

Chapter 2

Tools

Baryonic matter in the Universe, apart from extremely rare exceptions like planets and us, is in the *plasma state*,¹ not necessarily but possibly magnetized. The good and fascinating news is that the laws governing plasma physics are *scale invariant* (see e.g. Goedbloed and Poedts 2004). Plasma physics developed, understood and experimented with in laboratories, is thus an essential tool for Astrophysicists and Cosmologists. It is particularly essential for me, because I am directly interested in the question of the origin and evolution of cosmological magnetic fields. But also, generally speaking, it is often very fruitful to learn from another field what the relevant methods are. In Sect. 7.2 and beyond, I will give an example of this fact, as I will explore gravitational fragmentation, without considering magnetic fields (yet), in the lines of studies performed in the plasma literature. But finally, I must add that in fact plasma physics is worth studying for its own intrinsic beauty. More precisely, my personal interest in it comes from the fact that it is a field of physics which is both very intuitive, because we are familiar, to some extent, with the quantities involved, and yet, the more we study plasmas, the more we discover that they may be extremely surprising and rich of subtleties. I also enjoy the fact that we can (in general) visualize the phenomena at play, and make the link between the equations and what we see, which is far less evident in fields like quantum and particle physics for instance.

Let us then consider that baryonic matter in the Universe is a plasma, which is largely driven by *gravity*, and in particular by its own gravitation. Therefore, to understand the Universe, it is crucial to master as well as possible the formal tools to describe self-gravitating plasmas, i.e. magnetic fields *and* gravity. To this day, the most general theoretical framework to describe gravity, and thus to model the Universe, is that of General Relativity. However, for the questions I will address

¹Throughout my manuscript, I take the liberty to use the term plasma loosely to designate interchangeably ionized and neutral fluids except, of course, in cases where the ionization state is essential.

in this manuscript, the relevant range of parameters is such that my study does not require this general framework. Indeed, although the distances considered here are essentially cosmological, typically of the order of one Mpc, they remain small compared to the curvature of the Universe. Even to consider the expansion of the Universe, a Newtonian approach, complemented with the use of the scale factor and comoving coordinates, is perfectly relevant. Also, densities are small enough to only weakly curved space-time (no black hole physics for instance) and velocities are small compared to the speed of light so that we neither need to consider gravitational waves nor any special relativistic effect (as opposed to studies of AGN jets for instance). This study will thus be conducted using Newtonian dynamics.

Which approach are we going to adopt here? Due to their intrinsic limitations, numerical simulations are not able to capture fully the breadth of time and length scales involved in structure formation, especially in diluted, numerically under-sampled regions of space. The analytical approach, adopted here, is crucial for understanding fully the underlying physics, and is complementary to numerical simulations. Finally, just like the personal reason why I am focusing on plasma physics evoked above, choosing the analytical approach is also worth for the sheer pleasure of it.

2.1 Electromagnetism

Any vector field, that vanishes suitably quickly at infinity, is entirely determined by its divergence and curl.² The divergence and curl of the electric and magnetic fields are determined in a coupled manner, constituting Maxwell's equations for electromagnetism. In Gaussian (CGS) units, they read

$$\left\{ \begin{array}{ll} \vec{\nabla} \times \vec{E} = -\frac{1}{c} \partial_t \vec{B} & \text{(Maxwell-Faraday)} \\ \vec{\nabla} \times \vec{B} = \frac{1}{c} \partial_t \vec{E} + \frac{4\pi}{c} \vec{J} & \text{(Maxwell-Ampère)} \\ \vec{\nabla} \cdot \vec{E} = 4\pi \rho_q & \text{(Maxwell-Gauss)} \\ \vec{\nabla} \cdot \vec{B} = 0 & \text{(No Magnetic Monopoles)} \end{array} \right. \quad (2.1)$$

while in SI units, they read

$$\left\{ \begin{array}{ll} \vec{\nabla} \times \vec{E} = -\partial_t \vec{B} & \text{(Maxwell-Faraday)} \\ \vec{\nabla} \times \vec{B} = \frac{1}{c^2} \partial_t \vec{E} + \mu_0 \vec{J} & \text{(Maxwell-Ampère)} \\ \vec{\nabla} \cdot \vec{E} = \frac{\rho_q}{\epsilon_0} & \text{(Maxwell-Gauss)} \\ \vec{\nabla} \cdot \vec{B} = 0 & \text{(No Magnetic Monopoles)} \end{array} \right. \quad (2.2)$$

