CHAPTER 2

Double- and Single-Sided Risk Measures

2.1 Double-Sided Measures of Volatility, Variance, and Beta

Traditional specifications of volatility, variance, covariance, and beta form the basis of the CAPM and related branches of finance.[1] Since portfolios are typically constructed with multiple assets or asset classes, it is useful to speak of covariance between returns on an asset (or an entire asset class) and market-wide returns:

$$\sigma(a,m) = \mathrm{cov}(a,m) = E[(x_a - \mu_a)(x_m - \mu_m)] = \langle (x_a - \mu_a)(x_m - \mu_m) \rangle$$

where a indicates the asset or asset class, m indicates the market as a whole, x_p indicates returns on either the asset-specific or the market-wide portfolio ($p \in \{a, m\}$), and μ_p indicates mean returns on either portfolio. For compactness in notation, I shall henceforth use angle brackets, $\langle\,\rangle$, to express the expectation operator, or mean:

$$\langle f(x) \rangle = E[f(x)] = \mu_{f(x)}$$

The variance of a single distribution can be understood as a special case of covariance, where the two variables are identical:

$$\sigma(p,p) = \sigma_p^2 = \mathrm{cov}(p,p) = \left\langle \left(x_p - \mu_p\right)\left(x_p - \mu_p\right) \right\rangle = \left\langle \left(x_p - \mu_p\right)^2 \right\rangle$$

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Volatility, or standard deviation, is the positive square root of variance:

$$\sigma_p = \sqrt{\left\langle \left(x_p - \mu_p\right)^2 \right\rangle}$$

Normalizing covariance according to the product of asset-specific and market-wide volatility yields the correlation between those returns:

$$\rho(a,m) = \frac{\text{cov}(a,m)}{\sigma_a \sigma_m} = \frac{\langle (x_a - \mu_a)(x_m - \mu_m) \rangle}{\sqrt{\left\langle (x_a - \mu_a)^2 \right\rangle \left\langle (x_m - \mu_m)^2 \right\rangle}}$$

Squaring the correlation yields the coefficient of determination, more popularly known as r^2, or r-squared:

$$r^2 = \rho(a,m)^2 = \frac{\langle (x_a - \mu_a)(x_m - \mu_m) \rangle^2}{\left\langle (x_a - \mu_a)^2 \right\rangle \left\langle (x_m - \mu_m)^2 \right\rangle}$$

The product of (1) the ratio of asset-specific volatility to market-wide volatility and (2) the correlation between returns on that asset and market-wide returns is the beta of that asset:

$$\beta_a = \frac{\sigma_a}{\sigma_m} \rho(a,m) = \frac{\sigma_a}{\sigma_m} \cdot \frac{\text{cov}(a,m)}{\sigma_a \sigma_m} = \frac{\text{cov}(a,m)}{\sigma_m^2} = \frac{\langle (x_a - \mu_a)(x_m - \mu_m) \rangle}{\left\langle (x_m - \mu_m)^2 \right\rangle}$$

When beta is broken down into these components, it is readily understood as *correlated relative volatility*.[2]

2.2 Single-Sided Risk Measures

Traditional, two-tailed risk measures give dangerous guidance during bear markets because they implicitly assume that returns are normally distributed and because they treat upside and downside volatility as equal constituents of risk.[3] Skewed returns and fatter-than-normal tails reveal departures from normality in, respectively, the third and fourth moments of the distribution of returns.[4]

Beyond this basic descriptive case against the conventional CAPM, a persuasive behavioral model of finance begins with the recognition that risk on either side of expected returns may exert pressure of different sorts and unequal magnitudes on investor psychology.[5] Investors "are subject to sentiment," or "belief[s] about future cash flows and investment risks that [are] not justified by the facts at hand."[6]

