

Chapter 2

Theoretical Background

Chapter 1 discussed how high temperature plasmas may be formed in the laboratory in ICF and high intensity laser-plasma experiments. The purpose of this chapter is to introduce the various theoretical descriptions of these systems.

Firstly, in Sect. 2.1, we outline the salient results of kinetic theory, which describe the way in which charged particles interact in a plasma. Beginning with the description that involves the fewest approximations, our discussion gradually works towards those that are more confined in their validity but are analytically more tractable. These results form the basis of the work presented in Chaps. 3–5 of this thesis.

As the temperature of a plasma approaches the electron rest mass, pair production and radiative effects also become important. In Sect. 2.2.1 the various pair producing and radiative QED processes that are important in high temperature plasmas are therefore discussed; these are pertinent to the work of Chap. 6. Our final topic of study is strong field QED, which we review in Sect. 2.2.2 due to its relevance to high intensity laser experiments.

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2.1 Kinetic Theory of Plasmas

Plasmas are many-body systems, in which the number of particles routinely reaches of order 10^{20} in high power laser experiments¹ and many orders of magnitude beyond in astrophysics. Describing the behaviour of such large, integrated and inherently non-linear systems necessarily involves various approximations. This is the aim of kinetic theory.

We begin in Sect. 2.1.1 by outlining the derivation of the Klimontovich equation, which is an exact description of the microscopic behaviour of a classical plasma. By taking ensemble averages of this equation, we then in Sect. 2.1.2 introduce the Boltzmann equation, which is widely used in this thesis. Sect. 2.1.3 introduces the Fokker-Planck collision operator and its use in describing charged particle interactions. Collisions cause a plasma to tend towards equilibrium and, in Sect. 2.1.4, the distribution that describes relativistic particles in equilibrium is introduced. In Sect. 2.1.5, we outline how the distribution function and collision operator can be expanded as a means to solve the Boltzmann equation. Finally, in Sect. 2.1.6, an introduction to the linear transport theory of plasmas is provided.

As we are interested in plasmas in which the electron's thermal velocity approaches $c \approx 2.998 \times 10^8 \text{ ms}^{-1}$, the kinetic theories we focus on are all relativistically correct.

2.1.1 The Klimontovich Equation

We begin by considering the i th particle in a plasma containing N particles of species a . According to classical electrodynamics, the particle's dynamics are determined by

$$m_a \frac{d\mathbf{u}_i}{dt} = q_a [\mathbf{E}^M(\mathbf{r}_i, t) + \mathbf{v}_i \times \mathbf{B}^M(\mathbf{r}_i, t)], \quad (2.1)$$

$$\gamma_i \frac{d\mathbf{r}_i}{dt} = \mathbf{u}_i, \quad (2.2)$$

where $\mathbf{r}_i(t)$ and $\mathbf{u}_i(t)$ are its instantaneous position and momentum per unit mass at time t , m_a and q_a are the mass and charge of the particle, $\gamma_i = (1 + |\mathbf{u}_i^2|/c^2)^{1/2}$ its Lorentz factor, $\mathbf{v}_i$ its velocity, and $\mathbf{E}^M(\mathbf{r}_i, t)$ and $\mathbf{B}^M(\mathbf{r}_i, t)$ are the microscopic electric field strength and magnetic flux density at the particle; due to all other particles in the system as well as any external fields. Given N individual particle trajectories $\mathbf{r}_i$ and $\mathbf{u}_i$, we can write the *density functional* of the system as [2]

$$F_a(\mathbf{r}, \mathbf{u}, t) = \sum_{i=1}^N \delta[\mathbf{r} - \mathbf{r}_i(t)] \delta[\mathbf{u} - \mathbf{u}_i(t)], \quad (2.3)$$

¹For example, in a NIF hohlraum, ionised gas densities (10^{27} m^{-3}) are produced over of order 10^{-7} m^3 [1].

where $\delta(\mathbf{x})$ is the Dirac delta function. F_a records the position and momenta of each particle as a function of time, and its normalisation satisfies

$$n_a(\mathbf{r}, t) = \int F_a(\mathbf{r}, \mathbf{u}, t) d^3\mathbf{u} = \sum_{i=1}^N \delta[\mathbf{r} - \mathbf{r}_i(t)], \quad (2.4)$$

where n_a is the density of species a .

The electric field $\mathbf{E}^M$ and magnetic field $\mathbf{B}^M$ that appear in Eq. (2.1) are determined by Maxwell's equations:

$$\begin{aligned} \nabla \cdot \mathbf{E}^M &= \frac{\rho^M}{\epsilon_0}, & (\text{Gauss's law}) \\ \nabla \cdot \mathbf{B}^M &= 0, & (\text{No magnetic monopoles}) \\ \nabla \times \mathbf{E}^M &= -\frac{\partial \mathbf{B}^M}{\partial t}, & (\text{Faraday's law}) \\ \nabla \times \mathbf{B}^M &= \mu_0 \mathbf{J}^M + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}^M}{\partial t}, & (\text{Ampère's law}) \end{aligned}$$

in which ρ^M is the microscopic charge density, $\mathbf{J}^M$ is the microscopic current density, and ϵ_0 and μ_0 are the permittivity and permeability of free space respectively.

The complexity and non-linearity of a plasma stem from the fact that the charge and current densities are functions of the individual particle trajectories $\mathbf{r}_i$ and $\mathbf{v}_i$,

$$\rho_a^M(\mathbf{r}, t) = q_a n_a(\mathbf{r}, t) = q_a \sum_{i=1}^N \delta[\mathbf{r} - \mathbf{r}_i(t)], \quad (2.5)$$

$$\mathbf{J}_a^M(\mathbf{r}, t) = q_a \sum_{i=1}^N \mathbf{v}_i \delta[\mathbf{r} - \mathbf{r}_i(t)], \quad (2.6)$$

which, as we have seen, are themselves functions of $\mathbf{E}^M$ and $\mathbf{B}^M$.

