Generalized Cubic Hermite Interpolation Based on Perturbed *Padé* Approximation

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Abstract. Generalized cubic Hermite interpolation was constructed by using perturbed *Padé* approximation in this paper. We generalize our method to the $2n+1$ times Hermite interpolation of $n+1$ points and study its barycentric form. Numerical example is given to show the effectiveness of our method. Finally, we further generalize the proposed method to generalized cubic Hermite interpolation based on perturbed Chebyshev-*Padé* approximation.

Keywords: Perturbed *Padé* approximation · Chebyshev-*Padé* approximation · Cubic hermite interpolation · Barycentric form

1 Introduction

The interpolation method plays an important pole in numerical analysis. The interpolation have been widely applied to the numerical approximation, and graphics image processing, image processing. Some scholars have been studying their application in image processing, numerical integration, engineering technology, curves/surface construction. The Hermite interpolation problem is a special polynomial interpolation, and has been widely studied. It is well known that the Hermite interpolation in the Chebychev nodes succeeds where Lagrange interpolation failed, that is, for every continuous function the Hermite interpolation polynomials in the Chebychev nodes converge to the function [1]. The classical formula given by Hermite interpolation takes prefixed values as well as its consecutive derivatives at some fixed points. Several researchers have studied this problem and have obtained interesting results concerning the convergence to the interpolant function [1, 2]. Corless studied Hermite interpolant occur naturally in the context of the numerical solution of initial value problems for ordinary differential equations [2]. Xie studied rational cubic Hermite interpolation spline and its approximation properties [3]. *Padé* approximation is an effective tool for rational approximation. When the function has a convergent power series in the interval [0, 1], Khodier proposed perturbation *Padé* approximation method, the accuracy of the introduced approximation increases as the order increases [4]. Based on *Padé* approximation and Lagrange

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interpolation, Zhao studied generalize blending rational interpolation [5]. Zhao also constructed the barycentric rational Hermite interpolant [6]. Zou studied generalize barycentric Lagrange rational interpolation [7]. Then, Zhang studied the general frames of barycentric blending rational interpolation [8].

Our contribution in this paper is to obtain a new type of cubic Hermite interpolation which combine cubic Hermite interpolation with perturbation *Padé* approximation, when the function has a convergent power series in the interval $[0, 1]$. The organization of the paper is as follows. In Sect. 2, we solve generalized cubic Hermite interpolation on equally spaced nodes based on perturbed *Padé* approximation. As an application of the preceding results, we give examples to show the effectiveness of our algorithm. In Sect. 3, we generalize our method to solve the $2n+1$ times Hermite interpolation problem with $n+1$ points and give its barycentric case. In Sect. 4 we point out that the generalize cubic Hermite interpolation and the case of $2n+1$ times cubic Hermite interpolation of $n+1$ points based on perturbed *Padé* approximation can be generalized to construct generalize cubic Hermite interpolation based on Chebyshev-*Padé* approximation.

2 Generalized Cubic Hermite Interpolation Based on Perturbed *Padé* Approximation

2.1 The Construction of Generalized Cubic Hermite Interpolation Based on Perturbed Padé Approximation

In this section we propose and solve generalized cubic Hermite interpolation based on perturbation *Padé* approximation.

Assume that $f(x)$ is a function which has the convergent power series form at $x = x_k$,

$$f(x) = \sum_{i=0}^{\infty} c_i^{(k)} (x - x_k) \quad x \in [0, 1], c_0^{(k)} \neq 0, (k = 0, 1, \cdots, n). \tag{1}$$

The (m, n) type perturbed *Padé* approximation $R_{m,n}^{(k)}(x)$ of the function $f(x)$ at $x = x_k$ is a rational function of the form [8]

$$R_{m,n}^{(k)}(x) = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m}{b_0 + b_1 x + b_2 x^2 + \cdots + b_n x^n} \tag{2}$$

where a_i, b_j can be defined with the method in the paper [8]. Without loss of generality, we restrict our description to $(1, 1)$ type perturbed *Padé* approximation.