Theories of behavioral finance become necessary only in the presence of uninformed investors and noise traders.[7] A "market composed solely of information traders" is a market "where price efficiency and the CAPM hold," where "[r]isk premia are determined solely by beta and distribution of returns on the market portfolio," and where option prices[8] and the term structure of bonds[9] follow mathematically beautiful models reflecting comparably rational assumptions about those corners of the financial marketplace.[10] Although "noise trad[ing] weaken[s] the relation between security returns and beta," it also "create[s] a positive conditional correlation between abnormal returns and beta."[11] Betting against noise traders through arbitrage is theoretically possible, but expensive, risky, and often ineffective in practice.[12] As behavioral anomalies exert "steady and forceful" pressure upon "the twin paradigms of price efficiency and the CAPM," a corresponding need arises for a "behavioral theory of capital asset prices and the volume of trade."[13]

As an outgrowth of Louis Jean-Baptiste Bachelier's early twentieth-century work,[14] mathematical finance has served as the leading embodiment of econophysics and its deepest reservoir of scientific insights. The earliest approaches to mathematical finance assumed that asset pricing proceeds according to the random walk, that Brownian motion and the Wiener process suffice to describe the cross section of stock prices.[15] Fama and French identified predictable departures from such beautiful but brittle modeling of complex financial behavior. In varying degrees of departure from Eugene Fama's own efficient market hypothesis, temporally and spatially imperfect diffusion of market information may generate significant but systematic violations of the random walk in ways that carefully bifurcated, "baryonic" subcomponents of beta can measure. Mindful that the distinct psychology of upside gain and downside loss may imply distinct relationships of risk to asset pricing on either side of expected returns, I now turn to a consideration of single-sided risk measures.

The conventional CAPM's descriptive and behavioral pitfalls have been recognized, if not effectively addressed, since the earliest days of mathematical finance.[16] In grudging acceptance of the computational limitations of their time, the pioneers of mathematical finance adopted traditional, two-tailed risk measurements as a statistical shortcut.[17] Vastly improved computation and greater mathematical sophistication have created multiple ways to measure risk on either side of a target return.

Contemporary financial literature speaks freely of semivariance, semideviation, semicovariance, semicodeviation, and single-sided beta. Leading contributions include those by William Hogan and James Warren (1974),[18] Vijay Bawa and Eric Lindenberg (1977),[19] and W.V. Harlow and Ramesh Rao (1989).[20] *Postmodern Portfolio Theory: Navigating Abnormal Markets and Investor Behavior*, another book in this series, devotes greater attention to diverse descriptions of single-sided risk.[21]

This book's baryonic model of beta will adopt Javier Estrada's 2002 specification of conditional covariance on the downside of expected return as the product of *two* conditional shortfalls: that of returns on an asset relative to its mean, and that of the market-wide portfolio relative to expected market-wide returns:[22]

$$\sigma_{-}(a, m) = \langle (x_a - \mu_a \mid x_a < \mu_a) \cdot (x_m - \mu_m \mid x_m < \mu_m) \rangle$$

where $\langle\,\rangle$ again is the expectation operator. Estrada's definition of downside covariance satisfies the reflexive property that characterizes ordinary variance: the covariance of returns on a specific asset and market-wide returns is equal to the covariance of market-wide returns and returns on that asset, $\text{cov}(a, m) = \text{cov}(m, a)$.[23] This specification avoids the "problematic" suggestion "that covariance between securities *i* and *j* is different from" covariance "between securities *j* and *i*."[24]

To facilitate the calculation of conditional, single-sided versions of deviation, variance, covariance, correlation, and beta, I adopt the organizational logic of Javier Estrada's downside risk framework,[25] as extended by Andrew Ang's recognition of "relative upside beta" and "relative downside beta."[26] Specifying semivariance and semideviation carries the added benefit of generalizing conventional CAPM and reducing that model into a special case of mean-semivariance analysis.[27] At a minimum, projections based on semivariance and semideviation can do no worse than those based on conventional CAPM as a special case.[28] We define upside and downside covariances between two portfolios, *p* and *q*, as conditional functions:

$$\sigma_{+}(p, q) = \text{cov}\left(p, q \mid x_p > \mu_p, x_q > \mu_q\right)$$
$$\sigma_{-}(p, q) = \text{cov}\left(p, q \mid x_p < \mu_p, x_q < \mu_q\right)$$

In the case of downside covariance, risk increases only when *both* portfolios fail to meet their mean returns: $x_p < \mu_p$, $x_q < \mu_q$.[29] Downside covariance as a measure of risk increases only when asset-specific returns fall below their mean *and* returns on the market as a whole falls below the market-wide mean.

