(t)$;

end

Algorithm 1: Power management with inelastic energy demands

2.2.3 Algorithmic Properties

In this subsection, we summarize the properties of our proposed algorithm as follows.

Theorem 2.1 *Assume that* $G_l^{max} + G_b^{max} \geq A_{ine}^{max}$*, then for any parameter* V_{ine} *satisfying* $0 < V_{ine} \leq V_{ine}^{max}$ *for all* $t \in \{0, 1, 2, \ldots\}$*, where*

$$V_{ine}^{max} \triangleq \frac{B_{max} - D_{max} - G_b^{max} - S_{max}}{C_{max} - C_{min}}, \tag{2.15}$$

our Algorithm 3 has the following properties:

1. The queue $X(t)$ *is always lower and upper bounded for all slots t as follows*

$$X(t) \geq -V_{ine}C_{max} - D_{max},$$

and

$$X(t) \le B_{max} - V_{ine}C_{max} - D_{max}.$$

2. *All control decisions are feasible.*
3. *If $S(t)$, $C(t)$ and $A_{ine}(t)$ are i.i.d. over slots, then the time-average expected cost under our algorithm is within bound B_1/V_{ine} of the optimal value, i.e.,*

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C(t)(G_l(t) + G_b(t))\} \le P_1^* + B_1/V_{ine}, \tag{2.16}$$

where B_1 is a constant given by

$$B_1 \triangleq \frac{[(G_b^{max} + S_{max})^2, D_{max}^2]}{2}. \tag{2.17}$$

Proof See Appendix 2.4.2.

2.3 Elastic Energy Demand

While the previous section deals with inelastic energy demand, some household appliances in the smart home can be made "smart", meaning that they can be controlled to adjust the times of their operations and the amount of their energy usage. In other words, as long as their energy requirements are met within certain deadlines, the residential customers will be satisfied. Some typical examples include dish washer, water heater, air conditioning, and charging of PHEVs. A schematic for the power management with elastic energy demands is shown in Fig. 2.3.

We assume that the amount of elastic energy demands requested at slot t is $A_{ela}(t)$. These energy demands are stored in an energy demand queue. In every slot t, the energy discharged from the battery is denoted as $D(t)$ and the energy amount drawn directly from the power grid is denoted as $G_l(t)$, which are determined from the buffered energy demands. The energy demands are served in a First-In-First-Out (FIFO) manner.

Let $Q(t)$ denote the total energy demands in the queue for time slot t, we have the following queuing equation:

$$Q(t+1) = max\{Q(t) - D(t) - G_l(t), 0\} + A_{ela}(t). \tag{2.18}$$

As long as the waiting time of any buffered energy demand in this demand queue does not exceed a certain maximum deadline δ_{max}, the utility perceived by customers does not decrease. We use the same battery model (2.3) as in the case for inelastic energy demands. Our problem here is to minimize the time-average expected

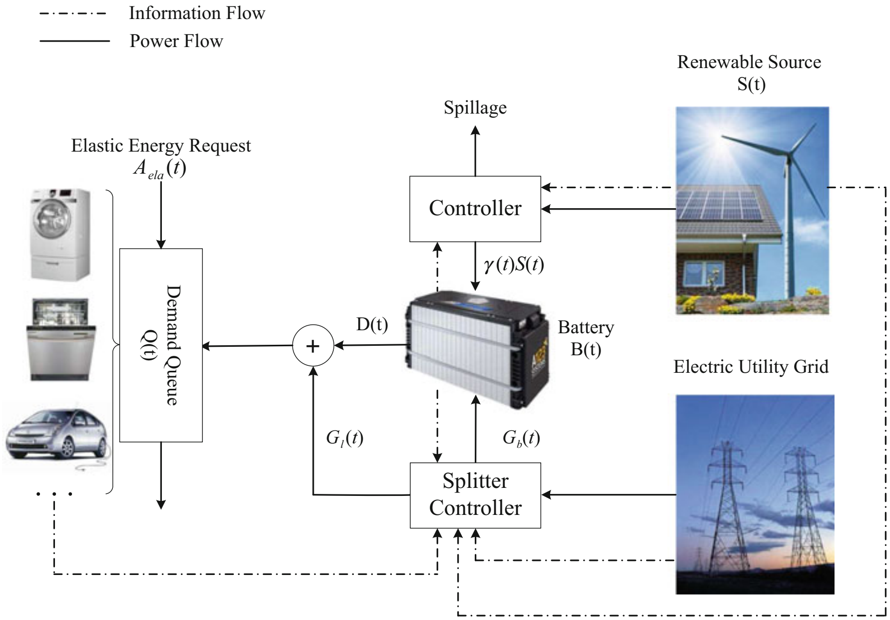

Fig. 2.3 A schematic for power management with elastic energy demands in the smart grid

electricity cost subject to all constraints discussed before and to ensure finite average delay for any buffered energy demand, which can be stated below and is called **Problem Four**:

$$\min_{D(t),G_b(t),G_l(t),\gamma(t)} P_2 = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C(t)(G_l(t) + G_b(t))\} \tag{2.19}$$

subject to

$$\begin{aligned} B(t+1) = B(t) - D(t) + \gamma(t)S(t) + G_b(t), \\ D(t) \leq B(t) \leq B_{max}, \\ Q(t+1) = max\{Q(t) - D(t) - G_l(t), 0\} + A_{ela}(t), \\ 0 \leq D(t) \leq D_{max}, \\ 0 \leq G_b(t) \leq G_b^{max}, 0 \leq G_l(t) \leq G_l^{max}, \\ 0 \leq \gamma(t) \leq 1, \\ \overline{Q} < \infty, \end{aligned} \tag{2.20}$$

where $\overline{Q}$ is the time-average expected energy demand queue backlog defined as:

$$\overline{Q} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q(t)\}.$$

2.3.1 Relaxed Problem

Similar to the case for the inelastic energy demands, we define a relaxed problem, called **Problem Five**, which can be stated as follows:

$$\min_{D(t),G_b(t),G_l(t),\gamma(t)} P_2 = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C(t)(G_l(t) + G_b(t))\} \tag{2.21}$$

subject to

$$\overline{D} = \overline{\gamma S} + \overline{G_b},$$
$$Q(t+1) = max\{Q(t) - D(t) - G_l(t), 0\} + A_{ela}(t),$$
$$0 \le D(t) \le D_{max},$$
$$0 \le G_b(t) \le G_b^{max}, 0 \le G_l(t) \le G_l^{max},$$
$$0 \le \gamma(t) \le 1,$$
$$\overline{Q} < \infty.$$