²For additional details, see for instance the following discussion by Kirk T. McDonald on the Helmholtz decomposition: <http://puhep1.princeton.edu/~kirkmcd/examples/helmholtz.pdf>.

where \vec{J} is the total current density and ρ_q the total charge density. The fundamental constants μ_0 , ϵ_0 and c are linked by $\mu_0\epsilon_0 = \frac{1}{c^2}$. Note that my main plasma physics reference during the first part of my Ph.D. has been Krall and Trivelpiece (1973) who privilege Gaussian units, while later it has been (Goedbloed and Poedts, 2004) and (Goedbloed et al., 2010) who work in SI units. This is the reason why my work on magnetogenesis, presented in part I, is formulated in Gaussian units, while the part mentioning magnetic fields in my work on gravitational fragmentation, in part II, is formulated in SI units. This should not be a difficulty for the reader since the equations are the same, up to multiplicative constants. As a reminder, as far as charge density, electric and magnetic fields are concerned we have

$$\rho_{q,\text{cgs}} = \frac{\rho_{q,\text{SI}}}{\sqrt{4\pi\epsilon_0}}, \quad \vec{E}_{\text{cgs}} = \sqrt{4\pi\epsilon_0}\vec{E}_{\text{SI}}, \quad \vec{B}_{\text{cgs}} = \sqrt{\frac{4\pi}{\mu_0}}\vec{B}_{\text{SI}}. \quad (2.3)$$

The four Eqs. (2.1) or (2.2) do not have the same nature: The two relations governing the curl of \vec{E} and \vec{B} are dynamical, corresponding to evolution equations, while those on the divergences should be seen as initial conditions. Indeed, taking the divergence of the Maxwell-Ampère and Maxwell-Faraday equations, together with the local charge conservation equation $\vec{\nabla} \cdot \vec{J} + \partial_t \rho_q = 0$, gives (in SI units)

$$\begin{cases} \partial_t \left(\vec{\nabla} \cdot \vec{E} - \frac{\rho_q}{\epsilon_0} \right) = 0, \\ \partial_t \left(\vec{\nabla} \cdot \vec{B} \right) = 0. \end{cases} \quad (2.4)$$

Hence, due to charge conservation, if the Maxwell-Gauss equation is satisfied initially, then it remains so during the whole evolution, and similarly for $\vec{\nabla} \cdot \vec{B} = 0$. In that sense they constitute initial conditions for the evolution equations Maxwell-Faraday and Maxwell-Ampère.

In the non relativistic limit, displacement currents, corresponding to the term $c^{-1}\partial_t \vec{E}$, are negligible, so that in this manuscript we will neglect this term. The resulting set of equations is usually called ‘Pre-Maxwell equations’, because historically the displacement current term was introduced by J. C. Maxwell to ensure local charge conservation.

While the electromagnetic field may propagate in vacuum ($\rho_q = 0$ and $\vec{J} = \vec{0}$), we are interested in electric and magnetic fields evolving with matter. The quantities ρ_q and \vec{J} acting as sources in the above Maxwell equations are themselves governed by fluid equations in which \vec{E} and \vec{B} intervene. All these equations together constitute the MHD equations, describing the complex intertwining of matter and electromagnetic fields, and that we shall now have a look at.

2.2 Magneto-Fluid Dynamics

I want to introduce the ideal MHD equations from a rather fundamental description, that of kinetic theory, because it will be the starting point of part I, but also because it is intellectually satisfying to have an idea of their fundamental origin rather than simply admitting them. However, the derivation presented below is a straight-to-the-point one. I omit a certain number of details that are, in my opinion, very important to be clear about the meaning and validity of the equations that we are dealing with, but that are unnecessary to expose here. For a precise discussion of the present implicit averages and unmentioned assumptions see for instance the very good Chaps. 2 and 3 of Krall and Trivelpiece (1973).