It may be easier to understand semicovariances in terms of expected values:

$$\sigma_+(p,q) = \left\langle \max\left[\left(x_p - \mu_p\right),0\right] \cdot \max\left[\left(x_q - \mu_q\right),0\right]\right\rangle$$
$$\sigma_-(p,q) = \left\langle \min\left[\left(x_p - \mu_p\right),0\right] \cdot \min\left[\left(x_q - \mu_q\right),0\right]\right\rangle$$

Upside or downside semivariance within a single portfolio is merely a special case of the corresponding form of semicovariance:

$$\sigma^2_{p,+} = \mathrm{var}\left(p \mid x_p > \mu_p\right) = \left\langle \max\left[\left(x_p - \mu_p\right),0\right]^2\right\rangle$$
$$\sigma^2_{p,-} = \mathrm{var}\left(p \mid x_p < \mu_p\right) = \left\langle \min\left[\left(x_p - \mu_p\right),0\right]^2\right\rangle$$

Taking the square root of these values immediately yields upside and downside semideviations:

$$\sigma_{p,+} = \sqrt{\mathrm{var}\left(p \mid x_p > \mu_p\right)} = \sqrt{\left\langle \max\left[\left(x_p - \mu_p\right),0\right]^2\right\rangle}$$
$$\sigma_{p,-} = \sqrt{\mathrm{var}\left(p \mid x_p < \mu_p\right)} = \sqrt{\left\langle \min\left[\left(x_p - \mu_p\right),0\right]^2\right\rangle}$$

Upside and downside semicovariances between asset-specific and market-wide portfolios are other special cases of general semicovariance:

$$\begin{aligned}\sigma_+(a,m) &= \mathrm{cov}(a,m \mid x_a > \mu_a, x_m > \mu_m)\\ &= \langle \max[(x_a - \mu_a),0] \cdot \max[(x_m - \mu_m),0]\rangle\\ \sigma_-(a,m) &= \mathrm{cov}(a,m \mid x_a < \mu_a, x_m < \mu_m)\\ &= \langle \min[(x_a - \mu_a),0] \cdot \min[(x_m - \mu_m),0]\rangle\end{aligned}$$

Dividing each form of semicovariance among asset-specific and market-wide portfolios by the product of the corresponding form of semideviation produces upside and downside semicorrelations:

$$\rho_+(a,m) = \frac{\sigma_+(a,m)}{\sigma_{a,+}\,\sigma_{m,+}} = \frac{\mathrm{cov}_+(a,m)}{\sigma_{a,+}\,\sigma_{m,+}}$$
$$\rho_+(a,m) = \frac{\langle \max[(x_a - \mu_a),0] \cdot \max[(x_m - \mu_m),0]\rangle}{\sqrt{\left\langle \max[(x_a - \mu_a),0]^2\right\rangle \cdot \left\langle \max[(x_m - \mu_m),0]^2\right\rangle}}$$

$$\rho_-(a,m) = \frac{\sigma_-(a,m)}{\sigma_{a,-}\,\sigma_{m,-}} = \frac{\text{cov}_-(a,m)}{\sigma_{a,-}\sigma_{m,-}}$$

$$\rho_-(a,m) = \frac{\langle \min[(x_a - \mu_a), 0] \cdot \min[(x_m - \mu_m), 0] \rangle}{\sqrt{\left\langle \min[(x_a - \mu_a), 0]^2 \right\rangle \cdot \left\langle \min[(x_m - \mu_m), 0]^2 \right\rangle}}$$