Denote the optimal objective value of **Problem Four** and **Problem Five** as P_2^* and $P_{2,rel}^*$, respectively. Similarly, any feasible solution to **Problem Four** is also a feasible solution to **Problem Five**, therefore, $P_{2,rel}^* \le P_2^*$. Similar to Lemma 2.1, we have the following result:

Lemma 2.2 *If* $\{A_{ela}(t), C(t), S(t)\}$ *are i.i.d. over slots, then there exists a stationary and randomized policy that takes control decisions* $\hat{D}_{ela}(t)$, $\hat{G}_{l,ela}(t)$, $\hat{G}_{b,ela}(t)$ *and* $\hat{\gamma}_{ela}(t)$ *every slot t purely as a (possibly randomized) function of current system states* $\{A_{ela}(t), C(t), S(t)\}$ *while satisfying the constraints above and providing the following guarantees:*

$$\mathbb{E}\{\hat{D}_{ela}(t)\} = \mathbb{E}\{\hat{\gamma}_{ela}S(t)\} + \mathbb{E}\{\hat{G}_{b,ela}(t)\}, \tag{2.22}$$

$$\mathbb{E}\{\hat{D}_{ela}(t) + \hat{G}_{l,ela}(t)\} \ge \mathbb{E}\{A_{ela}(t)\}, \tag{2.23}$$

$$\mathbb{E}\{C(t)(\hat{G}_{l,ela}(t) + \hat{G}_{b,ela}(t))\} = P_{2,rel}^*, \tag{2.24}$$

where the expectations are w.r.t. the stationary distribution of $\{A_{ela}(t), C(t), S(t)\}$ *and the randomized control decisions.*

The proof is similar to that in [1, 9] and follows directly from the framework of Lyapunov optimization in [6, 7], which is omitted here for brevity.

Note that the constraint (2.20) only ensures finite average delay without any guarantee for the worst-case delay. In the following, we use the technique of ε-persistent queue [10] to guarantee the finite worst-case delay for any buffered energy demand in the queue.

2.3.2 Delay-Aware Virtual Queue

We use the following virtual queue $Z(t)$ to provide the worst-case delay guarantee on any buffered energy demand in $Q(t)$:

$$Z(t+1) = \max\{Z(t) - D(t) - G_l(t) + \varepsilon 1_{\{Q(t)>0\}}, 0\}, \tag{2.25}$$

where $1_{\{Q(t)>0\}}$ is an indicator function that is 1 if $Q(t) > 0$ or 0 otherwise, and ε is a fixed positive parameter to be specified later. The intuition behind this virtual queue is that since $Z(t)$ has the same service process as $Q(t)$, but has an arrival process that adds ε whenever the actual backlog is nonempty, this ensures that $Z(t)$ grows if there is energy demand in the queue $Q(t)$ that has not been serviced for a long time. The following lemma shows that if we can control the system to ensure that the queues $Q(t)$ and $Z(t)$ have finite upper bounds, then any buffered energy demand is served within the worst-case delay.

Lemma 2.3 *Suppose we can control the system to ensure that* $Z(t) \leq Z_{max}$ *and* $Q(t) \leq Q_{max}$ *for all slots t, where* Z_{max} *and* Q_{max} *are some positive constants, then the worst-case delay for all buffered energy demand is upper bounded by* δ_{max} *slots where*

$$\delta_{max} \triangleq \lceil \frac{(Q_{max} + Z_{max})}{\varepsilon} \rceil. \tag{2.26}$$

Proof The proof follows directly from the framework of Lyapunov optimization [7] and is given in Appendix 3.5.2 for completeness.
We will show that there indeed exist such constants Z_{max} and Q_{max} later.

2.3.3 Our Proposed Algorithm

Before presenting our algorithm, we define another variable $X_{ela}(t)$ as a shifted version of battery SOC level $B(t)$ for time slot t as follows:

$$X_{ela}(t) = B(t) - \Theta_{max} - D_{max}, \tag{2.27}$$

where Θ_{max} is a positive constant to be specified. $X_{ela}(t)$ is also used to ensure that the constraint (2.4) of battery SOC level is satisfied in our algorithm. According to Eq. (2.3), we obtain the same update equation for $X_{ela}(t)$,

$$X_{ela}(t+1) = X_{ela}(t) - D(t) + \gamma(t)S(t) + G_b(t). \tag{2.28}$$

The algorithm for elastic energy demands is shown in Algorithm 4. Similarly, the algorithm only makes use of the current system states $(X_{ela}(t), Q(t), Z(t))$, $C(t)$, $S(t)$ and $A_{ela}(t)$, and does not require any knowledge on the statistics of the renewable energy generation, the electricity price, and the energy demand arrival process.

foreach *Time slot t* **do**

1. Measure system states $(X_{ela}(t), Q(t), Z(t))$, $C(t)$, $S(t)$ and $A_{ela}(t)$;
2. Choose control decisions $D^*_{ela}(t)$, $G^*_{l,ela}(t)$, $G^*_{b,ela}(t)$ and $\gamma^*_{ela}(t)$ as the solution to the following optimization problem, called **Problem Six**:
   min $[V_{ela}C(t) + X_{ela}(t)]G_b(t) + [X_{ela}(t)S(t)]\gamma(t) + [V_{ela}C(t) - Z(t) - Q(t)]G_l(t) - [X_{ela}(t) + Z(t) + Q(t)]D(t)$
   s.t.
   $$0 \le D(t) \le D_{max},$$
   $$0 \le G_l(t) \le G_l^{max}, 0 \le G_b(t) \le G_b^{max},$$
   $$0 \le \gamma(t) \le 1;$$
3. $X(t+1) = X(t) - D^*_{ela}(t) + \gamma^*_{ela}(t)S(t) + G^*_{b,ela}(t)$;
4. $Z(t+1) = \max[Z(t) - D^*_{ela}(t) - G^*_{l,ela}(t) + \varepsilon 1_{\{Q(t)>0\}}, 0]$;
5. $Q(t+1) = \max[Q(t) - D^*_{ela}(t) - G^*_{l,ela}(t), 0] + A_{ela}(t)$;

end

Algorithm 2: Power management with elastic energy demands

2.3.4 Algorithmic Properties

In this subsection, we summarize the properties of our proposed algorithm as follows.

