Non-dominated Sorting and Crowding Distance Based Multi-objective Chaotic Evolution

Yan Pei[1](✉) and Jia Hao[2]

[1] Computer Science Division, University of Aizu,
Aizu-wakamatsu, Fukushima 965-8580, Japan
peiyan@u-aizu.ac.jp

[2] School of Mechanical Engineering, Beijing Institute of Technology,
Beijing 100081, China
haojia632@bit.edu.cn
http://web-ext.u-aizu.ac.jp/~peiyan/

Abstract. We propose a new evolutionary multi-objective optimization (EMO) algorithm based on chaotic evolution optimization framework, which is called as multi-objective chaotic evolution (MOCE). It extends the optimization application of chaotic evolution algorithm to multi-objective optimization field. The non-dominated sorting and tournament selection using crowding distance are two techniques to ensure Pareto dominance and solution diversity in EMO algorithm. However, the search capability of multi-objective optimization algorithm is a serious issue for its practical application. Chaotic evolution algorithm presents a strong search capability for single objective optimization due to the ergodicity of chaotic system. Proposed algorithm is a promising multi-objective optimization algorithm that composes a search algorithm with strong search capability, dominant sort for keeping Pareto dominance, and tournament selection using crowding distance for increasing the solution diversity. We evaluate our proposed MOCE by comparing with NSGA-II and an algorithm using the basic framework of chaotic evolution but different mutation strategy. From the evaluation results, the MOCE presents a strong optimization performance for multi-objective optimization problems, especially in the condition of higher dimensional problems. We also analyse, discuss, and present some research subjects, open topics, and future works on the MOCE.

Keywords: Chaotic evolution · Multi-objective chaotic evolution · Evolutionary multi-objective optimization · Chaos theory · Chaotic optimization

1 Introduction

In the conventional mathematical optimization field, the optimization problems are categorized into continuous optimization and discrete optimization according to the domain of optimized variables. The canonical form of objective function for continuous optimization is defined as a single objective function with

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some equality or inequality constraint functions. The mathematical programming methods, such as Karush-Kuhn-Tucker conditions [3], are efficient algorithms that can solve this set of problems when the objective functions and constraints have some perfect characteristics, such as continuous, differentiable, etc. However, most of the real world problem's objectives are not singleton. They have multiple objectives, i.e., multiple objective (multi-objective) functions. If the objectives are more than four, they also are referred as to many objectives problem [2]. This leads us to think whether the single objective problem or singleton concept only exist in our human brain. The multi-objective optimization problem is formally defined as in Eq. (1), where it defines a m ($m \geq 2$) objectives optimization function, and the optimized variable is vector $\mathbf{x}$ in a search space Ω.

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_m(\mathbf{x})) \\ \text{subject to} \quad & \mathbf{x} \in \Omega. \end{aligned} \tag{1}$$

Evolutionary multi-objective optimization (EMO) algorithms are efficient and effective to handle the multi-objective optimization problem. From the methodological viewpoint, the class of EMO algorithms belongs to stochastic optimization, whose theoretical explanation lies on probability theory. The EMO keeps the multiple objective functions independently and uses Pareto-based ranking schemes to maintain the feasible solution, rather than the determinative programming methods use the scalarization method that needs to transfer multiple objectives into one objective. Most of the state-of-the-art studies in EMO concentrate on Pareto dominance handling in objective space, which ties to generate solutions to approximate the Pareto frontiers [4]. However, the main disadvantage of EMO are its worse optimization capability and non guarantee of Pareto optimality, which cannot be perfectly solved by Pareto dominance studies in EMO.

In this paper, we propose a non-dominated sorting and tournament selection using crowding distance based chaotic evolution (CE) algorithm for the multi-objective optimization problem. This work extends the fundamental optimization framework of CE in the EMO field. The non-dominated sorting method handles the Pareto dominance issue, and tries to keep the non-dominated solution into the next generation, and tournament selection using crowding distance technique maintains the diversity of solution. The CE algorithm implements the search function in optimization process. We do not only handle the Pareto dominance and solution diversity issues, but also compose an efficient optimization algorithm in proposed algorithm. Because the CE has better optimization capability in a variety of problems, the proposed non-dominated sorting and tournament selection using crowding distance based CE can therefore appeal its optimization capability in multi-objective optimization problem. It presents one of originalities and contributions in this work.

