

Chapter 2

Background

In this chapter, we present the mathematical foundations of quantum logic and computation which are required to make this book self-contained.¹ The respective descriptions are kept brief; readers wishing an in-depth introduction are referred to the respective literature containing more comprehensive descriptions such as e.g. [NC00, Mer07].

We begin in Sect. 2.1 by providing an introduction to Boolean logic with a particular focus on reversible logic, i.e. reversible Boolean functions. The latter play an important role for quantum computation. Afterwards, the basics of quantum logic and circuits are provided in Sects. 2.2 and 2.3, respectively.

2.1 Boolean Logic

Logic computations can be defined as functions over Boolean variables. More precisely:

Definition 2.1 A *Boolean function* over the standard Boolean algebra $\mathbb{B} = \{0, 1\}$ is a mapping $f: \mathbb{B}^n \rightarrow \mathbb{B}$ with $n \in \mathbb{N}$, where $\mathbb{B}^n$ is the n -fold Cartesian product of $\mathbb{B}$, i.e. $\mathbb{B}^n = \mathbb{B} \times \dots \times \mathbb{B}$. A function f is defined over its *primary input* variables $X = \{x_1, x_2, \dots, x_n\}$ and, hence, is also denoted by $f(x_1, x_2, \dots, x_n)$.

Definition 2.2 A *multi-output Boolean function* is a mapping $f: \mathbb{B}^n \rightarrow \mathbb{B}^m$ with $n, m \in \mathbb{N}$. More precisely, it is a system of Boolean functions $f_i(x_1, x_2, \dots, x_n)$.

¹The reader is assumed to be familiar with basic concepts of set theory and linear algebra. Additionally, some basic knowledge about group theory is required to completely understand the proofs in Sect. 7.2.3.

The respective functions f_i ($1 \leq i \leq m$) are also denoted as *component functions* or *primary outputs*. The set of all Boolean functions with n inputs and m outputs is denoted by $\mathcal{B}_{n,m} = \{f \mid f: \mathbb{B}^n \rightarrow \mathbb{B}^m\}$.

Multi-output functions can also be denoted as n -input, m -output functions or $n \times m$ functions. The precise mapping defining a Boolean function or multi-output Boolean function can be described by a truth table or in terms of Boolean algebra with expressions formed over the variables from X and operations like $\wedge$ (AND), $\vee$ (OR), $\oplus$ (XOR), and $\neg$ (NOT).

An important subset of multi-output Boolean functions highly relevant to quantum computing is constituted by reversible functions which are defined as follows:

Definition 2.3 A multi-output function $f \in \mathcal{B}_{n,m}$ is termed *reversible* if it is a bijection, i.e.

- its number of inputs is equal to the number of outputs (i.e. $n = m$) and
- it maps each input pattern to a unique output pattern (i.e. performs a permutation of the set of input patterns).

A Boolean function that is not reversible is termed *irreversible*.

Example 2.1 Table 2.1(a) shows the truth table of a 3-input, 2-output function representing a 1-bit adder. This function is irreversible, since $n \neq m$. The function in Table 2.1(b) is also irreversible since, although the number n of inputs is equal to the number m of outputs, the mapping is not a permutation; e.g. both inputs 000 and 001 map to the output 000. In contrast, the 3×3 function shown in Table 2.1(c) is reversible, since each input pattern maps to a unique output pattern.

The input/output mapping of a (reversible) Boolean function can also be represented in terms of a characteristic function.

Definition 2.4 The characteristic function for a Boolean function $f \in \mathcal{B}_{n,m}$ is defined as $\chi_f: \mathbb{B}^n \times \mathbb{B}^m \rightarrow \mathbb{B}$ where $\chi_f(x, y) = 1$ if, and only if, f maps the input pattern determined by the first n inputs of χ_f to the output pattern determined by the remaining m inputs, i.e. $f(x) = y$.

Table 2.1 Sample Boolean functions

(a) Irrev. (Adder)					(b) Irreversible						(c) Reversible					
x_1	x_2	x_3	f_1	f_2	x_1	x_2	x_3	f_1	f_2	f_3	x_1	x_2	x_3	f_1	f_2	f_3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	1	0	1	0	0	1	0	0	0	0	0	1	1	0	1
0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0
0	1	1	1	0	0	1	1	0	1	1	0	1	1	0	1	1
1	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	1
1	0	1	1	0	1	0	1	1	0	1	1	0	1	0	0	0
1	1	0	1	0	1	1	0	1	1	1	1	1	0	1	1	0
1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1