From orbit, to kinetic, to fluid Consider a collection of particles of various species, hereafter tagged by a symbol α , and characterized by their charge q_α and mass m_α . They are for example electrons or protons. The equation of motion of each of these charged, non-relativistic particles evolving in an electric field \vec{E} , magnetic field \vec{B} and gravitational potential ϕ , is given by Newton's second law (Gaussian units)

$$\frac{d\vec{v}}{dt} = \frac{q_\alpha}{m_\alpha} \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) - \vec{\nabla}\phi. \quad (2.5)$$

This many-body description, in which the motion of every single particle is taken into account, is in general not tractable. However, in the vast majority of situations, it is in fact not necessary for answering our questions. Indeed, because we are interested in systems with extremely large numbers of particles, say of the order of the Avogadro number, a statistical description of the system, in terms of macroscopic variables (like density, temperature, pressure, etc.), is perfectly relevant and sufficient. It is however good to keep in mind that by our choice of description, we are leaving behind some information, so that some plasma properties and phenomena are absent in the formalism we will adopt here.

The study of the trajectory of a single isolated charged particle is called *orbit theory* and is well understood. The difficulty comes from the fact that a plasma is a collection of a large number of *interacting* particles. Collective effects are well described statistically. The point is to partition the system into volumes that are large enough to neglect statistical fluctuations due to the discreteness of the particles they contain and treat the medium as a continuum, but small enough to use differential calculus and talk about fluid elements. In *kinetic theory*, the information on both the (probable) number of particles and their velocity distribution is retained by working with the distribution function $f_\alpha(t, \vec{r}, \vec{v})$. By definition, the probable number of particles of type α at position \vec{r} with velocity \vec{v} in the volume element $d^3\vec{r}d^3\vec{v}$ is equal to $f_\alpha(t, \vec{r}, \vec{v})d^3\vec{r}d^3\vec{v}$. Note that volume elements in this description are six-dimensional since points are described in the six-dimensional space (\vec{r}, \vec{v}) called phase space. Liouville's theorem states that in the absence of binary interactions between particles, density in phase space is constant in time ($\frac{df_\alpha}{dt} = 0$). Now collisions, for instance,

modify the distribution function because it is a process that changes the velocity of particles. The evolution of f_α is governed by the following equation

$$\frac{df_\alpha}{dt} = \partial_t f_\alpha|_s, \quad (2.6)$$

which we will refer to as the Boltzmann equation. On the right hand side, the source term is usually the term modeling collisions, but this term corresponds to any process which sources the distribution function. For example in Chap. 3, we will model photoionization processes as a source term in this equation since it also modifies the velocity of particles, and in fact modifies the number of particles too. Now, in the six-dimensional phase space, by definition of the total time derivative, we have

$$\frac{df_\alpha(t, \vec{r}, \vec{v})}{dt} \equiv \partial_t f_\alpha + \frac{d\vec{r}}{dt} \cdot \frac{\partial f_\alpha}{\partial \vec{r}} + \frac{d\vec{v}}{dt} \cdot \frac{\partial f_\alpha}{\partial \vec{v}} \quad (2.7)$$

so that, using Newton's second law (2.5) for each species α , the Boltzmann equation may be explicitated as

$$\partial_t f_\alpha + \vec{v} \cdot \frac{\partial f_\alpha}{\partial \vec{r}} + \left[\frac{q_\alpha}{m_\alpha} \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) - \vec{\nabla} \phi \right] \cdot \frac{\partial f_\alpha}{\partial \vec{v}} = \partial_t f_\alpha|_s. \quad (2.8)$$

This equation contains a lot of information, and often too much for our purposes. The *fluid description* consists in leaving behind the information about the whole distribution of velocities, by averaging, inside each volume element, on the velocity variable. This is called the fluid reduction, because we are reducing the amount of information carried in the equations we are manipulating.

