Preface

The book presents new methods of asymptotic analysis for nonlinearly perturbed semi-Markov processes with finite phase spaces. These methods are based on special time-space screening procedures for sequential reduction of phase spaces for semi-Markov processes combined with the systematical use of operational calculus for Laurent asymptotic expansions.

We compose effective recurrent algorithms for the construction of Laurent asymptotic expansions, without and with explicit upper bounds for remainders, for power moments of hitting times for nonlinearly perturbed semi-Markov processes. We also illustrate the above results by getting asymptotic expansions for stationary and conditional quasi-stationary distributions of nonlinearly perturbed semi-Markov processes, in particular for birth-death-type semi-Markov processes, which play an important role in various applications.

It is worth noting that asymptotic expansions are a very effective instrument for studies of perturbed stochastic processes. The corresponding first terms in expansions give limiting values for properly normalized functionals of interest. The second terms let one estimate the sensitivity of models to small parameter perturbations. The subsequent terms in the corresponding expansions are usually neglected in standard linearization procedures used in studies of perturbed models. This, however, cannot be acceptable in the cases where values of perturbation parameter are not small enough. Asymptotic expansions let one take into account high-order terms in expansions, and in this way, to improve accuracy of the corresponding numerical procedures.

Semi-Markov processes are a natural generalization of discrete and continuous time Markov chains. These jump processes possess Markov property at moments of jumps and can have arbitrary distributions concentrated on a positive half-line for inter-jump times. In fact, this combination of basic properties makes semi-Markov processes a very flexible and effective tool for the description of queuing, reliability, and some biological systems, financial and insurance processes, and many other stochastic models.

As for Markov chains, a very important role in the theory of semi-Markov processes is played by hitting times and their moments. These random functionals are also known under such names as first-rare-event times, first passage times, and absorption times, in theoretical studies, and as lifetimes, failure times, extinction times, etc., in applications.

Expectations of hitting times play the key role in ergodic theorems and limit theorems of the law of large numbers type, due to dual relations connecting the corresponding stationary distributions and expectations of return times. Second moments appear in asymptotic results such as a central limit theorem. High-order moments appear in asymptotic results related to rates of convergence and asymptotical expansions in the above limit theorems as well as in large-deviation-type theorems. The moments of hitting times can also be effectively used for the estimation of tail probabilities for hitting times.

Models of perturbed Markov chains and semi-Markov processes, in particular for the most difficult cases of perturbed processes with absorption and so-called singularly perturbed processes, attracted attention of researchers in the middle of the twentieth century. An interest to these models has been stimulated by applications to control and queuing systems, information networks, epidemic models and models of population dynamics and mathematical genetics. We give references to related publications in the methodological and bibliographical remarks included in the book.

Markov-type processes with singular perturbations appear as natural tools for mathematical analysis of multicomponent systems with weakly interacting components. Asymptotics of moments for hitting-time type functionals and stationary distributions for corresponding perturbed processes play an important role in studies of such systems.

The role of perturbation parameters can be played by small failure probabilities or intensities in queuing and reliability systems and small mutation, extinction, or migration probabilities or intensities in biological systems. Perturbation parameters can also appear as artificial regularization parameters for decomposed systems, for example, as so-called damping parameters in information networks, etc.

In many cases, transition characteristics of the corresponding perturbed semi-Markov processes, in particular transition probabilities (of embedded Markov chains), and moments of transition times are nonlinear functions of a perturbation parameter, which admit asymptotic expansions with respect to this parameter.

Such Taylor and Laurent asymptotic expansions, respectively, for transition probabilities (of embedded Markov chains) and moments of transition times for perturbed semi-Markov processes, play the role of initial perturbation conditions in our studies. Two variants of these expansions are considered, with remainders given in the standard form $o(\cdot)$ and with explicit upper bounds for remainders.

We consider models with non-singular and singular perturbations, where the phase space for embedded Markov chains of pre-limiting perturbed semi-Markov processes is one class of communicative states, while the phase space for the limiting embedded Markov chain can possess an arbitrary communicative structure, i.e., can consist of one or several closed classes of communicative states and, possibly, a class of transient states.

The corresponding computational algorithms presented in the book have a universal character. They can be applied to perturbed semi-Markov processes with an arbitrary asymptotic communicative structure of phase spaces and are computationally effective due to the recurrent character of computational procedures.

The book includes six chapters, an appendix, and a bibliography.

In the introductory Chapter 1, we present in an informal form main problems, methods, and algorithms developed in the book, describe contents of the book by chapters, and give additional information for potential readers. Chapter 2 presents a calculus of Laurent asymptotic expansions, which serves as a basic analytic tool for asymptotic perturbation analysis of nonlinearly perturbed semi-Markov processes. Chapter 3 plays the key role in the book. We present here new algorithms of sequential phase space reduction for the construction of Laurent asymptotic expansions, without and with explicit upper bounds for remainders, for moments of hitting times for perturbed semi-Markov processes. In Chapter 4, the above asymptotic results are applied for getting asymptotic expansions for stationary distributions and related functionals for nonlinearly perturbed semi-Markov processes. In Chapter 5, the results presented in Chapter 4 are illustrated by the corresponding asymptotic results for nonlinearly perturbed birth-death-type semi-Markov processes, which play an important role in applications. In Chapter 6, we present some numerical examples illustrating results of previous chapters and give a brief survey of applied perturbed queuing systems, stochastic networks, and stochastic models of biological nature. In Appendix A, we make some methodological and bibliographical remarks and comment on new results presented in the book.

We hope that the publication of this new book related to asymptotic problems for perturbed stochastic processes will be a useful contribution to the continuing intensive studies in this area.

In addition to its use for research and reference purposes, the book can also be used in special courses on the subject and as a complementary reading in general courses on stochastic processes. In this respect, it may be useful for specialists as well as doctoral and advanced undergraduate students.

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