Squaring these values produces the coefficient of determination, or *r*-squared, above and below the mean return:

$$r_+^2 = \rho_+(a,m)^2 = \frac{\langle \max[(x_a - \mu_a), 0] \cdot \max[(x_m - \mu_m), 0] \rangle^2}{\left\langle \max[(x_a - \mu_a), 0]^2 \right\rangle \cdot \left\langle \max[(x_m - \mu_m), 0]^2 \right\rangle}$$

$$r_-^2 = \rho_-(a,m)^2 = \frac{\langle \min[(x_a - \mu_a), 0] \cdot \min[(x_m - \mu_m), 0] \rangle^2}{\left\langle \min[(x_a - \mu_a), 0]^2 \right\rangle \cdot \left\langle \min[(x_m - \mu_m), 0]^2 \right\rangle}$$

Reassembling these single-sided measures into upside and downside beta is now a straightforward exercise in applying the basic definition of beta as correlated relative volatility:[30]

$$\beta_a = \frac{\sigma_a}{\sigma_m}\rho(a,m) = \frac{\sigma_a}{\sigma_m} \cdot \frac{\text{cov}(a,m)}{\sigma_a \sigma_m} = \frac{\text{cov}(a,m)}{\sigma_m^2} = \frac{\langle (x_a - \mu_a)(x_m - \mu_m) \rangle}{\left\langle (x_m - \mu_m)^2 \right\rangle}$$

Multiplying upside and downside semicorrelations by the ratio of upside or downside semideviation for the asset-specific portfolio to upside or downside semideviation for the entire market, as appropriate, produces upside and downside beta:

$$\beta_+ = \frac{\sigma_{a,+}}{\sigma_{m,+}}\rho_+(a,m) = \frac{\sigma_{a,+}}{\sigma_{m,+}} \cdot \frac{\sigma_+(a,m)}{\sigma_{a,+}\sigma_{m,+}} = \frac{\text{cov}_+(a,m)}{\sigma_{m,+}^2}$$

$$\beta_+ = \frac{\langle \max[(x_a - \mu_a), 0] \cdot \max[(x_m - \mu_m), 0] \rangle}{\left\langle \max[(x_m - \mu_m), 0]^2 \right\rangle}$$

$$\beta_- = \frac{\sigma_{a,-}}{\sigma_{m,-}}\rho_-(a,m) = \frac{\sigma_{a,-}}{\sigma_{m,-}} \cdot \frac{\sigma_-(a,m)}{\sigma_{a,-}\sigma_{m,-}} = \frac{\text{cov}_-(a,m)}{\sigma_{m,-}^2}$$

$$\beta_- = \frac{\langle \min[(x_a - \mu_a), 0] \cdot \min[(x_m - \mu_m), 0] \rangle}{\left\langle \min[(x_m - \mu_m), 0]^2 \right\rangle}$$

2.3 The Trigonometry of Semideviation

A surprisingly easy and elegant generalization connects measures of semideviation to trigonometry.[31] Some sources misleadingly describe the relationship between upside and downside volatility as one of the simple arithmetic although "the lower semideviation" is equal to "half the standard deviation" in a purely symmetrical distribution of returns.[32] Proper specification of semivariance and semideviation demonstrates otherwise. Upside and downside semideviations are related to standard deviation according to the Pythagorean theorem.

Recall the general definitions of upside and downside semicovariances between two portfolios, p and q:

$$\sigma_+(p,q) = \mathrm{cov}\big(p,q \mid x_p > \mu_p, x_q > \mu_q\big)$$
$$\sigma_-(p,q) = \mathrm{cov}\big(p,q \mid x_p < \mu_p, x_q < \mu_q\big)$$