Following this introductory section, we briefly make an overview of the CE and NSGA-II in Sect. 2. In Sect. 3, we present the proposed multi-objective CE algorithm and its framework. We explain the optimization principle of the proposal and its optimization framework. The evaluation and discussion of the proposed method are presented in Sects. 4 and 5. We report the performance of proposed algorithm, and address how and why the proposed algorithm has a better optimization performance by comparing with NSGA-II. Finally, we conclude the whole work, and discuss the future works, open topics in Sect. 6.

2 An Overview on Chaotic Evolution and NSGA-II

2.1 Chaotic Evolution

The fundamental of chaotic evolution (CE) simulates the chaotic motion when generating new search points, which is a critical implementation determining the exploration and exploitation capabilities [5]. For example, if we have k individuals (vectors) in the $n-1th$ generation, we can directly generate k mutant vectors in nth generation using different chaotic systems. This idea is motivated by some natural phenomenons that can be well modeled and explained with chaotic systems while deterministic or stochastic methods fail. Therefore, we believe that the evolution of a set of individuals can be modeled by chaotic systems. The feasibility and performance of CE are guaranteed by the ergodicity property of chaotic systems, which means iterative variables of the systems can approach to any location in a search space with an arbitrary accuracy [6].

2.2 NSGA-II

Non-dominant sorting genetic algorithm II (NSGA-II) was firstly introduced in reference [1], and it is an improved version of NSGA [7]. NSGA is one of the EMO algorithms, which have the capability of finding multiple Pareto optimal solutions with a single simulation running [1]. Although NSGA contributed to the EMO community, it also suffered from several criticisms, including high computational cost, lack of elitism and requirement for the setting of sharing parameter. The NSGA-II succeed in solving all the above three issues at once by introducing fast non-dominated sorting and tournament selection using crowding distance.

3 Multi-objective Chaotic Evolution Using Non-dominated Sorting and Tournament Selection with Crowding Distance

There are two research issues and subjects in Pareto-based EMO. The one is Pareto dominance problem, most of studies try to define new dominant concepts to apply in the selection and survive processing for maintaining performance of Pareto dominance. The other is diversity issue that EMO should support a variety of Pareto solutions to be selected. This research issue also leads to another EMO related subject that how to select one Pareto solution from Pareto frontier for a real world application in practice. The canonical approaches to cope with these two issues are non-dominated sorting and tournament selection using crowding distance, respectively. However, the optimization performance of EMO does not only lie on the solutions of Pareto dominance and solution diversity, but also depends on the optimization capability of search algorithm. The design of EMO algorithm should consider three aspects of issue, i.e., optimization capability of search algorithm, Pareto dominance, and solution diversity. This paper contributes a new EMO algorithm that implements a better optimization capability in EMO, i.e., multi-objective chaotic evolution (MOCE).

The proposed MOCE composes three design elements in its optimization framework, i.e., optimization capability of search algorithm, Pareto dominance, and solution diversity. CE algorithm implements the basic search function that ensures a strong optimization capability, non-dominant sorting solves the Pareto dominance issue, and tournament selection using crowing distance keeps the solution diversity when selecting the dominant solutions. We can design a variety of MOCE algorithms by implementing these three techniques. From the single objective optimization results of CE, the basic performance of search and optimization can be maintained [5]. The non-dominant sorting and crowding distance selection are two canonical methods that can solve the issues of Pareto dominance and solution diversity [1]. The proposed non-dominant sorting and tournament selection using crowding distance based MOCE, therefore, be considered as an efficient and effective implementation of EMO algorithm. It is one of the implementations of MOCE algorithms as well.

Table 1. Abbreviations of algorithms in evaluation.

Abbreviations	Meaning
Multi-CE	MOCE based on non-dominated sort and crowding distance tournament selection
NSGA-II	non-dominant sort genetic algorithm
Multi-Rand	MOCE implemented by a uniform distribution parameter generator based on non-dominated sort and crowding distance tournament selection

4 Numerical Evaluations

We use five multi-objective benchmark problems to evaluate our proposed algorithm. The abbreviations of investigated algorithms are in Table 1. The population size of all evaluated algorithms is set to 100, we evaluate each algorithm using 1000 generations. 30 running trails are conducted to be applied in statistical test. For each running trail, each evaluated algorithm is set with the same initialization for a comparison. The benchmark problems [9], dimensional setting, search range, and Pareto frontier are listed in Table 2. The NSGA-II is also applied to these problems for a comparison. The crossover rate and mutation

Table 2. Multi-objective benchmark function used in evaluation. All of the Pareto frontier are $g(x) = 1$.

