

Preface

Mathematical models of the most physical phenomena are governed by initial and boundary value problems for partial differential equations (PDEs). Inverse problems governed by these equations arise naturally in almost all branches of science and engineering. The main objective of this textbook is to introduce students and researchers to inverse problems arising in PDEs. This book presents a systematic exposition of the main ideas and methods in treating inverse problems for PDEs arising in basic mathematical models, though we make no claim to cover all of the topics. More detailed additional information related to each chapter can be found in the following books/lecture notes/monographs of Aster, Borchers, and Thurber [2], Bal [6], Baunmeister [8], Beilina and Klibanov [9], Belishev and Blagovestchenskii [12], Chadan and Sabatier [17], Colton and Kress [19], Engl, Hanke, and Neubauer [23], Groetsch [31], Hào [36], Hofmann [43], Isakov [45, 46], Itou and Jin [48], Kabanikhin [50], Kaipio and E. Somersalo [51], Kirsch [54], Lavrentiev [58], Lavrentiev, Romanov, and Shishatski [59], Lavrentiev, Romanov, and Vasiliev [60], Lebedev, Vorovich, and Gladwell [61], Louis [63], Morozov [68], Nakamura and Potthast [72], Natterer [74], Ramm [84], Romanov [85, 86], Romanov and Kabanikhin [87], Schuster, Kaltenbacher, Hofmann, and Kazimierski [90], Tarantola [92], Tikhonov and Arsenin [97], Tikhonov, Concharsky, Stepanov, and Yagola [98], Vogel [102].

In Introduction, we discuss the nature of ill-posedness in differential and integral equations based on well-known mathematical models. Further, we pursue an in-depth analysis of a reason of ill-posedness of an inverse problem governed by integral operator. We tried to answer the question “why this problem is ill-posed?”, by arriving to the physical meaning of the mathematical model, on one hand, and then explaining this in terms of compact operators, on the other hand. Main notions and tools, including best approximation, Moore–Penrose (generalized) inverse, singular value decomposition, regularization strategy, Tikhonov regularization for linear inverse problems, and Morozov’s discrepancy principle, are given in Chap. 1. In Chap. 2, we tried to illustrate an implementation of all these notions and tools to inverse source problems with final overdetermination for evolution equations and to the backward problem for the parabolic equation, including some numerical

reconstructions. The reason for the choice of this problem stems from the fact that historically, one of the first successful applications of inverse problems was Tikhonov's work [93] on inverse problem with final overdetermination for heat equation.

The second part of the book consists of almost independent six chapters. The choice of these chapters is motivated by the fact that the inverse coefficient and source problems considered here are based on the basic and commonly mathematical models governed by PDEs. These chapters describe not only these inverse problems, but also main inversion methods and techniques. Since the most distinctive features of any inverse problem related to PDEs are hidden in the properties of corresponding direct problems solutions, special attention is paid to the investigation of these properties.

Chapter 3 deals with some inverse problems related to the second-order hyperbolic equations. Starting with the simplest inverse source problems for the wave equation with the separated right-hand side containing spatial or time-dependent unknown source functions, we use the reflection method to demonstrate a method for finding the unknown functions based on integral equations. The next and more complex problem here is the problem of recovering the potential in the string equation. The direct problem is stated for semi-infinite string with homogeneous initial data and non-homogeneous Dirichlet data at the both ends. The Neumann output is used as an additional data for recovering an unknown potential. Using the method the successive approximations for obtained system of integral equations, we prove a local solvability for the considered inverse problem. Note that the typical situation for nonlinear inverse problems is that the only local solvability can be proved (see [88]). Nevertheless, for the considered inverse problem, the uniqueness and global stability estimates of solutions are derived. As an application, inverse coefficient problems for layered media are studied in the final part of Chap. 3.