It should be evident from this definition that upside and downside covariances are straightforwardly *additive*.[33] In other words, overall covariance is the sum of upside and downside covariances:

$$\mathrm{cov}(p,q) = \sigma(p,q) = \sigma_+(p,q) + \sigma_-(p,q)$$

Since the variance of a single distribution is merely a special case of covariance, where both variables are the same, the same additive relationship holds for upside and downside semivariances:

$$\mathrm{cov}(p,p) = \sigma_p^2 = \sigma_{p,+}^2 + \sigma_{p,-}^2$$

Volatility in any of its guises is the positive square root of the corresponding form of variance. This insight confirms what should be evident from the foregoing equation: The relationship between upside and downside *semideviations* is exactly that of the legs of a right triangle to the hypotenuse under the Pythagorean theorem. The sum of the squares of the upside and downside semideviations is equal to the square of standard deviation, or overall variance. Or more simply:

$$\sigma^2 = \sigma_-^2 + \sigma_+^2$$

$$\sigma = \sqrt{\sigma_-^2 + \sigma_+^2}$$

The applicability of the Pythagorean theorem to semideviation subjects single-sided measures of volatility to the entire apparatus of trigonometry. This property proves extremely useful for evaluating asymmetrical financial returns. Trigonometric tools enable us to evaluate the relationship between upside (σ_+) and downside (σ_-) semideviations in interesting and useful ways. To the extent that financial returns are negatively skewed,[34] we may expect downside semideviation to exceed its upside counterpart.

The ratio of upside to downside semideviation provides a crude gauge of asymmetry in volatility: σ_+ / σ_-. The Pythagorean relationship between standard deviation and upside and downside semideviations enables us to express asymmetry in volatility in angular terms. The angle formed by the downside semideviation and the standard deviation, θ, can be derived from the ratio between semideviation and standard deviation:

$$\cos\theta = \frac{\sigma_-}{\sigma}$$

$$\theta = \cos^{-1}\left(\frac{\sigma_-}{\sigma}\right)$$

Equivalently, in terms making use of upside semideviation:

$$\theta = \sin^{-1}\left(\frac{\sigma_+}{\sigma}\right) = \tan^{-1}\left(\frac{\sigma_+}{\sigma_-}\right)$$

2.4 The Behavioral Implications of Single-Sided Risk Measures

The development of single-sided risk measures—upside and downside volatility, covariances, and correlations—facilitates the testing of hypotheses reflecting market and behavioral factors that may change when returns cross above *or* below critical thresholds. Although the original impetus for devising single-sided risk measures arose from the intuition that downside risk is the true driver of investor expectations, these measures apply with equal force to either side of mean returns.

To name just one possibility, the explicit specification of upside volatility, covariance, and correlation in § 2.2 supports research into the risks that lurk when returns *exceed* investor expectations. Perhaps surprisingly, upside potential poses a behavioral risk in its own right. Investor preferences for positively skewed investments offering lottery-like payouts pose a particularly treacherous pitfall.[35] Tennessee Williams called this phenomenon "the catastrophe of success."[36] Investors and institutions shred their investment plans in the presence (or even the mere anticipation) of upside gain.[37]

Other variations on the theme of the catastrophe of success abound. Properly measured upside volatility may improve portfolio performance. For instance, one source promotes upside semideviation as a measure of active portfolio managers' performance that does not punish performance exceeding a benchmark rate of return.[38] This criticism is often leveled at the "information ratio," which is "the ratio of the expected annual residual return to the annual volatility of the residual return."[39] In an exercise that may be colorfully described as beta hedging, other sources encourage portfolio managers to combine stocks with relatively low downside beta (to temper exposure to declining markets) and stocks with relatively high upside beta (to capture potential gains in rising markets).[40]

Econophysics is not explicitly behavioral. At least in the first instance, econophysics seeks answers in economic fundamentals before embracing explanations resting exclusively on human psychology. Nevertheless, certain applications of econophysics do have behavioral implications. Financial markets are descriptively abnormal, and the investors whose preferences drive those markets are notoriously irrational. We should not expect the conventional capital asset pricing model, or any of its symmetrically specified components, to provide an accurate description of financial markets. *A fortiori*, any expectation that symmetrical beta models irrational investor behavior is even more forlorn.