Theorem 2.2 *Assume that* $G_l^{max} \ge \max[A_{ela}^{max}, \varepsilon]$. *If* $Q(0) = Z(0) = 0$, *then for any fixed parameter* $0 \le \varepsilon \le \mathbb{E}\{A(t)\}$ *and a parameter* V_{ela} *such that* $0 < V_{ela} \le V_{ela}^{max}$ *for all* $t \in \{0, 1, 2, \ldots\}$, *where*

$$V_{ela}^{max} \triangleq \frac{B_{max} - A_{ela}^{max} - \varepsilon - D_{max} - G_b^{max} - S_{max}}{C_{max} - C_{min}}, \tag{2.29}$$

our Algorithm 4 has the following properties:

1. *The queues $Q(t)$ and $Z(t)$ are deterministically upper bounded by Q_{max} and Z_{max} at every slot, where:*

$$Q_{max} \triangleq V_{ela}C_{max} + A_{ela}^{max}, \quad Z_{max} \triangleq V_{ela}C_{max} + \varepsilon. \tag{2.30}$$

Further, $Q(t) + Z(t)$ are upper bounded by Θ_{max} where

$$\Theta_{max} \triangleq V_{ela}C_{max} + A_{ela}^{max} + \varepsilon. \tag{2.31}$$

2. *The worst-case delay of any buffered energy demand is given by:*

$$\delta_{max} = \lceil \frac{2V_{ela}C_{max} + A_{ela}^{max} + \varepsilon}{\varepsilon} \rceil. \tag{2.32}$$

3. *The queue $X_{ela}(t)$ is always lower and upper bounded for all slots t by the following:*

$$-\Theta_{max} - D_{max} \leq X_{ela}(t) \leq B_{max} - \Theta_{max} - D_{max}.$$

4. *All control decisions are feasible.*
5. *If $S(t)$, $C(t)$, and $A_{ela}(t)$ are i.i.d. over slots, then the time-average expected electricity cost under our algorithm is within bound B_2/V_{ela} of the optimal value, i.e.,*

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C(t)(G_l(t) + G_b(t))\} \leq P_2^* + B_2/V_{ela}, \tag{2.33}$$

where B_2 is a constant given by

$$\begin{aligned} B_2 \triangleq\ & \frac{[(D_{max} + G_l^{max})^2 + A_{ela,max}^2]}{2} \\ & + \frac{\max[(D_{max} + G_l^{max})^2, \varepsilon^2]}{2} \\ & + \frac{\max[(S_{max} + G_b^{max})^2, D_{max}^2]}{2}. \end{aligned} \tag{2.34}$$

Proof See Appendix 2.4.2.

2.4 Performance Evaluation

In this section, we evaluate the proposed algorithms using practical data sets of electricity price and renewable energy generation. We consider a single household with a battery, a PV panel and various appliances subject to real-time pricing.

2.4.1 Simulation Setup

The data set of electricity price we use is from the California Independent System Operator (CAISO) [11] for Los Angeles area, which consists of 5 min interval average spot market price $C(t)$. Meanwhile, we use the 5 min interval average solar irradiance data for Los Angels area from the Measurement and Instrumentation Data center (MIDC) [12] at National Renewable Energy Laboratory. The period we consider in this paper is half year from January 1, 2011 to June 30, 2011. In total, this duration includes 181 days or 52, 128 5 min slots. The control interval is chosen to be 5 min. A portion of average hourly spot market electricity price and solar irradiance during the first week of January 2011 are plotted in Figs. 2.4 and 2.5, respectively.

We execute our algorithms in 5 min time slots and experiment with different values of parameters V, B_{max}, and ε. In our simulation, we set the energy demand arrival, either elastic or inelastic, during each time slot t as uniformly distributed from $[1, 24]$ KW-slot based on the practical home appliance usage in [5]. We fix the parameters $D_{max} = 30$ KW-slot, $G_l^{max} = 30$ KW-slot, and $G_b^{max} = 20$ KW-slot.

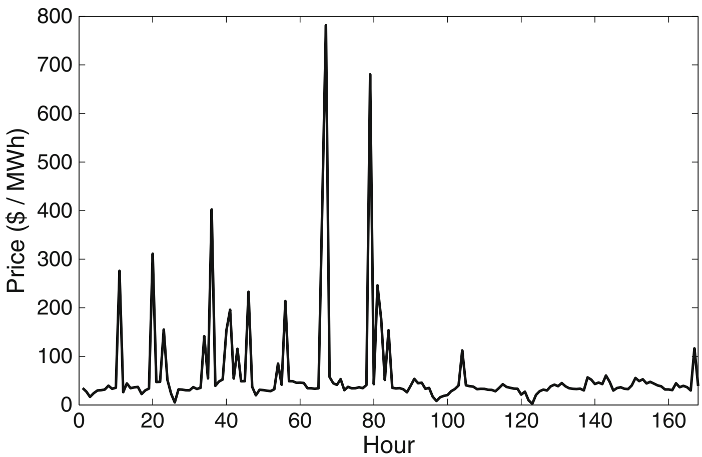

Fig. 2.4 Average hourly spot market price during the week of 01/01/2011 to 01/07/2011 at LA [11]

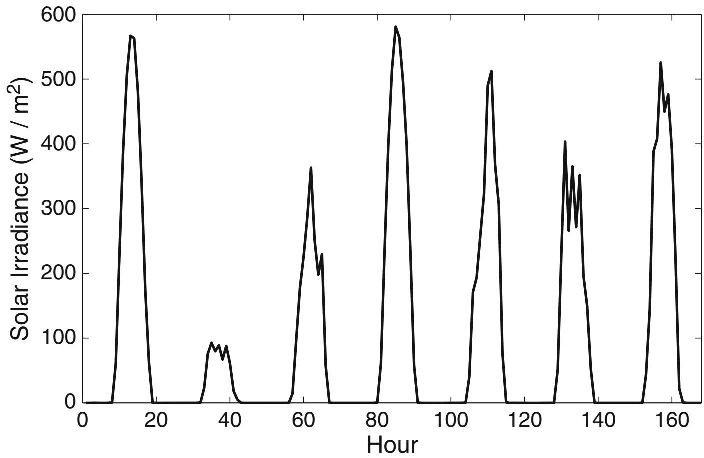

Fig. 2.5 Average hourly solar irradiance profile during the week of 01/01/2011 to 01/07/2011 at LA [12]

