

Chapter 2

Continuous Time Methodology and Mathematical Models in Search of Optimum Investment Strategy in Thermal Plants and Combined Heat and Power Plants

Abstract This chapter presents an original continuous time methodology and mathematical models applied for analyzing the effectiveness of technical and economic aspects of the operation of heat and electricity sources.

Keywords Investment strategy • Mathematical modeling • NPV • IRR • DPBP measures • Specific cost of heat production • Continuous time notation

2.1 Introduction

The development of heat and electricity production in combined cycles was rightly stated as one of the objectives of the energy policy of the EU. An expression of this has taken the form of the Directive 2004/8/EC. The promotion of the combined heat and power guided by the demand for heat offers considerable economic and ecological benefits due to the savings in the use of the chemical energy of primary fuels (called *PES, Primary Energy Saving* [1]). In addition, Directive 2012/27/UE passed on by the European Parliament on October 25 in the area of energy efficiency considers cogeneration as one of the measures used to realize the fulfillment of the EU requirements with regard to the energy and climate policy. In Poland, as early as in 1950s, it was recognized that considerable advantages can be derived from the development of applying combined heat and power systems. This was reflected in the amendment to the Power Law passed in April 2014, which extends into the end of 2018 the support for production of electricity in high-efficiency cogeneration installations. Hence, Poland is one of the few countries in which the ratio of heat production in combined heat and power systems in the overall heat production in centralized systems remains at a relatively high level. In 2011, this ratio was equal to 64% of overall heat production, and leads to a reduction of coal combustion by over a dozen per cent on the scale of the domestic economy. In absolute numbers, this means the use of around ten million tons of coal less. Despite the fact that this figure is considerable, there is still a great deal more work to be done, and it is obligatory to do.

The obligation to reduce greenhouse gas emissions by the Polish economy, which is based on hard coal and lignite, is a very costly process and can potentially lead to its considerable weakening. At this point, we need to note, that it is important that a country with huge coal resources applies this resource as the basic fuel for the domestic power industry, as it rightly guarantees its energy security. Therefore, it is important to lead the energy policy of the country in such a way as to bear the lowest possible cost of adapting to the European climate policy requirements. The support for the generation of electricity in high-efficiency combined heat and power installations is meant to support the building and commissioning of new, high-efficiency sources operating in cogeneration and modernization of the existing ones. On the basis of the 'Poland's energy policy until 2030' resolution adopted on November 10, 2009 by the Council of Ministers [2], the volume of electricity production in high-efficiency cogeneration will nearly double from the level of 24.4 TWh in 2006 to 47.9 TWh in 2030. The ratio of this energy in its total production will then be equal to 22% (compared to 16.2% in 2006). The projected aim needs to be realized as a result of the construction of new cogeneration sources and modernization of the existing ones as well as by replacement of all existing communal and industrial thermal plants with the ones working in the new technology until 2030. Chapter 3 contains the methodology and mathematical models with the continuous time of analyzing the technical and economic effectiveness of modernizing the existing thermal plant and combined heat and power plants.

The meaning of these facts that is not recognized *expressis verbis* demonstrates the need for the development of a methodology and mathematical models accounting for the functional space of the technical and economic phenomena occurring during the processes of combined heat and power production with the purpose of their further analysis. Thus, it is necessary to establish an answer to the following fundamental questions. What is the economically justified cogeneration factor, i.e., relation between the level of electricity and heat production in its sources depending on the price relations between coal, gas, electricity and heat? We can note at this time that the cogeneration factor assumes different values in the various technologies, whereas the particular technologies differ in terms of the exploitation cost (in particular, fuel cost) and investment, and the cost of investment as well. Hence, what is the optimum value of the cogeneration factor from the economic perspective, depending on the above price relations and the capital cost? Equally importantly, to what degree is this value affected by the variations in the price relations between energy carriers, coal, gas, electricity, and heat as well as values and fluctuations in the specific tariffs on the use of the environment and CO₂ emission allowances introduced as part of the energy and climate policy by the EU? What technologies of combined heat and electricity production are and will be most justified from the economic perspective? In the market economy the economic criterion, and the objective to gain profit and its maximization decides on the application of a given technical solution and the analysis of the economic profitability offers the basis of the decisions regarding investments. Hence, the economic criterion is superior to the technical one. Another question regards the justification of the existing combined heat and power sources accounting for the

necessary purchase of CO₂ emission allowances in the economic calculations. If the decision is positive, what technologies should be applied? Alternatively, should we focus on the construction of new, zero-emission sources including *Carbon Capture and Storage* installations (CCS), including the facilities for capturing, transporting and storing CO₂ in deep geological formations?

2.2 Continuous Time Methodology in Search of an Optimum Investment Strategy in Heat Sources

Gaining profits forms the objective of any business. The answer to the question regarding the profitability of an investment in heat and electricity sources, and more general, in any activity, is offered by the discounting measures of the economic effectiveness of an investment, including: *NPV* (*Net Present Value*), *IRR* (*Internal Rate of Return*), *DPBP* (*Discounted Payback Period*). The calculation of these measures applies discounted rates, i.e., the account of the variable value of money over time. The discount rate also offers grounds for the conversion of the value of money from a given period into any other arbitrary period. The conversion of the value of money from a given period needs to involve a reduction to a common denominator, i.e., to the same instant in time since a comparison is possible only in this case. The current instant (present time, $t = 0$, Fig. 2.1), forms the most justified instant for such a reduction and comparison since we know the value of money only at the given point in time. We are familiar with its purchasing power and we know the prices of goods and services. However, we are also not capable of reliably answering the questions regarding the prices in a few years and a dozen of years and the value of money at that time. Such potential answers are only guesses, not to say that they belong to wishful thinking and therefore, the values of *NPV*, *IRR*, and *DPBP* measures are calculated for a given instant, i.e., for $t = 0$. This motion of the instant in time is illustrated in Fig. 2.1, which presents a timeline on the basis of which *NPV*, *IRR*, and *DPBP* measures are defined.

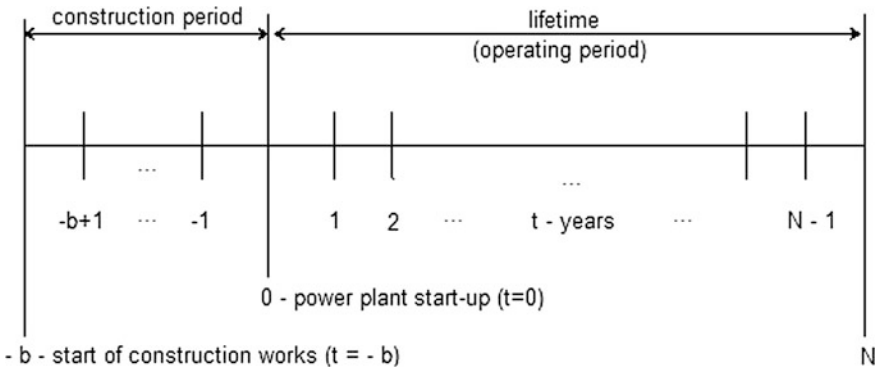


Fig. 2.1 Timeline representing the course of an investment enterprise

The total *NPV* gained over the successive years of exploitation of a thermal plant and CHP plant converted for the present instant $t = 0$, Fig. 2.1, interest rate (*IRR*) that is gained on the invested capital J and its dynamic payback period (*DPBP*) are expressed in the discreet notations, i.e., by means of series and such forms have been solely applied to this time. As a result, the total net profit was defined by the formula [3, 4]:

$$NPV = \sum_{t=1}^N \frac{CF_{t,net}}{(1+r)^t} - J_0, \quad (2.1)$$

and by means of it and assuming that $NPV = 0$, it is possible to define *IRR* and *DPBP* measures

$$\sum_{t=1}^N \frac{CF_{t,gross}}{(1+IRR)^t} = J_0, \quad (2.2)$$

$$\sum_{t=1}^{DPBP} \frac{CF_{t,net}}{(1+r)^t} = J_0, \quad (2.3)$$

where

$CF_{t,net}$	annual net cash flow in the successive years, which is given by the difference between revenues S_A from the sales of products (e.g., electricity and heat) and expenses (exploitation cost K_e , and corporate tax P paid on the annual gross profit. The cost K_e evidently does not include depreciation since it does not occur during the exploitation; depreciation in the formulae (2.1–2.3) is equal to J_0); $CF_{t,net} = S_A - K_e - P$ [3, 4],
$CF_{t,gross}$	gross cash flow; this cash flow does not include corporate tax; $CF_{t,gross} = S_A - K_e$,
J_0	investment J , discounted forward, i.e., at the instant of starting exploitation of a CHP plant, i.e., $t = 0$ incurred in connection with the construction in the years b (investment J_0 , formula (2.4) obviously needs to be repaid, i.e., depreciated), Fig. 2.1,
N	calculation period of the heating plant, or CHP plant lifetime expressed in years,
r	discount rate (interest rate on capital J),
$1/(1+r)^t$	discreet factor used for discounting of cash flows in formulae (2.1–2.3) back in time, i.e., at the instant $t = 0$,
t	successive years of the exploitation of heating plant, CHP plant, $t = 1, 2, \dots, N$.

On the basis of the definition of formula *IRR*, the discounted expenditure J_0 on the right hand of formula (2.2) also forms a function of *IRR* (whereas it is not the

function of the rate r) as in the formulae (2.1) and (2.3). It is derived from the formula [3, 4]

$$J_0 = zJ = \sum_{t=-b}^{t=0} \Delta J_t (1+r)^{|t|} = J \frac{(1+r)^{b+1} - 1}{(b+1)r} \quad (2.4)$$

where

- $(1+r)^{|t|}$ disreet factor used to discount forward the cost of investment J_t incurred in the successive year of the duration of the construction period t , for the instant $t = 0$; $\sum_{t=-b}^{t=0} \Delta J_t = J$,
- b construction period of the thermal plant and CHP plant expressed in years (Fig. 2.1),
- J investment; it is relative to the applied technology of the combined heat and electricity production,
- z freezing coefficient on the investment J at the instant of completion of the investment, $z > 1$; this coefficient accounts for the undesired effect of frozen investment during the construction period, since it does not bring any profit at this time, while the interest on the capital J increases

$$z = \frac{(1+r)^{b+1} - 1}{(b+1)r}. \quad (2.5)$$

2.2.1 Continuous Time Mathematical Model in Search of an Optimum Investment Strategy in Combined Heat and Electricity Sources

The notations of the measures NPV , IRR , and $DPBP$ in terms of series (2.1–2.3) form their considerable drawback since the time and space-consuming process of calculating the values of the particular terms in series in the successive years $t = 1, 2, \dots, N$ and subsequent addition of the terms does not provide an easy analysis of the variability of their values. This drawback can be overcome by assuming (as it is done in [4] that cash flow CF , including prices of energy carriers and environmental charges) are constant over the analyzed period. On the basis of this assumption, geometrical series of measures NPV , IRR , $DPBP$ can be expressed by means of a formula with the sum N of their initial terms and, consequently, this notation is convenient for further analysis [4]. However, if the formula for NPV , IRR , $DPBP$ are assumed to include constant cash flows over the entire period of N years, there is not possibility of optimizing the investment strategy for the case of the variability of energy carrier prices and environmental charges in the successive years. Such problems are not part of the notation of NPV , IRR , $DPBP$ in the continuous time, i.e., for the case when they are written in the form of integrals (2.6, 2.8, 2.9).