Single-sided risk measures alleviate the pressure on mathematical finance to predict asset prices and to anticipate (if not neutralize) investor psychology. Devising distinct measures of upside and downside volatility, covariances, and correlations facilitates the testing of hypotheses reflecting market and behavioral factors that may change as returns cross-critical thresholds.

Risk measures with clear physical interpretations provide readily understandable, easily quantifiable, and statistically verifiable support or contradiction for intuitions about risk management and portfolio design. The range of potential applications includes the identification and containment of systemic risk among interrelated financial institutions.[41] A quest for the psychological roots of financial behavior requires the elaboration of even more comprehensive approaches, such as prospect theory and a generalized higher-moment capital asset pricing model. Examining the psychophysics of finance thus leads to a more complete formulation of the econophysics of beta.

Remarkably, this book's approach to econophysics provides persuasive, perhaps even compelling, explanations for anomalies such as Fama and French's three-factor model, the low-volatility anomaly, the equity premium puzzle, and short-term price continuation anomalies such as momentum and post-earnings announcement drift without resort to more sophisticated tools such as a generalized higher-moment capital asset pricing model,[42] the Yilmaz-Diebold model of volatility transmission,[43] error correction through cointegration,[44] or wavelet analysis.[45] The econophysics of baryonic beta opens the door to more explicitly behavioral accounts of abnormal markets and irrational investors, such as prospect theory,[46] SP/A theory,[47] and behavioral portfolio theory.[48] Baryonic beta also offers considerable value in its own right. Bifurcating beta into its constituent "subatomic" particles offers great explanatory power merely through a fuller specification of a two-moment model known to all academic experts, requiring no more algorithmic firepower than is available to most financial professionals.

Notes

1. See, e.g., Fischer Black, *Capital Market Equilibrium with Restricted Borrowing*, 45 J. Bus. 444–455 (1972); Fischer Black, Michael C. Jensen & Myron S. Scholes, *The Capital Asset Pricing Model: Some Empirical Tests*, *in* Studies in the Theory of Capital Markets 79–122 (Michael C. Jensen ed., 1972); John Lintner, *Security Prices, Risk and Maximal Gains from Diversification*, 20 J. Fin. 587–615 (1965); John Lintner, *The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets*, 73 Rev. Econ. & Stats. 13–37 (1965); Jan Mossin, *Equilibrium in a Capital Asset Market*, 34 Econometrica 768–783 (1966); William F. Sharpe, *Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*, 19 J. Fin. 425–442 (1964); Jack L. Treynor, *Toward a Theory of Market Value of Risky Assets*, *in* Asset Pricing and Portfolio Performance: Models, Strategy and Performance Metrics 15–22 (Robert A. Korajczyk ed., 1999); Jack L. Treynor & Fischer Black, *Corporate Investment Decisions*, *in* Modern Developments in Financial Management 310–327 (Stewart C. Myers ed., 1976). See generally Bernell K. Stone, Risk, Return, and Equlilibrium: A General Single-Period Theory of Asset Selection and Capital Market Equilibrium (1970); Eugene F. Fama & Kenneth R. French, *The Capital Asset Pricing Model: Theory and Evidence*, 18:3 J. Econ. Persp. 25–46 (Summer 2004).