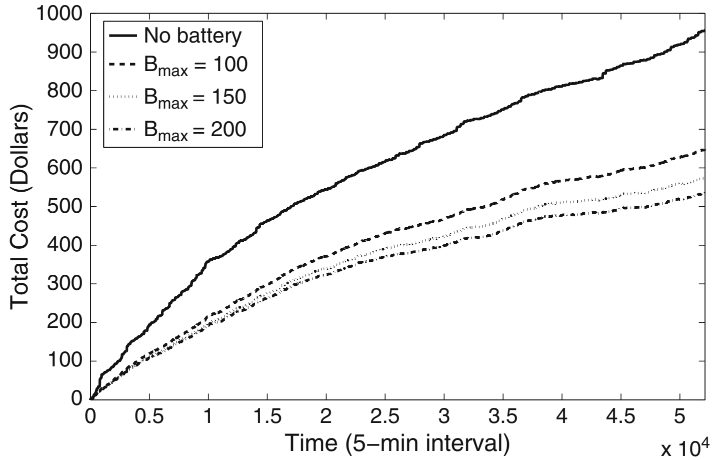

Fig. 2.6 Total Cost with i.i.d. A(t) and different battery capacity B_{max} for inelastic energy demands

2.4.2 Results and Analysis

First, we consider the impact of storage capacity B_{max} on cost saving in the case for inelastic energy demands. We compare our algorithm against a simple algorithm without storage. The simple algorithm uses the renewable energy generation to meet the demand as much as possible; in the case of insufficiency, it would draw some power from the power grid to meet the energy demand. The result is illustrated in Fig. 2.6 during 6-month period with $B_{max} = \{100, 150, 200\}$ KW-slot and

$V = V_{ine}^{max}$. From the figure, it is clear that the larger the battery is, the more saving our proposed algorithm can obtain. The saving comes from two aspects: one is by storing excessive renewable energy generated in current time slot for use at later time when renewable energy generation is insufficient; the other is by charging the battery when price is low while discharging it when price is high.

Next, we compare the cases for the inelastic energy demand and the elastic one when $B_{max} = 100$ KW-slot, $\varepsilon = 1$, and $V = V_{ine}^{max}$. The result is shown in Fig. 2.7. The case when the battery is used in conjunction with elastic energy demand provides more spaces to optimize the cost saving, as illustrated in the figure. This result is intuitive as some elastic energy demands can be delayed to time when free renewable energy is sufficient or the electricity price is low.

Finally, we consider the impact of ε on the performance of our algorithm for the case for elastic energy demands. As explained before, ε is related to the worst-case delay of queued energy demands. Smaller ε implies larger delay. We set $B_{max} = 100$ KW-slot and $V = V_{ela}^{max}$, and select different $\varepsilon \in \{0, 0.3, 0.6, 1\}$. As observed in Fig. 2.8, the decrease in ε gives lower cost with the tradeoff that the worst-case delay is increased.

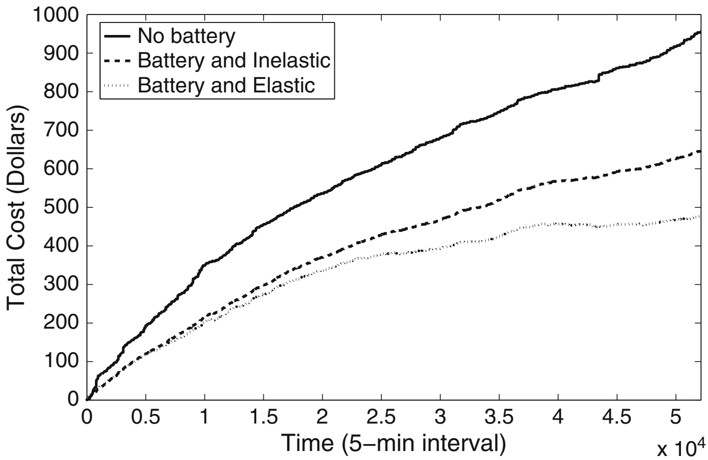

Fig. 2.7 Comparison of total cost for the cases when there is no battery, there is battery with inelastic energy demands, and there is battery with elastic energy demands

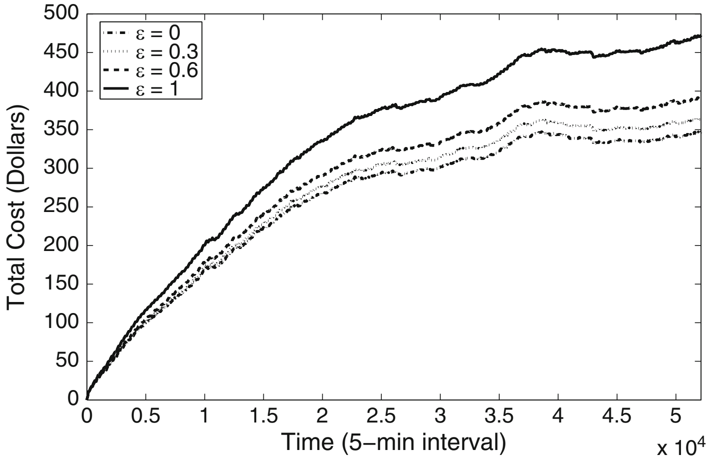

Fig. 2.8 Comparison of total cost for different ε with elastic energy demands

Appendix

Proof of Theorem 2.1

Here we prove Theorem 2.1.

Proof 1). It is obvious that the optimal solution to **Problem Three** has the following properties:

- If $X_{ine}(t) > -V_{ine}C_{min}$, $G^*_{b,ine}(t) = 0$ and $D^*_{ine}(t) = \min\{A_{ine}(t), D_{max}\}$.
- If $X_{ine}(t) < -V_{ine}C_{max}$, $G^*_{b,ine}(t) = G^{max}_b$ and $D^*_{ine}(t) = \max\{0, A_{ine}(t) - G^{max}_l\}$.

We now use induction to prove this result. When $t = 0$, as $X_{ine}(0) = B_0 - V_{ine}C_{max} - D_{max}$ and $0 \leq B_0 \leq B_{max}$, we have $-V_{ine}C_{max} - D_{max} \leq X_{ine}(0) \leq B_{max} - V_{ine}C_{max} - D_{max}$.