In formulae (2.6, 2.8, 2.9) for all sub-integral values, we are able to adopt any functions along with their variability in time, e.g., to assume any scenarios involving the variability in time of energy carriers and specific rates per emissions into the environment [3, 4]. Continuous notations (2.6, 2.8, 2.9) demonstrate an unparalleled advantage over the discreet notations (2.1–2.3). They can be applied to analyze the variability of the *NPV* easily and quickly so that the highest possible value is identified. Moreover, they are applicable in the study of the variability of functions *NPV*, *IRR*, *DPBP* and for development of diagrams with the use of the differential calculus, as it can demonstrate a number of additional important details that could not be accounted for without the use of them. These notations offer an explicit evaluation of the impact of initial values on the final results as well as identification of an optimum solution as well as an area with solutions that are close to this optimum. Additionally, they can be used to demonstrate the nature of changes. Therefore, discussion and analysis of the results is possible. In technology, economics and their applications this plays a considerable role. Moreover, mathematical models with the continuous time offer the statement of general conclusions, whereas only a way from the general to the specific is correct and provides the possibility of making generalizations out of a discussion. In contrast, the way from the specific to general is most commonly (not to say most often) does not demonstrate to be true.

In the notation with the continuous time, the value *NPV* [formula (2.1)] is expressed by the relation [3, 4]

$$NPV = \int_0^T [S_A - K_e - F - R - (S_A - K_e - F - A)p]e^{-rt} dt, \quad (2.6)$$

where

- A depreciation rate, $A = J_0/T$,
- e^{-rt} continuous factor used to discount the resources in formula (2.6) backwards in time, i.e., at instant $t = 0$,
- F time variable interest (financial cost) on investment capital J_0 ; interest F forms an unknown function of the variable in time installments R ; $F = F[R(t)]$,
- K_e variable in time annual exploitation cost,
- p variable in time corporate income tax rate,
- r variable in time discount rate (interest on capital J),
- R variable in time installment for loan repayment, (in practice, $R = J_0/T = \text{const}$),
- S_A variable in time annual revenue,
- t time,
- T calculated exploitation period of the thermal plant and CHP plant expressed in years.

The time variable measures associated with exploitation costs K_e include: the cost of fuel K_{coal} , cost of supplementing water in system circulation K_{sw} , cost of remuneration including overheads K_{sal} , cost of maintenance and overhaul K_{serv} , cost of non-energy resources and supplementary materials K_m , cost associated with the use of the environment K_{env} (including: tariff charges for emission of flue gases into the atmosphere, waste disposal, waste storage, etc.), taxes, local charges and insurance K_t , cost of purchasing CO₂ emission allowances K_{CO_2} and the cost of electricity K_{pump} for driving pumps used to circulate the network hot water and supplementary pumps in the heat distribution network used to maintain the required static pressure of network hot water [2–5]

$$K_e = K_{\text{coal}} + K_{\text{sw}} + K_{\text{sal}} + K_{\text{serv}} + K_m + K_{\text{env}} + K_t + K_{\text{CO}_2} + K_{\text{pump}} \quad (2.7)$$

The cost K_{CO_2} , which results from the climate policy implemented by the old 15 member states of the EU leads to an increase in the exploitation cost K_e of the combined heat and power plant. The total of the cost $K_{\text{sw}} + K_m + K_{\text{pump}}$ and $K_{\text{sal}} + K_t$ in formula (2.7) can be accounted for as a result of a subsequent increasing the cost K_{coal} by several percent and the cost of maintenance and overhaul K_{serv} by a dozen or so percent. The calculation of the cost K_{pump} is only possible when we are familiar with the thermal power and length of the heat distribution network.

From the formula in (2.6) on condition that NPV = 0, we are able to derive further measures of the economic effectiveness of the investment in the notation with continuous time: value of *IRR* rate, that is obtained on the invested capital J and its payback period *DPBP* expressed in years

$$\int_0^T (S_A - K_e) e^{-\text{IRR}t} dt = \int_0^T [F(\text{IRR}) + R(\text{IRR})] e^{-\text{IRR}t} dt, \quad (2.8)$$

$$\int_0^{\text{DPBP}} [S_A - K_e - (S_A - K_e - F - A)p] e^{-rt} dt = \int_0^T (F + R) e^{-rt} dt \quad (2.9)$$

The measure *IRR* [formula (2.8)] on the basis of its definition [3, 4] is derived on the basis of an assumption that the corporate income tax P is equal to zero: $P = (S_A - K_e - F - A)p = 0$. This is justified since *IRR* forms the maximum interest rate on the investment capital J that can be obtained with the purpose of funding the construction of the thermal plant and CHP plant. Hence, *IRR* rate is calculated when the profit gained on the operation of the thermal plant or CHP plant for the rate r ($r < \text{IRR}$) is converted into the increase of the capital cost (interest) exceeding this cost for the rate equal to r . The notations $F(\text{IRR})$ and $R(\text{IRR})$ in formula (2.8) also mean that the cost of finance F and installment on the loan R are functions of the rate *IRR*, whereas in formulae (2.6 and 2.9) they are functions of the rate r along with depreciation rate A [3, 4]. The right-hand sides of Eqs. (2.8 and 2.9)

represent the discounted investment J_0 (formula (2.4) for the present moment $t = 0$ (Fig. 2.1) [3, 4].

The models *NPV*, *IRR*, *DPBP* presented here allow the engineers not only to draw conclusions regarding the economic determinants of implementing particular technologies and selection of the most effective ones from the economic perspective [3, 4]. They can also lead offer the economic calculation with the justified price relations between energy carriers and values of the tariffs on the use of the environment. We can make a statement that these relations could be (or should be) derived on the basis of a criterion of minimizing the specific cost of heat production, which is calculated for the value of the *NPV* equal to zero for the technologically and technically established power installations that are in common use. Apart from this, the presented model offers an analysis of the impact of the discussed above price relations between energy carriers and levels of environmental charges, as well as examples of such parameters as the influence of heat and electricity demand (represented by the production of electricity and heat) on the optimum investment strategy.

The selection of an optimum investment strategy should be undertaken for the condition

$$NPV \rightarrow \max, \quad (2.10)$$

where for the case of combined energy sources

$$\begin{aligned} NPV = & \left\{ E_{el,A} e^{t=0} \frac{1}{a_{el} - r} [e^{(a_{el}-r)T} - 1] + Q_A e_h^{t=0} \frac{1}{a_h - r} [e^{(a_h-r)T} - 1] \right. \\ & - \frac{E_{el,A} + Q_A}{\eta_{CHP}} (1 + x_{sw,m,was}) e_{coal}^{t=0} \frac{1}{a_{coal} - r} [e^{(a_{coal}-r)T} - 1] \\ & - \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{CO_2} p_{CO_2}^{t=0} \frac{1}{a_{CO_2} - r} [e^{(a_{CO_2}-r)T} - 1] \\ & - \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{CO} p_{CO}^{t=0} \frac{1}{a_{CO} - r} [e^{(a_{CO}-r)T} - 1] \\ & - \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{NO_x} p_{NO_x}^{t=0} \frac{1}{a_{NO_x} - r} [e^{(a_{NO_x}-r)T} - 1] \\ & - \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{SO_2} p_{SO_2}^{t=0} \frac{1}{a_{SO_2} - r} [e^{(a_{SO_2}-r)T} - 1] \\ & - \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{dust} p_{dust}^{t=0} \frac{1}{a_{dust} - r} [e^{(a_{dust}-r)T} - 1] \\ & - \frac{E_{el,A} + Q_A}{\eta_{CHP}} (1 - u) \rho_{CO_2} e_{CO_2}^{t=0} \frac{1}{b_{CO_2} - r} [e^{(b_{CO_2}-r)T} - 1] \\ & \left. - J(1 - e^{-rT})(1 + x_{sal,t,ins}) \frac{\delta_{serv}}{r} - J_0 \left(\frac{1 - e^{-rT}}{T} + 1 \right) \right\} (1 - p) \end{aligned} \quad (2.11)$$

where

$a_{el}, a_h, a_{coal}, a_{CO_2},$ $a_{CO}, a_{SO_2}, a_{NO_x},$ a_{dust}, b_{CO_2}	controls representing variable electricity, fuel prices, etc. (exponential values, e.g., $e_{el}(t) = e_{el}^{t=0} e^{a_{el}t}$) [3, 4],
$e_{el}^{t=0}, e_h^{t=0}, e_{coal}^{t=0}, e_{CO_2}^{t=0},$ $p_{CO_2}^{t=0},$ etc.	initial values of the prices of electricity, heat, fuel, CO ₂ emission allowances, tariffs on the use of the environment,
$E_{el,A}$ Q_A η_{CHP}	annual net production of electricity in a CHP plant, MWh/a, annual net production of heat in a CHP plant, GJ/a, net energy efficiency of heat and electricity production (this value is relative to the applied technology of combined heat and electricity production),
u	ratio of chemical energy of fuel to its total use, for which it is not necessary to purchase CO ₂ emission allowances,
$p_{CO_2}, p_{CO},$ $p_{NO_x}, p_{SO_2}, p_{dust}$ $X_{sw,m,was}$	specific tariff charges on CO ₂ , CO, NO _x , SO ₂ , and particulate matter emissions, PLN/kg, coefficient accounting for the cost of supplementing water, auxiliary materials, waste disposal, slag storage and waste (in practice, the value of $x_{sw,m,was}$ is equal to around 0.25),
$X_{sal,t,ins}$	coefficient accounting for the cost of remuneration, taxes, insurance, etc., in practice, the value of $x_{sal,t,ins}$ is equal to around 0.02).
δ_{serv}	annual rate of fixed cost dependent on the value of the investment (cost of maintenance, equipment overhaul; in practice we assume that $\delta_{serv} \approx 3\%$).
$\rho_{CO_2}, \rho_{CO}, \rho_{NO_x},$ ρ_{SO_2}, ρ_{dust}	CO ₂ , CO, NO _x , SO ₂ emission per unit of chemical energy of the fuel, kg/GJ (these values are relative to the applied fuel).