The Klimontovich equation can be derived by taking the partial and total derivatives of Eq. (2.3) with respect to time, which, after some manipulation [2], yields

$$\frac{\partial F_a(\mathbf{r}, \mathbf{u}, t)}{\partial t} + \mathbf{v} \cdot \frac{\partial F_a}{\partial \mathbf{r}} + q_a (\mathbf{E}^M + \mathbf{v} \times \mathbf{B}^M) \cdot \frac{\partial F_a}{\partial \mathbf{u}} = 0. \quad (2.7)$$

Within the confines of classical electrodynamics, the Klimontovich equation is an exact description, although it is very difficult to deal with analytically due to the highly singular nature of the density functional F . Despite this, it may be solved numerically under various approximations using the particle-in-cell (PIC) method [3]. PIC simulations operate by discretising Eq. (2.1) and Maxwell's equations in time and space, and “pushing” pseudo-particles (each typically representing more than one real particle) along their trajectories, whilst self-consistently updating the electromagnetic fields. Modern supercomputers are able to perform these calculations with systems described by over 10^{10} pseudo-particles [4].

Although PIC simulations have been found to accurately describe the physics of various systems (particularly intense laser interactions in low density gases), such a description requires modifications to account for atomic physics, non-ideal equations of state and—of pertinence to this work—radiative and pair production effects.

2.1.2 The Boltzmann Equation

As we have seen, the Klimontovich equation is exact though unwieldy. In order to proceed analytically, we must smooth the discontinuities by taking an *ensemble average* of the density functional over infinite realisations of the system. Doing so yields

$$\overline{F_a} = f_a(\mathbf{r}, \mathbf{u}, t), \quad (2.8)$$

where $f_a(\mathbf{r}, \mathbf{u}, t)$ is the *one-particle probability distribution function* (PDF) of species a [5]. The PDF can be defined as follows [6]: an observer $\mathcal{O}$, who measures a system of N particles at time t , finds

$$N f(\mathbf{r}, \mathbf{u}, t) d^3\mathbf{r} d^3\mathbf{u} \quad (2.9)$$

in a volume element $d^3\mathbf{r}$ located at $\mathbf{r}$, having a momentum $\mathbf{u}$ within a range $d^3\mathbf{u}$. A second observer $\mathcal{O}'$, who is travelling with a relativistic velocity with respect to $\mathcal{O}$, will measure another distribution, f' . As was first proven by van Kampen [6],

$$f(\mathbf{r}, \mathbf{u}, t) = f'(\mathbf{r}', \mathbf{u}', t') \quad (2.10)$$

where $(\mathbf{r}, \mathbf{u}, t)$ and $(\mathbf{r}', \mathbf{u}', t')$ are related by Lorentz transformations; that is, the one-particle PDF is a Lorentz invariant. It has further been shown that [6]

$$\int f(\mathbf{r}, \mathbf{u}, t) d^3\mathbf{r} d^3\mathbf{u} = \int f'(\mathbf{r}', \mathbf{u}', t') d^3\mathbf{r}' d^3\mathbf{u}', \quad (2.11)$$

such that both f and f' satisfy the normalisation condition simultaneously. Note that the distribution function is necessarily defined in momentum rather than velocity space when describing relativistic particles, as otherwise it is not a Lorentz invariant.

We can define the statistical fluctuations δF_a of the density functional about the ensemble average as follows:

$$F_a(\mathbf{r}, \mathbf{u}, t) = f_a(\mathbf{r}, \mathbf{u}, t) + \delta F_a(\mathbf{r}, \mathbf{u}, t), \quad (2.12)$$

and, in a similar way, we may also express the electromagnetic fields as

$$\mathbf{E}^M(\mathbf{r}, \mathbf{u}, t) = \mathbf{E}(\mathbf{r}, \mathbf{u}, t) + \delta \mathbf{E}(\mathbf{r}, \mathbf{u}, t), \quad (2.13)$$

$$\mathbf{B}^M(\mathbf{r}, \mathbf{u}, t) = \mathbf{B}(\mathbf{r}, \mathbf{u}, t) + \delta\mathbf{B}(\mathbf{r}, \mathbf{u}, t), \quad (2.14)$$

where $\overline{\mathbf{E}^M} = \mathbf{E}$ and $\overline{\mathbf{B}^M} = \mathbf{B}$ are the macroscopic (smoothed) fields. On substituting this representation into Eq. (2.7) and taking an ensemble average, we arrive at

$$\begin{aligned} \frac{\partial f_a}{\partial t} + \mathbf{v} \cdot \frac{\partial f_a}{\partial \mathbf{r}} + q_a (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_a}{\partial \mathbf{u}} &= -q_a \overline{\left((\delta\mathbf{E} + \mathbf{v} \times \delta\mathbf{B}) \cdot \frac{\partial(\delta F_a)}{\partial \mathbf{u}} \right)} \\ &= C^{a/b}, \end{aligned} \quad (2.15)$$

where we have used $\overline{\delta\mathbf{E}} = \mathbf{0}$, $\overline{\delta\mathbf{B}} = \mathbf{0}$ and $\overline{\delta F_a} = 0$ (i.e., the mean value of statistical fluctuations is zero). This can be recognised as a form of the *Boltzmann equation* and bears similarity to Eq. (2.7), with the notable exception that the right-hand side is here given by the *collision term* $C^{a/b}$.