				Inputs										
				x_3	x_2	x_1	000	001	010	011	100	101	110	111
Outputs	x_3	x_2	x_1	000	0	0	0	0	0	0	1	0	0	
	001	0	0	0	0	0	0	0	1	0	0	0	0	
	010	0	0	1	0	0	0	0	0	0	0	0	0	
	011	0	0	0	1	0	0	0	0	0	0	0	0	
	100	1	0	0	0	0	0	0	0	0	0	0	0	
	101	0	1	0	0	0	0	0	0	0	0	0	0	
	110	0	0	0	0	0	0	0	0	0	1	0	0	
	111	0	0	0	0	0	0	0	0	0	0	1	1	

Fig. 2.1 Matrix representation of the reversible function from Table 2.1(c)

Finally, permutation matrices are a common representation for reversible Boolean functions, especially in the context of quantum computation.

Definition 2.5 A reversible Boolean function with n variables describes a permutation σ of the set $\{0, \dots, 2^n - 1\}$. This permutation can also be described using a *permutation matrix*, i.e. a $2^n \times 2^n$ matrix $\mathbf{P} = [p_{i,j}]_{2^n \times 2^n}$ with $p_{i,j} = 1$ if $i = \sigma(j)$ and 0 otherwise, for all $i, j = 0, \dots, 2^n - 1$. Each column (row) of the matrix represents one possible input pattern (output pattern) of the function. If $p_{i,j} = 1$, then the input pattern corresponding to column j maps to the output pattern corresponding to row i .

Example 2.2 The reversible Boolean function defined by the truth table from Table 2.1(c) is also represented by the permutation matrix shown in Fig. 2.1.

The above concepts can readily be extended from the Boolean domain to the multiple-valued case. Here, variables can take values in $\{0, \dots, r - 1\}$, where r is called the radix. Then, a truth table for a multiple-valued function over n variables has r^n rows. The concept of reversibility remains the same, i.e. a multiple-output multiple-valued function is reversible if it has the same number of inputs and outputs and each input pattern maps to a unique output pattern. Such a function can be represented by an $r^n \times r^n$ permutation matrix. The entries of the matrix are 0's and 1's since they denote correspondence in the mapping and not logical values.

2.2 Quantum Logic

Next we consider the preliminaries on quantum logic. We begin by introducing the means for describing and measuring quantum systems in Sect. 2.2.1. This is followed by an introduction of the mathematical description of quantum operations and their

effect on quantum systems in Sect. 2.2.2. Finally, we consider the realization of complex quantum operations in terms of quantum circuits and review gate libraries proposed for this purpose in Sect. 2.3.

2.2.1 Qubits and Measurement

Quantum systems are composed of *qubits*. Analogously to conventional bits, a qubit can represent a (Boolean) 0 or 1, but also superpositions of the two. More formally:

Definition 2.6 A *qubit* is a two-level quantum system, described by a two-dimensional Hilbert space. A basis of this *state space* is given by the two orthogonal quantum states $|0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ which are used to represent the Boolean values 0 and 1. Thus, the state of a qubit may always be written as

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle,$$

where α_0 and α_1 are complex numbers with $|\alpha_0|^2 + |\alpha_1|^2 = 1$, i.e. as a so-called *superposition* of the (computational) *basis states* $|0\rangle$ and $|1\rangle$.

The quantum state of a single qubit is denoted by the vector $\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$. The state space of larger quantum systems composed of $n > 1$ qubits is given by the tensor product of the state spaces of the individual qubits. Accordingly, there are 2^n basis states ($|0 \dots 00\rangle, |0 \dots 01\rangle, \dots, |1 \dots 11\rangle$), and the system can be in an arbitrary superposition² of these states

$$|\psi\rangle = \sum_{k=0}^{2^n-1} \alpha_k |k\rangle$$

for complex numbers $\alpha_0, \dots, \alpha_{2^n-1}$ with $\sum_{k=0}^{2^n-1} |\alpha_k|^2 = 1$. The corresponding state vector $(\alpha_k)_{0 \leq k \leq 2^n-1}$ has dimension 2^n . Note, that two states $|\psi_1\rangle$ and $|\psi_2\rangle$ actually represent the same physical state of a quantum system if, and only if, there is a complex number $\phi \neq 0$, such that $|\psi_1\rangle = e^{i\phi} \cdot |\psi_2\rangle$. In this case, we say that $|\psi_1\rangle$ and $|\psi_2\rangle$ differ by a *global phase shift*.³

Due to physical limitations there is no possibility to precisely read-out the state of a quantum system, i.e. to obtain the so-called *amplitudes* α_k . In fact, it is only possible to perform a *measurement* which destroys the quantum state of the system and causes it to collapse to some basis state where the probability for measuring basis state $|k\rangle$ is given by $|\alpha_k|^2$.