Equation (2.11) was derived in the identical manner as the equation representing NPV for the power plant in [3]. It applies the same notations and scenarios representing the variability of exploitation and capital cost as well as revenues, just as in [3, 4]:

- function of the revenue gained from heat and electricity production

$$S_A(t) = E_{el,A} e_{el}^{t=0} e^{a_{el}t} + Q_A e_h^{t=0} e^{a_h t} + E_{CHP,A} e_{CHP}^{t=0} e^{a_{CHP}t} \quad (2.12)$$

whereas $E_{CHP,A} e_{CHP}^{t=0} e^{a_{CHP}t}$ denotes the revenue from the property rights exercised on the basis of certificates of origin for electricity coming from high-efficiency cogeneration, where:

a_{CHP} control (exponential values $e_{CHP}^{t=0} e^{a_{CHP}t}$) representing the variability in time of certificates of origin for electricity produced in high-efficiency cogeneration,

- $e_{\text{CHP}}^{t=0}$ initial price of the of certificate of origin for electricity produced in high-efficiency cogeneration,
 $E_{\text{CHP},A}$ annual production of electricity coming from high-efficiency cogeneration calculated on the basis of [5],

- function of the fuel cost

$$K_{\text{coal}}(t) = E_{\text{ch},A} e_{\text{coal}}^{t=0} e^{a_{\text{coal}} t} \quad (2.13)$$

whereas the annual use of the chemical energy of fuel is expressed by the formula

$$E_{\text{ch},A} = \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \quad (2.14)$$

- function of the cost associated with the use of the environment

$$K_{\text{env}}(t) = E_{\text{ch},A} [\rho_{\text{CO}_2} p_{\text{CO}_2}(t) + \rho_{\text{CO}} p_{\text{CO}}(t) + \rho_{\text{NO}_x} p_{\text{NO}_x}(t) + \rho_{\text{SO}_2} p_{\text{SO}_2}(t) + \rho_{\text{dust}} p_{\text{dust}}(t)], \quad (2.15)$$

whereas the variability in time of the specific tariff charges (per unit of mass) for CO_2 , CO, NO_x , SO_2 and particulate matter emissions are defined by the formulae

$$p_{\text{CO}_2}(t) = p_{\text{CO}_2}^{t=0} e^{a_{\text{CO}_2} t} \quad (2.16)$$

$$p_{\text{CO}}(t) = p_{\text{CO}}^{t=0} e^{a_{\text{CO}} t} \quad (2.17)$$

$$p_{\text{NO}_x}(t) = p_{\text{NO}_x}^{t=0} e^{a_{\text{NO}_x} t} \quad (2.18)$$

$$p_{\text{SO}_2}(t) = p_{\text{SO}_2}^{t=0} e^{a_{\text{SO}_2} t} \quad (2.19)$$

$$p_{\text{dust}}(t) = p_{\text{dust}}^{t=0} e^{a_{\text{dust}} t} \quad (2.20)$$

- function of purchasing additional CO_2 emission allowances

$$K_{\text{CO}_2}(t) = E_{\text{ch},A} (1 - u) \rho_{\text{CO}_2} e_{\text{CO}_2}^{t=0} e^{b_{\text{CO}_2} t} \quad (2.21)$$

- function of the cost of maintenance and overhauls

$$K_{\text{serv}} = \delta_{\text{serv}} J \quad (2.22)$$

- function of financial cost F

$$F(t) = r[J_0 - (t - 1)R] \quad (2.23)$$

After substituting the above relations into Eqs. (2.8 and 2.9) and their integration, we obtain the relations representing IRR and $DPBP$:

- internal rate of return IRR

$$\begin{aligned}
& Q_A e_h^{t=0} \frac{1}{a_h - \text{IRR}} [e^{(a_h - \text{IRR})T} - 1] + E_{\text{el},A} e_{\text{el}}^{t=0} \frac{1}{a_{\text{el}} - \text{IRR}} [e^{(a_{\text{el}} - \text{IRR})T} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} (1 + x_{\text{sw},m,\text{was}}) e_{\text{coal}}^{t=0} \frac{1}{a_{\text{coal}} - \text{IRR}} [e^{(a_{\text{coal}} - \text{IRR})T} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{CO}_2} p_{\text{CO}_2}^{t=0} \frac{1}{a_{\text{CO}_2} - \text{IRR}} [e^{(a_{\text{CO}_2} - \text{IRR})T} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{CO}} p_{\text{CO}}^{t=0} \frac{1}{a_{\text{CO}} - \text{IRR}} [e^{(a_{\text{CO}} - \text{IRR})T} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{NO}_x} p_{\text{NO}_x}^{t=0} \frac{1}{a_{\text{NO}_x} - \text{IRR}} [e^{(a_{\text{NO}_x} - \text{IRR})T} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{SO}_2} p_{\text{SO}_2}^{t=0} \frac{1}{a_{\text{SO}_2} - \text{IRR}} [e^{(a_{\text{SO}_2} - \text{IRR})T} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{dust}} p_{\text{dust}}^{t=0} \frac{1}{a_{\text{dust}} - \text{IRR}} [e^{(a_{\text{dust}} - \text{IRR})T} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} (1 - u) \rho_{\text{CO}_2} e_{\text{CO}_2}^{t=0} \frac{1}{b_{\text{CO}_2} - \text{IRR}_p^{\text{IPP}}} [e^{(b_{\text{CO}_2} - \text{IRR}_p^{\text{IPP}})T} - 1] \\
& - J(1 + x_{\text{sal},t,\text{ins}}) \frac{\delta_{\text{serv}}}{\text{IRR}} (1 - e^{-\text{IRR}T}) = J \frac{(1 + \text{IRR})^{b+1} - 1}{(b+1)\text{IRR}} \left(1 + \frac{1 - e^{-\text{IRR}T}}{T} \right)
\end{aligned} \tag{2.24}$$

- dynamic payback period (DPBP)

$$\begin{aligned}
& \left\{ Q_A e_h^{t=0} \frac{1}{a_h - r} [e^{(a_h - r)\text{DPBP}} - 1] + E_{\text{el},A} e_{\text{el}}^{t=0} \frac{1}{a_{\text{el}} - r} [e^{(a_{\text{el}} - r)\text{DPBP}} - 1] \right. \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} (1 + x_{\text{sw},m,\text{was}}) e_{\text{coal}}^{t=0} \frac{1}{a_{\text{coal}} - r} [e^{(a_{\text{coal}} - r)\text{DPBP}} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{CO}_2} p_{\text{CO}_2}^{t=0} \frac{1}{a_{\text{CO}_2} - r} [e^{(a_{\text{CO}_2} - r)\text{DPBP}} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{CO}} p_{\text{CO}}^{t=0} \frac{1}{a_{\text{CO}} - r} [e^{(a_{\text{CO}} - r)\text{DPBP}} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{NO}_x} p_{\text{NO}_x}^{t=0} \frac{1}{a_{\text{NO}_x} - r} [e^{(a_{\text{NO}_x} - r)\text{DPBP}} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{SO}_2} p_{\text{SO}_2}^{t=0} \frac{1}{a_{\text{SO}_2} - r} [e^{(a_{\text{SO}_2} - r)\text{DPBP}} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} \rho_{\text{dust}} p_{\text{dust}}^{t=0} \frac{1}{a_{\text{dust}} - r} [e^{(a_{\text{dust}} - r)\text{DPBP}} - 1] \\
& - \frac{E_{\text{el},A} + Q_A}{\eta_{\text{CHP}}} (1 - u) \rho_{\text{CO}_2} e_{\text{CO}_2}^{t=0} \frac{1}{b_{\text{CO}_2} - r} [e^{(b_{\text{CO}_2} - r)\text{DPBP}} - 1] \\
& \left. - J(1 - e^{-r\text{DPBP}}) (1 + x_{\text{sal},t,\text{ins}}) \frac{\delta_{\text{serv}}}{r} \right\} (1 - p) - J_0 \left[1 + \frac{1}{T} - e^{-r\text{DPBP}} \left(1 + \frac{1}{T} - \frac{\text{DPBP}}{T} \right) \right] p \\
& = J_0 \left(1 + \frac{1 - e^{-rT}}{T} \right).
\end{aligned} \tag{2.25}$$

The calculation of the values of *IRR* and *DPBP* from Eqs. (2.24 and 2.25) requires the application of the successive approximation method.

An equivalent criterion to $NPV \rightarrow \max$ during search for an optimum investment strategy [3, 4] in combined heat and power sources is the one based on the search for the minimum specific value of the heat production cost

$$k_h \rightarrow \min. \quad (2.26)$$

This cost is derived from the relations in Eq. (2.11) on condition that $NPV = 0$

$$\begin{aligned} Q_A k_h^{t=0} \frac{1}{a_h - r} [e^{(a_h - r)T} - 1] = & \frac{E_{el,A} + Q_A}{\eta_{CHP}} (1 + x_{sw,m,was}) e_{coal}^{t=0} \frac{1}{a_{coal} - r} [e^{(a_{coal} - r)T} - 1] \\ & + \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{CO_2} p_{CO_2}^{t=0} \frac{1}{a_{CO_2} - r} [e^{(a_{CO_2} - r)T} - 1] \\ & + \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{CO} p_{CO}^{t=0} \frac{1}{a_{CO} - r} [e^{(a_{CO} - r)T} - 1] \\ & + \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{NO_x} p_{NO_x}^{t=0} \frac{1}{a_{NO_x} - r} [e^{(a_{NO_x} - r)T} - 1] \\ & + \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{SO_2} p_{SO_2}^{t=0} \frac{1}{a_{SO_2} - r} [e^{(a_{SO_2} - r)T} - 1] \\ & + \frac{E_{el,A} + Q_A}{\eta_{CHP}} \rho_{dust} p_{dust}^{t=0} \frac{1}{a_{dust} - r} [e^{(a_{dust} - r)T} - 1] \\ & + \frac{E_{el,A} + Q_A}{\eta_{CHP}} (1 - u) \rho_{CO_2} e_{CO_2}^{t=0} \frac{1}{b_{CO_2} - r} [e^{(b_{CO_2} - r)T} - 1] \\ & + J(1 - e^{-rT})(1 + x_{sal,t,ins}) \frac{\delta_{serv}}{r} + J_0 \left(\frac{1 - e^{-rT}}{T} + 1 \right) \\ & - E_{el,A} e_{el}^{t=0} \frac{1}{a_{el} - r} [e^{(a_{el} - r)T} - 1] \end{aligned} \quad (2.27)$$

and for $a_h = 0$, we obtain the mean specific cost of heat production

$$\begin{aligned} k_{h,av} = & \frac{\sigma_A + 1}{\eta_{CHP}(1 - e^{-rT})} \left\{ (1 + x_{sw,m,was}) e_{coal}^{t=0} \frac{r}{a_{coal} - r} [e^{(a_{coal} - r)T} - 1] \right. \\ & + \rho_{CO_2} p_{CO_2}^{t=0} \frac{r}{a_{CO_2} - r} [e^{(a_{CO_2} - r)T} - 1] \\ & + \rho_{CO} p_{CO}^{t=0} \frac{r}{a_{CO} - r} [e^{(a_{CO} - r)T} - 1] + \rho_{NO_x} p_{NO_x}^{t=0} \frac{r}{a_{NO_x} - r} [e^{(a_{NO_x} - r)T} - 1] \\ & + \rho_{SO_2} p_{SO_2}^{t=0} \frac{r}{a_{SO_2} - r} [e^{(a_{SO_2} - r)T} - 1] + \rho_{dust} p_{dust}^{t=0} \frac{r}{a_{dust} - r} [e^{(a_{dust} - r)T} - 1] \\ & \left. + (1 - u) \rho_{CO_2} e_{CO_2}^{t=0} \frac{r}{b_{CO_2} - r} [e^{(b_{CO_2} - r)T} - 1] \right\} + \frac{i(1 + x_{sal,t,ins})\delta_{serv}}{\tau_s} \\ & + \frac{rzi}{\tau_s(1 - e^{-rT})} \left[(1 - e^{-rT}) \frac{1}{T} + 1 \right] - \frac{r\sigma_A e_{el}^{t=0}}{(a_{el} - r)(1 - e^{-rT})} [e^{(a_{el} - r)T} - 1] \end{aligned} \quad (2.28)$$

where:

- i specific investment in a CHP plant (per unit of power output),
 $i = J / \dot{Q}_{h\max}^{\text{CHP}}$, (its value is relative to the applied technology of combined heat and electricity production);
- $\sigma_A = \frac{E_{el,A}}{Q_A} \geq 0$ annual cogeneration factor (its value is relative to the applied technology of combined heat and electricity production); it assumes the highest values for communal CHP plants operating in the gas-steam technology; $\sigma_A^{G-S} \cong 4.1$ [6]; we can note that the value of the cogeneration factor for instantaneous loads, defined as the ratio of instantaneous electricity output to instantaneous thermal power $\sigma = N_{el}^{\text{CHP}} / \dot{Q}_h^{\text{CHP}}$ in the communal CHP plants operating according to the annual scheduled chart based on the demand for heat [6] is variable depending on the ambient temperature; σ assumes its greatest value in the summer, i.e., when CHP plants operate with a constant thermal power $\dot{Q}_{h\min}^{\text{CHP}} = \dot{Q}_{l\text{nhw}}^{\text{CHP}}$ only for the purposes of producing network hot water, and with the maximum electricity output $N_{el\max}^{\text{CHP}}$ due to the small volume of heating steam extraction from turbine exhausts, and σ assumes its lowest value when CHP plants operate with the maximum thermal output $\dot{Q}_{h\max}^{\text{CHP}}$, i.e., with the minimum electrical capacity $N_{el\min}^{\text{CHP}}$; e.g., for CHP plants in the gas-steam technology $\sigma_{\max}^{G-S} = N_{el\max}^{\text{CHP}} / \dot{Q}_{l\text{nhw}}^{\text{CHP}} \cong 20$, $\sigma_{\min}^{G-S} = N_{el\min}^{\text{CHP}} / \dot{Q}_{h\max}^{\text{CHP}} \cong 1.2$ [3]),
- τ_s annual period when the CHP plant reaches its maximum (rated, peak) power output $\dot{Q}_{h\max}^{\text{CHP}}$ [6].

