

Contents

- 1 A Tutorial Introduction to Mathematica** 1
 - 1.1 A Quick Tour of Mathematica 2
 - 1.2 Tutorial One: The Basics (One Hour) 4
 - 1.3 Tutorial Two: Plots and Differential Equations
(One Hour) 6
 - 1.4 The Manipulate Command and Simple
Mathematica Programs 8
 - 1.5 Hints for Programming 11
 - 1.6 Mathematica Exercises 12
 - Bibliography 15

- 2 Differential Equations** 17
 - 2.1 Simple Differential Equations and Applications 18
 - 2.2 Applications to Chemical Kinetics 27
 - 2.3 Applications to Electric Circuits 31
 - 2.4 Existence and Uniqueness Theorem 37
 - 2.5 Mathematica Commands in Text Format 40
 - 2.6 Exercises 41
 - Bibliography 44

- 3 Planar Systems** 45
 - 3.1 Canonical Forms 46
 - 3.2 Eigenvectors Defining Stable and Unstable Manifolds 51
 - 3.3 Phase Portraits of Linear Systems in the Plane 54
 - 3.4 Linearization and Hartman’s Theorem 58
 - 3.5 Constructing Phase Plane Diagrams 59
 - 3.6 Mathematica Commands 68
 - 3.7 Exercises 69
 - Bibliography 72

4 Interacting Species	73
4.1 Competing Species	73
4.2 Predator-Prey Models	76
4.3 Other Characteristics Affecting Interacting Species	82
4.4 Mathematica Commands	84
4.5 Exercises	84
Bibliography	87
5 Limit Cycles	89
5.1 Historical Background	90
5.2 Existence and Uniqueness of Limit Cycles in the Plane	93
5.3 Nonexistence of Limit Cycles in the Plane	100
5.4 Perturbation Methods	102
5.5 Mathematica Commands	112
5.6 Exercises	113
Bibliography	116
6 Hamiltonian Systems, Lyapunov Functions, and Stability	119
6.1 Hamiltonian Systems in the Plane	119
6.2 Lyapunov Functions and Stability	124
6.3 Mathematica Commands	131
6.4 Exercises	131
Bibliography	133
7 Bifurcation Theory	135
7.1 Bifurcations of Nonlinear Systems in the Plane	136
7.2 Normal Forms	142
7.3 Multistability and Bistability	146
7.4 Mathematica Commands	149
7.5 Exercises	150
Bibliography	153
8 Three-Dimensional Autonomous Systems and Chaos	155
8.1 Linear Systems and Canonical Forms	156
8.2 Nonlinear Systems and Stability	160
8.3 The Rössler System and Chaos	164
8.4 The Lorenz Equations, Chua's Circuit, and the Belousov-Zhabotinski Reaction	169
8.5 Mathematica Commands	176
8.6 Exercises	179
Bibliography	182

9 Poincaré Maps and Nonautonomous Systems	
in the Plane	185
9.1 Poincaré Maps	186
9.2 Hamiltonian Systems with Two Degrees of Freedom	192
9.3 Nonautonomous Systems in the Plane	195
9.4 Mathematica Commands	204
9.5 Exercises	205
Bibliography	208
10 Local and Global Bifurcations	209
10.1 Small-Amplitude Limit Cycle Bifurcations	210
10.2 Gröbner Bases	215
10.3 Melnikov Integrals and Bifurcating Limit Cycles from a Center	221
10.4 Bifurcations Involving Homoclinic Loops	223
10.5 Mathematica Commands	225
10.6 Exercises	227
Bibliography	229
11 The Second Part of Hilbert's Sixteenth Problem	231
11.1 Statement of Problem and Main Results	232
11.2 Poincaré Compactification	235
11.3 Global Results for Liénard Systems	241
11.4 Local Results for Liénard Systems	250
11.5 Mathematica Commands	251
11.6 Exercises	252
Bibliography	254
12 Delay Differential Equations	257
12.1 Introduction and the Method of Steps	258
12.2 Applications in Biology	264
12.3 Applications in Nonlinear Optics	270
12.4 Other Applications	274
12.5 Exercises	278
Bibliography	281
13 Linear Discrete Dynamical Systems	285
13.1 Recurrence Relations	286
13.2 The Leslie Model	291
13.3 Harvesting and Culling Policies	296
13.4 Mathematica Commands	300
13.5 Exercises	300
Bibliography	304

