

Preface

The modern mathematical control theory emerged in the 1950s as a result of demand from numerous real-world applications, particularly in space engineering. Since then it has significantly developed both in engineering (including bioengineering) and applied mathematics communities.

A typical control problem studies the evolution processes that can be affected by a certain parameter, called “control.” The goal of such problems is to steer the process at hand from a given initial state to a desirable target state by selecting a control parameter within the set of available options. If this is possible, then one usually wishes to achieve the above while optimizing a given criterion (a functional). This setup represents what is called *an optimal control problem*.

In its turn, the *controllability theory* focuses on the steering capabilities of the controlled evolution processes. Namely, given *any* initial state, it studies the richness of the range of the mapping: *control* $\rightarrow$ *the state of the process* at some moment of time. The controllability theory was initially developed in the 1960s for linear ordinary differential equations, governed by additive controls.

The modern mathematical controllability theory of linear and semilinear partial differential equations with additive controls emerged about 50 years ago. Since then, many powerful mathematical methods have been introduced and/or adopted to deal with a wide variety of applied problems along the so-called duality approach and the Hilbert uniqueness method, the techniques of harmonic and nonharmonic analysis, unique continuation, the multiplier method, Carleman estimates, microlocal analysis, and others (we refer to the Bibliography for further references).

The reader can note that the typical mathematical models considered within this theory make use of either boundary or internal locally distributed additive controls or sensors to describe respectively the effect of external controlling actions on the process at hand or the methods to collect available data. The intrinsic characteristic of such controls and sensors is that they act in their “*full-dimensional capacity*,” namely, either in an open part of the boundary of a system’s spatial domain or in an open set within this domain and, thus, are *infinite-dimensional* at every moment of time. Namely, if the spatial dimensionality of the original system is n , the actions

of the former “devices” are represented by the functions of $n - 1$ spatial variables, while the latter “devices” are represented by the functions of n spatial variables at every moment of time. An example of such controls might be a source/sink in a heat/mass transfer process placed inside the spatial domain or on its boundary.

The goal of this monograph is to present a concise controllability theory of partial differential equations in cases when they are equipped with actuators and/or sensors that are *finite-dimensional* at every moment of time. Typical examples here could be the *point* “devices,” static or mobile, that are designed to act at isolated spatial points. Such finite-dimensional designs of controlling actions seem to be better suited for many real-world applications. On the other hand, for the same reason, their mathematical capabilities are principally reduced relative to their infinite-dimensional counterparts, and thus require special consideration.

The subjects of interest in this monograph are the issues of controllability, observability and stabilizability for the parabolic and hyperbolic partial differential equations, and some related applied questions, such as the problem of localization of unknown pollution sources based on the information obtained from point sensors, arising in environmental monitoring.

This monograph is based on research conducted by the author in [36–58] in the area of controllability theory of partial differential equations.

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