2. See Martin L. Leibowitz, Anthony Bova & P. Brett Hammond, The Endowment Model of Investing: Return, Risk, and Diversification 14 (2010) (defining beta as "the correlation between the asset (or portfolio) return and the market return, multiplied by the ratio of their volatilities"); Michael B. Miller, Mathematics and Statistics for Financial Risk Management 198, 213, 292 (2nd ed. 2014) (defining beta as the product of correlation between the returns on two assets and the ratio of their volatilities); Shannon P. Pratt & Roger J. Grabowski, Cost of Capital: Applications and Examples 305–306 (4th ed. 2010).
3. See James S. Ang & Jess H. Chua, *Composite Measures for the Evaluation of Investing Performance*, 14 J. Fin. & Quant. Analysis 361–384 (1979); Robert C. Klemkosky, *The Bias in Composite Performance Measures*, 8 J. Fin. & Quant. Analysis 505–514 (1973); Hendrik Scholz, *Refinements to the Sharpe Ratio: Comparing Alternatives for Bear Markets*, 7 J. Asset Mgmt. 347–357 (2007).
4. See *supra* § 1.2, text accompanying notes 61–62.
5. See generally, e.g., Brian R. Bruce, *Reflections on 25 Years of Behavioral Finance*, 26:1 J. Investing 131–135 (Spring 2017); Nicholas Barberis & Richard H. Thaler, *A Survey of Behavioral Finance*, *in* Handbook of the Economics of Finance 1052–1090 (George M. Constantinides, Milton Harris & René M. Stulz eds., 2003).
6. Malcolm Baker & Jeffrey Wurgler, *Investor Sentiment in the Stock Market*, 21 J. Econ. Persp. 129–151, 129 (2007). See generally J. Bradford DeLong, Andrei Shleifer, Lawrence H. Summers & Robert J. Waldmann, *Noise Trader Risk in Financial Markets*, 98 J. Pol. Econ. 703–738 (1990).
7. See Hersh Shefrin & Meir Statman, *Behavioral Capital Asset Pricing Theory*, 29 J. Fin. & Quant. Analysis 323–349, 323 (1994). See generally Fischer Black, *Noise*, 41 J. Fin. 529–543 (1986).
8. See generally Fischer Black & Myron S. Scholes, *The Pricing of Options and Corporate Liabilities,* 81 J. Pol. Econ. 637–654 (1973); Robert C. Merton, *The Theory of Rational Option Pricing*, 4 Bell J. Econ. 141–183 (1973).
9. See generally John C. Cox, Jonathan E. Ingersoll, Jr. & Stephen A. Ross, *A Theory of the Term Structure of Interest Rates*, 53 Econometrica 385–408 (1985); Stephen J. Brown & Philp H. Dybvig, *The Empirical Implications of the Cox, Ingersoll, Ross Theory of the Term Structure of Interest Rates*, 41 J. Fin. 617–630 (1986); Roger H. Brown & Stephen M. Schaefer, *The Term Structure of Real Interest Rates and the Cox, Ingersoll, and Ross Model*, 35 J. Fin. Econ. 3–42 (1994).
10. Shefrin & Statman, *Behavioral Capital Asset Pricing Theory*, *supra* note 7, at 323.

11. Id. at 346. See generally Allan W. Kleidon, *Anomalies in Financial Economics: Blueprint for Change?* 59 J. Bus. 469–499 (1986).
12. See generally Andrei Shleifer & Robert Vishny, *The Limits of Arbitrage*, 52 J. Fin. 35–55 (1997).
13. Shefrin & Statman, *Behavioral Capital Asset Pricing Theory*, *supra* note 7, at 323.
14. See, e.g., Louis Jean-Baptiste Bachelier, Théorie de la Spéculation (1900); Louis Jean-Baptiste Bachelier, Calcul des Probabilités (1912); Louis Jean-Baptiste Bachelier, Le Jeu, la Chance, et le Hasard (1914); cf. James Owen Weatherall, The Physics of Wall Street: A Brief History of Predicting the Unpredictable 10–11 (2013) (reporting that Bachelier's thesis at La Sorbonne was poorly received because he was trying to apply mathematics to a field with which mathematicians of his time were unfamiliar).
15. See generally Peter Richmond, Jürgen Mimkes & Stefan Hutzler, Econophysics and Physical Economics §§ 5.1–5.4, at 46–51 (2013); id. § 7.7, at 74–75; Sitabhra Sinha, Arnab Chatterjee, Anirban Chakraborti & Bikar K. Chakrabarti, Econophysics: An Introduction §§ 2.1.4–2.1.5, at 23–27 (2011); M.F.M. Osborne, *Brownian Motion in the Stock Market*, Operations Research, March/April 1959, at 145–173, *reprinted in* The Random Character of Stock Prices 100–128 (Paul H. Cootner ed., 1964).
16. See generally James Ming Chen, Postmodern Portfolio Theory: Navigating Abnormal Markets and Investor Behavior §§ 3.1–3.4, at 27–38 (2016).
17. See, e.g., Fred D. Arditti, *Risk and the Required Return in Equity*, 22 J. Fin. 19–36 (1967) (analyzing the relationship between expected return and skewnewss in the distribution of returns); Merton H. Miller & Myron S. Scholes, *Rates of Return with Relation to Risk: A Reexamination of Some Recent Findings*, *in* Studies in the Theory of Capital Markets, *supra* note 1, at 47–78 (subjecting one capital asset pricing model to testing in response to asymmetry in the distribution of returns).
18. William W. Hogan & James M. Warren, *Toward the Development of an Equilibrium Capital-Market Model Based on Semivariance*, 9 J. Fin. & Quant. Analysis 1–11 (1974).
19. Vijay S. Bawa & Eric B. Lindenberg, *Capital Market Equilibrium in a Mean-Lower Partial Moment Framework*, 5 J. Fin. Econ. 189–200 (1977).
20. See W.V. Harlow & Ramesh K.S. Rao, *Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence*, 24 J. Fin. & Quant. Analysis 285–311 (1989).