Now suppose that the above bound holds for time slot t. We need to prove that it also holds for time slot $t+1$. First, suppose $-V_{ine}C_{max} - D_{max} \leq X_{ine}(t) < -V_{ine}C_{max}$, then $X_{ine}(t+1) = X_{ine}(t) + G^*_{b,ine}(t) + S(t) - D^*_{ine}(t) \geq -V_{ine}C_{max} - D_{max} + G^{max}_b - \max\{0, A_{ine}(t) - G^{max}_l\}$. As $G^{max}_l + G^{max}_b \geq A^{max}_{ine}$, we have $X_{ine}(t+1) \geq X_{ine}(t) \geq -V_{ine}C_{max} - D_{max}$. Moreover, $X_{ine}(t+1) \leq X_{ine}(t) + G^{max}_b + S_{max} \leq -V_{ine}C_{min} + G^{max}_b + S_{max} \leq B_{max} - V_{ine}C_{max} - D_{max}$, where we have used

$$V_{ine} \leq \frac{B_{max} - D_{max} - G^{max}_b - S_{max}}{C_{max} - C_{min}}.$$

Second, suppose $-V_{ine}C_{max} \leq X_{ine}(t) \leq -V_{ine}C_{min}$, then $-V_{ine}C_{max} - D_{max} \leq X_{ine}(t) - D_{max} \leq X_{ine}(t+1) \leq X_{ine}(t) + G_b^{max} + S_{max} \leq -V_{ine}C_{min} + G_b^{max} + S_{max} \leq B_{max} - V_{ine}C_{max} - D_{max}$, where we have used the same bound of V_{ine} as the case above. Third, suppose $-V_{ine}C_{min} < X_{ine}(t) \leq 0$, then $G^*_{b,ine}(t) = 0$. It is obvious that $X_{ine}(t+1) \geq X_{ine}(t) - D_{max} > -V_{ine}C_{max} - D_{max}$. Moreover, $X_{ine}(t+1) \leq X_{ine}(t) + S_{max} \leq S_{max} \leq B_{max} - V_{ine}C_{max} - D_{max}$, where we have used the upper bound of V_{ine} and $G_b^{max} \geq V_{ine}^{max}C_{min}$. Finally, suppose $0 < X_{ine}(t) \leq B_{max} - V_{ine}C_{max} - D_{max}$, then $G^*_{b,ine}(t) = \gamma^*_{ine}(t) = 0$. Hence $-V_{ine}C_{max} - D_{max} \leq X_{ine}(t+1) \leq X_{ine}(t) \leq B_{max} - V_{ine}C_{max} - D_{max}$. From the induction, we conclude the proof of 1).

2). From 1) and the definition (2.13) of $X_{ine}(t)$, it follows immediately that $0 \leq B(t) \leq B_{max}$ holds for any time slot t. Further, we make our decisions to satisfy all constraints in **Problem Three**. Combining them together, all constraints of **Problem One** are satisfied. Therefore, our control decisions are feasible to **Problem One**.

3). We make use of the Lyapunov optimization techniques [7] to derive the performance bound for our algorithm. Define the Lyapunov function as $L(X_{ine}(t)) = \frac{1}{2}X^2_{ine}(t)$ and the conditional 1-slot Lyapunov drift as follows:

$$\triangle(X_{ine}(t)) = \mathbb{E}\{L(X_{ine}(t+1)) - L(X_{ine}(t))|X_{ine}(t)\}.$$

From Eq. (2.14), squaring both sides, we obtain

$$\frac{X^2_{ine}(t+1) - X^2_{ine}(t)}{2} = \frac{(D(t) - \gamma(t)S(t) - G_b(t))^2}{2} - X_{ine}(t)(D(t) - \gamma(t)S(t) - G_b(t)).$$

As $0 \leq D(t) \leq D_{max}$ and $0 \leq \gamma(t)S(t) + G_b(t) \leq G_b^{max} + S_{max}$, we have

$$\frac{(D(t) - \gamma(t)S(t) - G_b(t))^2}{2} \leq \frac{1}{2}\max[(G_b^{max} + S_{max})^2, D^2_{max}].$$

Therefore, we can obtain the following upper bound on the Lyapunov drift for $X_{ine}(t)$:

$$\frac{X^2_{ine}(t+1) - X^2_{ine}(t)}{2} \leq \frac{1}{2}\max[(G_b^{max} + S_{max})^2, D^2_{max}] - X_{ine}(t)(D(t) - \gamma(t)S(t) - G_b(t)) \quad .$$

Taking expectation w.r.t. $X_{ine}(t)$ and adding the penalty term $V_{ine}\mathbb{E}\{C(t)(G_l(t) + G_b(t))|X_{ine}(t)\}$ to both sides of the inequality above, we obtain the following inequality:

$$\triangle(X_{ine}(t)) + V_{ine}\mathbb{E}\{C(t)(G_l(t) + G_b(t))|X_{ine}(t)\} \leq B_1$$

$$- X_{ine}(t)\mathbb{E}\{D(t) - \gamma(t)S(t) - G_b(t)|X_{ine}(t)\}$$
$$+ V_{ine}\mathbb{E}\{C(t)(G_l(t) + G_b(t))|X_{ine}(t)\},$$

where B_1 is defined as

$$B_1 \triangleq \frac{[(G_b^{max} + S_{max})^2, D_{max}^2]}{2}.$$

Comparing with the objective of **Problem Three**, it is obvious that our algorithm always attempts to greedily minimize the right hand side (R.H.S.) of the inequality above for each time slot t over all possible feasible control policies including the optimal, stationary policy given in Lemma 2.1. Plugging this policy $(\hat{D}_{ine}(t), \hat{G}_{l,ine}(t), \hat{G}_{b,ine}(t), \hat{\gamma}_{ine}(t))$ into the R.H.S. of the inequality above and using the fact that this policy is independent of queue state $X_{ine}(t)$, we obtain the following:

$$\begin{aligned}\triangle(X_{ine}(t)) + V_{ine}\mathbb{E}\{C(t)(G_l(t) + G_b(t))|X_{ine}(t)\} \leq B_1 \\ - X_{ine}(t)\mathbb{E}\{\hat{D}_{ine}(t) - \hat{\gamma}_{ine}(t)S(t) - \hat{G}_{b,ine}(t)|X_{ine}(t)\} \\ + V_{ine}\mathbb{E}\{C(t)(\hat{G}_{l,ine}(t) + \hat{G}_{b,ine}(t))|X_{ine}(t)\} \\ \leq B_1 + V_{ine}P_{1,rel}^* \leq B_1 + V_{ine}P_1^*,\end{aligned}$$

where the following facts have been used:

$$\mathbb{E}\{\hat{D}_{ine}(t) - \hat{\gamma}_{ine}(t)S(t) - \hat{G}_{b,ine}(t)|X_{ine}(t)\} = 0,$$
$$\mathbb{E}\{C(t)(\hat{G}_{l,ine}(t) + \hat{G}_{b,ine}(t))|X_{ine}(t)\} = P_{1,rel}^*.$$