Func	Dim	Definition	Search Range
ZDT1	30	$f_1(x) = x_1$	$x_i \in [0, 1]$
		$f_2(x) = g(x)[1 - \sqrt{\frac{x_1}{g(x)}}]$	
		$g(x) = 1 + 9\frac{\sum_{i=2}^{n} x_i}{n-1}$	
ZDT2	30	$f_1(x) = x_1$	$x_i \in [0, 1]$
		$f_2(x) = g(x)[1 - (\frac{x_1}{g(x)})^2]$	
		$g(x) = 1 + 9\frac{\sum_{i=2}^{n} x_i}{n-1}$	
ZDT3	30	$f_1(x) = x_1$	$x_i \in [0, 1]$
		$f_2(x) = g(x)[1 - \sqrt{\frac{x_1}{g(x)}} - \frac{x_1}{g(x)} \sin(10\pi x_1)]$	
		$g(x) = 1 + 9\frac{\sum_{i=2}^{n} x_i}{n-1}$	
ZDT4	10	$f_1(x) = x_1$	$x_1 \in [0, 1]$, $x_i \in [-5, 5]$, $i = 2, 3, ..., 10$
		$f_2(x) = g(x)[1 - \sqrt{\frac{x_1}{g(x)}}]$	
		$g(x) = 1 + 10(n-1) + \sum_(i = 2)^n [x_i^2 - 10\cos(4\pi x_i)]$	
ZDT6	10	$f_1(x) = 1 - \exp(-4\pi x_1)\sin^6(6\pi x_1)$	$x_i \in [0, 1]$
		$f_2(x) = g(x)[1 - (\frac{x_1}{g(x)})^2]$	
		$g(x) = 1 + 9[\frac{\sum_{i=2}^{n} x_i}{n-1}]^{0.25}$	

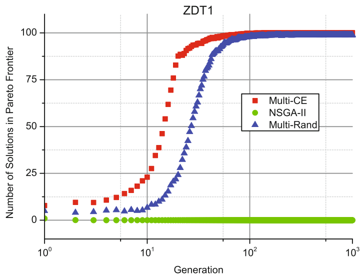
rate of NSGA-II are set to 0.9 and 0.1, respectively. The chaotic system used in CE is the logistic map (Eq. (2)) with the $\mu = 4$, and the initial number of that is $random[0, 1]$ not equal to $\{0, 0.25, 0.5, 0.75, 1\}$. We also implement a random parameter based search algorithm that uses a uniform distribution parameter generator in CE algorithm to investigate the performance of chaotic parameter implementation with a comparison. Concretely, we use $CP_{i,j} = rand(0, 1)$ to replace $CP_{i,j} = ChaoticSystem(CP_{i,j})$ in CE algorithm to implement this compared algorithm. Figure 1 shows the average number of Pareto frontier solution of 30 running trails. Based on these evaluation results, we apply some statistical tests on these data, and discuss and analyse proposed MOCE algorithm.

$$x(i + 1) = \mu x(i)(1 - x(i)) \tag{2}$$

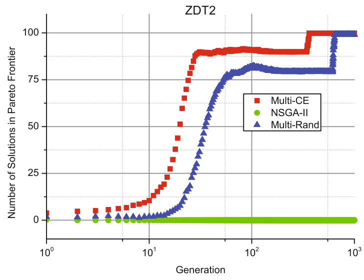
5 Discussions and Analyses

5.1 Discussion on the Number of Pareto Frontier Solution

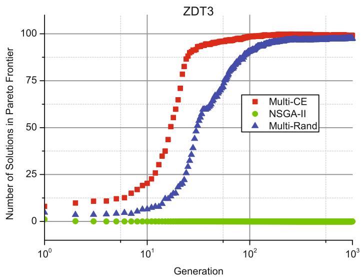
Pareto dominance and solution diversity are two evaluation metrics to compare EMO algorithms. Because most EMO algorithms use Pareto-based ranking



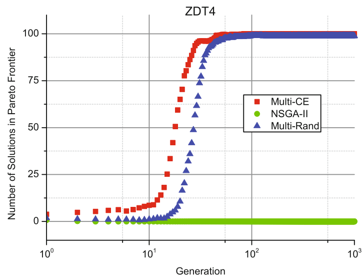
(a). ZDT1



(b). ZDT2



(c). ZDT3



(d). ZDT4

Fig. 1. The average number of Pareto frontier solution in each generation, the Wilcoxon signed rank test results present that there is a significant difference between Multi-CE and Multi-Rand.

schemes, the number and the distribution of solutions in Pareto frontier can present the optimization performance of EMO algorithms. From the Fig. 1, we can conclude that the number of Pareto frontier of Multi-CE and Multi-Rand are more than that of NSGA-II. We collect and calculate the number of Pareto frontier solution in 500 generation from 30 running trails, and apply a Friedman test to rank these numbers, and a Bonferroni-Dunn tests in the significant level of $\alpha = 0.05$ for a statistical evaluation. Figure 2 is a result of critical difference calculation. From Fig. 2, we observe that there is a significant difference between NSGA-II and Multi-CE/Multi-Rand in the aspect of the number of Pareto frontier solution, i.e., the number of Pareto frontier solution from our proposed Multi-CE is significantly more than that from NSGA-II.