We can get $(1, 1)$ type perturbed *Padé* approximation from [4]

$$R_{1,1}^{(k)}(x) = \frac{c_0^{(k)} + [c_1^{(k)} + c_0^{(k)}(\varepsilon^{(k)} - c_2^{(k)})/c_1^{(k)}](x - x_k)}{1 + [(\varepsilon^{(k)} - c_2^{(k)})/c_1^{(k)}](x - x_k)}. \tag{3}$$

where $\varepsilon^{(k)}$ are the perturbation parameters and

$$\varepsilon^{(k)} = \frac{(x-x_k)[c_n^{(k)}c_2^{(k)}(x-x_k)^{n-2} + \sum_{i=0}^{n-3}(c_{i+2}^{(k)}c_2^{(k)} - c_{i+3}^{(k)}c_1^{(k)})(x-x_k)^i]}{\sum_{i=0}^{n-1} c_{i+1}^{(k)}(x-x_k)^i}. \tag{4}$$

Note that if we put $\varepsilon^{(k)}= 0$ we get the classical *Padé* approximation $R_{1,1}(x)$ Then $R_{1,1}^{(k)}(x)$ agrees with the power series of $f(x)$ to a certain order n for all $x \in [0,1]$ with the perturbation parameter $\varepsilon^{(k)}$.

Assume that $f(x)$ is a function which has the convergent power series form at $x = x_k$.

Lemma 1. If $f(x)$ has the convergent power series form (1) and the perturbed parameters $\varepsilon^{(k)}$ are determined from (4), then the truncation error induced by using the perturbed *Padé* approximation $R_{1,1}^{(k)}(x)$ is of order n + 1.

Now we will get the perturbed *Padé* approximation $R_{1,1}^{(k)}(x)$ of derivatives. Differentiating the Eq. (1), we can get

$$f'(x) = \sum_{j=0}^{\infty} \frac{(1+j)!}{j!} c_{1+j}^{(k)}(x-x_k)^j = \sum_{j=0}^{\infty} \tilde{c}_j^{(k)}(x-x_k)^j. \tag{5}$$

where $\tilde{c}_j^{(k)} = \frac{(1+j)!}{j!}c_{1+j}^{(k)}$.

From [4] we can get perturbed *Padé* approximation of $f'(x)$ at point $x = x_k$

$$\tilde{R}_{1,1}^{(k)}(x) = \frac{\tilde{c}_0^{(k)} + [\tilde{c}_1^{(k)} + \tilde{c}_0^{(k)}(\tilde{\varepsilon}^{(k)} - \tilde{c}_2^{(k)})/\tilde{c}_1^{(k)}](x-x_k)}{1 + [(\tilde{\varepsilon}^{(k)} - \tilde{c}_2^{(k)})/\tilde{c}_1^{(k)}](x-x_k)}. \tag{6}$$

Given the points x_0, x_1, and its function $f^{(s)}(x_i) = f_i^{(s)}, s = 0,1;\ i = 0,1$. We can construct a generalized cubic Hermite interpolation based on perturbed *Padé* approximation

$$\begin{aligned}\bar{H}_3(x) = {} & R_{1,1}^{(0)}(x)\left(1 - 2\frac{x-x_0}{x_0-x_1}\right)\left(\frac{x-x_1}{x_0-x_1}\right)^2 + R_{1,1}^{(1)}(x)\left(1 - 2\frac{x-x_1}{x_1-x_0}\right)\left(\frac{x-x_0}{x_1-x_0}\right)^2 \\ & + \tilde{R}_{1,1}^{(0)}(x)(x-x_0)\left(\frac{x-x_1}{x_0-x_1}\right)^2 + \tilde{R}_{1,1}^{(1)}(x)(x-x_1)\left(\frac{x-x_0}{x_1-x_0}\right)^2.\end{aligned} \tag{7}$$

where $R_{1,1}^{(0)}(x)$ and $\tilde{R}_{1,1}^{(0)}(x)$ are the $(1,1)$ type perturbed *Padé* approximation of $f(x)$ at x_0 and the $(1,1)$ type perturbed *Padé* approximation of $f'(x)$ at x_0. $R_{1,1}^{(1)}(x)$ and $\tilde{R}_{1,1}^{(1)}(x)$ are the $(1,1)$ type perturbed *Padé* approximation of $f(x)$ at x_1 and the $(1,1)$ type perturbed *Padé* approximation of $f'(x)$ at x_1 respectively.

From [4], we know that

$$R_{1,1}^{(0)}(x_0) = f(x_0), \tag{8}$$

$$\tilde{R}_{1,1}^{(0)\prime}(x_0) = f'(x_0), \tag{9}$$

$$R_{1,1}^{(1)}(x_1) = f(x_1), \tag{10}$$

$$\tilde{R}_{1,1}^{(1)\prime}(x_1) = f'(x_1), \tag{11}$$

So we have

$$\bar{H}_3(x_i) = f(x_i), H_3'(x_i) = f'(x_i), i = 0, 1. \tag{12}$$

That is to say, generalized cubic Hermite interpolation based on perturbed *Padé* approximation satisfies interpolation conditions.