$C^{a/b}$ includes the effects of microscopic charge fluctuations at each particle and describes collisional behaviour. By contrast, the left-hand side of the Boltzmann equation deals with smoothed fields and describes collective behaviour. (Note that collisional and collective phenomena are both captured in the left-hand side of the Klimontovich equation, Eq. (2.7).) The transition between these two types of behaviour takes place at approximately the *Debye length* $\lambda_D = (\epsilon_0 k_B T_e / n_e e^2)^{1/2}$; the interaction between charged particles manifests itself as collisions for distances $r < \lambda_D$, and as collective behaviour for $r > \lambda_D$.²

In the case where $C^{a/b} = 0$, Eq. (2.15) reduces to the *Vlasov equation* [8], which is used extensively to describe collisionless plasmas. However, in this work we are interested in particle collisions,³ which—in the case of the Coulomb interaction—are typically described using the *Fokker-Planck collision operator*, which is the subject of the next section. A full (and lengthy) derivation of how the Fokker-Planck operator may be derived from the right-hand side of Eq. (2.15) is given in Ref. [2].

2.1.3 The Fokker-Planck Collision Operator

In a plasma, distant encounters dominate over close encounters [9] and the motion of a particular particle (which we will call a *test particle*) is the result of small deflections due to its interaction with many other particles (referred to as *field particles*). The resulting behaviour can be likened to Brownian motion in a fluid, and a similar mathematical treatment can be applied to describe the kinetics of both types of system [9, 10].

²This interpretation relies on the fact that an individual charge is screened at distances of order λ_D due to Debye shielding, which takes of order $1/\omega_p$ to be established [7]; we assume this to be the case.

³Collisionless processes are ignored throughout this work.

Landau was the first to derive a Fokker-Planck collision operator describing small-angle interactions in a plasma, which is given by the general form [11]

$$C^{a/b} = -\frac{\Gamma^{a/b}}{2} \frac{\partial}{\partial \mathbf{u}} \cdot \int \mathbf{U}(\mathbf{u}, \mathbf{u}') \cdot \left(f_a(\mathbf{u}) \frac{\partial f_b(\mathbf{u}')}{\partial \mathbf{u}'} - \frac{m_a}{m_b} f_b(\mathbf{u}') \frac{\partial f_a(\mathbf{u})}{\partial \mathbf{u}} \right) d^3 \mathbf{u}', \quad (2.16)$$

with

$$\Gamma^{a/b} = \frac{n_b q_a^2 q_b^2 \ln \Lambda^{a/b}}{4\pi \epsilon_0^2 m_a^2},$$

where n_b is the density of the background, q_a and q_b are the species charges, $\ln \Lambda^{a/b}$ is the *Coulomb logarithm* and ϵ_0 is the permittivity of free space.

The Coulomb logarithm is an artefact of an integration over angles from the minimum scattering angle $\theta_{\min}$ to π , and is given by [12]

$$\ln \Lambda^{a/b} = \ln \left(\frac{1}{\tan(\theta_{\min}/2)} \right). \quad (2.17)$$

Integrating over all angles (i.e., taking $\theta_{\min} = 0$) leads to a divergent expression; a fact that can be attributed to the slow fall off of the Coulomb potential [7], or the assumption that interactions take place instantaneously [13]. To avoid this, we set $\theta_{\min}$ to be a physically motivated, non-zero value. Many authors have set it on the basis of Debye shielding, which in effect results in the Coulomb force on the test particle being cut off at distances greater than the Debye length λ_D [7, 9, 13]. Others determine $\theta_{\min}$ by the minimum momentum transfer—that of the excitation of a single plasmon—which, for a relativistic plasma, gives $\theta_{\min} \sim \hbar \omega_p / k_B T$, where ω_p is the plasma frequency (Eq. 1.5) and $\hbar$ is Planck's constant [14, 15]. It is, of course, important to calculate the logarithm as accurately as possible, although its exact form is not considered further in this work.

In the relativistic case, the kernel $\mathbf{U}(\mathbf{u}, \mathbf{u}')$ of Eq. (2.16) was first found by Beliaev and Budker [16] and is given by⁴

$$\mathbf{U}(\mathbf{u}, \mathbf{u}') = \frac{r^2}{\gamma \gamma' (r^2 - 1)^{3/2} c^3} [(r^2 - 1) c^2 \mathbf{l} - \mathbf{u} \mathbf{u} - \mathbf{u}' \mathbf{u}' + r(\mathbf{u} \mathbf{u}' + \mathbf{u}' \mathbf{u})], \quad (2.18)$$

where $\gamma = (1 + |\mathbf{u}|^2/c^2)^{1/2}$, $\gamma' = (1 + |\mathbf{u}'|^2/c^2)^{1/2}$, $r = \gamma \gamma' - \mathbf{u} \cdot \mathbf{u}'/c^2$, $\mathbf{l}$ is the unit diagonal second-order tensor and

$$\frac{\partial}{\partial \mathbf{u}'} \cdot \mathbf{U}(\mathbf{u}, \mathbf{u}') = \frac{2r^2}{\gamma \gamma' (r^2 - 1)^{3/2}} (r \mathbf{u} - \mathbf{u}'). \quad (2.19)$$

For cases in which $|\mathbf{v} \cdot \mathbf{v}'| \ll c^2$, the kernel reduces to the widely-used result of Landau, $\mathbf{U} = (|\mathbf{s}|^2 \mathbf{l} - \mathbf{s} \mathbf{s})/|\mathbf{s}|^3$, where $\mathbf{s} = \mathbf{v} - \mathbf{v}'$ is the relative velocity [11]. Note that, even

⁴Akama separately obtained this expression using a quantum electrodynamical treatment [12].

though $\mathbf{U}$ is not a simple function of $\mathbf{v} - \mathbf{v}'$ in the relativistic case, $\mathbf{U} \cdot (\mathbf{v} - \mathbf{v}') = 0$ remains satisfied, as is required for the collision operator to vanish in equilibrium [17].