14 Nonlinear Discrete Dynamical Systems	305
14.1 The Tent Map and Graphical Iterations	306
14.2 Fixed Points and Periodic Orbits	312
14.3 The Logistic Map, Bifurcation Diagram, and Feigenbaum Number	318
14.4 Gaussian and Hénon Maps	325
14.5 Applications	330
14.6 Mathematica Commands	332
14.7 Exercises	334
Bibliography	337
15 Complex Iterative Maps	339
15.1 Julia Sets and the Mandelbrot Set	340
15.2 Boundaries of Periodic Orbits	344
15.3 The Newton Fractal	348
15.4 Mathematica Commands	349
15.5 Exercises	350
Bibliography	351
16 Electromagnetic Waves and Optical Resonators	353
16.1 Maxwell's Equations and Electromagnetic Waves	354
16.2 Historical Background	356
16.3 The Nonlinear SFR Resonator	361
16.4 Chaotic Attractors and Bistability	363
16.5 Linear Stability Analysis	366
16.6 Instabilities and Bistability	370
16.7 Mathematica Commands	375
16.8 Exercises	376
Bibliography	379
17 Fractals and Multifractals	381
17.1 Construction of Simple Examples	382
17.2 Calculating Fractal Dimensions	389
17.3 A Multifractal Formalism	395
17.4 Multifractals in the Real World and Some Simple Examples	400
17.5 Mathematica Commands	408
17.6 Exercises	410
Bibliography	413
18 Image Processing and Analysis with Mathematica	417
18.1 Image Processing and Matrices	418
18.2 The Fast Fourier Transform	421
18.3 The Fast Fourier Transform on Images	425
18.4 Exercises	426
Bibliography	428

19 Chaos Control and Synchronization 429

19.1 Historical Background 430

19.2 Controlling Chaos in the Logistic Map 434

19.3 Controlling Chaos in the Hénon Map 438

19.4 Chaos Synchronization 442

19.5 Mathematica Commands 447

19.6 Exercises 448

Bibliography 450

20 Neural Networks 453

20.1 Introduction 454

20.2 The Delta Learning Rule and Backpropagation 460

20.3 The Hopfield Network and Lyapunov Stability 465

20.4 Neurodynamics 474

20.5 Mathematica Commands 478

20.6 Exercises 480

Bibliography 483

21 Binary Oscillator Computing 485

21.1 Brain Inspired Computing 485

21.2 Oscillatory Threshold Logic 491

21.3 Applications and Future Work 496

21.4 Mathematica Commands 501

21.5 Exercises 503

Bibliography 505

22 An Introduction to Wolfram SystemModeler 509

22.1 Introduction 510

22.2 Electric Circuits 513

22.3 A Mechanical System 515

22.4 Causal (Block Based) Modeling 517

22.5 Exercises 520

Bibliography 522

23 Coursework and Examination-Type Questions 523

23.1 Examples of Coursework Questions 524

23.2 Examination 1 533

23.3 Examination 2 537

23.4 Examination 3 541

24 Solutions to Exercises 547

24.1 Chapter 1 547

24.2 Chapter 2 548

24.3 Chapter 3 549

24.4 Chapter 4 551

24.5 Chapter 5 553

24.6 Chapter 6 555

24.7 Chapter 7 556

24.8 Chapter 8 558

24.9 Chapter 9 559

24.10 Chapter 10..... 560

24.11 Chapter 11..... 560

24.12 Chapter 12..... 562

24.13 Chapter 13..... 562

24.14 Chapter 14..... 564

24.15 Chapter 15..... 566

24.16 Chapter 16..... 567

24.17 Chapter 17..... 567

24.18 Chapter 18..... 568

24.19 Chapter 19..... 568

24.20 Chapter 20..... 569

24.21 Chapter 21..... 569

24.22 Chapter 22..... 570

24.23 Chapter 23..... 571

Index 575

Dynamical Systems with Applications Using
Mathematica®

Lynch, S.

2017, XVI, 585 p. 270 illus., 98 illus. in color., Hardcover

ISBN: 978-3-319-61484-7

A product of Birkhäuser Basel