²More precisely, we say that a system is in *superposition* if more than one α_k is non-zero.

³Strictly speaking, the state spaces are *projective* Hilbert spaces where each state $|\psi\rangle$ is represented by a *ray* of vectors, i.e. a set $\{c \cdot |\psi\rangle \mid c \in \mathbb{C}, |c| = 1\}$.

Example 2.3 Consider the following three different states of a qubit: $|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $|\psi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $|\psi_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ i \end{pmatrix}$. For all three states, the probability of measuring $|0\rangle$ is the same as measuring $|1\rangle$: $|\alpha_0|^2 = |\alpha_1|^2 = \frac{1}{2}$.

In this regard, the amplitudes are often represented in *polar coordinates* by $\alpha_k = p_k \cdot e^{i\theta_k}$, i.e. as a decomposition into the *modulus* $p_k = |\alpha_k| \in [0, 1]$ (determining the probability of measuring the corresponding basis state $|k\rangle$) and the so-called *phase (factor)* $\theta_k \in [0, 2\pi)$.

Example 2.4 Consider again the three qubit states from Example 2.3. While the modulus is always the same ($\frac{1}{\sqrt{2}}$), we observe the three different phases 0 ($e^{i \cdot 0} = 1$), $\frac{\pi}{2}$ ($e^{i \cdot \pi/2} = i$), and π ($e^{i \cdot \pi} = -1$). Moreover, $|\psi_3\rangle = i \cdot |\psi_1\rangle$ which means that $|\psi_1\rangle$ and $|\psi_3\rangle$ are equal up to global phase and, thus, there is no way to find out which of them is actually present. In contrast, $|\psi_2\rangle$ —though having the same moduli—can in principle be distinguished from these states as shown later on in Example 2.6.

2.2.2 Quantum Operations

According to the postulates of quantum mechanics, the time-dependent evolution of a quantum system (so-called *Hamiltonians*) can be described by linear transformation operations on the state space satisfying the following

Definition 2.7 A *quantum operation* over n qubits can be represented by a unitary transformation matrix, i.e. a $2^n \times 2^n$ matrix $\mathbf{U} = [u_{i,j}]_{2^n \times 2^n}$ with

- each entry $u_{i,j}$ assuming a complex value and
- the inverse $\mathbf{U}^{-1}$ of $\mathbf{U}$ being the *adjoint* $\mathbf{U}^\dagger$, i.e. the conjugate transpose matrix of $\mathbf{U}$, such that $\mathbf{U} \cdot \mathbf{U}^\dagger = \mathbf{U}^\dagger \cdot \mathbf{U}$ yields the identity matrix.

According to this definition, every quantum operation is reversible since the matrix $\mathbf{U}$ defining any quantum operation is invertible and $\mathbf{U}^{-1} = \mathbf{U}^\dagger$ describes the inverse quantum operation.

Commonly used quantum operations include the *Hadamard* operation H (setting a qubit into a balanced superposition) as well as the *rotation* operations R_x , R_y , and R_z (parametrized by a rotation angle θ). Important special cases of the latter are the *NOT* operation $X = R_x(\pi)$ which flips the basis states $|0\rangle$ and $|1\rangle$, and the *phase shift* operations T , $S = T^2$, and $Z = S^2 = T^4$ which correspond to $R_z(\theta)$ rotations with the rotation angles $\theta = \frac{\pi}{4}$, $\frac{\pi}{2}$, and π , respectively.⁴

⁴More precisely, the equalities only hold up to *global phase*, i.e. a multiplicative scalar factor.

The corresponding transformation matrices are defined as

$$\begin{aligned} \mathbf{H} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{R}_x(\theta) = \begin{pmatrix} \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix}, \\ \mathbf{R}_y(\theta) &= \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ +\sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix}, \quad \mathbf{R}_z(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}, \text{ and} \\ \mathbf{X} &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad \mathbf{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned}$$

In any case, the columns of the transformation matrix correspond to the output state vectors that result when applying the respective operation to basis states as inputs. Thus, the entry $u_{i,j}$ in the i th row and j th column of the matrix describes the amplitude mapping from the input basis state $|j\rangle$ to the output basis state $|i\rangle$.