5.2 Discussion on Comparison of Chaotic and Random Generators

The difference of Multi-CE and Multi-Rand lies on the mutation generator. For comparing generators from a chaotic system and a uniform distribution random system, we use two generators to conduct a comparison study. One is from a chaotic system (the logistic map in this paper), the other is from a uniform distribution. Whether the optimization performance is significantly influenced by

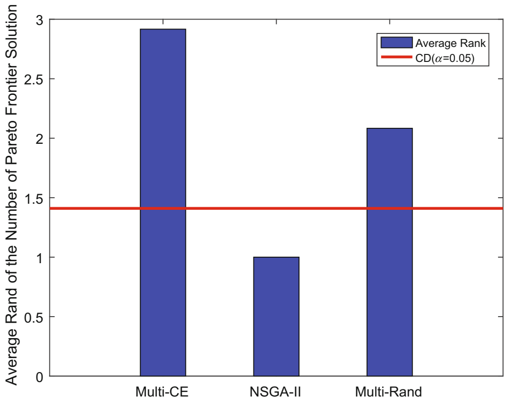


Fig. 2. Bonferroni-Dunn test in the significant level of $\alpha = 0.05$, we observe that there is a significant difference of the number of Pareto frontier solution between NSGA-II and Multi-CE/Multi-Rand. Critical difference (CD) used in Bonferroni-Dunn test is $CD = q_{\alpha}\sqrt{\frac{k(k+1)}{6*N}}$, and $k = 3$ and $N = 5$, q is equal to $q_{\alpha}(0.05) = 2.242$ from Appendix Table B.16 of [8].

mutation generator? We investigate this issue using a statistical test. The number of Pareto frontier solution in each generation in Multi-CE and Multi-Rand is pairwise related, and we do not know the normality of the data distribution, so we apply a Wilcoxon signed-rank test for five benchmark functions with the number of Pareto frontier solution in 500th generation. The five p-values from the tests are all less than 0.05, and we check Fig. 1, which indicates that the number of Pareto frontier solution from Multi-CE is significantly more than that from Multi-Rand, i.e., the optimization performance is significantly influenced by the mutation generator, and chaotic generator presents a better optimization performance than that of the random generator.

The primary difference of chaotic generator and random generator lies on the distribution of the generated numbers from the statistical viewpoint. The random generator has the uniform distribution of generated numbers among the whole range of (0,1). It have been investigated that the distribution characteristic of the logistic map is that most generated numbers are in the ranges of (0,0.1) and (0.9,1) [6], which presents the exploitation and exploration search capabilities of CE algorithm. The above analysis proves that this characteristic also influences the optimization results of the MOCE. This is one of discoveries in this work.

6 Conclusion

In this paper, we proposed a new EMO algorithm using the optimization framework of CE and two canonical techniques from NSGA-II. The proposed algorithm uses non-dominated sort to keep the Pareto dominance, applies the tournament selection using crowding distance to increasing solution diversity, and composes the strong optimization capability of chaotic evolution due to the ergodicity of chaotic system. The proposed MOCE is a promising multi-objective optimization because of the three technique aspects of the algorithm. From the evaluation results, it indicates that the proposed MOCE has a stronger optimization performance than that of the NSGA-II. It is useful in both lower dimensional problems and higher dimensional problems. Especially for the higher dimensional problems, it significantly outperforms NSGA-II. We compared the optimization performance of proposed MOCE and an algorithm using the framework of CE and a uniform distribution mutation generator. The results present the advantage of chaotic generator, it indicates that chaotic system and theory have the promising potential to be applied in optimization algorithms due to characteristic of ergodicity.

The Pareto dominant and solution diversity issues are two research subjects in EMO field. In the future, we will implement MOCE algorithm using other techniques to solve these two problems. The chaotic generator presents an advantage for evolution optimization. Why the chaotic generator can results in the better optimization performance but the uniform random distribution generator cannot, we need to investigate this study subject in both aspects of theory and application. These two research subjects will be involved in our future work. Another research subject is the application of the proposed algorithm. We will also apply it in a variety of applications to benefit our society.

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