21. See generally Chen, Postmodern Portfolio Theory, *supra* note 16, §§ 5.1–5.5, at 59–78.
22. See Javier Estrada, *Systematic Risk in Emerging Markets: The D-CAPM*, 3 Emerging Mkts. Rev. 365–377, 368 (2002).
23. See id. at 369–370 & n.2; Javier Estrada, *Mean-Semivariance Behavior: Downside Risk and Capital Asset Pricing*, 16 Int'l Rev. Econ. & Fin. 169–185, 174 (2007); Hsin-Jung Tsai, Ming-Chi Chen & Chih-Yuan Yang, *A Time-Varying Perspective on the CAPM and Downside Betas*, 29 Int'l Rev. Econ. & Fin. 440–454, 441 (2014).
24. Don U.A. Galagedera, *An Alternative Perspective on the Relationship Between Downside Beta and CAPM Beta*, 8 Emerging Mkts. Rev. 4–19, 7 (2007).
25. See Estrada, *Downside Risk and Capital Asset Pricing*, *supra* note 22, at 171; Estrada, *Systematic Risk in Emerging Markets*, *supra* note 23, at 367.
26. See Andrew Ang, Joseph Chen & Yuhang Xing, *Downside Risk*, 19 Rev. Fin. Stud. 1191–1239, 1199–1200 (2006) (introducing "two additional measures" beyond ordinary beta: relative upside beta and relative downside beta).
27. See Estrada, *Systematic Risk in Emerging Markets*, *supra* note 22, at 366; Bawa & Lindenberg, *supra* note 19, at 191, 198.
28. See Javier Estrada, *Mean-Semivariance Behaviour: An Alternative Behavioural Model*, 3 J. Emerging Mkt. Fin. 231–248, 242 (2004) (validating this analytical observation through empirical data).
29. See Estrada, *Downside Risk and Capital Asset Pricing*, *supra* note 23, at 173; Estrada, *Systematic Risk in Emerging Markets*, *supra* note 23, at 369–370.
30. See sources cited *supra* note 2.
31. For further discussion of trigonometric extensions of single-sided risk measures, see Chen, Postmodern Portfolio Theory, *supra* note 16, §§ 6.4–6.7, at 85–93.
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33. See, e.g., Estrada, *An Alternative Behavioural Model*, *supra* note 28, at 231, 237 (contrasting the straightforwardly additive nature of semivariance from the slightly more complicated relationship of upside to downside semideviation); Estrada, *Downside Risk and Capital Asset Pricing*, *supra* note 23, at 177 n.4 (same).
34. See generally § 1.3 (especially sources cited *supra* Chap. 1 note 61).
35. See James Ming Chen, *Momentary Lapses of Reason: The Psychophysics of Law and Behavior*, 2016 Michigan St. L. Rev. 607–642, 626–636.

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