We can select the perturbed parameters $\varepsilon^{(k)}, \tilde{\varepsilon}^{(k)} (k = 0, 1)$ which are determined from (4), so that the truncation error induced by using the perturbed *Padé* approximation $R_{1,1}^{(k)}(x)$, $\tilde{R}_{1,1}^{(k)}(x)$ is of order n + 1.

2.2 Numerical Example

In this Section, we will give two examples to show the effectiveness of our method.

Example 1. Given $f(x) = e^x$, $x_0 = 0, x_1 = 1$, then we can get the former power series of the function $f(x)$ at point $x = x_0, x = x_1$:

$$f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \cdots,$$
$$f(x) = e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 + \frac{e}{24}(x-1)^4 + \cdots.$$

From Eq. (3), we can get [9]

$$R_{1,1}^{(0)}(x) = \frac{1 + (\varepsilon^{(0)} + 0.5)x}{1 + (\varepsilon^{(0)} - 0.5)x}$$
$$R_{1,1}^{(1)}(x) = \frac{\mathrm{e} + (\mathrm{e} + \varepsilon^{(1)} - \mathrm{e}/2)(x-1)}{1 + (\varepsilon^{(1)} - \mathrm{e})/\mathrm{e}(x-1)}$$

where $\varepsilon^{(k)} (k = 0, 1)$ are the perturbed parameters given in Eq. (4)

We can get the derivative of $f(x)$

$$f'(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \cdots,$$

and

$$f'(x) = e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 + \frac{e}{24}(x-1)^4 + \cdots .$$

So we can get

$$\tilde{R}_{1,1}^{(0)}(x) = \frac{1 + (\tilde{\varepsilon}^{(0)} + 0.5)x}{1 + (\tilde{\varepsilon}^{(0)} - 0.5)x}$$
$$\tilde{R}_{1,1}^{(1)}(x) = \frac{\mathrm{e} + (\mathrm{e} + \tilde{\varepsilon}^{(1)} - \mathrm{e}/2)(x-1)}{1 + (\tilde{\varepsilon}^{(1)} - \mathrm{e})/\mathrm{e}(x-1)}$$

where $\tilde{\varepsilon}^{(k)}(k = 0, 1)$ are the perturbed parameters given in Eq. (4)

By using the constructed method in Sect. 2.1, we can get generalized cubic Hermite interpolation based on perturbed *Padé* approximation

$$\begin{aligned}\bar{H}_3(x) =& R_{1,1}^{(0)}(x)(1+2x)(x-1)^2 + R_{1,1}^{(1)}(x)(2(x-1))x^2 + \\ & \tilde{R}_{1,1}^{(0)}(x)x(x-1)^2 + \tilde{R}_{1,1}^{(1)}(x)(x-1)x^2\end{aligned}$$

It is easy to verify that

$$\bar{H}_3(x) = f(x)$$

and

$$\bar{H}_3(x) = f'(x).$$

Example 2. Given $f(x) = x^2 + \sin x$, $x_0 = 0, x_1 = \pi/4$, then we can get the former power series of the function $f(x)$ at point $x = x_0, x = x_1$:

$$f(x) = x + x^2 - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots$$
$$f(x) = \frac{\sqrt{2}}{2} + \frac{\pi^2}{16} + (\frac{\sqrt{2}}{2} + \frac{\pi}{2})(x - \frac{\pi}{4}) + (\frac{\sqrt{2}}{4} - 1)(x - \frac{\pi}{4})^2 + \frac{\sqrt{2}}{12}(x - \frac{\pi}{2})^3 + \cdots .$$

From Eq. (3), we can get [9]

$$R_{1,1}^{(0)}(x) = \frac{1}{1 + (\varepsilon^{(0)} - 1)x}$$
$$R_{1,1}^{(1)}(x) = \frac{\frac{\sqrt{2}}{2} + \frac{\pi^2}{16} + [\frac{\sqrt{2}}{2} + \frac{\pi}{2} + (\frac{\sqrt{2}}{2} + \frac{\pi^2}{16})(\varepsilon^{(1)} - (\frac{\sqrt{2}}{4} - 1))/(\frac{\sqrt{2}}{2} + \frac{\pi}{2})](x - \frac{\pi}{4})}{1 + (\varepsilon^{(1)} - (\frac{\sqrt{2}}{4} - 1))/(\frac{\sqrt{2}}{2} + \frac{\pi}{2})(x - \frac{\pi}{4})}$$

where $\varepsilon^{(k)}(k = 0, 1)$ is the perturbed parameter given in Eq. (4)

The absolute maximum error obtained by using the perturbed *Padé* approximation with $n = 3$ is 81E-4 instead of 16E-2 for classical *Padé* approximation. Absolute maximum errors obtained for other values of n at point $x = x_0$ are given in Table 1.