The duration of period τ_s for communal CHP plants is only a theoretical figure, as it denotes how many hours per year it needs to operate with the maximum thermal power in order to produce the volume of heat equal to Q_A . Importantly, the introduction of this time, together with σ_A , leads to the considerable reduction of the calculations of the specific cost $k_{h,\text{av}}$ in multi-variant scenarios, which would have to be performed to obtain a widely understood and clear form of a cost that could be applied for the analysis of an optimum investment strategy in the heat sources. Time τ_s and parameter σ_A eliminate Q_A and $E_{el,A}$ from the formula in Eq. (2.28), whose ranges can be very high, and almost unlimited.

For the communal CHP plant, the annual time τ_s is derived from the equation [6]

$$Q_A = \dot{Q}_{h\max}^{\text{CHP}} \tau_s = \dot{Q}_{h\text{zav}}^{\text{CHP}}|_0^{\tau_z} \tau_z + \dot{Q}_{l\text{nhw}}^{\text{CHP}} \tau_l, \quad (2.29)$$

Hence,

$$\tau_s = \frac{\dot{Q}_{h\text{zav}}^{\text{CHP}}|_0^{\tau_z}}{\dot{Q}_{h\max}^{\text{CHP}}} \tau_z + \frac{\dot{Q}_{l\text{nhw}}^{\text{CHP}}}{\dot{Q}_{h\max}^{\text{CHP}}} \tau_l = \xi \tau_z + \beta \tau_l, \quad (2.30)$$

where

$\dot{Q}_{h\max}^{\text{CHP}}, \dot{Q}_{h\text{zav}}^{\text{CHP}}|_{\tau_z}, \dot{Q}_{l\text{nhw}}^{\text{CHP}}$ maximum thermal (peak) power, i.e., rated capacity of the CHP plant, mean thermal power of the CHP plant in the peak season, and power needed for heating network hot water in the off-peak (summer) season, respectively,
 τ_l, τ_z duration of off-peak (summer) season, duration of the heating season of the CHP plant.

In practice, for communal CHP plants, the value of ξ is equal to around 0.5, whereas $\beta \in \langle 0.05; 0.15 \rangle$. The duration of the peak season τ_z depending on the climate zone (Poland consists of five zones) is in the range from $5040 \div 5400$ h/a, i.e., $210 \div 225$ days. The duration of the peak season τ_l is derived from the equation $\tau_A = \tau_z + \tau_l$, where the annual operating time of a CHP plant is on average equal to $\tau_A \cong 8424$ h/a (including around 2 weeks of downtime on holiday months). The annual time τ_s therefore assumes values in the range of only 2800–3000 h/a, which forms a clear advantage, since it reduces the number of necessary calculations so they can be developed in the form of nomograms; whereas the range of the variability in the value of heat Q_A is almost unlimited, as we know from the discussion above.

The operation of industrial CHP plants used to feed heat for technology processes, in contrast to the communal CHP plants, which operate according to annual scheduled chart based on the demand for heat [6], is based on individual charts with the heat demand and they are principally characterized by constant thermal power. Therefore, the annual duration of time τ_s assumes values equal to the annual operating times τ_A . Hence, such installations include backpressure steam turbines, extraction-backpressure turbines, rarely extraction-condensing steam turbines since they are more expensive in terms of investment; however, the electricity production is then independent of the demand for heat. The latter is not necessary for the case of the industrial CHP plants. However, if we could gain economic benefits from the operation of a CHP plant with a greater electrical capacity, for instance due to the installation of a condensing boiler, this solution should be applied. If the gas-steam technology was to be applied in the CHP plant (as it is characterized by the greatest electricity production), the annual cogeneration factor could assume the value $\sigma_A^{G-S} = \sigma_{\min}^{G-S} \cong 1.2$. For the steam CHP plant, this factor assumes two times bigger values $\sigma_A \cong 0.4 \div 0.6$. The price of the coal that is combusted in it is at present equal to $e_{\text{coal}}^{t=0} \cong 10.5$ PLN/GJ is three times smaller than the price of natural gas $e_{\text{gas}}^{t=0} \cong 32$ PLN/GJ applied in the gas-steam systems. For this reason, the gas-steam systems are currently economically unjustified despite the relatively low price of electricity. This is so despite the smallest necessary investment in them compared to other accessible technologies.

The equation in Eq. (2.28) could be restated in the dimensionless form by dividing its both sides, e.g., by the current specific price of electricity $e_{\text{el}}^{t=0}$. As a consequence, it represents the specific cost of heat production $k_{h,\text{av}}$ related to the specific price of electricity $e_{\text{el}}^{t=0}$, importantly, in the function of dimensionless

independent variables: σ_A , a_{el} , a_{coal} , a_{CO_2} , a_{CO} , a_{SO_2} , a_{NO_x} , a_{dust} , b_{CO_2} , u , η_{CHP} , $e_{dust}^{t=0}/e_{el}^{t=0}$, etc., $\rho_{CO_2} p_{CO_2}^{t=0}/e_{el}^{t=0}$, etc., $i/(\tau_s e_{el}^{t=0}) = J/(\dot{Q}_{h\max}^{CHP} \tau_s e_{el}^{t=0})$:

$$\begin{aligned} \frac{k_{h,av}}{e_{el}^{t=0}} = & \frac{\sigma_A + 1}{\eta_{CHP}(1 - e^{-rT})} \left\{ (1 + x_{sw,m,was}) \frac{e_{coal}^{t=0}}{e_{el}^{t=0}} \frac{r}{a_{coal} - r} [e^{(a_{coal}-r)T} - 1] \right. \\ & + \rho_{CO_2} \frac{p_{CO_2}^{t=0}}{e_{el}^{t=0}} \frac{r}{a_{CO_2} - r} [e^{(a_{CO_2}-r)T} - 1] \\ & + \rho_{CO} \frac{p_{CO}^{t=0}}{e_{el}^{t=0}} \frac{r}{a_{CO} - r} [e^{(a_{CO}-r)T} - 1] + \rho_{NO_x} \frac{p_{NO_x}^{t=0}}{e_{el}^{t=0}} \frac{r}{a_{NO_x} - r} [e^{(a_{NO_x}-r)T} - 1] \\ & + \rho_{SO_2} \frac{p_{SO_2}^{t=0}}{e_{el}^{t=0}} \frac{r}{a_{SO_2} - r} [e^{(a_{SO_2}-r)T} - 1] + \rho_{dust} \frac{p_{dust}^{t=0}}{e_{el}^{t=0}} \frac{r}{a_{dust} - r} [e^{(a_{dust}-r)T} - 1] \\ & \left. + (1 - u) \rho_{CO_2} \frac{e_{CO_2}^{t=0}}{e_{el}^{t=0}} \frac{r}{b_{CO_2} - r} [e^{(b_{CO_2}-r)T} - 1] \right\} + (1 + x_{sal,t,ins}) \frac{i \delta_{serv}}{\tau_s e_{el}^{t=0}} \\ & + \frac{rzi}{\tau_s e_{el}^{t=0}(1 - e^{-rT})} \left[(1 - e^{-rT}) \frac{1}{T} + 1 \right] - \frac{r\sigma_A}{(a_{el} - r)(1 - e^{-rT})} [e^{(a_{el}-r)T} - 1]. \end{aligned} \quad (2.31)$$

The dimensionless form of the formula (2.31) is very convenient for the analysis of the variability of heat production prices. This is so as it takes on a general character, which provides the possibility of the analysis of the technical and economic effectiveness of the combined heat and electricity production regardless of the thermal power and the applied technology. In other words, this notation makes it possible to transfer the results of calculations to include sources with any technical and economic parameters.

The value of the dimensionless ratio $k_{h,av}/e_{el}^{t=0}$ can take on a negative value, since the cost $k_{h,av}$ can take on a negative value as a consequence of the avoided cost, which is equal to the revenue from the sales of electricity produced in a heat source with the negative sign: $-E_{el,A} e_{el}^{t=0} \frac{1}{a_{el}-r} [e^{(a_{el}-r)T} - 1]$ [formula (2.27)]. The peak value of this ratio should be considerably smaller than one. If the specific cost of heat were to be similar to the price of electricity, the system with the CHP plant would be completely unjustified for both economic and thermodynamic reasons.

The optimum technology would be the one for which the mean specific relative cost of heat production $k_{h,av}/e_{el}^{t=0}$ is the lowest. It is relative to the annual production of electricity $E_{el,A}$ in relation to the annual heat production Q_A , i.e., to the annual value of σ_A parameter (which, as mentioned above, depends on the applied technology) and to the price relations between energy carriers and their variability in time. This means that the cost is relative to the relations between fuel prices (coal, gas) and electricity price, and also to the specific charges on the use of the environment, including the cost of purchasing CO₂ emission allowances. The variability

of energy carrier prices in time and changing relations between them are inevitable for a number of reasons. One of them is associated with inflation, but it is founded to a greater degree on political disturbances.

2.2.2 *Methodology of Analyzing the Impact of Technical and Economic Parameters on the Specific Cost of Heat Production*

The value of the specific cost of heat production is relative to the technology in which it is generated. Technology determines the level of the investment necessary to build a combined heat and power plant, its energy efficiency and the annual period when its peak power can be utilized. This cost also depends on the interest rate on the investment capital, fuel prices, and the cost associated with the use of the environment, and importantly, on the variability of these values in time. The variability of energy carrier prices in time and the changing relations between them are inevitable for a number of reasons. One of them is associated with inflation, but to a greater degree by political instability.