Using integration by parts, the collision operator can be rewritten:

$$C^{a/b} = -\frac{\partial}{\partial \mathbf{u}} \cdot \mathbf{S} = -\frac{\partial}{\partial \mathbf{u}} \cdot \left(\frac{\mathbf{F}^{a/b}}{m_a} f_a - \mathbf{D}^{a/b} \cdot \frac{\partial f_a}{\partial \mathbf{u}} \right), \quad (2.20)$$

in which $\mathbf{S}$ represents the particle flux in momentum space due to collisions. This is expressed in terms of the force of dynamical friction $\mathbf{F}^{a/b}$ and diffusion tensor $\mathbf{D}^{a/b}$, which are given by

$$\mathbf{F}^{a/b}(\mathbf{u}) = -\frac{\Gamma^{a/b}}{2} \frac{m_a^2}{m_b} \ln \Lambda^{a/b} \int \left(\frac{\partial}{\partial \mathbf{u}'} \cdot \mathbf{U}(\mathbf{u}, \mathbf{u}') \right) f_b(\mathbf{u}') d^3 \mathbf{u}', \quad (2.21)$$

$$\mathbf{D}^{a/b}(\mathbf{u}) = \frac{\Gamma^{a/b}}{2} \ln \Lambda^{a/b} \int \mathbf{U}(\mathbf{u}, \mathbf{u}') f_b(\mathbf{u}') d^3 \mathbf{u}'. \quad (2.22)$$

$\mathbf{F}^{a/b}$ and $\mathbf{D}^{a/b}$ do not have any physical significance in themselves [7]. However, as well we shall see in Chap. 3, they can be related to the momentum loss $\langle \Delta \mathbf{u} \rangle / \Delta t$ and diffusion $\langle \Delta \mathbf{u} \Delta \mathbf{u} \rangle / \Delta t$ rates of a test particle as it moves within a plasma: $\mathbf{F}^{a/b}$ is a simple function of the two rates and $\mathbf{D}^{a/b}$ is trivially related to $\langle \Delta \mathbf{u} \Delta \mathbf{u} \rangle / \Delta t$. These relations allow us to restate the collision operator as

$$C^{a/b} = -\frac{\partial}{\partial \mathbf{u}} \cdot \frac{\langle \Delta \mathbf{u} \rangle}{\Delta t} f_a + \frac{1}{2} \frac{\partial^2}{\partial \mathbf{u} \partial \mathbf{u}} : \left(\frac{\langle \Delta \mathbf{u} \Delta \mathbf{u} \rangle}{\Delta t} f_a \right), \quad (2.23)$$

which is the *Fokker-Planck* form [18, 19]. We will not deal with this formulation explicitly, although it is useful to note that this is effectively a Taylor expansion in momentum space, which has been truncated at the second term. All higher order terms are neglected on the basis that deflections in momentum space are small or, equivalently, that these terms form integrals over scattering angles that are not logarithmically divergent (i.e., not proportional to $\ln \Lambda^{a/b}$). The Fokker-Planck description of interactions in a plasma is therefore of *logarithmic accuracy* and only well defined when $\ln \Lambda^{a/b}$ is large.

Dynamical friction and diffusion cause a system to relax towards statistical equilibrium over time. We now consider the one-particle distribution that describes a system of relativistic particles in equilibrium: *Jüttner's distribution*.

2.1.4 Jüttner's Distribution

The equilibrium distribution for relativistic particles is given by

$$f_J(u) = \frac{n \exp(-\gamma/\Theta)}{4\pi c^3 \Theta K_2(1/\Theta)}, \quad (2.24)$$

which was first found by Jüttner in 1911 through the use of the maximum entropy principle [20]. Here K_ν is the ν th-order Bessel function of the second kind (see Appendix B for further details of these functions). The average energy per particle can be obtained by logarithmic differentiation of the normalisation constant [5]:

$$\begin{aligned}\langle\gamma\rangle &= -\frac{\partial}{\partial(1/\Theta)} \ln \{\Theta K_2(1/\Theta)\} \\ &= 3\Theta + \frac{K_1(1/\Theta)}{K_2(1/\Theta)}.\end{aligned}\tag{2.25}$$

In the two limits of extreme temperature, this can be expanded to give

$$\lim_{\Theta \rightarrow 0} \langle\gamma\rangle = 1 + \frac{3}{2}\Theta + \frac{15}{8}\Theta^2 + \mathcal{O}(\Theta^3),\tag{2.26}$$

$$\lim_{\Theta \rightarrow \infty} \langle\gamma\rangle = 3\Theta + \frac{1}{2}\Theta^{-1} + \frac{1}{4}\Theta^{-3}[\gamma_E - \ln(2\Theta)] + \mathcal{O}(\Theta^{-5}),\tag{2.27}$$

where γ_E is Euler's constant. There has been some debate as to the accuracy of Eq. (2.24) over the past few decades, as it is not manifestly covariant (i.e., written in terms of four-vectors) [5]. However, recent numerical simulations in $d = 1, 2, 3$ dimensions have corroborated its validity as the relativistic generalisation to the Maxwell-Boltzmann distribution [21–23], and recently it has been rewritten in a manifestly covariant form [24]. Monte Carlo simulations presented in Chap. 5 of this thesis also show that a system of relativistic particles relaxes to the Jüttner distribution.