Now, since we are dealing with a system containing various species α , we may reduce the kinetic description to a fluid one, for *each* of these species. Doing so consists in working with

$$\begin{cases} n_\alpha = \int f_\alpha d^3 \vec{v} \\ \vec{V}_\alpha = \frac{1}{n_\alpha} \int \vec{v} f_\alpha d^3 \vec{v} \\ P_\alpha = m_\alpha \int (\vec{V}_\alpha - \vec{v}) (\vec{V}_\alpha - \vec{v}) f_\alpha d^3 \vec{v} \end{cases} \quad (2.9)$$

which are respectively the number density, the velocity and the pressure tensor of species α . These are called macroscopic quantities, because we are now only considering the averaged velocity \vec{V}_α inside each volume element rather than the microscopic details of the distribution of velocities carried by the full distribution function. To derive the equations governing these quantities, one has to evaluate the various

moments (i.e. first multiply the equation by powers of \vec{v} and then integrate over the entire velocity space) of the Boltzmann equation. This means evaluating

$$\int g_i \left[\frac{df_\alpha}{dt} - \partial_t f_{\alpha|s} \right] d^3\vec{v} = 0 \quad (2.10)$$

where for instance the first three moments are

$$\begin{aligned} g_0 &= 1 && 0^{\text{th}} \text{ moment: mass conservation} \\ g_1 &= m_\alpha \vec{v} && 1^{\text{st}} \text{ moment: momentum conservation} \\ g_2 &= \frac{1}{2} m_\alpha v^2 && 2^{\text{nd}} \text{ moment: energy conservation} \end{aligned} \quad (2.11)$$

and yield respectively the mass, the momentum and the energy conservation equations of species α . The system is then described as multiple interacting fluids. This description is thus called the *multi-fluid description*.

Keeping track of the individual properties of each species is not always necessary, and despite the existence of multiple components, the description is often further reduced to a *mono-fluid* description. This consists in working in the center-of-mass with the quantities

$$\begin{cases} \rho = \Sigma_\alpha n_\alpha m_\alpha \\ \vec{V} = \frac{1}{\rho} \Sigma_\alpha n_\alpha m_\alpha \vec{V}_\alpha \\ P = \Sigma_\alpha m_\alpha \int (\vec{V} - \vec{v}) (\vec{V} - \vec{v}) f_\alpha d^3\vec{v} \end{cases} \quad (2.12)$$

being respectively the mass density, the center-of-mass velocity and the total center-of-mass pressure tensor in the one-fluid. With the above information, one may write the fluid equations in full generality. However, as far as the fluid equations are concerned, in part II we will not manipulate them in full generality, as exposed in Krall and Trivelpiece (1973) for instance, but in the ideal MHD limit. For example, pressure is described by the above tensor, but when viscosity is small, as we shall assume in this manuscript, it becomes diagonal and proportional to the scalar pressure: $P = pI$ where I is the identity tensor. The starting point in part I however, will be the very general Boltzmann equation (2.8).

The zeroth moment gives the mass conservation equation

$$\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = 0. \quad (2.13)$$

As its name suggests, this equation simply states that mass is conserved: In a given volume element, if the amount of matter varies ($\partial_t \rho$), it necessarily comes from the imbalance of the incoming and outgoing matter (formally: the divergence operator) from neighboring volume elements (there is no source term here).

The first moment gives the following momentum conservation

$$\rho \frac{d\vec{v}}{dt} = \rho \left(\partial_t \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v} \right) = -\vec{\nabla} p + \vec{j} \times \vec{B} - \rho \vec{\nabla} \phi. \quad (2.14)$$

This corresponds to Newton's second law for a fluid element of the mono-fluid. It stems from Newton's law on single particles (2.5), but is fundamentally different from it: We are now considering fluid elements, so that quantities are per unit volume (mass and current densities), the concept of pressure arises due to the collection of particles, and also the acceleration is now either Lagrangian ($\frac{d\vec{v}}{dt}$, evaluated while moving with the fluid) or Eulerian ($\partial_t \vec{v}$, evaluated at a fixed position). By order of appearance, the terms on the far right hand side correspond to the force (density) due to pressure gradients, to the Lorentz force and finally to the gravitational force.