The above technical and economic parameters determine the value of the specific cost of electricity production to a different degree. Moreover, the possible of their variability in a given technology is limited, in contrast to the economic parameters, such as fuel prices and environmental charges. Therefore, it is necessary to undertake an analysis with the aim of getting an answer to the question concerning the impact of the above parameter on the value of the specific cost, and also, importantly, the degree to which the changes in their values offer a potential for cost reduction in the specific technologies. Such an analysis can be most conveniently conducted by the application of the differential calculus—which is possible due to the statement of mathematical models with the continuous time—by using the exact differential of the specific cost of heat production [formula (2.28)]

$$\begin{aligned}
 dk_{h,av} = & \frac{\partial k_{h,av}}{\partial a_{fuel}} \Delta a_{fuel} + \frac{\partial k_{h,av}}{\partial a_{CO_2}} \Delta a_{CO_2} + \frac{\partial k_{h,av}}{\partial a_{CO}} \Delta a_{CO} + \frac{\partial k_{h,av}}{\partial a_{NO_x}} \Delta a_{NO_x} \\
 & + \frac{\partial k_{h,av}}{\partial a_{SO_2}} \Delta a_{SO_2} + \frac{\partial k_{h,av}}{\partial a_{dust}} \Delta a_{dust} + \frac{\partial k_{h,av}}{\partial b_{CO_2}} \Delta b_{CO_2} + \frac{\partial k_{h,av}}{\partial t_s} \Delta t_s + \frac{\partial k_{h,av}}{\partial i} \Delta i \\
 & + \frac{\partial k_{h,av}}{\partial \sigma_A} \Delta \sigma_A + \frac{\partial k_{h,av}}{\partial \eta_{CHP}} \Delta \eta_{CHP} + \frac{\partial k_{h,av}}{\partial r} \Delta r + \frac{\partial k_{h,av}}{\partial a_{el}} \Delta a_{el}.
 \end{aligned} \tag{2.32}$$

The partial derivatives in formula (2.32) form the weights representing the impact of the particular technical and economic parameters on the variability of the specific cost $k_{h,av}$. These weights are expressed by the formulae

- weight representing the variability of fuel prices in time

$$\frac{\partial k_{h,av}}{\partial a_{fuel}} = \frac{(\sigma_A + 1)(1 + x_{sw,m,was})re_{fuel}^{t=0} e^{(a_{fuel}-r)T} [T(a_{fuel} - r) - 1] + 1}{\eta_{CHP}(1 - e^{-rT})} \frac{1}{(a_{fuel} - r)^2} \quad (2.33)$$

whereas the mean values representing the exponents of the variability in time of electricity, coal prices and specific charges on CO₂, CO, NO_x, SO₂ and particulate matter emission in the interval $\langle 0, T \rangle$ are given by the functions of the exponents: a_{el} , a_{fuel} , a_{CO_2} , a_{CO} , a_{SO_2} , a_{NO_x} , a_{dust} , b_{CO_2} . For instance, for the exponent representing the variability in time of the coal price $e_{fuel}(t) = e_{fuel}^{t=0} e^{a_{fuel}t}$, the mean value is expressed by the formula [3]:

$$e_{fuel}^{av} = \frac{e_{fuel}^{t=0}}{Ta_{fuel}} (e^{a_{fuel}T} - 1) \quad (2.34)$$

- weight representing the variability in time of tariff charges on carbon dioxide emission

$$\frac{\partial k_{h,av}}{\partial a_{CO_2}} = \frac{(\sigma_A + 1)r\rho_{CO_2}p_{CO_2}^{t=0} e^{(a_{CO_2}-r)T} [T(a_{CO_2} - r) - 1] + 1}{\eta_{CHP}(1 - e^{-rT})} \frac{1}{(a_{CO_2} - r)^2} \quad (2.35)$$

- weight representing the variability in time of charges on carbon monoxide emission

$$\frac{\partial k_{h,av}}{\partial a_{CO}} = \frac{(\sigma_A + 1)r\rho_{CO}p_{CO}^{t=0} e^{(a_{CO}-r)T} [T(a_{CO} - r) - 1] + 1}{\eta_{CHP}(1 - e^{-rT})} \frac{1}{(a_{CO} - r)^2} \quad (2.36)$$

- weight representing the variability in time of tariff charges on nitrogen oxide emission

$$\frac{\partial k_{h,av}}{\partial a_{NO_x}} = \frac{(\sigma_A + 1)r\rho_{NO_x}p_{NO_x}^{t=0} e^{(a_{NO_x}-r)T} [T(a_{NO_x} - r) - 1] + 1}{\eta_{CHP}(1 - e^{-rT})} \frac{1}{(a_{NO_x} - r)^2} \quad (2.37)$$

- weight representing the variability in time of tariff charges on sulfur dioxide emission

$$\frac{\partial k_{h,av}}{\partial a_{SO_2}} = \frac{(\sigma_A + 1)r\rho_{SO_2}p_{SO_2}^{t=0} e^{(a_{SO_2}-r)T} [T(a_{SO_2} - r) - 1] + 1}{\eta_{CHP}(1 - e^{-rT})} \frac{1}{(a_{SO_2} - r)^2} \quad (2.38)$$

- weight representing the variability in time of tariff charges on particulate matter emission

$$\frac{\partial k_{h,av}}{\partial a_{dust}} = \frac{(\sigma_A + 1)r\rho_{dust}p_{dust}^{t=0} e^{(a_{dust}-r)T} [T(a_{dust} - r) - 1] + 1}{\eta_{CHP}(1 - e^{-rT})} \frac{1}{(a_{dust} - r)^2} \quad (2.39)$$

- weight representing the variability in time of prices on purchase CO₂ emission allowances (the need to purchase of these, results from irrational climate and energy policy of the EU)

$$\frac{\partial k_{h,av}}{\partial b_{CO_2}} = \frac{(\sigma_A + 1)(1 - u)r\rho_{CO_2}e_{CO_2}^{t=0}e^{(b_{CO_2}-r)T}[T(b_{CO_2} - r) - 1] + 1}{\eta_{CHP}(1 - e^{-rT})(b_{CO_2} - r)^2} \quad (2.40)$$

- weight representing the value of the annual cogeneration factor σ_A

$$\begin{aligned} \frac{\partial k_{h,av}}{\partial \sigma_A} = & \frac{r}{\eta_{CHP}(1 - e^{-rT})} \left\{ (1 + x_{sw,m,was}) \frac{e_{fuel}^{t=0}}{a_{fuel} - r} [e^{(a_{fuel}-r)T} - 1] \right. \\ & + \frac{\rho_{CO_2}P_{CO_2}^{t=0}}{a_{CO_2} - r} [e^{(a_{CO_2}-r)T} - 1] \\ & + \frac{\rho_{CO}P_{CO}^{t=0}}{a_{CO} - r} [e^{(a_{CO}-r)T} - 1] + \frac{\rho_{NO_X}P_{NO_X}^{t=0}}{a_{NO_X} - r} [e^{(a_{NO_X}-r)T} - 1] \\ & + \frac{\rho_{SO_2}P_{SO_2}^{t=0}}{a_{SO_2} - r} [e^{(a_{SO_2}-r)T} - 1] + \frac{\rho_{dust}P_{dust}^{t=0}}{a_{dust} - r} [e^{(a_{dust}-r)T} - 1] \\ & \left. + (1 - u) \frac{\rho_{CO_2}e_{CO_2}^{t=0}}{b_{CO_2} - r} [e^{(b_{CO_2}-r)T} - 1] \right\} - \frac{r e_{el}^{t=0}}{(a_{el} - r)(1 - e^{-rT})} [e^{(a_{el}-r)T} - 1] \end{aligned} \quad (2.41)$$

- weight representing the variability of interest rate r on investment capital associated with the construction of a CHP plant

$$\begin{aligned} \frac{\partial k_{h,av}}{\partial r} = & \frac{(\sigma_A + 1)(1 + x_{sw,m,was})e_{fuel}^{t=0}}{\eta_{CHP}} \left\{ \frac{e^{(a_{fuel}-r)T}(1 - rT) - 1}{(a_{fuel} - r)(1 - e^{-rT})} - \frac{r[e^{(a_{fuel}-r)T} - 1]\{e^{-rT}[1 + T(a_{fuel} - r)] - 1\}}{(a_{fuel} - r)^2(1 - e^{-rT})^2} \right\} \\ & + \frac{(\sigma_A + 1)\rho_{CO_2}P_{CO_2}^{t=0}}{\eta_{CHP}} \left\{ \frac{e^{(a_{CO_2}-r)T}(1 - rT) - 1}{(a_{CO_2} - r)(1 - e^{-rT})} - \frac{r[e^{(a_{CO_2}-r)T} - 1]\{e^{-rT}[1 + T(a_{CO_2} - r)] - 1\}}{(a_{CO_2} - r)^2(1 - e^{-rT})^2} \right\} \\ & + \frac{(\sigma_A + 1)\rho_{CO}P_{CO}^{t=0}}{\eta_{CHP}} \left\{ \frac{e^{(a_{CO}-r)T}(1 - rT) - 1}{(a_{CO} - r)(1 - e^{-rT})} - \frac{r[e^{(a_{CO}-r)T} - 1]\{e^{-rT}[1 + T(a_{CO} - r)] - 1\}}{(a_{CO} - r)^2(1 - e^{-rT})^2} \right\} \\ & + \frac{(\sigma_A + 1)\rho_{NO_X}P_{NO_X}^{t=0}}{\eta_{CHP}} \left\{ \frac{e^{(a_{NO_X}-r)T}(1 - rT) - 1}{(a_{NO_X} - r)(1 - e^{-rT})} - \frac{r[e^{(a_{NO_X}-r)T} - 1]\{e^{-rT}[1 + T(a_{NO_X} - r)] - 1\}}{(a_{NO_X} - r)^2(1 - e^{-rT})^2} \right\} \\ & + \frac{(\sigma_A + 1)\rho_{SO_2}P_{SO_2}^{t=0}}{\eta_{CHP}} \left\{ \frac{e^{(a_{SO_2}-r)T}(1 - rT) - 1}{(a_{SO_2} - r)(1 - e^{-rT})} - \frac{r[e^{(a_{SO_2}-r)T} - 1]\{e^{-rT}[1 + T(a_{SO_2} - r)] - 1\}}{(a_{SO_2} - r)^2(1 - e^{-rT})^2} \right\} \\ & + \frac{(\sigma_A + 1)\rho_{dust}P_{dust}^{t=0}}{\eta_{CHP}} \left\{ \frac{e^{(a_{dust}-r)T}(1 - rT) - 1}{(a_{dust} - r)(1 - e^{-rT})} - \frac{r[e^{(a_{dust}-r)T} - 1]\{e^{-rT}[1 + T(a_{dust} - r)] - 1\}}{(a_{dust} - r)^2(1 - e^{-rT})^2} \right\} \\ & + \frac{(\sigma_A + 1)(1 - u)\rho_{CO_2}e_{CO_2}^{t=0}}{\eta_{CHP}} \left\{ \frac{e^{(b_{CO_2}-r)T}(1 - rT) - 1}{(b_{CO_2} - r)(1 - e^{-rT})} - \frac{r[e^{(b_{CO_2}-r)T} - 1]\{e^{-rT}[1 + T(b_{CO_2} - r)] - 1\}}{(b_{CO_2} - r)^2(1 - e^{-rT})^2} \right\} \\ & - \frac{\sigma_A e_{el}^{t=0}}{\eta_{CHP}} \left\{ \frac{e^{(a_{el}-r)T}(1 - rT) - 1}{(a_{el} - r)(1 - e^{-rT})} - \frac{r[e^{(a_{el}-r)T} - 1]\{e^{-rT}[1 + T(a_{el} - r)] - 1\}}{(a_{el} - r)^2(1 - e^{-rT})^2} \right\} \\ & + \frac{iz}{\tau_s} \left[\frac{1}{T} + \frac{1 - e^{-rT}(1 + rT)}{(1 - e^{-rT})^2} \right] \end{aligned} \quad (2.42)$$