The momenta of non-relativistic particles in equilibrium are described by the Maxwell-Boltzmann distribution,

$$f_M(v) = \frac{n \exp(-mv^2/2k_B T)}{(2\pi k_B T/m)^{3/2}}.\tag{2.28}$$

As we are interested in relativistic plasmas in this thesis, the terms *Jüttner's distribution* and *Maxwellian distribution* will be used interchangeably from this point onwards. In the next section we outline the expansion of the distribution function about equilibrium as a means to solve the Boltzmann equation.

2.1.5 Linearisation of the Boltzmann Equation

Directly solving the Boltzmann equation is extremely difficult, due in the main part to the complexity of the collision term, Eq. (2.20). In Chap. 3 we will see that in the particular case of a test particle interacting with a Maxwellian background, simple analytical expressions can be derived. However, in the more general case, it is necessary to make an expansion of the distribution function and collision operator, and one means of doing this is through the use of Cartesian tensors, e.g.,

$$f(\mathbf{r}, \mathbf{u}, t) = \sum_{l=0}^{\infty} \frac{\{\mathbf{f}_l(\mathbf{r}, u, t)\} \cdot_l \{\mathbf{u}^l\}}{u^l}, \quad (2.29)$$

where

$$\{\mathbf{f}_n(\mathbf{r}, u, t)\} \cdot_l \{\mathbf{u}^l\} = \sum_{ijk} f_{(l) \dots kji} u^i u^j u^k \dots$$

with i, j, k , etc. each referring to the x, y or z axes. The first three terms are given by

$$f(\mathbf{r}, \mathbf{u}, t) = f_0(\mathbf{r}, u, t) + \mathbf{f}_1(\mathbf{r}, u, t) \cdot \frac{\mathbf{u}}{u} + \mathbf{f}_2(\mathbf{r}, \mathbf{u}, t) : \frac{\mathbf{u}\mathbf{u}}{u^2} + \dots, \quad (2.30)$$

where $f_0(\mathbf{r}, u, t)$ is the isotropic part of the distribution that describes the temperature and number density of the plasma, $\mathbf{f}_1$ is the anisotropic part of the distribution that leads to the current and heat flow, and $\mathbf{f}_2$ accounts for the effects of pressure anisotropy on transport.

It is then possible to derive equations governing the time evolution of each of these terms; the f_0 and $\mathbf{f}_1$ equations are given by

$$\frac{\partial f_0}{\partial t} + \frac{v}{3} \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{f}_1 + \frac{q_e}{3m_e u^2} \frac{\partial}{\partial u} (u^2 \mathbf{E} \cdot \mathbf{f}_1) = C_0, \quad (2.31)$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + v \frac{\partial f_0}{\partial \mathbf{r}} + \frac{q_e}{m_e} \mathbf{E} \frac{\partial f_0}{\partial u} + \frac{q_e}{\gamma m_e} \mathbf{B} \times \mathbf{f}_1 = \mathbf{C}_1, \quad (2.32)$$

where we have neglected terms in $\mathbf{f}_2$. Note that these equations are coupled; to determine the evolution of f_0 , we must also know that of $\mathbf{f}_1$.

The collision terms C_0 and $\mathbf{C}_1$ are complex, particularly in the relativistic case [25], and so we defer providing their explicit forms until required in Chap. 4. At this point, it is sufficient to note that this formulation relies on the linearisation of the self-collision operator, e.g., in the case of the self-interaction of species a [26]:

$$\begin{aligned} C_{\text{lin}}^{a/a}(f_{a0} + \mathbf{f}_{a1} \cdot \mathbf{u}/u, f_{a0} + \mathbf{f}_{a1} \cdot \mathbf{u}/u) &= C^{a/a}(f_{a0}, f_{a0}) + C^{a/a}(f_{a0}, \mathbf{f}_{a1} \cdot \mathbf{u}/u) \\ &\quad + C^{a/a}(\mathbf{f}_{a1} \cdot \mathbf{u}/u, f_{a0}) + C^{a/a}(\mathbf{f}_{a1} \cdot \mathbf{u}/u, \mathbf{f}_{a1} \cdot \mathbf{u}/u). \end{aligned}$$

It is also possible to expand the distribution and collision operator in spherical harmonics, e.g.,

$$f(\mathbf{r}, \mathbf{u}, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^l f_l^m(\mathbf{r}, u, t) P_l^{|m|}(\cos \theta) e^{im\phi}, \quad (2.33)$$

where $f_l^{-m} = (f_l^m)^*$. This is formally equivalent to the Cartesian tensor expansion [27], and we shall use both formulations in this thesis.

In the next section, we will consider one of the main applications of this; the development of *classical transport theory*.

2.1.6 Classical Transport Theory

By taking moments of the $\mathbf{f}_1$ equation (Eq. 2.32) it is possible to derive the classical transport relations of a plasma, which describe the current and heat flow in terms of the applied electromagnetic fields and local thermodynamic gradients.

In this work we adopt the notation of Braginskii [28], who obtained the following expressions for the electric field $\mathbf{E}$ (Ohm's law) and total heat flow $\mathbf{q}$ (heat flow equation):

$$en\mathbf{E} = -\nabla p + \mathbf{j} \times \mathbf{B} + \boldsymbol{\alpha} \cdot \mathbf{j}/ne - nk_B\boldsymbol{\beta} \cdot \nabla T, \quad (2.34)$$

$$\mathbf{q} = -\boldsymbol{\kappa} \cdot \nabla T - \boldsymbol{\beta}' \cdot \mathbf{j}k_BT/e, \quad (2.35)$$

where p is the scalar pressure, T is the temperature, e is the elementary charge, $\mathbf{j}$ is the electric current and $\mathbf{B}$ is the magnetic flux density. $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\kappa}$ are the electrical resistivity, thermoelectric and thermal conductivity tensors, respectively, and, for a non-relativistic plasma,

$$\boldsymbol{\beta}' = \boldsymbol{\beta} + \frac{5}{2}\mathbf{I}, \quad (2.36)$$

where $\mathbf{I}$ is the unit diagonal second-order tensor.