The equations above follow from Lemma 2.1. Taking the expectation on both sides, using the law of iterative expectation, and summing over $t \in \{0, 1, 2, \ldots, T-1\}$, we obtain

$$\begin{aligned}V_{ine}\sum_{t=0}^{T-1}\mathbb{E}\{C(t)(G_l(t) + G_b(t)) \leq B_1T + V_{ine}TP_1^* \\ - \mathbb{E}\{L(X_{ine}(T))\} + \mathbb{E}\{L(X_{ine}(0))\}.\end{aligned}$$

Dividing both sides by T, letting $T \to \infty$, and using the facts that $E\{L(X_{ine}(0))\}$ are finite and $E\{L(X_{ine}(t))\}$ are nonnegative, we finally arrive at the following:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{C(t)(G_l(t) + G_b(t))\} \leq P_1^* + B_1/V_{ine},$$

where P_1^* is the optimal objective value, B_1 is a constant given by Eq. (2.17), and V_{ine} is a control parameter which has a maximum value given by Eq. (2.15). This completes the proof of 3).

Proof of Lemma 3.1

Here we prove Theorem 3.1.

Proof Consider any slot t for which $A_{ela}(t) > 0$. We will show that this energy request $A_{ela}(t)$ is served on or before time $t + \delta_{max}$ by contradiction. Suppose not, then during slots $\tau \in \{t + 1, \ldots, t + \delta_{max}\}$ it must be that $Q(\tau) > 0$, otherwise, the energy request $A_{ela}(t)$ would have been served before τ. Therefore, $1_{Q(t)>0} = 1$, and from the update Eq. (3.25) of $Z(t)$, we have for all $\tau = \{t + 1, \ldots, t + \delta_{max}\}$:

$$Z(\tau + 1) \geq Z(\tau) - D(t) - G_l(t) + \varepsilon.$$

Summing the above over $\tau = \{t + 1, \ldots, t + \delta_{max}\}$ yields:

$$Z(t + \delta_{max} + 1) - Z(t + 1) \geq - \sum_{\tau=t+1}^{t+\delta_{max}} [D(t) + G_l(t)] + \delta_{max}\varepsilon.$$

Rearranging the terms and using the facts that $Z(t+1) \geq 0$ and $Z(t+\delta_{max}+1) \leq Z_{max}$ yields:

$$\sum_{\tau=t+1}^{t+\delta_{max}} [D(t) + G_l(t)] \geq \delta_{max}\varepsilon - Z_{max}. \tag{2.35}$$

Since the request $A_{ela}(t)$ are queued in a FIFO manner and $Q(t + 1) \leq Q_{max}$, it would be served on or before time $t + \delta_{max}$ whenever there are at leat Q_{max} units of energy served during $\tau \in \{t+1, \ldots, t+\delta_{max}\}$. As we have assumed that the request $A_{ela}(t)$ are not served by time $t+\delta_{max}$, it must be that $\sum_{\tau=t+1}^{t+\delta_{max}} [D(t)+G_l(t)] < Q_{max}$. Comparing this inequality with (3.41) yields:

$$Q_{max} > \delta_{max}\varepsilon - Z_{max},$$

which implies that $\delta_{max} < (Q_{max} + Z_{max})/\varepsilon$, contradicting the definition of δ_{max} in (3.26).

Proof of Theorem 2.2

Here we prove Theorem 2.2.

Proof 1). First, we prove $Q(t) \le Q_{max}$ for every time slot t. Once again, we will use induction method. Obviously, $Q(0) \le Q_{max}$. Suppose it holds at time slot t, we need to show that it also holds at time slot $t+1$. As $Q(t+1) = \max[Q(t)-D(t)-G_l(t), 0] + A_{ela}(t)$, if $Q(t) \le V_{ela}C_{max}$, then the maximum amount of energy demand arrival is A_{ela}^{max}, we have $Q(t+1) \le V_{ela}C_{max} + A_{ela}^{max}$. If $V_{ela}C_{max} < Q(t) \le V_{ela}C_{max} + A_{ela}^{max}$, then $V_{ela}C(t) - Z(t) - Q(t) < 0$. According to **Problem Six**, our algorithm will choose $G_{l,ela}^*(t) = G_l^{max}$. If $Q(t) - D_{ela}^*(t) - G_l^{max} > 0$, then, in time slot t the amount of energy demand being served is at least G_l^{max}, which is larger than the maximum amount of arrival during time slot t. Hence, the queue cannot increase, i.e., $Q(t+1) \le Q(t) \le V_{ela}C_{max} + A_{ela}^{max}$. If $Q(t) - D_{ela}^*(t) - G_l^{max} \le 0$, then $Q(t+1) = A_{ela}(t) \le A_{ela}^{max} \le V_{ela}C_{max} + A_{ela}^{max}$. Therefore, we have proved $Q(t) \le V_{ela}C_{max} + A_{ela}^{max}$.

Next, we prove $Z(t) \le Z_{max}$ for every time slot t. Obviously, $Z(0) \le Z_{max}$. Suppose it holds for time slot t, we need to show that it also holds in time slot $t+1$. As $Z(t+1) = \max[Z(t)-D(t)-G_l(t)+\varepsilon 1_{Q(t)>0}, 0]$, if $Z(t) \le V_{ela}C_{max}$, then the maximum amount of queuing increase is ε, we have $Z(t+1) \le V_{ela}C_{max} + \varepsilon$; if $V_{ela}C_{max} < Z(t) \le V_{ela}C_{max} + \varepsilon$, then $V_{ela}C(t) - Z(t) - Q(t) < 0$. According to **Problem Six**, our algorithm will choose $G_{l,ela}^*(t) = G_l^{max}$. If $Z(t) - D_{ela}^*(t) - G_l^{max} > 0$, then, in time slot t the amount of energy demand being served is at least G_l^{max}, which is larger than the maximum amount of arrival ε during time slot t. Hence, the queue cannot increase, i.e., $Z(t+1) \le Z(t) \le V_{ela}C_{max} + \varepsilon$. If $Z(t) - D_{ela}^*(t) - G_l^{max} \le 0$, then $Z(t+1) \le \varepsilon \le V_{ela}C_{max} + \varepsilon$. Therefore, we have proved that $Z(t) \le V_{ela}C_{max} + \varepsilon$.