Table 1. Absolute maximum error of perturbed *Padé* approximation

n	Absolute maximum error	
	Example 1	Example 2
5	16E-04	20E-05
7	28E-06	27E-07
9	30E-08	25E-09
11	23E-10	16E-11
13	12E-12	76E-14
15	51E-16	28E-16

We can get the derivative of $f(x)$,

$$f'(x) = 1 + 2x - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cdots,$$

and

$$f'(x) = \frac{\pi}{2} + \frac{\sqrt{2}}{2} + (2 - \frac{\sqrt{2}}{2})(x - \frac{\pi}{4}) - \frac{\sqrt{2}}{4}(x - \frac{\pi}{4})^2 + \frac{\sqrt{2}}{12}(x - \frac{\pi}{4})^3 + \cdots$$

So we can get

$$\tilde{R}^{(0)}_{1,1}(x) = \frac{1 + (\frac{\tilde{\varepsilon}^{(0)}}{2} + \frac{9}{4})x}{1 + (\frac{\tilde{\varepsilon}^{(0)}}{2} + \frac{1}{4})x}$$

$$\tilde{R}^{(1)}_{1,1}(x) = \frac{\frac{\pi+\sqrt{2}}{2} + [\frac{2-\sqrt{2}}{2} + (\frac{\pi+\sqrt{2}}{2} + (\tilde{\varepsilon}^{(1)} + \frac{\sqrt{2}}{4})/\frac{2-\sqrt{2}}{2}](x - \frac{\pi}{4})}{1 + (\tilde{\varepsilon}^{(1)} + \frac{\sqrt{2}}{4})/\frac{2-\sqrt{2}}{2}(x - \frac{\pi}{4})}$$

where $\tilde{\varepsilon}^{(k)}(k = 0, 1)$ are the perturbed parameters given in Eq. (4)

By using the proposed method in Sect. 2.1, we can get generalized cubic Hermite interpolation based on perturbed *Padé* approximation

$$\begin{aligned}\bar{H}_3(x) = {} & R^{(0)}_{1,1}(x)(1 + \frac{8x}{\pi})(\frac{4x - \pi}{\pi})^2 + R^{(1)}_{1,1}(x)(1 - \frac{8x - 2\pi}{\pi})(\frac{4x}{\pi})^2 + \\ & \tilde{R}^{(0)}_{1,1}(x)x(\frac{4x - \pi}{\pi})^2 + \tilde{R}^{(1)}_{1,1}(x)(x - \frac{\pi}{4})(\frac{4x}{\pi})^2\end{aligned}$$

It is easy to verify that

$$\bar{H}_3(x) = f(x)$$

and

$$\bar{H}_3(x) = f'(x).$$

As can be seen from Table 1, we can select the perturbed parameters $\varepsilon^{(k)}, \tilde{\varepsilon}^{(k)} (k = 0, 1)$ which are determined from (4), so that the truncation error induced by using the perturbed *Padé* approximation $R_{1,1}^{(k)}(x)$ and $\tilde{R}_{1,1}^{(k)}(x)$ is of order $n+1$.

From the above numerical examples, we can see our method gives higher accuracy. Generally, image interpolation can be widely used in digital image processing field, it is important to construct an efficient interpolation function to deal with image processing. Because image interpolation is sensitive to the image contours or edges, which are the high frequency information regions, interpolation kernel function should be more accurately approximate to the ideal interpolation (the Sinc function) in space domain and frequency domain. Hence the rational interpolating kernel is a good choice to process the image edge and texture regions. The Hermite rational interpolation can better preserve the geometric regularity around the color edges and thus generate interpolant images with higher visual quality.

3 *2n + 1* Times Hermite Interpolation of *n + 1* Point Based on Perturbed *Padé* Approximation

3.1 Construction of *2n + 1* Times Hermite Interpolation of *n + 1* Point Based on Perturbed *Padé* Approximation

Further, we can extend our method to $2n+1$ times Hermite interpolation of $n+1$ point based on perturbed *Padé* approximation.