The derivation of the transport equations is dealt with in detail in Chap. 4, for the case in which the electrons are relativistic. Here we note that obtaining Eqs. (2.34) and (2.35) is contingent upon us neglecting the time derivative of $\mathbf{f}_1$ (which becomes $\partial\mathbf{j}/\partial t$ in the generalised Ohm's law [29]), and we are thus dealing with steady-state quantities. In any real plasma, it will take a finite time for this state to be reached, as shown explicitly through Monte Carlo simulations in Chap. 5.

The transport coefficients that appear in Braginskii's theory, which we denote in general by the symbol φ , are typically expressed in terms of their components relative to the magnetic field vector $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$ and driving force $\mathbf{s}$ by using

$$\varphi \cdot \mathbf{s} = \varphi_{\parallel} \mathbf{b}(\mathbf{b} \cdot \mathbf{s}) + \varphi_{\perp} \mathbf{b} \times (\mathbf{s} \times \mathbf{b}) \pm \varphi_{\wedge} \mathbf{b} \times \mathbf{s}, \quad (2.37)$$

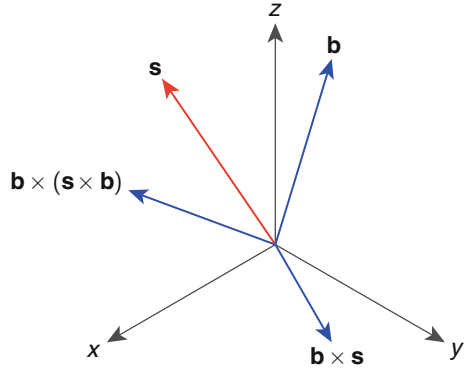
where $\varphi \in \{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\kappa}\}$, $\mathbf{s} \in \{\mathbf{E}, \nabla T\}$ and the negative sign applies only in the case $\varphi = \boldsymbol{\alpha}$. This geometry is shown in Fig. 2.1.

For reference, we note that the notation typically used for non-magnetised plasmas is that of Spitzer and Härm [30], whose transport relations are

$$\mathbf{j} = \sigma \mathbf{E} + A \nabla T, \quad (2.38)$$

$$\mathbf{q} = -B \mathbf{E} - K \nabla T, \quad (2.39)$$

Fig. 2.1 The geometry used in our description of transport, as per Eq. (2.37). The tensorial transport coefficients are described by their components in the $\mathbf{b}$ ($\parallel$), $\mathbf{b} \times (\mathbf{s} \times \mathbf{b})$ ($\perp$) and $\mathbf{b} \times \mathbf{s}$ ($\wedge$) directions



where σ and K are the electrical and thermal conductivities respectively, and A and B are the thermoelectric coefficients. (The uppercase A and B are used to distinguish these from the Braginskii coefficients). A simple relationship exists between the two sets of coefficients:

$$\begin{aligned} \sigma &= \frac{n^2 q_e^2}{\alpha_{\parallel}}, & A &= n^2 q_e k_B \frac{\beta_{\parallel}}{\alpha_{\parallel}}, \\ B &= n^2 q_e k_B T \frac{\beta'_{\parallel}}{\alpha_{\parallel}}, & K &= \kappa_{\parallel} + n^2 k_B^2 T \frac{\beta_{\parallel} \beta'_{\parallel}}{\alpha_{\parallel}}. \end{aligned}$$

where the subscript $\parallel$ denotes the tensorial component parallel to the magnetic field. Spitzer's effective thermal conductivity K' [30] is equivalent to $\kappa_{\parallel}$.

Although we do not consider it explicitly in this work, the next level of approximation, and one that is very frequently used, is *magnetohydrodynamics* (MHD). The MHD equations can be derived by taking moments of the Boltzmann equation, corresponding to the conservation of mass, momentum and energy of the plasma [28, 31]. This method of taking moments is performed under the assumption of local thermodynamic equilibrium, that is, the distribution function is described by a Maxwellian at each point in space. However, in order to close the MHD equations it is necessary to specify a heat flow. Assuming the distribution is not much perturbed from equilibrium, we can use the approach above and, indeed, this is the main use of classical transport theory.

2.2 Quantum Electrodynamics

Having reviewed the various classical descriptions of a plasma, in this section we consider the quantum electrodynamical processes relevant to this work. These can be categorised into linear or perturbative QED processes, which are discussed in

Sect. 2.2.1, and the QED processes that become accessible in a strong electromagnetic field, which are the subject of Sect. 2.2.2.

2.2.1 QED Processes in High-Temperature Plasmas

Many low temperature plasmas are classical and their behaviour is therefore accurately described by the theory outlined in the previous section.

However, as the temperature of a plasma is increased, radiative processes become increasingly important, because: (a) the cross sections for both electron-nucleus and electron-electron bremsstrahlung scale strongly with the energy of the particles when $E \lesssim m_e c^2$ [32]⁵; and (b) for plasmas in the presence of a magnetic field, the total power radiated by synchrotron processes scales as γ^2 [33]. A radiation field is therefore an inherent component of a high temperature plasma.