Example 2.5 Applying the Hadamard operation H to the input basis state $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, i.e. computing $\mathbf{H} \cdot |1\rangle$ yields a new quantum state

$$|\psi'\rangle = \mathbf{H} \cdot |1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

for which $\alpha_0 = -\alpha_1 = \frac{1}{\sqrt{2}}$, i.e. the input amplitude of $|1\rangle$ is split up equally to the output amplitude of $|0\rangle$ and $|1\rangle$. Measuring the qubit would either lead to a Boolean 0 or a Boolean 1 with a probability of $|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$ each. This computation represents one of the simplest quantum computers—a single-qubit random number generator.

Example 2.6 Applying H to the basis state $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, i.e. computing $\mathbf{H} \cdot |0\rangle$ yields the state $|\psi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ from Example 2.3. Similarly, we obtain that $\mathbf{H} \cdot |\psi_1\rangle = |0\rangle$ and $\mathbf{H} \cdot |\psi_2\rangle = |1\rangle$. This means that we can apply the Hadamard operation in order to distinguish between $|\psi_1\rangle$ and $|\psi_2\rangle$ which is not possible with a direct measurement (cf. Example 2.4).

Besides these operations that work on a single *target qubit*, there are also controlled operations on multiple qubits. The state of the additional *control qubits* determines which operation is performed on the target qubit. More precisely, the operation on the target qubit is executed if, and only if, the control qubits with a *positive* control are in the $|1\rangle$ -state and the ones with a *negative* control are in the $|0\rangle$ -state.

Example 2.7 A very prominent example for multiple-qubit operations is the *controlled NOT* (CNOT) operation on two qubits which applies the NOT operation to the target if, and only if, the control qubit is in the $|1\rangle$ -state (positive control). On the matrix level, it is defined by

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

which can be expressed as $\mathbf{I} \oplus \mathbf{X}$ using the direct matrix product notation “ $\oplus$ ”.

The above concepts can readily be extended to the multiple-valued case of so-called *qudits* where there are r basis states $|0\rangle, \dots, |r-1\rangle$ and the state of a qudit can be written as $|x\rangle = \sum_{i=0}^{r-1} \alpha_i |i\rangle$ with $\sum_{i=0}^{r-1} |\alpha_i|^2 = 1$. The transformations in this case are described by $r^n \times r^n$ unitary matrices.

2.3 Quantum Circuits and Gate Libraries

Complex quantum operations (e.g. described by a quantum algorithm) are usually realized by a set of elementary, low-level quantum operations or quantum computational instructions (e.g. represented in terms of *quantum gates* g_i) that are performed in a predetermined serial fashion as a *quantum circuit* $G = g_1 \dots g_l$ with $1 \leq i \leq l$. On the matrix level, such a composition of quantum gates can be expressed by a direct matrix multiplication of the corresponding gate matrices.

Example 2.8 Consider the 3-qubit quantum circuit shown in Fig. 2.2. It realizes a 2-controlled NOT operation known as the *Toffoli gate* (depicted in Fig. 2.3). More precisely, the basis states of the third qubit are swapped if, and only if, the first and second qubits are in the $|1\rangle$ -state. Following the established conventions, horizontal lines represent qubits. Quantum operations $\boxed{H}$ (Hadamard operation, cf. Example 2.5), $\boxed{T}$ (phase shift by $\frac{\pi}{4}$), $\bullet \oplus$ (*CNOT* with a positive control), etc. are applied successively from left to right.

Example 2.9 With the concepts introduced above, we are now able to understand the details of the circuit from Fig. 1.1 (cf. Example 1.1 on page 5). Recall, that it

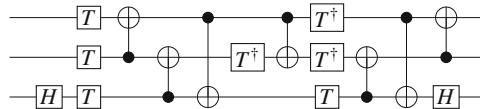


Fig. 2.2 A quantum circuit

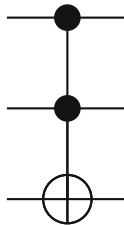


Fig. 2.3 The Toffoli gate

allows for teleporting qubit states over potentially large distances. Let us follow the precise evolution of the quantum state.