- weight representing the variability of the annual duration of the period τ_s when the maximum thermal capacity is utilized by a CHP plant

$$\frac{\partial k_{h,av}}{\partial \tau_s} = \frac{rzi}{\tau_s^2(e^{-rT} - 1)} \left(\frac{1 - e^{-rT}}{T} + 1 \right) - \frac{i(1 + x_{sal,t,ins})\delta_{serv}}{\tau_s^2} \quad (2.43)$$

- weight representing the variability of the specific investment made in the construction of a CHP plant

$$\frac{\partial k_{h,av}}{\partial i} = \frac{(1 + x_{sal,t,ins})\delta_{serv}}{\tau_s} + \frac{rz}{\tau_s(1 - e^{-rT})} \left(\frac{1 - e^{-rT}}{T} + 1 \right) \quad (2.44)$$

- weight representing the variability in time of the electricity price

$$\frac{\partial k_{h,av}}{\partial a_{el}} = \frac{\sigma_A r e_{el}^{t=0}}{(e^{-rT} - 1)} \frac{e^{(a_{el}-r)T} [T(a_{el} - r) - 1] + 1}{(a_{el} - r)^2} \quad (2.45)$$

- weight representing the variability of the efficiency η_{CHP} of heat production

$$\begin{aligned} \frac{\partial k_{h,av}}{\partial \eta_{CHP}} = & \frac{(\sigma_A + 1)r}{\eta_{CHP}^2(e^{-rT} - 1)} \left\{ (1 + x_{sw,m,was}) \frac{e_{fuel}^{t=0}}{a_{fuel} - r} [e^{(a_{fuel}-r)T} - 1] \right. \\ & + \frac{\rho_{CO_2} p_{CO_2}^{t=0}}{a_{CO_2} - r} [e^{(a_{CO_2}-r)T} - 1] \\ & + \frac{\rho_{CO} p_{CO}^{t=0}}{a_{CO} - r} [e^{(a_{CO}-r)T} - 1] + \frac{\rho_{NO_x} p_{NO_x}^{t=0}}{a_{NO_x} - r} [e^{(a_{NO_x}-r)T} - 1] \\ & + \frac{\rho_{SO_2} p_{SO_2}^{t=0}}{a_{SO_2} - r} [e^{(a_{SO_2}-r)T} - 1] + \frac{\rho_{dust} p_{dust}^{t=0}}{a_{dust} - r} [e^{(a_{dust}-r)T} - 1] \\ & \left. + (1 - u) \frac{\rho_{CO_2} e_{CO_2}^{t=0}}{b_{CO_2} - r} [e^{(b_{CO_2}-r)T} - 1] \right\}. \end{aligned} \quad (2.46)$$

Figures 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10, 2.11, 2.12, 2.13 and 2.14 present the results of calculations involving the impact of the particular technical and economic parameters on the value of the specific cost $k_{h,av}$ of heat production in a CHP plant. These calculations applied formulae (2.33–2.46).

The spread value of the results from marginal to very high, just as expected, indicates that the fuel prices (a few dozen per cent), sales price of electricity and its volume (represented by the value of the annual cogeneration factor $\sigma_A = E_{el,A}/Q_A$) along with the political support instruments for particular power engineering technologies have the largest impact on the value of this cost. It indicates that the value of the cost $k_{h,av}$ is affected by the prices of fuel and electricity to the greatest extent—Figs. 2.2 and 2.3.

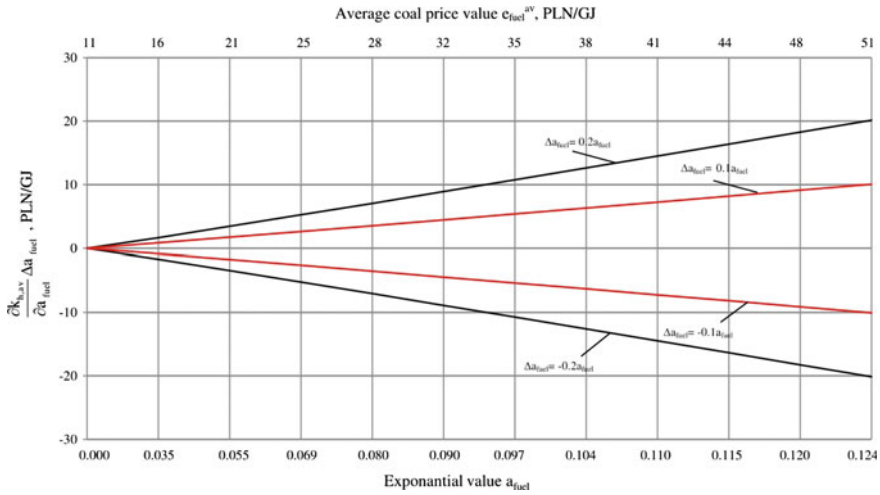


Fig. 2.2 Increase of specific cost of heat production in the function of coal price $c_{fuel}(t) = c_{fuel}^{t=0} e^{a_{fuel}t}$

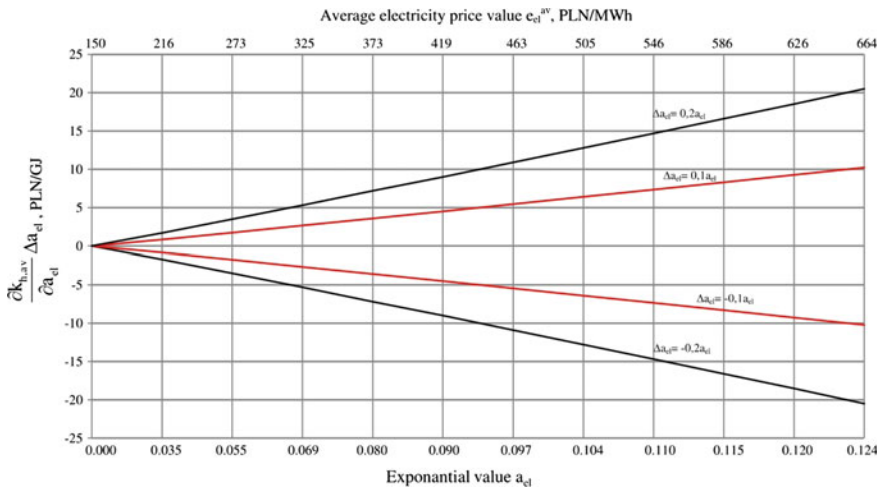


Fig. 2.3 Increase of specific cost of heat production in the function of electricity price $c_{el}(t) = c_{el}^{t=0} e^{a_{el}t}$

The value of the cost $k_{h,av}$ is only marginally affected by the rates: p_{CO_2} , p_{CO} , p_{NO_x} , p_{SO_2} , p_{dust} on CO_2 , CO , NO_x , SO_2 and particulate matter emissions, respectively—Figs. 2.4, 2.5, 2.6, 2.7 and 2.8.

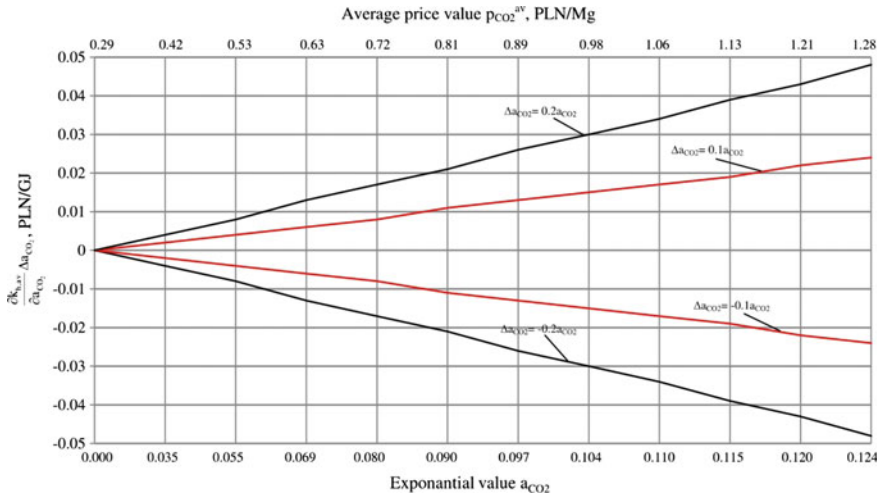


Fig. 2.4 Increase of the value of specific cost of heat production in the function of the level of tariff charges on CO₂ emission $p_{CO_2}(t) = p_{CO_2}^{t=0} e^{a_{CO_2} t}$

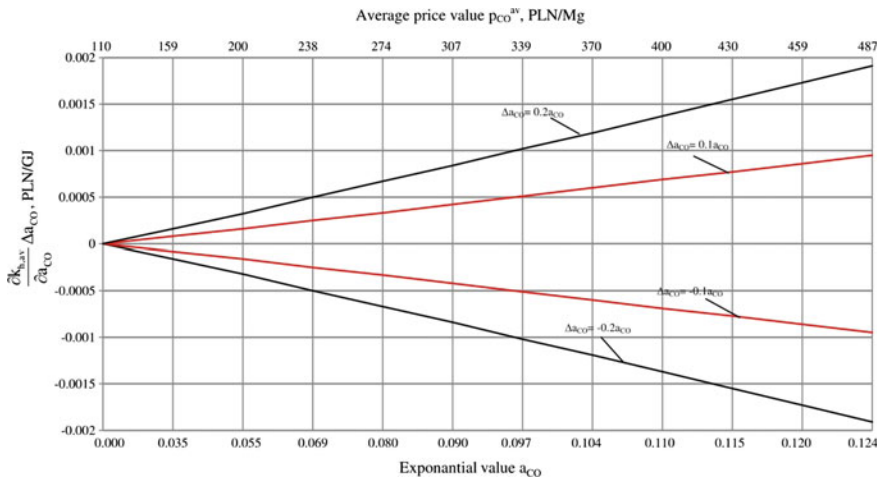


Fig. 2.5 Increase of the value of specific cost of heat production in the function of the level of tariff charges on CO emission $p_{CO}(t) = p_{CO}^{t=0} e^{a_{CO} t}$

The cost $k_{h,av}$ is considerably affected by the price CO₂ emission allowances calculated per ton of its emission—Fig. 2.9. This price is also dependent on politicians.