Given $x_0 < x_1 < \cdots < x_n, f^{(s)}(x_i) = f_i^{(s)}, \; s = 0, 1; i =, 1, 2, \cdots, n,$ we can get the formula from [9]

$$\begin{aligned}\tilde{H}(x) = & \sum_{k=0}^{n} (R_{1,1}^{(k)}(x)(1 - 2(x - x_k) l_k'(x)) l_k^2(x) \\ & + \sum_{k=0}^{n} \tilde{R}_{1,1}^{(k)}(x)(x - x_k) l_k^2(x)).\end{aligned} \tag{13}$$

where $l_k(x)$ is the Lagrange basic function based on $x_i (i = 0, 1, 2, \cdots, n)$. $R_{1,1}^{(k)}(x)$ is perturbed *Padé* approximation of $f(x)$ at point $x = x_k$. $\tilde{R}_{1,1}^{(k)}(x)$ is perturbed *Padé* approximation of $f'(x)$ at point $x = x_k$.

Similar to the method in Sect. 2, we can get

$$\tilde{H}(x_i) = f(x_i), \tilde{H}'(x_i) = f'(x_i), i = 0, 1, 2, \cdots, n. \tag{14}$$

3.2 The Barycentric Form

Generally, the barycentric rational interpolations have more accuracy than the polynomial interpolation in computation, and the barycentric rational interpolation has more advantage than polynomial interpolation, for example, easy to calculate, the information concerning the existence and location of poles of the interpolation detecting the unattainable points of the interpolation, good numerical stability.

Then, the following Hermite interpolating polynomial [10, 11] can be obtained.

$$P(x) = l^2(x)\sum_{k=0}^{n} \frac{\omega_k^2}{x - x_k}\left(\left(\frac{1}{x - x_k} - \mu_k\omega_k\right)f_k + f'_k\right). \tag{15}$$

With $\omega_k = (l'(x_k))^{-1}, \mu_k = l''(x_k)$, we can get the barycentric version of (15)

$$P(x) = \frac{\sum_{k=0}^{n} \frac{\omega_k^2}{x-x_k}\left(\left(\frac{1}{x-x_k} - \mu_k\omega_k\right)f_k + f'_k\right)}{\sum_{k=0}^{n} \frac{\omega_k^2}{x-x_k}\left(\frac{1}{x-x_k} - \mu_k\omega_k\right)}. \tag{16}$$

Then, we can get $2n+1$ times barycentric Hermite interpolation of $n+1$ point based on perturbed *Padé* approximation

$$P(x) = \frac{\sum_{k=0}^{n} \frac{\omega_k^2}{x-x_k}\left(\left(\frac{1}{x-x_k} - \mu_k\omega_k\right)R_{1,1}^{(k)}(x) + \tilde{R}_{1,1}^{(k)}(x)\right)}{\sum_{k=0}^{n} \frac{\omega_k^2}{x-x_k}\left(\frac{1}{x-x_k} - \mu_k\omega_k\right)} \tag{17}$$

Similarly, we can select the perturbed parameters $\varepsilon^{(k)}, \tilde{\varepsilon}^{(k)}(k = 0, 1)$ which are determined from (4), so that the truncation error induced by using the perturbed *Padé* approximation $R_{1,1}^{(k)}(x)$, $\tilde{R}_{1,1}^{(k)}(x)$ is of order n + 1.

4 Conclusion

In this paper, we propose a method for computing the generalized cubic Hermite interpolation based on perturbed *Padé* approximation. We generalize the method to $2n+1$ times Hermite interpolation of $n+1$ points based on perturbed *Padé* approximation and its barycentric form.

We can also construct new kinds of generalized cubic Hermite blending rational interpolation based on *Padé* approximation and *Padé*-type approximation with the proposed method. If the Chebyshev series of the given function is given, we can construct the perturbation of Chebyshev-*Padé* approximation [12, 13]. By using Chebyshev-*Padé* instead of the perturbed *Padé* approximation, we can construct a new generalized cubic Hermite blending rational interpolation and $2n+1$ times Hermite interpolation of $n+1$ point and its barycentric case.

The Hermite rational interpolation can better preserve the geometric regularity around the color edges and thus generate interpolated images with higher visual quality, so we will construct a novel image magnification scheme based on the proposed method in future.

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