The first two gates transform the lower pair of qubits from $|00\rangle$ to $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$ and finally to $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Note that this state has an important property: measuring a $|0\rangle$ on one qubit enforces a $|0\rangle$ on the other one (and analogously for $|1\rangle$). Thus, the two gates create *entanglement* between the two qubits, since measuring one of it completely determines the state of the other. The third gate of the circuit encodes the information of the first qubit on the entangled pair (thereby entangling all qubits of the system). More precisely, if $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$, we obtain for the state of the entire system:

$$\alpha_0|0\rangle \cdot \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) + \alpha_1|1\rangle \cdot \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$$

Applying the final Hadamard gate to this state yields

$$\frac{\alpha_0}{2}(|0\rangle + |1\rangle) \cdot (|00\rangle + |11\rangle) + \frac{\alpha_1}{2}(|0\rangle - |1\rangle) \cdot (|10\rangle + |01\rangle).$$

Now, in order to establish ψ on the third qubit, we have to destroy the entanglement by measuring the other qubits. More precisely, measuring the first qubit results in the quantum state

$$\frac{\alpha_0}{\sqrt{2}} \cdot (|00\rangle + |11\rangle) \pm \frac{\alpha_1}{\sqrt{2}} \cdot (|10\rangle + |01\rangle),$$

where the minus occurs only for the case that $M_1 = |1\rangle$ is measured. This possible phase shift is taken care of with the final controlled Z gate. Finally, we observe that measuring a $|0\rangle$ on the second qubit yields the desired state on the third qubit (modulo the phase shift), while measuring a $|1\rangle$ leads to $\alpha_0|1\rangle \pm \alpha_1|0\rangle$. In the latter case, the basis states $|0\rangle$ and $|1\rangle$ have to be swapped. This is achieved by the controlled NOT that is applied depending on the result of M_2 . Overall, the circuit demonstrates how to transfer the state of one qubit to another qubit without a direct interaction between them. Even more, the third qubit could have possibly been separated from the others after applying the first two gates and only the measurement results M_1 and M_2 , i.e. two classical bits (indicated by double lines), are required to establish the desired state on the distant qubit. In fact, the two controlled quantum gates that are applied for this purpose do not depend on $|\psi\rangle$, but only (!) on the measurement results.

Note that inverting a given quantum circuit is a relatively easy task in most cases. In fact, most quantum gates are self-inverse (like the Hadamard, NOT, and CNOT gate) or determining the inverse is straight-forward (e.g. for rotations by taking the negated rotation angle). Consequently, the inverse quantum circuit is obtained by (1) reversing the gate order and (2) replacing each gate by its inverse.

An important sub-class of quantum circuits is constituted by *reversible circuits*. These circuits are composed of (classical) reversible gates, such that the

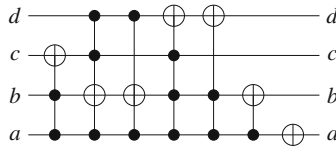


Fig. 2.4 A reversible circuit

transformation matrix for each gate and the entire circuit is a permutation matrix. The most established type of reversible gate is the *multiple-controlled Toffoli* (MCT) gate. MCT gates consist of a possibly empty set of control qubits and a single target qubit on which a NOT gate is performed if, and only if, all control qubits carry the appropriate value. Note that the MCT gate library also includes the special cases of NOT (empty set of controls) and *controlled NOT* (CNOT) gates (singleton set of controls). For historical reasons and for brevity, we simply use the term *Toffoli gate* to refer to the 2-controlled Toffoli gate.

Example 2.10 Consider the reversible circuit shown in Fig. 2.4 that realizes a modulo 10 counter. More precisely, if the input (taken as a binary number $dcb a_2$) is less than the (decimal) number 10, then the output is incremented and taken modulo 10, i.e. the output is $((dcb a + 1) \% 10)_2$. For binary numbers greater than or equal to 10, the circuit does not behave according to this formula. However, it is clear that—due to reversibility—the output also has to be greater than or equal to 10.

It is a common phenomenon that, as in the previous example, reversible circuits have a meaningful output only for a subset of the input patterns. This is because many reversible functions are obtained from irreversible functions by adding extra inputs/outputs in order to ensure a bijective mapping, as already seen in Example 2.1. This process is called *embedding* and is discussed in more detail later in Sect. 7.1.1.

Several libraries of elementary quantum operations have been proposed in the literature. From a theoretical point of view, the set of arbitrary one-qubit gates (unitary 2×2 matrices) and a single 2-qubit gate, namely the controlled NOT (CNOT) gate, is sufficient to approximate any quantum operation to an arbitrary precision [BBC+95]. Libraries with this property are called *universal*. To this end, it has been shown that a much smaller set of operations, comprising only CNOT, H , and T operations (forming the so-called *Clifford+T library*), is also universal [BMP+00]. Moreover, these quantum operations can in principle be implemented in a fault-tolerant fashion [BMP+00]—a crucial property since quantum computing is inherently very sensitive to environmental factors such as radiation and, hence, fault-tolerance is even more important than for conventional systems. However, the technologies that are actually used for the physical realization of quantum circuits support a small subset of quantum operations only. This is discussed in more detail in Sect. 6.1.

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Logic

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