Finally, we prove $Q(t) + Z(t) \le \Theta_{max}$. Obviously, $Q(0) + Z(0) \le \Theta_{max}$. Suppose $Q(t) + Z(t) \le \Theta_{max}$ holds for time slot t. If $Q(t) + Z(t) \le V_{ela}C_{max}$, then, according to the queuing equations of $Q(t)$ and $Z(t)$, the maximum increase during one slot is $A_{ela}^{max} + \varepsilon$. If $VC_{max} < Q(t) + Z(t) \le V_{ela}C_{max} + A_{ela}^{max} + \varepsilon$, then $V_{ela}C(t) - Z(t) - Q(t) < 0$. According to **Problem Six**, our algorithm will choose $G_{l,ela}^*(t) = G_l^{max}$. Using the proof above, $U(t+1)$ and $Z(t+1)$ cannot increase. Hence, $U(t+1) + Z(t+1) \le U(t) + Z(t) \le \Theta_{max}$. This completes the proof for 1).

2). This is straightforward from Lemma 3.1.

3). It is obvious that the optimal solution to **Problem Six** has the following properties:

- If $X(t) > -V_{ela}C_{min}$, $G_{b,ela}^*(t) = 0$.
- If $X(t) < -[Q(t) + Z(t)]_{max} = -\Theta_{max}$, $D_{ela}^*(t) = 0$.

In the following, we prove this result by induction. When $t = 0$, as $X_{ela}(0) = B_0 - \Theta_{max} - D_{max}$ and $0 \le B_0 \le B_{max}$, we have $-\Theta_{max} - D_{max} \le X(0) \le B_{max} - \Theta_{max} - D_{max}$.

Now suppose that the above bound holds for time slot t. We need to show that it also holds for time slot $t+1$. First, suppose $0 < X_{ela}(t) \leq B_{max} - \Theta_{max} - D_{max}$, then $G^*_{b,ela}(t) = \gamma^*_{ela}(t) = 0$. As there is no recharge to the battery and the maximum discharge rate during one time slot is D_{max}, we have $-\Theta_{max} - D_{max} < -D_{max} < X_{ela}(t+1) \leq X_{ela}(t) \leq B_{max} - \Theta_{max} - D_{max}$. Second, suppose $-V_{ela}C_{min} < X_{ela}(t) \leq 0$, then $G^*_{b,ela}(t) = 0$ and the maximum recharge and discharge rate for the battery are S_{max} and D_{max}, respectively. Hence, $-\Theta_{max} - D_{max} < -V_{ela}C_{min} - D_{max} < X_{ela}(t+1) \leq X_{ela}(t) + S_{max} \leq B_{max} - \Theta_{max} - D_{max}$, where we have used

$$V_{ela} \leq \frac{B_{max} - A^{max}_{ela} - \varepsilon - D_{max} - G^{max}_{b} - S_{max}}{C_{max} - C_{min}},$$

and $V^{max}_{ela}C_{min} \leq G^{max}_{b}$. Third, suppose $-\Theta_{max} \leq X_{ela}(t) \leq -V_{ela}C_{min}$. As $X_{ela}(t) - D_{max} \leq X_{ela}(t+1) \leq X_{ela}(t) + S_{max} + G^{max}_{b}$, we have $-\Theta_{max} - D_{max} \leq X_{ela}(t+1) \leq -V_{ela}C_{min} + S_{max} + G^{max}_{b} \leq B_{max} - (V_{ela}C_{max} + A^{max}_{ela} + \varepsilon) - D_{max} = B_{max} - \Theta_{max} - D_{max}$, where we use the same bound of V_{ela} as the case before. Last, suppose $-\Theta_{max} - D_{max} \leq X(t) < -\Theta_{max}$, from **Problem Six**, we have $D^*_{ela}(t) = 0$, then $-\Theta_{max} - D_{max} \leq X_{ela}(t) \leq X_{ela}(t+1) < -\Theta_{max} + S_{max} + G^{max}_{b} \leq B_{max} - \Theta_{max} - D_{max}$. This completes the proof of 3).

4). From 3) and the definition (2.27) of $X_{ela}(t)$, it follows immediately that $0 \leq B(t) \leq B_{max}$ holds all slots t. Further, we choose our decisions to satisfy all constraints in **Problem Six**. Combining them together, all constraints of **Problem Four** are satisfied. Therefore, our control decisions are feasible to **Problem Four**.

5). Here, we make use of Lyapunov optimization techniques to derive this result. Denote queue states $\mathbf{K}(t) \triangleq (Q(t), Z(t), X_{ela}(t))$. Define the Lyapunov function as $L(\mathbf{K}(t)) = \frac{1}{2}(Q^2(t) + Z^2(t) + X^2_{ela}(t))$ and the conditional 1-slot Lyapunov drift as follows:

$$\triangle(\mathbf{K}(t)) = \mathbb{E}\{L(\mathbf{K}(t+1)) - L(\mathbf{K}(t))|\mathbf{K}(t)\}.$$

From the update Eq. (2.28), squaring both sides, we obtain

$$\frac{X^2_{ela}(t+1) - X^2_{ela}(t)}{2} = \frac{(D(t) - \gamma(t)S(t) - G_b(t))^2}{2} - X_{ela}(t)(D(t) - \gamma(t)S(t) - G_b(t)).$$

As $0 \leq D(t) \leq D_{max}$ and $0 \leq \gamma(t)S(t) + G_b(t) \leq G^{max}_{b} + S_{max}$, we have

$$\frac{(D(t) - \gamma(t)S(t) - G_b(t))^2}{2} \leq \frac{1}{2}\max[(G^{max}_{b} + S_{max})^2, D^2_{max}].$$

Therefore, we can get the following upper bound for the Lyapunov drift for $X(t)$:

$$\frac{X_{ela}^2(t+1) - X_{ela}^2(t)}{2} \leq \frac{1}{2}\max[(G_b^{max} + S_{max})^2, D_{max}^2] \\ - X_{ela}(t)(D(t) - \gamma(t)S(t) - G_b(t)).$$