The specific cost $k_{h,av}$ of heat production is equally significantly affected by the duration of the period t_s [6] when the maximum (rated, peak) production capacity is

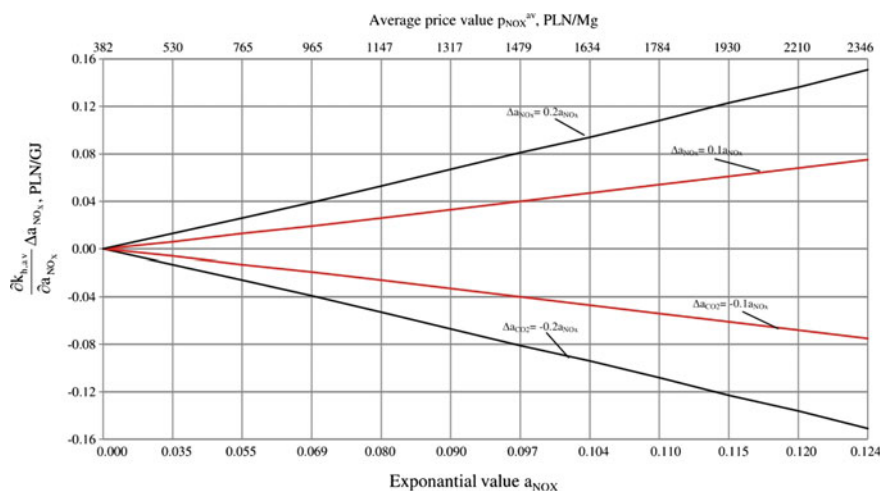


Fig. 2.6 Increase of the value of specific cost of heat production in the function of the level of tariff charges on NO_x emission $p_{\text{NO}_x}(t) = p_{\text{NO}_x}^{t=0} e^{a_{\text{NO}_x} t}$

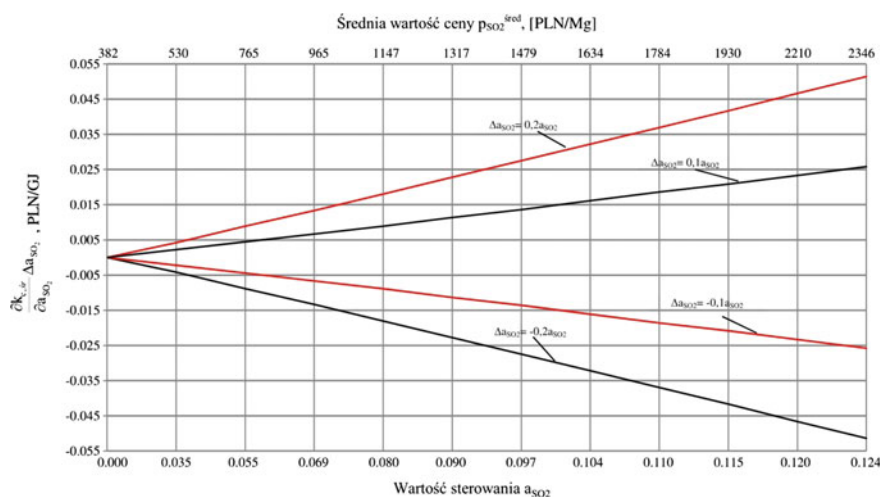


Fig. 2.7 Increase of the value of specific cost of heat production in the function of the level of tariff charges on SO_2 emission $p_{\text{SO}_2}(t) = p_{\text{SO}_2}^{t=0} e^{a_{\text{SO}_2} t}$

reached by the CHP plant—Fig. 2.10. However, this impact of this cost is lesser than the one associated with fuel prices.

The cost $k_{h,av}$ is considerably affected by the specific investment, cogeneration factor related to heat production, energy efficiency of heat production and to a lower extent—by the interest rate on capital investment—Figs. 2.11, 2.12, 2.12 and 2.14.

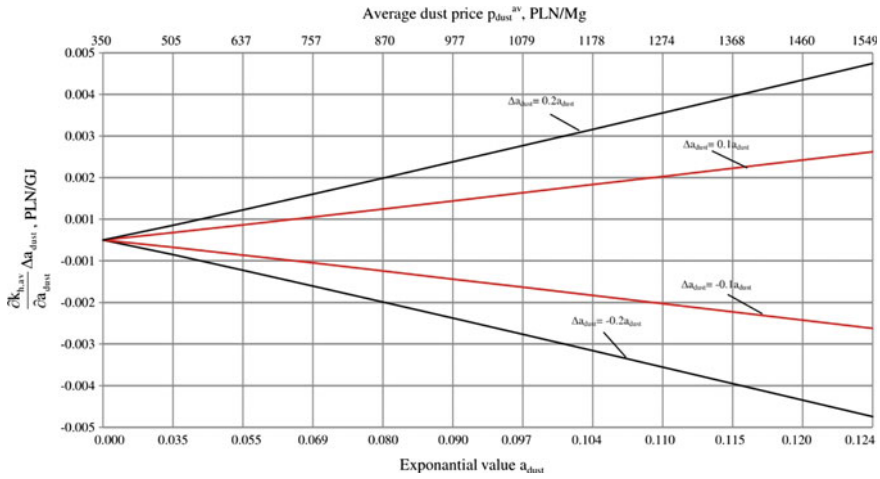


Fig. 2.8 Increase of the value of specific cost of heat production in the function of level of the tariff charge on particulate matter emission $p_{\text{dust}}(t) = p_{\text{dust}}^{t=0} e^{a_{\text{dust}} t}$

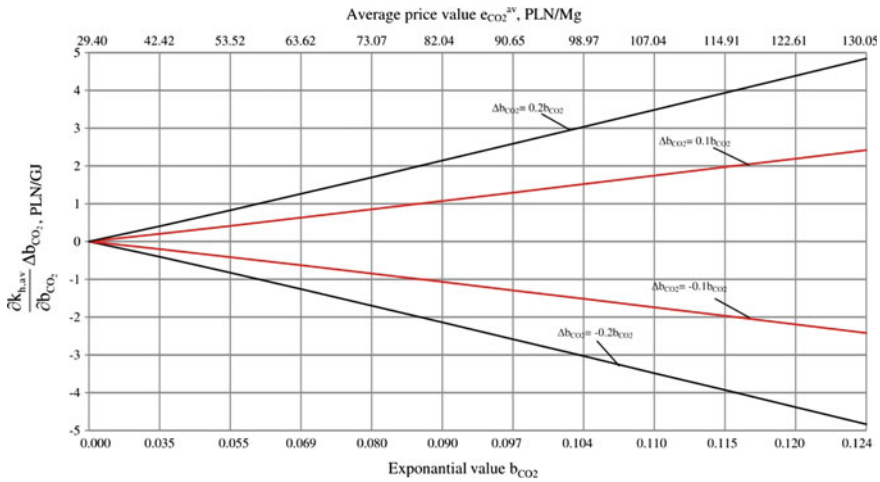


Fig. 2.9 Increase of the value of specific cost of heat production in the function of the purchase price of CO₂ emission allowances $e_{\text{CO}_2}(t) = e_{\text{CO}_2}^{t=0} e^{b_{\text{CO}_2} t}$

The lower the energy efficiency of a CHP plant η_{CHP} , the greater the degree to which its increase leads to the decrease of the specific cost of heat production $k_{h,\text{av}}$. The increase of the energy efficiency of equipment in each technology cannot result in the considerable increase of efficiency η_{CHP} . This is due to the fact that the potential increase of the equipment energy efficiency in the range of a few percentage points does not result in a considerable increase of the value η_{CHP} .

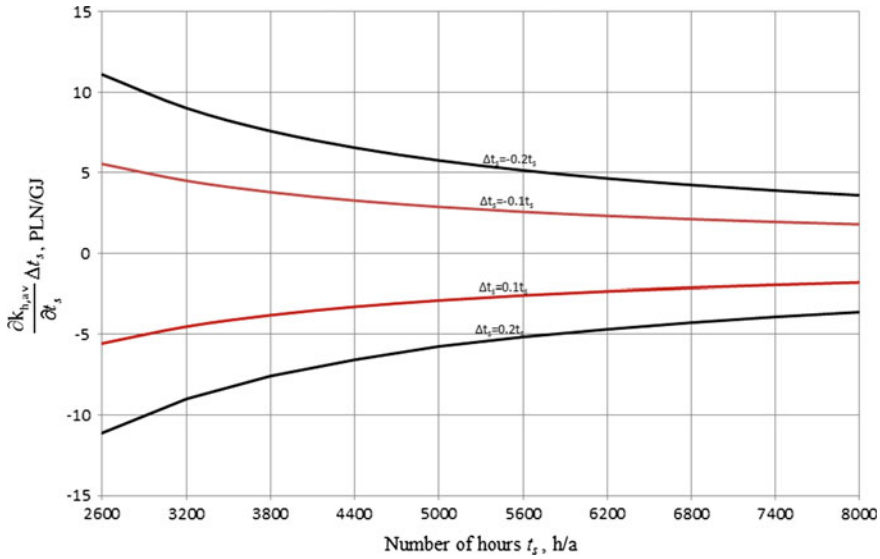


Fig. 2.10 Increase of the value of specific cost of heat production in the function of the number of hours t_s when maximum (peak, rated) thermal capacity is obtained by the CHP plant annually t_s

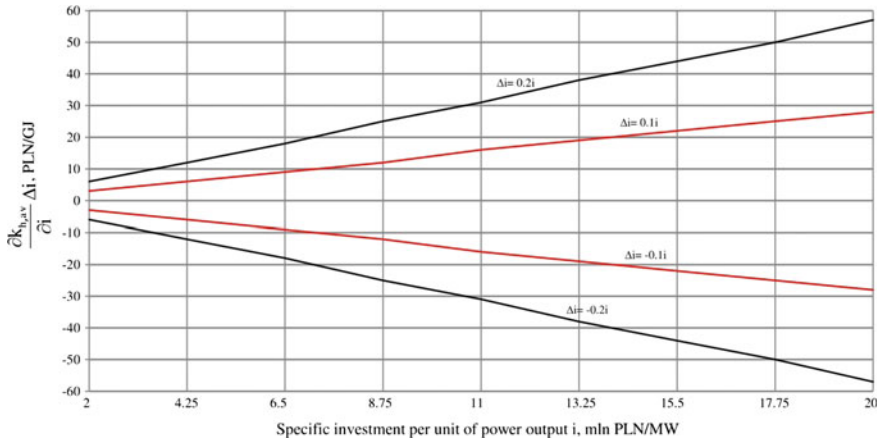


Fig. 2.11 Increase of the value of specific cost of heat production in the function of the specific investment i

An increase of the value of cogeneration factor $\sigma_A = E_{el,A}/Q_A$ has a considerable impact on the decrease of the specific cost $k_{h,av}$. In such a case, the avoided cost of heat production in a CHP plant increases in terms of its absolute value. The increase in the electricity production in a CHP plant is possible as a result of the application of a hierarchical single-fuel gas-steam system [6]. However, in order to

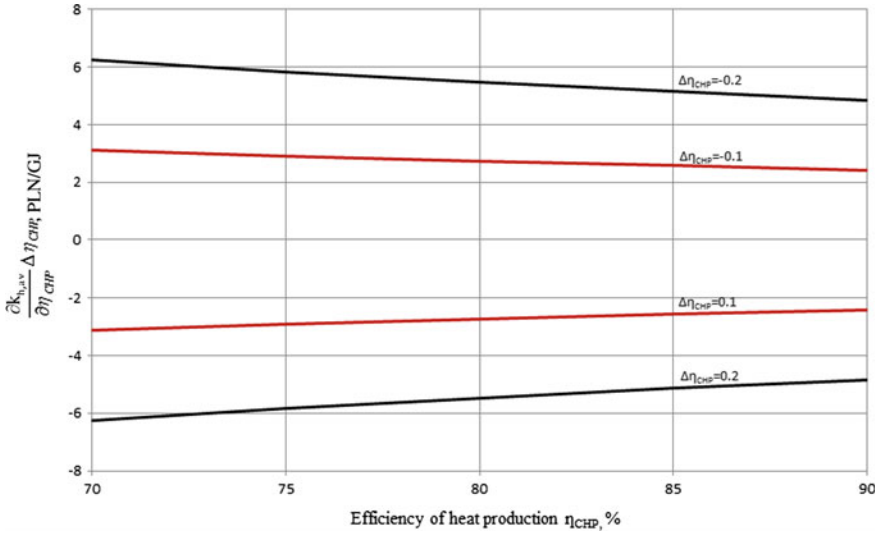


Fig. 2.12 Increase of the value of specific cost of heat production in the function of efficiency of a CHP plant η_{CHP}

