

Preface

In SERC School, or Science and Engineering Research Council (SERC) of the Department of Science and Technology, Government of India, New Delhi (DST), research orientation program is a regular feature at the Centre for Mathematical and Statistical Sciences India (CMSS). It is a four- to five-week intensive program run in April–May of every year at CMSS. Lecture notes, in the form of research level books, are produced every year. The summary of the notes from the first five schools at CMSS was printed by Springer, New York, in 2008 under the title “*Special Functions for Applied Scientists*.” The theme of those five schools was Special Functions and Their Applications.

The theme of the five SERC Schools from 2008 to 2012 was “Multivariable and matrix variable calculus with applications in model building, optimization, fractional calculus, statistical distributions, and astrophysics problems.” The summary of the notes from the main lecturers in the five SERC Schools is put in the form of a book. This represents the present manuscript.

Chapter 1 starts with the properties of Mittag-Leffler and Wright’s functions, which are associated with the solutions of fractional order differential and integral equations. Then, an exposure into the essentials of fractional calculus is given with reference to Cauchy problem, signaling problem, reaction-diffusion problems, relaxation, etc. The leading researchers in the area such as the late Prof. Dr. Anatoly A. Kilbas of Belarus, Prof. Dr. Rudolf Gorenflo of Germany, Prof. Dr. Francesco Mainardi of Italy, and Prof. Dr. R.K. Saxena of India have given lectures on this topic at CMSS in the SERC Schools in various years. The material in Chapter 1 is taken from the lectures of Prof. Dr. Francesco Mainardi of Italy.

Chapter 2 gives a basic introduction to multivariable calculus geared to applicable analysis. Some three people had lectured on this topic in different years at CMSS SERC Schools. We have included the lecture notes from Prof. Dr. D.V. Pai of IIT (Indian Institute of Technology) Bombay and IIT Gandhinagar.

Chapter 3 deals with deterministic models. Some three people had given lectures in different years on deterministic models, linear and nonlinear analysis, etc. We have included the introductory materials from the lectures of Prof. Dr. A.M. Mathai,

the Director of SERC Schools, and Emeritus Professor of Mathematics and Statistics at McGill University, Canada.

Chapter 4 gives details of some non-deterministic models or models catering to random phenomena. Here, regression type prediction models are given extensive treatment. Concepts of regression, correlation, and the bases of prediction models are explained and illustrated properly. Some matrix-variate calculus in the form of vector and matrix derivatives and their applications into optimization problems leading into popular multivariate techniques such as principal component analysis, canonical correlation analysis, linear and nonlinear least squares, etc, are also described in this chapter. The material here is based on the lectures of Prof. A.M. Mathai.

Chapter 5 leads into some optimal designs in the area of Design of Experiments and Analysis of Variance. The material is based on the lectures of Prof. Dr. Stratis Kounias of the University of Athens, Greece.

Other topics covered in the SERC Schools of 2008–2012 include order statistics, time series analysis, optimization and game theory, astrophysics, stellar and solar models, and wavelet analysis. Since there were no extensive or deeper coverage of these topics, they are not included in the present manuscript. It is hoped that some of the topics presented in this manuscript will be of use to researchers working in the interface between different disciplines or in the area of applicable mathematics or applications of mathematical techniques to other disciplines such as differential equations, fractional calculus, statistical distributions, stochastic processes, theoretical physics, reaction-diffusion, input–output type problems, and model building in general.

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Mathai, A.M.; Haubold, H.

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