From the update Eq. (3.25), we have

$$Z(t+1) \leq \max[Z(t) - D(t) - G_l(t) + \varepsilon, 0],$$

then,

$$Z^2(t+1) \leq (Z(t) - D(t) - G_l(t) + \varepsilon)^2,$$

and we obtain the following inequality:

$$\begin{aligned} &\frac{Z^2(t+1) - Z^2(t)}{2} \\ &\leq \frac{(\varepsilon - D(t) - G_l(t))^2}{2} + Z(t)(\varepsilon - D(t) - G_l(t)) \\ &\leq \frac{\max[(D_{max} + G_l^{max})^2, \varepsilon^2]}{2} + Z(t)(\varepsilon - D(t) - G_l(t)). \end{aligned}$$

From the update Eq. (2.18), squaring both sides and using the following inequality:

$$(\max[Q(t) - D(t) - G_l(t), 0] + A_{ela}(t))^2 \leq A_{ela}^2(t) + \\ Q^2(t) + (D(t) + G_l(t))^2 + 2Q(t)(A_{ela}(t) - D(t) - G_l(t)),$$

we obtain

$$\frac{Q^2(t+1) - Q^2(t)}{2} \leq \frac{[(D_{max} + G_l^{max})^2 + A_{ela,max}^2]}{2} \\ + Q(t)(A_{ela}(t) - D(t) - G_l(t)).$$

Combining these three bounds together and taking the expectation w.r.t. $\mathbf{K}(t)$ on both sides, we arrive at the following inequality:

$$\begin{aligned} \triangle(\mathbf{K}(t)) \leq & B_2 + \mathbb{E}\{Z(t)(\varepsilon - D(t) - G_l(t))|\mathbf{K}(t)\} \\ & + \mathbb{E}\{Q(t)(A_{ela}(t) - D(t) - G_l(t))|\mathbf{K}(t)\} \\ & - \mathbb{E}\{X_{ela}(t)(D(t) - \gamma(t)S(t) - G_b(t))|\mathbf{K}(t)\}, \end{aligned}$$

where $B_2 = \frac{\max[(S_{max}+G_b^{max})^2, D_{max}^2]}{2} + \frac{\max[(D_{max}+G_l^{max})^2, \varepsilon^2]}{2} + \frac{[(D_{max}+G_l^{max})^2+A_{ela,max}^2]}{2}$. Adding penalty term $V_{ela}\mathbb{E}\{C(t)(G_l(t)+G_b(t))|\mathbf{K}(t)\}$ to both sides of the above inequality, we obtain the following inequality:

$$\begin{aligned}\triangle(\mathbf{K}(t)) + V_{ela}\mathbb{E}\{C(t)(G_l(t) + G_b(t))|\mathbf{K}(t)\} \le B_2 \\ - X_{ela}(t)\mathbb{E}\{D(t) - \gamma(t)S(t) - G_b(t)|\mathbf{K}(t)\} \\ + Z(t)\mathbb{E}\{\varepsilon - D(t) - G_l(t)|\mathbf{K}(t)\} \\ + Q(t)\mathbb{E}\{A_{ela}(t) - D(t) - G_l(t)|\mathbf{K}(t)\} \\ + V_{ela}\mathbb{E}\{C(t)(G_l(t) + G_b(t))|\mathbf{K}(t)\}.\end{aligned}$$

Comparing with the objective of **Problem Six**, it is obvious that our algorithm always attempts to greedily minimize the R.H.S. of the above inequality at each time slot t over all feasible control policies including the optimal, stationary policy given in Lemma 2.2. Plugging this policy $(\hat{D}_{ela}(t), \hat{G}_{l,ela}(t), \hat{G}_{b,ela}(t), \hat{\gamma}_{ela}(t))$ into the R.H.S. of the inequality above and using the fact that this policy is independent of queue state $\mathbf{K}(t)$, we obtain the following:

$$\begin{aligned}\triangle(\mathbf{K}(t)) + V_{ela}\mathbb{E}\{C(t)(G_l(t) + G_b(t))|\mathbf{K}(t)\} \le B_2 \\ - X_{ela}(t)\mathbb{E}\{\hat{D}_{ela}(t) - \hat{\gamma}_{ela}(t)S(t) - \hat{G}_{b,ela}(t)|\mathbf{K}(t)\} \\ + Z(t)\mathbb{E}\{\varepsilon - \hat{D}_{ela}(t) - \hat{G}_{l,ela}(t)|\mathbf{K}(t)\} \\ + Q(t)\mathbb{E}\{A_{ela}(t) - \hat{D}_{ela}(t) - \hat{G}_{l,ela}(t)|\mathbf{K}(t)\} \\ + V_{ela}\mathbb{E}\{C(t)(\hat{G}_{l,ela}(t) + \hat{G}_{b,ela}(t))|\mathbf{K}(t)\} \\ \le B_2 + V_{ela}P^*_{2,rel} \le B_2 + V_{ela}P^*_2,\end{aligned}$$

where the following facts have been used:

$$\mathbb{E}\{\hat{D}_{ela}(t) - \hat{\gamma}_{ela}(t)S(t) - \hat{G}_{b,ela}(t)|\mathbf{K}(t)\} = 0, \tag{2.36}$$

$$\mathbb{E}\{A_{ela}(t) - \hat{D}_{ela}(t) - \hat{G}_{l,ela}(t)|\mathbf{K}(t)\} \le 0, \tag{2.37}$$

$$\mathbb{E}\{\varepsilon - \hat{D}_{ela}(t) - \hat{G}_{l,ela}(t)|\mathbf{K}(t)\} \le 0. \tag{2.38}$$

The first two equations follow from Lemma 2.2 and the last one follows from (2.37) together with $\varepsilon \le \mathbb{E}\{A_{ela}(t)\}$. Taking the expectation on both sides, using the law of iterative expectation, and summing over $t \in \{0, 1, 2, \ldots, T-1\}$, we obtain

$$\begin{aligned}V_{ela}\sum_{t=0}^{T-1}\mathbb{E}\{C(t)(G_l(t) + G_b(t)) \le B_2T + V_{ela}TP^*_2 \\ - \mathbb{E}\{L(\mathbf{K}(T))\} + \mathbb{E}\{L(\mathbf{K}(0))\}.\end{aligned}$$

Dividing both sides by T, letting $T \to \infty$, and using the facts that $E\{L(\mathbf{K}(0))\}$ are finite and $E\{L(\mathbf{K}(t))\}$ are nonnegative, we arrive at the following result for our algorithm:

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{C(t)(G_l(t) + G_b(t))\} \leq P_2^* + B_2/V_{ela},$$

where P_2^* is the optimal objective value, B_2 is a constant given by Eq. (2.34), and V_{ela} is a tunable control parameter which has a maximum value given by Eq. (2.29). This completes the proof.

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