In addition, for temperatures at which the centre-of-mass (CM) energy of an interaction can exceed $2m_e c^2$, the nature of a plasma fundamentally changes due to the possibility of pair production. Electron-electron, electron-photon and photon-photon collisions can all lead to the production of electron-positron pairs and the final of these, the Breit-Wheeler process ($\gamma\gamma' \rightarrow e^+e^-$), is that of particular interest in the latter part of this work. In the context of a relativistic plasma, this can cause highly non-linear dynamics, and processes between photons, electrons, positrons and ions must be accounted for in order to obtain an accurate description of the plasma.

Table 2.1 lists the relevant QED processes of order α^2 and α^3 in the coupling constant (as well as double pair production, which is of order α^4) for an electron-positron plasma. Electrons, positrons and photons are referred to here as e^- , e^+ and γ respectively. A similar listing is provided for QED interactions with a nuclear field Z in Table 2.2. For very high temperature plasmas, pair production processes between protons may also become significant [34].

The double-ended arrows highlight the fact that these interactions can take place in either direction. In fact, neglecting the backward reactions prevents thermalisation of the plasma [35], although these are much more difficult to describe theoretically; the relevant reaction rates have only been calculated for a few of these cases (see, e.g., Refs. [36, 37]). In general they can only be determined through Kirchoff's law [38], which relates the emission and absorption of a body which is in local thermal equilibrium [39].

Much work has been done by the astrophysical community at describing the interplay of these processes in a relativistic plasma (see, e.g., [14, 34, 40–43]). In general, this is not a trivial problem, as the dominant processes depend on the temperature and optical depth of the system [41].

In the present work, we consider instead two isolated problems. Firstly, we study the effect of radiative processes, namely bremsstrahlung (Sect. 3.4) and synchrotron

⁵In fact, as we will see in Chap. 6, it is bremsstrahlung that is the dominant energy loss mechanism for an electron at very high energies.

Table 2.1 QED processes in an electron-positron plasma. Adapted from Ref. [40]

Binary interactions	Radiative variants	Pair-producing variants
Coulomb $e^\pm e^\pm \longleftrightarrow e^\pm e^\pm$ $e^+ e^- \longleftrightarrow e^+ e^-$	Bremsstrahlung $e^\pm e^\pm \longleftrightarrow e^\pm e^\pm \gamma$ $e^+ e^- \longleftrightarrow e^+ e^- \gamma$	Double pair production $e^\pm e^\pm \longleftrightarrow e^\pm e^\pm e^+ e^-$ $e^+ e^- \longleftrightarrow e^+ e^- e^+ e^-$
Single Compton $e^\pm \gamma \longleftrightarrow e^\pm \gamma$	Double Compton $e^\pm \gamma \longleftrightarrow e^\pm \gamma \gamma$	Triplet pair production $e^\pm \gamma \longleftrightarrow e^\pm e^+ e^-$
Pair production and annihilation $\gamma \gamma \longleftrightarrow e^+ e^-$	Radiative pair production and three-photon annihilation $\gamma \gamma \longleftrightarrow e^+ e^- \gamma$ $e^+ e^- \longleftrightarrow \gamma \gamma \gamma$	Double pair production (real photons) $\gamma \gamma \longleftrightarrow e^+ e^- e^+ e^-$

Table 2.2 QED processes involving a nuclear field in a high temperature plasma

Binary interactions	Radiative variants	Pair-producing variants
Coulomb $Ze^\pm \longleftrightarrow Ze^\pm$	Bremsstrahlung $Ze^\pm \longleftrightarrow Ze^\pm \gamma$	Trident pair production $Ze^\pm \longleftrightarrow Ze^\pm e^+ e^-$
Delbrück scattering $Z\gamma \longleftrightarrow Z\gamma$	Photon splitting $Z\gamma \longleftrightarrow Z\gamma \gamma$	Bethe-Heitler pair production $Z\gamma \longleftrightarrow Ze^+ e^-$

radiation (Sect. 4.5), on the interaction of charged particles in a high temperature plasma, under the assumption that the system is optically thin. Secondly, in Chap. 6, we explore the possibility of observing the Breit-Wheeler process in the laboratory for the first time. This includes considerations for many of the processes listed above (albeit not in a plasma environment), as they also represent the simplest processes in QED.

A final point of note is that it is somewhat fortuitous that the Coulomb interaction conforms to the Fokker-Planck description; a fact that arises due to the peculiar nature of the inverse square force [10]. In order to consider these other quantum electrodynamical processes, however, we find it necessary to drop this approximation.

In fact, much of the mathematical complexity in the Fokker-Planck approach is due to the need to sum over a background distribution. As our treatment of QED processes in this work is primarily numerical, we are able to consider instead just two particles. This simplifies the discussion significantly. The Lorentz-invariant reaction rate between two particle beams a and b is given by [44, 45]

$$\frac{dN^{a/b}}{dt} = \begin{cases} \frac{n_b \sigma(r^2 - 1)^{1/2} c}{\gamma \gamma'} & \text{for } m_a, m_b \neq 0, \\ n_b \sigma(1 - \mathbf{v} \cdot \mathbf{v}'/c^2) c & \text{for } m_a = 0 \text{ or } m_b = 0, \end{cases} \quad (2.40)$$

where $r = \gamma \gamma' - \mathbf{u} \cdot \mathbf{u}'/c^2$ as before and primed quantities refer to particle b . Note that the relative velocity $v_{\text{rel}} = (r^2 - 1)^{1/2} c/r$, such that the upper equation can be rewritten $N^{a/b} = n_b \sigma v_{\text{rel}} r / \gamma \gamma'$. In the rest frame of particle b , this reduces to the simple expression $N^{a/b} = n_b \sigma v_{\text{rel}}$. For the case of two photons scattering, the lower

equation reduces to $dN^{a/b}/dt = n_b \sigma c(1 - \cos \xi)$, where ξ is the angle between their initial momentum vectors.