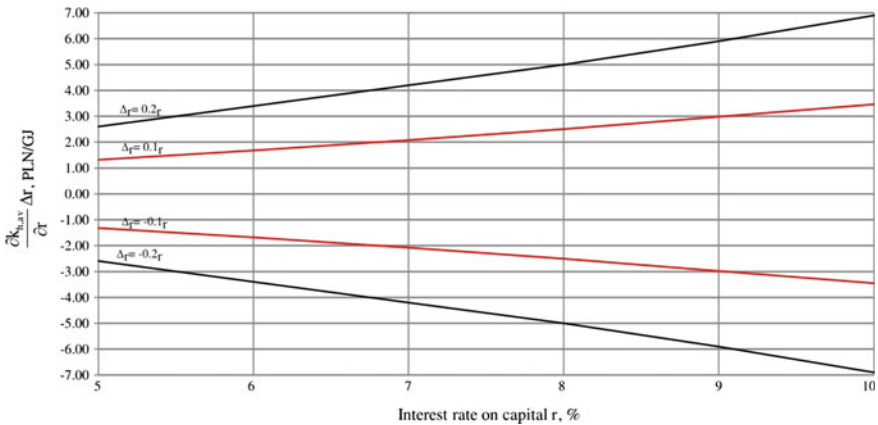


Fig. 2.13 Increase of the value of specific cost of heat production in the function of the interest rate on capital investment r

ensure the economic justification of the gas-steam system, the price of electricity needs to be at an adequately high level in relation to the price of the natural gas, which is used to power a gas turbine [4, 6]. At present, electricity price is too low in relation to the expensive gas and the hierarchical single-fuel and dual-fuel gas-steam systems (In-Series Hot Windbox System or Parallel System [6]) are unprofitable. The profitability can be improved when this relation will be higher. However, this is not all. The profitability can also be boosted by reducing CO_2

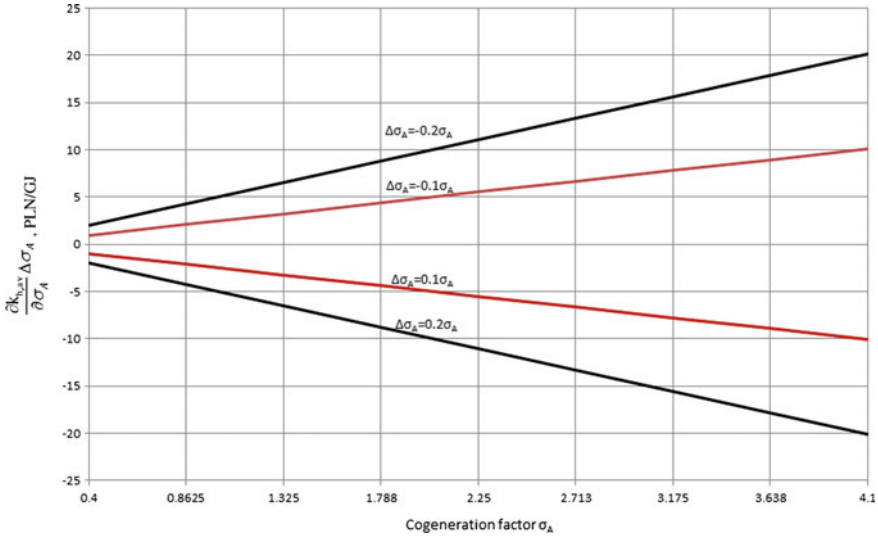


Fig. 2.14 Increase of the value of specific cost of heat production in the function of the cogeneration factor σ_A

emission from a CHP plant, which is possible as a result of gas combustion in it and by increasing the purchase price of emission allowances. Consequently, the increase of this price will lead to a smaller profitability of a CHP plant, in which the relatively cheap coal is applied (which costs 3 times less than natural gas calculated per unit of energy; the price of gas is around 32 PLN/GJ; whereas the price of coal is 10.5 PLN/GJ). CO₂ emission from natural gas combustion is two times smaller than the emission from coal combustion calculated per unit of the chemical energy of the fuel combustion (for hard coal, CO₂ emission is equal to $\rho_{\text{CO}_2}^{\text{coal}} \approx 95 \text{ kg}_{\text{CO}_2}/\text{GJ}$, whereas for natural gas it is $\rho_{\text{CO}_2}^{\text{gas}} \approx 55 \text{ kg}_{\text{CO}_2}/\text{GJ}$). On condition of the same volume of heat production, the cost K_{CO_2} incurred during gas combustion in a CHP plant will then be consequently lower. A question that arises concerns about the degree to which this cost will be lower. This is relative to the purchase price of CO₂ emission allowances. There must be a boundary value of this price, above which for a given electricity and fuel prices, the decrease of CO₂ emission will lead to the economic profitability of the application of the gas-steam systems.

The models developed and presented in this chapter provide a comprehensive analysis of the price relations between fuel prices, electricity price, and purchase price of CO₂ emission allowances, for which it will be profitable to apply a hierarchical single-fuel gas-steam system as well as dual-fuel gas-steam systems [6].

Section 3.3.1 presents the energy balances for the particular technologies of combined heat and electricity production. As a result, we can represent the efficiency η_{CHP} by means of the efficiency of the facilities used in the process in formula (2.28) and, subsequently, analyze their impact on the value $k_{h,\text{av}}$.

2.3 Conclusions

After multi-variant calculations are performed by application of the formula in (2.31), we will be able to elaborate the data by presenting them in a graphical form of nomograms of the dimensionless ratio $k_{h,av}/e_{el}^{t=0}$ with dimensionless measures, $e_{fuel}^{t=0}/e_{el}^{t=0}$, etc., $\rho_{CO_2} p_{CO_2}^{t=0}/e_{el}^{t=0}$, etc., $i/(\tau_s e_{el}^{t=0}) = J/(\dot{Q}_{h,max}^{CHP} \tau_s e_{el}^{t=0})$, etc., as the parameters. As a consequence, we will be able to find an answer to the question regarding the optimum technology of the heat and electricity production in a CHP plant from the economic viewpoint, regardless of its thermal power for the current prices of energy carriers and environmental charges and independently of the forecasts with regard to their variability in time.

Equations (2.24, 2.25, 2.27, 2.28 and 2.31) as well as the equations in Chaps. 3 and 4 do not account for the revenues from the sales of rights exercised on the basis of certificates of origin for electricity coming from high-efficiency cogeneration [see formula (2.12) [1, 7]]. They were introduced as an incentive to support for generation of electricity in cogeneration sources as a consequence of implementing Directive 2004/8/EC of the European Parliament to the national Power Law. The number of certificates is determined by calculating the parameter called *PES* (*Primary Energy Saving*), which accounts for the savings in the use of the primary fuels [1]. The goal of obtaining these certificates is achieved by recording savings in the economy based on cogeneration, which need to be greater by a minimum of 10% compared with the use of fuels for the same volume of heat and electricity production in the separate system. The additional revenue from these certificates was meant to support a theoretical goal of reducing the specific cost of heat $k_{h,av}$. In practice, we can forecast that the reverse will be the case and, therefore, the economic analysis the question of potential subsidies was deliberately avoided. All artificial and politically motivated elements of financial support for cogeneration systems, whose cost is imposed on the tax payers, not politicians, only lead to various degenerations, not to say pathologies, and the cost is ultimately paid by the customers using heat. The subsidies will form the profit of the fuel suppliers, who can then increase their prices (when there are subsidies, why would not they want to gain profit in this way?!), manufacturers of energy equipment, newly appointed clerks employed to divide subsidies and corrupted politicians). Therefore, subsidies aggravate the upward price spiral and lead to heat price increase above the value above the normal level. Besides, the grace of the politicians is a conjunctural matter, as it may occur at one time and vanish later. Besides, cogeneration system, as well as others, should be able to defend their viability in the market economy. Moreover, the elements of financial support create a distorted image of thermodynamic processes and power technologies despite the fact that they should serve the purposes of their rationalization. An example of a highest degree of falsification and even pathology is associated with the promotion and subsidies to renewable energy sources (*RES*), wind turbogenerators, and photovoltaic installations (in Poland

subsidies to RES in the form of green certificates are estimated to cost 78 billion PLN over 15 years from 2006 to 2020; in Germany this cost is 28 billion €/year and will increase over the coming years to 31 billion €/year). The volume of electricity production in them is small due to the short operating times of these installations (the annual duration of work of wind turbines is 1500–2000 h/a in Poland), in particular this issue concerns photovoltaics (with the annual duration of work in the range of 750 h). When this is coupled with the huge investment necessary in such installations, the electricity produced in them is an expensive, even luxury products [8]. We have to agree with the opinions of scientists, including Germans, that energy derived from RES is useless and the use of wind turbines and photovoltaic panels should be abandoned [9]. For instance, whereas in Germany the installed capacity in the wind turbogenerators is 36,000, and 38,000 MW in photovoltaic installations, i.e., their total capacity is almost equal to the one in all German power plants (coal-fired, nuclear and gas-fired), the annual electricity output from them is only equal to few per cent of the annual production. The remaining part (above 90%) has to be produced by the existing power plants. For the reason of the use of *RES*, the power bills of the German customers is almost twice as much as the ones paid by the French, who derived their power from nuclear sources. For this reason, as well as environmental burden from these installations, German experts and scientists demand that wind turbogenerators and photovoltaic cells should be taken away [9].

In summary, the presented methodology and mathematical models applied for the technical and economic analysis in search of an optimum investment strategy in power sources has both cognitive aspects and can extend our knowledge regarding the investment strategies. They can offer grounds for activities aimed at the extensive application of the results.

References

1. Bartnik, R., Buryn, Z.: Conversion of coal-fired power plants to cogeneration and combined-cycle: thermal and economic effectiveness. Springer-Verlag, London (2011)
2. Energy Policy of Poland until 2030. Document adopted by the Council of Ministers on 10 Nov 2009
3. Bartnik, R., Bartnik, B., Hnydiuk-Stefan, A.: Optimum investment strategy in the power industry. Mathematical Models. Wydawnictwo Springer, New York (2016)
4. Bartnik, R., Bartnik, B.: Economic calculus in power industry, (Wydawnictwo Naukowe Techniczne WNT), Warszawa 2014 (in Polish)
5. Ordinance of the Economy Minister of 26 July 2011 on the method of calculation of data specified in the application for a certificate of origin of cogenerated energy and the requirement for obtaining and redeeming such certificates, paying the substitution feed, and the obligation to confirm the data on the volume of electricity produced in high-efficiency cogeneration. Journal of Laws no. 176/2011, item 1052
6. Bartnik, R.: Combined cycle power plants. Thermal and economic effectiveness. WNT, Warszawa 2009 (reprint 2012) (in Polish)

7. Buryn, Z.: Quasi-unsteady CHP operation of power plants. Thermal and economic effectiveness. Springer-Verlag, London (2016)
8. Bartnik, R., Hnydiuk-Stefan, A.: Economic analysis of LCOE calculated for various generation technologies, *Energetyka*, nr 5, 2016 (in Polish)
9. Strupczewski, A.: Both analysis and practice confirm: atom is the best choice for Poland, *PostępyTechnikiJądrowej*, vol. 59, z. 1, Warszawa 2016 (in Polish)

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Mathematical Models

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