# Preface

Numerical dynamics is concerned with how well a numerical scheme applied to a differential equation replicates the dynamical behaviour of the dynamical system generated by the differential equation, in particular its long-term or asymptotic behaviour. This essentially involves the comparison of the dynamical behaviour of a continuous-time dynamical system with that of a corresponding discrete-time dynamical system. There are two broad classes of systems of particular interest: dissipative systems, which have an attractor, and non-dissipative systems, such as Hamiltonian systems, which preserve some structural feature or quantity.

This work focusses on the preservation of attractors and saddle points of ordinary differential equations under discretisation. Key results for autonomous ODEs were obtained in the 1980s by Beyn for saddle points and Kloeden and Lorenz for attractors. One-step numerical schemes with a constant step size were considered, so the resulting discrete-time dynamical system was also autonomous. Autonomous dynamical systems with a saddle point may not be dissipative, but the results are nevertheless relevant for dissipative systems as they apply to what may happen inside an attractor.

The theory of non-autonomous dynamical systems has undergone intensive development during the past 20 years, with the introduction of two kinds of non-autonomous attractor: pullback and forward attractors. In principle, a non-autonomous dynamical system can vary quite arbitrarily in time, but to obtain approximation results some sort of uniformity is required. One of the main aims of this book is to present new results on the discretisation of dissipative non-autonomous dynamical systems that have been obtained in recent years, in particular work on the properties of non-autonomous omega limit sets and their approximations by numerical schemes.

These results are also of interest for autonomous dynamical systems that are approximated by a numerical scheme with variable time steps, and thus by a discrete-time non-autonomous dynamical system.

The emphasis here is on the finite-dimensional case, i.e. on ordinary differential equations. Some similar results are known for the infinite dimensional case, e.g., systems generated by partial differential equations, but this case requires more sophisticated technical tools.

The “autonomous” part of this book is based on lecture notes given over many years by the second author in Frankurt am Main and later in Wuhan. The “non-autonomous” part is much more recent and is based on papers published in various mathematical journals.

Auburn, Wuhan Xiaoying Han
May 2017 Peter Kloeden

Springer

http://www.springer.com/978-3-319-61933-0

Attractors Under Discretisation

Han, X.; Kloeden, P.

2017, XI, 122 p. 23 illus., 14 illus. in color., Softcover

ISBN: 978-3-319-61933-0