

Preface

The subject of harmonic analysis dates to the days of Bernoulli and Euler, but it was first formalized by Joseph Fourier in the early nineteenth century. In the intervening 200 years we have seen the subject grow in many directions. The Fourier transform, Fourier analysis on locally compact abelian groups, group representations, Fourier integral operators, and many other fields find their genesis in the early work of Fourier.

It is safe to say that harmonic analysts are always seeking new venues in which to ply their craft. One of the newest of these is the context of the function theory of several complex variables. There were a few rather tentative early papers of Alberto Calderón and Antoni Zygmund in the 1950s. Explorations of nonlocal solvability of partial differential equations and of subelliptic estimates for the $\bar{\partial}$ -Neumann problem and of properties of plurisubharmonic functions certainly anticipated some of the key ideas that would become more explicit later. Papers of Lewy, Kohn, and Hörmander describe some of these results.

One of the notable developments in the modern theory is the work of Korányi in 1969. He showed that the study of the boundary behavior of holomorphic functions will be dramatically different in the several complex variable context (as opposed to the one complex variable context). Shortly thereafter, in 1972, Elias M. Stein wrote his seminal tract on the boundary behavior of holomorphic functions. This was the first coherent treatment of several aspects of the harmonic analysis of several complex variables—including maximal functions, area functions, the boundary behavior of Hardy space functions, and other topics. Stein introduced a number of important and original techniques that are still used today.

One of the notable features of Stein's book mentioned in the last paragraph is its terseness. It is *very hard work* to read that book, and to fill in all the details. In the meantime, there have been many research papers about the Bergman and Szegő kernels, area integrals, boundary behavior of holomorphic and meromorphic functions, singular integrals, and many other aspects of harmonic analysis in the several complex variables context. There have been some books on the Bergman theory, at least one in the several complex variable arena; but there has been no attempt to address the harmonic analysis of several complex variables taken as a coherent whole.

That is the purpose of the current book. Assuming a very basic background in the complex analysis of one and several complex variables, together with the basics of real and functional analysis, we develop the harmonic analysis of several complex variables from first principles. Every effort has been made to render this book as self-contained as possible. On the one hand, the Bibliography is encyclopedic. On the other hand, we do not want the reader to be frequently rushing off to the library to seek additional information and background.

This book is written with the student in mind. There are many examples, copious explanations, and exercise sets at the end of each chapter. Each chapter begins with a Prologue, to introduce the reader to the subject matter that is about to be presented. Each section begins with a Capsule, to give a spirited launch to that unit. Each major result (theorem or proposition) is preceded by a Prelude to help put the idea in context. The text is also sprinkled with Exercises for the Reader; these provide encouragement for the neophyte to pick up his/her pencil and get to work. Finally, we indulge in the conceit of repeating frequently used material, just to make many chapters more self-contained.

This author has been involved in the development of the harmonic analysis of several complex variables almost since its inception. He has contributed a number of key ideas to the subject. So he feels well qualified to engage in this exposition. Our goal is to plant a flag for the subject and to lay the foundation for future work.

It is a pleasure to thank my teacher E. M. Stein for teaching me the foundations of this subject. The publisher engaged several insightful reviewers who contributed many useful suggestions and corrections. My thanks to them. Jerry Folland read a great part of the book and contributed many insights, corrections, and suggestions. His edits were both insightful and incisive. I give him my sincere thanks.

My many collaborators and students (too numerous to mention by name) have also taught me a great deal. I also thank my Editor Elizabeth Loew for her unwavering encouragement and support. I of course assume all responsibilities for any errors or inaccuracies in the text. I look forward to hearing from readers of the book.

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