Chapter 4 deals with inverse problems for the electrodynamic equations. These problems are motivated by geophysical applications. In the considered physical model, we assume that the space \mathbb{R}^3 is divided in the two half-spaces $\mathbb{R}^3_- =: \{x \in \mathbb{R}^3 | x_3 < 0\}$ and $\mathbb{R}^3_+ =: \{x \in \mathbb{R}^3 | x_3 > 0\}$. The domain \mathbb{R}^3_- is filled by homogeneous non-conductive medium (air), while the domain \mathbb{R}^3_+ contains a ground which is a non-homogeneous medium with variable permittivity, permeability, and conductivity depending on the variable x_3 only. The tangential components of the electromagnetic field are assumed to be continuous across the interface $x_3 = 0$. The direct problem for electrodynamic equations with zero initial data and a dipole current applied at the interface is stated. The output data here is a tangential component of the electrical field given on the interface as a function of time $t > 0$. A method for reconstruction of one of the unknown coefficients (permittivity, permeability, or conductivity) is proposed when two others are given. This method is based on the Riemannian invariants and integral equations which lead to a well-convergent successive approximations. For the solutions of these inverse problems, stability estimates are stated.

Coefficient inverse problems for parabolic equations are studied in Chap. 5. First of all, the relationship between the solutions of direct problems corresponding to the parabolic and hyperbolic equations is derived. This relationship allows to show the similarity between the outputs corresponding inverse problems for parabolic and hyperbolic equations. Since this relationship is a special Laplace transform, it is invertible. Then it is shown that inverse problems for parabolic equations can be reduced to corresponding problems for hyperbolic equations, studied in Chap. 3. This, in particular, allows to use uniqueness theorems obtained for hyperbolic inverse problem. Further it is shown that the inverse problem of recovering the potential is closely related to the well-known inverse spectral problem for the Sturm–Liouville operator.

Chapter 6 deals with inverse problems for the elliptic equations. Here the inverse scattering problem for stationary Schrodinger equation is considered. For the sake of simplicity, we study this problem in Born approximation using the scattering amplitude for recovering a potential, following the approach proposed by R. Novikov [66]. Moreover, we study also the inverse problem which output is a measured value on a closed surface S of a trace of the solution of the problem for the Schrodinger equation with point sources located at the same surface. The later problem is reduced to the X-ray tomography problem. In the final part of this chapter, we define the Dirichlet-to-Neumann operator which originally has been introduced by J. Sylvester and G. Uhlmann [86]. Using this operator, we study the inverse problem of recovering the potential for an elliptic equation in the Born approximation and reduce it to the moment problem which has a unique solution.

In Chap. 7, inverse problems related to the transport equation without scattering are studied. We derive here the stability estimate for the solution of X-ray tomography problem and then inversion formula.

The inverse kinematic problem is studied in the final Chap. 8. On one hand, this is a problem of reconstructing the wave speed inside a domain from given travel times between arbitrary boundary points. On the other hand, this is also the problem of recovering conformal Riemannian metric via Riemannian distances between boundary points, which plays very important role in geophysics. It suffices to recall that much of our knowledge about the internal structure of the Earth are based on solutions of this problem. We derive first the equations for finding rays and fronts going from a point source. Then we consider one-dimensional and two-dimensional inverse problems and derive a stability estimates for the solutions of these problems.

For the convenience of the reader, a short review related to invertibility of linear, in particular compact, operators is given in Appendix A. Some necessary energy estimates for a weak and regular weak solutions of a one-dimensional parabolic equation are given in Appendix B.

The presentation of the material, especially Introduction and Part I, is self-contained and is intended to be accessible also to undergraduate and beginning graduate students, whose mathematical background includes only basic courses in advanced calculus, PDEs, and functional analysis. The book can be used as a

backbone for a lecture on inverse and ill-posed problems for partial differential equations.

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We would be most grateful if you could send your suggestions and list of mistakes to the email addresses: alemdar.hasanoglu@gmail.com; romanov0511@gmail.com.

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