The cross sections σ of the processes relevant to this thesis are listed in full in Appendix C.

2.2.2 Non-linear QED in Strong Electromagnetic Fields

Of tangential interest to this work are *multi-photon* QED processes, which become significant in the presence of strong electromagnetic fields. Such fields are thought to exist naturally in the vicinity of radio pulsars [46] and neutron stars [47] and, as we saw in the Introduction, it is now becoming possible to create them in the laboratory using high intensity lasers. We focus here on the latter case.

Firstly, we note that *non-linear* effects become significant in a laser when the strength parameter of the wave

$$\xi = \frac{eE_0}{\omega_0 m_e c} \quad (2.41)$$

approaches or exceeds unity, where E_0 is the electric field amplitude and ω_0 the angular frequency [48]. Classically, the parameter ξ can be interpreted as the energy provided by the field on a probe particle during one laser period ($T_0 = 2\pi/\omega_0$) in units of $m_e c^2$. Clearly for $\xi \gtrsim 1$, an electron will become relativistic in one period, in which case the electric and magnetic components of the Lorentz force are comparable, resulting in highly non-linear dynamics.

One may also interpret ξ as the work performed by the laser on an electron over a reduced Compton wavelength $\lambda_c = \hbar/m_e c$, in units of the laser photon energy $\hbar\omega_0$. Therefore, for $\xi \ll 1$, the most probable processes are those involving only one photon from the laser, whereas, for $\xi \gtrsim 1$, the probabilities of processes involving multiple photons become significant and these probabilities have a non-linear dependence on the strength of the field.

The magnitude of *quantum non-linear* processes can be determined under consideration of the strength of the laser field in the appropriate frame of reference. Effectively, we require the laser field to be comparable to the critical field, $E_c = m_e^2 c^3 / e \hbar \approx 1.323 \times 10^{18} \text{ Vm}^{-1}$, for these processes to become significant. This corresponds to the point at which the vacuum spontaneously breaks down to form an electron-positron pair; the so-called *Schwinger effect* [49]. To reach this in the laboratory frame, an intensity of order 10^{29} Wcm^{-2} is needed, which is approximately seven orders of magnitude greater than that currently achievable. In order to overcome this deficit, modern strong-field QED experiments employ the use of highly relativistic probe particles, such that the electric field in the rest frame of the particle (or the CM frame in the case of an incident photon) is increased to $E \sim \gamma E_0$, where γ is the relative Lorentz factor between the two frames.

When E_0 is much weaker than the critical field E_c , non-linear QED processes in the interaction of a relativistic electron and a laser field are determined by two invariants: ξ , as we have already seen, and

$$\chi = \frac{p_0 E_0}{m_e c E_c}, \quad (2.42)$$

where p_0 is the initial momentum of the electron. χ is the *quantum non-linearity parameter* and represents the ratio E_0/E_c in the electron rest frame [48]. As E_c is the electric field for which an electron-positron pair gains an energy $2m_e c^2$ over a Compton wavelength, $\chi \sim 1$ corresponds to the point at which trident pair production ($e^- + n\omega_0 \rightarrow e^- e^+ e^-$) becomes permissible. (It is exponentially suppressed below this point.) Other processes, such as non-linear Compton scattering ($e^- + n\omega_0 \rightarrow e^- \gamma$), also become significant at this point. For $\xi < 1$, non-linear QED effects are maximised when $\chi \sim \xi$, whereas, for $\xi \gtrsim 1$, these are maximised when $\chi \sim 1$.

Analogously, in the case of an energetic gamma-ray interacting with a high intensity laser, non-linear QED processes are a function of ξ and

$$\varkappa = \frac{k_0 E_0}{m_e c E_c}, \quad (2.43)$$

where k_0 is the momentum of the incoming photon and represents E_0/E_c in the centre-of-mass frame. This is of particular relevance to the multi-photon Breit-Wheeler process ($\gamma + n\omega_0 \rightarrow e^+ e^-$) [50], which is discussed in relation to the SLAC E-144 experiment in Sect. 6.1.⁶

It is worth noting that the various mechanisms for pair production can be thought of as analogous to the various mechanisms for ionisation. Firstly, we may relate the Breit-Wheeler process ($\gamma\gamma' \rightarrow e^+ e^-$)—that of interest in Chap. 6 of this work—to the photoelectric effect; each corresponds to the case $n = 1$ and involves a strict energy threshold, below which it is not possible for the process to take place. The multi-photon Breit-Wheeler process is analogous to multi-photon ionisation, for which $n \gtrsim 1$ and the contribution of each photon can be included perturbatively. This regime is characterised by $\xi \ll 1$ in the case of pair production, or $\gamma \gg 1$ in the case of ionisation, where $\gamma = \sqrt{Im_e}/eE_0$ is the Keldysh parameter and I the ionisation energy (c.f. Eq. 2.41) [55]. In the opposite limit ($\xi \gg 1$ or $\gamma \ll 1$), tunnelling processes—namely the Schwinger effect and tunnelling or field ionisation—dominate and the rate for each has an exponential dependence on the strength of the applied field. This effectively corresponds to the limit $n \rightarrow \infty$ and, in the case of pair production, a *non-perturbative* approach is required to accurately describe the dynamics of the system [56].

⁶The multi-photon Breit-Wheeler process has been referred to simply as the Breit-Wheeler process by various authors [51–54]. In this work, the Breit-Wheeler process or Breit-Wheeler pair production always refer to the two-photon process.

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