Ohm's law and induction equation From these equations, we may derive an extremely important relation, namely the equation governing the current density in the plasma, called Ohm's law. I defer its presentation in full generality to Sect. 3.2.2, where it will be at the center of the discussion. For now, let us admit here its simplest form (cf. e.g. Shu 1992)

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}), \quad (2.15)$$

where σ is the conductivity. Plugging it into Ampère's law, we obtain the equation governing the evolution of the magnetic field, called the induction equation

$$\partial_t \vec{B} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \eta \Delta \vec{B}. \quad (2.16)$$

The first term is the convective term, resulting from the interaction between the fluid and the magnetic field, and $\eta = (\sigma \mu_0)^{-1}$ is the magnetic diffusivity, assumed to be a constant. How efficient is magnetic diffusion in the cosmological context? Consider the diffusive limit, in which the convective term is negligible. Then \vec{B} obeys a diffusion equation. In terms of orders of magnitude it reads $\frac{1}{t_D} \sim \frac{\eta}{L^2}$, where t_D is a characteristic timescale and L a characteristic length scale of this diffusion. Since cosmic magnetic fields are spread on the largest scales of the Universe, we can try and estimate how much time it would take a magnetic field created at some point to reach such scales by simple diffusion. Taking for L the Hubble radius $\frac{c}{H_0} \sim 4 \times 10^{18}$ m and characterizing the intergalactic medium by a typical magnetic diffusivity of $\eta \sim 10^{-6} \Omega.m$, we obtain a diffusion time of the order

$$t_D \sim \frac{L^2}{\eta} \sim 10^{31} \text{ years}, \quad (2.17)$$

which is much more than the age of the Universe of $t_H \sim 10^{10}$ years. This means that due to the high conductivity (small η) of the intergalactic medium and the large scales involved, once a magnetic field is created somewhere, it does not diffuse. We say that it is 'frozen' into matter. Therefore, in the rest of this manuscript, we will not take this diffusion term into account.

So how does \vec{B} evolve with the expansion of the Universe? Let us look at a simple example: Consider a sphere of plasma of radius r undergoing a uniform and isotropic contraction. Since the \vec{B} field is frozen into matter, by mass conservation

in this volume and flux conservation through its surface, we have that ρr^3 and Br^2 are constant (cf. e.g. Kulsrud 2005). Thus

$$\frac{B}{\rho^{\frac{2}{3}}} = \text{constant}, \quad (2.18)$$

and in the Standard Model of Cosmology $\rho \propto a^{-3}$ (cf. Chap. 1) so that

$$B \propto a^{-2} \quad (2.19)$$

that is $B \propto (1+z)^2$. This evolution of \bar{B} is called the adiabatic dilution and is valid only for the largest scales since it is derived in the FLRW homogeneous and isotropic framework.

Higher moments: A need for Closure The zeroth moment of the Boltzmann equation (2.8) yields a relation (mass conservation) between the zeroth moment of the distribution function (density) and the first moment (velocity). The first moment of the Boltzmann equation yields a relation (momentum conservation) between the zeroth, the first, and the second moment (pressure) of the distribution function. The second moment of the Boltzmann equation yields a relation (energy conservation) between the zeroth, first, second and third moment (heat flux) of the distribution function. The pattern that emerges turns out to be general: The equation governing the n th moment always contains the $(n+1)$ th moment too. This is problematic because it means that the resulting system of equations is never closed this way. Everytime we add an equation, we add a new variable. Hence, rather than pursuing taking higher and higher moments, the usual procedure consists in stopping at the equation on the second moment, because only the three first moments have a simple physical interpretation, and to close the system with an additional equation, other than the following moment, dictated by physical arguments. This additional equation is called a *closure relation*. The choice of this relation strongly impacts the relevance and domain of validity of the model. In that sense, there are as many fluid models as closure relations. In this manuscript we will consider a classical model, namely that of polytropic fluids, which is interesting for its large domain of validity. Let us now see where it comes from.

2.3 Thermodynamics

In this manuscript, the choice of closure relations is as follows. In the most general case, pressure and density are related by an equation of state, corresponding to a relation of the form $p = p(\rho, s)$ or $p = p(\rho, T)$, where s and T are respectively the specific entropy (entropy per unit mass) and the temperature. Now, let us consider that baryons form a fluid that is an *ideal gas*, i.e. such that

$$p = \frac{\rho k_B T}{m} \quad (2.20)$$

where m is the mass of a single particle, T is the temperature and k_B is Boltzmann's constant. A first simple case corresponds to that of an isothermal fluid, for which temperature is uniform. Then

$$p = \kappa \rho \quad (2.21)$$

where $\kappa = \frac{k_B T}{m}$ is spatially constant. More generally, one can show (cf. Appendix F.2 of Binney and Tremaine 2008) that for an ideal gas the specific entropy s is linked to the number q of internal degrees of freedom of the particles by the relation

$$s = \frac{k_B}{m} \ln \left(\frac{T^{(q+3)/2}}{\rho} \right) + \text{constant}. \quad (2.22)$$

This relation brings in another interesting special case, namely the case in which the entropy is uniform (isentropic fluid). Then

$$\rho \propto T^{(q+3)/2} = T^{\frac{1}{\gamma-1}} \quad (2.23)$$

where

$$\gamma = \frac{q+5}{q+3} \quad (2.24)$$

is called the polytropic exponent. Combining this with (2.20), we obtain the following *polytropic* equation of state

$$p = \kappa \rho^\gamma \quad (2.25)$$

where κ is a constant that depends on the specific entropy. We see from (2.21) that the isothermal equation of state corresponds to that of a polytrope with $\gamma = 1$.

Note that in these cases the equation of state is of the form $p = p(\rho)$. A fluid having this property is said to be *barotropic*. In fact, for a non magnetized fluid to be at rest in a gravitational field (which will be the case of study in Chap. 6), it must necessarily be barotropic. Indeed, the hydrostatic equilibrium, marked with the subscripts 0, is then given by Eq. (2.14) with vanishing velocity and magnetic field, that is

$$\vec{\nabla} p_0 + \rho_0 \vec{\nabla} \phi_0 = \vec{0}. \quad (2.26)$$

Taking the curl of this relation gives $\vec{\nabla} \times (\rho_0^{-1} \vec{\nabla} p_0) = \vec{0}$ and thus at every position we have

$$\vec{\nabla} \rho_0 \times \vec{\nabla} p_0 = \vec{0}. \quad (2.27)$$

This means that the gradient of density and the gradient of pressure are aligned everywhere, which implies that surfaces of constant density need to coincide with

surfaces of constant pressure for a static solution to exist. Therefore, an unmagnetized fluid at rest in a gravitational field necessarily satisfies $p_0 = p_0(\rho_0)$, i.e. is barotropic.

It is thus particularly natural to consider a polytropic equation of state for the equilibrium. However the choice of equation of state, used as closure relation here, for the *out of equilibrium* fluid, is not evident. As we will see in Sect. 9.2, when studying the evolution of perturbations in a fluid, depending on the timescales of evolution of the perturbations, there may or may not be time for heat transfer to happen. The relevant closure relation for the perturbed fluid may then differ from that of the equilibrium fluid. This difference gives rise to buoyancy, and thus to g-modes (using the stellar physics terminology) and convection. This stresses the importance of the choice of closure. Except in Sect. 9.2 devoted to it, we will in this manuscript deliberately switch-off convection by considering Eq. (2.25), both for the equilibrium state and for out of equilibrium perturbations.

2.4 Gravitation

As far as gravity is concerned, in the Cosmology and Astrophysics literature, equilibrium states are generally discussed in terms of gravitational potentials (Φ) rather than in terms of gravitational accelerations (\vec{g}). We usually say that ‘baryons fall in the potential wells induced by Dark Matter’ for instance. However, as we will discuss in Sect. 7.1.3, in this manuscript I will describe perturbations in terms of forces, rather than in terms of energies and potentials. In that sense \vec{g} will turn out to be a more natural variable to discuss perturbations. Hence, I will here use both Φ and \vec{g} , though in essence both descriptions contain the same information, since one is (minus) the gradient of the other

$$\vec{g} = -\vec{\nabla}\Phi. \quad (2.28)$$

Note that because of this definition, the vector field \vec{g} is irrotational

$$\vec{\nabla} \times \vec{g} = \vec{0}. \quad (2.29)$$

As we will see in Sect. 8.2, the linearized version of this constraint will be a key ingredient in our study. The gravitational acceleration \vec{g} is governed by

$$\vec{\nabla} \cdot \vec{g} = -4\pi G\rho \quad (2.30)$$

so that, with definition (2.28), the gravitational potential Φ is governed by

$$\Delta\Phi = 4\pi G\rho \quad (2.31)$$

where G is Newton’s constant. These equations are called *Poisson equation*, respectively for the gravitational acceleration and for the gravitational potential. Physically, the form of Eq. (2.31) is very meaningful. Indeed, Newtonian gravity corresponds

to a double limit of Einstein's theory of General Relativity: the weak field and non-relativistic limits. More precisely (cf. e.g. Barrau and Grain 2011), linearizing Einstein's field equations around a flat space-time (weak field) results in a wave equation, sourced by the energy content of the Universe.³ This propagation of space-time perturbations corresponds to gravitational waves, which travel at the speed of light because it is governed by the d'Alembert operator $c^{-2}\partial_t^2 - \Delta$. Then, when taking the non-relativistic limit, corresponding formally to an infinite speed of light, the d'Alembert operator makes way to the Laplace operator that appears in (2.31). In other words, Eq. (2.31) states that gravity is instantaneous in the Newtonian regime considered here. Finally, note that it is common to find in the literature an opposite choice of sign in the definition of ϕ and \vec{g} and thus in Poisson equations (e.g. Goldreich and Lynden-Bell 1965; İbanoğlu 2000). The convention-independent quantity is the sign of the gravitational force term in the momentum conservation.

2.5 Self-gravitating Magnetized Structures

Gathering the material introduced above, we may state that the set of equations governing the dynamics of a self-gravitating ideal polytropic magnetized fluid reads

$$\left\{ \begin{array}{ll} \partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = 0 & \text{(Mass conservation)} \\ \rho \left(\partial_t \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v} \right) = -\vec{\nabla} p + \vec{j} \times \vec{B} - \rho \vec{\nabla} \phi & \text{(Momentum Conservation)} \\ \vec{j} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} & \text{(Maxwell-Ampère)} \\ p = \kappa \rho^\gamma & \text{(Polytrope)} \\ \partial_t \vec{B} = \vec{\nabla} \times (\vec{v} \times \vec{B}) & \text{(Induction Equation)} \\ \Delta \phi = 4\pi G \rho & \text{(Poisson Equation)} \end{array} \right. \quad (2.32)$$

This is our starting point, from which we are going to do the two following things.

First, there is one other absolutely crucial ingredient for the Astrophysical and Cosmological context that I have not mentioned so far: *Radiation*. Radiation is of course essential to probe the Universe because it is the main element we can directly collect and analyze, but it also plays, in many situations, an important dynamical role. The most evident ones are radiation pressure, and heating or cooling, by evacuating energy through radiative processes. But in the first part of this manuscript, I will reveal a more subtle role radiation may play: It can generate magnetic fields! Formally speaking, I will expose how a radiation field may modify the induction equation in the system (2.32) above, by acting as a source term in the Vlasov equation (2.8) and thus as a source term in the induction equation.

³The right hand side of (2.31) is the 'residue' of this source term once the non-relativistic limit is taken in addition.

Second, in the Universe everything moves, rotates, merges, accretes, etc. Nothing is at rest. Therefore, structures are permanently subject to perturbations which either make them *oscillate* or, under some circumstances, expose them to *instabilities*. Apprehending precisely how, when and where instabilities may occur is a key to understand the shaping of the Universe. The second part of this manuscript is thus dedicated to studying how sensitive structures are to the perturbations they are subject to, i.e. how instabilities may develop. Formally speaking, we will linearize the set of Eqs. (2.32) and perform a normal mode analysis, in the light of the so-called spectral theory.

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