

Chapter 2

Setting the Stage: Review of Previous Results

2.1 Introduction

It is well-known that general relativity has some peculiar features, introducing difficulties (these are not encountered in other field theories, e.g., non-Abelian gauge theories) when we try to derive the Einstein's equations from the Einstein-Hilbert action principle (see, e.g. Chap. 6 of [1]). Based on the usual theoretical prejudice [2], it is often conjectured that the field equations which are second order in derivatives of the coordinates, are obtained from an action which is quadratic in the first derivatives of the dynamical variables. However, the situation is entirely different in the case of gravity. Due to equivalence principle, the generally covariant Lagrangian for gravity is forced to have at least second order derivatives of the metric. The usual and the simplest choice, R , has a special structure (viz. linearity in second derivatives), which allows one to obtain the field equations which still involves only second derivatives of the metric, if (and only if) we fix metric *and* its derivatives on the boundary. This arises since one can separate out the second derivatives in the Lagrangian leading to a total divergence, which becomes a surface term in the action. Variation of the surface term does not contribute to the field equations however affects the boundary value problem and hence we need to set both the variations of the metric *and* its derivatives to be zero on the boundary.

The main conceptual difficulty with this program—which makes the gravitational action different from those in other field theories—is that we need to fix both the dynamical variable and its derivative at the boundary in order to arrive at the field equations. This procedure encounters a few difficulties at the conceptual level. Let the spacetime region we are considering is between two spacelike surfaces and also assume that all quantities of interest fall off at spatial infinities. If we now fix both the metric and its derivative on the earlier spacelike hypersurface, the Einstein's equations should tell us the corresponding values on the latter hypersurface. Thus it is clear that we do not really have the freedom to fix arbitrary values for the metric and its derivatives on both the boundaries. Another option is to add a boundary term to the gravitational action, such that the variation of this term exactly cancels the terms

with variation of the normal derivative. An well-known example of such a boundary term is the Gibbons-Hawking-York counter-term [3, 4], even though it is not a unique choice [5]. We should emphasize that addition of a boundary term, though leads to a generally covariant gravitational action but it turns out to be foliation dependent.

So far we have been concerned about general relativity alone. However it is well known that unlike the kinematics (which can be described in an elegant manner using the principles of equivalence and principle of general covariance), there is no natural principle to determine the field equations for gravity. Hence it seems reasonable to study all possible field equations, which are second order in the independent variable, the metric. The criteria—field equations should be second order differential equations in the dynamical variable—automatically fixes the gravitational Lagrangian to be Lanczos-Lovelock Lagrangian (this was first pointed out by Lanczos and Lovelock [6–10]). The field equations for Lanczos-Lovelock gravity also follow from an action in higher dimensions, which is polynomial in the curvature tensor, reducing to Einstein-Hilbert Lagrangian—*uniquely*—when $D = 4$. In higher dimensions the Lanczos-Lovelock Lagrangian has additional higher curvature terms and hence differ from the Einstein-Hilbert Lagrangian. Due to existence of higher curvature terms, the Lanczos-Lovelock Lagrangian also exhibits a rich structure in comparison to the Einstein-Hilbert Lagrangian.

In this chapter, we will mainly review earlier results and shall collect all the essential relations that will be used extensively in the subsequent chapters in relation to both Einstein-Hilbert and Lanczos-Lovelock gravity.

2.2 A Fresh Look at the Einstein-Hilbert Action

Use of g_{ab} and its conjugate momentum as the dynamical variables, leads to the conclusion—the surface variation *involves only the variation of the canonical momenta*. This is a non-trivial result and arises *only because* the surface and bulk terms of the Einstein-Hilbert Lagrangian are intertwined with each other. This, in turn, implies that one might be able to treat the Einstein-Hilbert action as a momentum space action. This suggests that it may be better to deal with gravitational physics in the space of the canonical momenta corresponding to the metric rather than in the space of the metric itself. It was first demonstrated in [11], that this program will also work with a new dynamical variable, a tensor density, $f^{ab} = \sqrt{-g}g^{ab}$. The variable $f^{ab} = \sqrt{-g}g^{ab}$ has already been used in earlier literatures [12–15], but have not attracted much attention in recent years. In this section we will content ourselves with introduction of these new canonically conjugate variables in general relativity and their various geometric properties following [11]. In particular we will present Einstein-Hilbert action as a momentum space action in terms of these variables. We will also discuss Noether current and gravitational momentum for the Einstein-Hilbert action [16] and its relation to this set of canonically conjugate variables.

2.2.1 On the Structure of the Einstein-Hilbert Action

In this section, we shall rapidly review various existing ideas and relations present in the literature associated with the action principle for general relativity, which corresponds to the Einstein-Hilbert action and is given by

$$A_{\text{EH}} \equiv \int_{\mathcal{V}} d^4x \sqrt{-g} R. \quad (2.1)$$

where R corresponds to the Ricci scalar and $\mathcal{V}$ stands for the four-dimensional space-time volume of interest. It is useful to define a quantity Q_{ab}^{cd} such that,

$$Q_{cd}^{ab} = \frac{1}{2} \delta_{cd}^{ab}; \quad \delta_{cd}^{ab} = \delta_c^a \delta_d^b - \delta_d^a \delta_c^b \quad (2.2)$$

where δ_{cd}^{ab} stands for the completely antisymmetric determinant tensor. Using this tensor Q_{cd}^{ab} , it is possible to write the Einstein-Hilbert action (see, Eq. (2.1)) in an equivalent form as:

$$A_{\text{EH}} \equiv \int_{\mathcal{V}} d^4x \sqrt{-g} Q_a^{bcd} R^a_{bcd} \quad (2.3)$$

$$= \int_{\mathcal{V}} d^4x \sqrt{-g} (L_{\text{quad}} + L_{\text{sur}}) \quad (2.4)$$

where we have defined the bulk Lagrangian (since this is quadratic in the connection, it is also referred to as the quadratic Lagrangian) and the surface Lagrangian respectively as

$$L_{\text{quad}} \equiv 2Q_a^{bcd} \Gamma_{dk}^a \Gamma_{bc}^k; \quad L_{\text{sur}} \equiv \frac{2}{\sqrt{-g}} \partial_c [\sqrt{-g} Q_a \Gamma_{bd}^a]. \quad (2.5)$$

The advantage of writing the Einstein-Hilbert action in this particular form is that it can be readily generalized to gravitational theories in more than 4 dimensions (for more details see Sect. 2.3). One of the striking feature of the above action stems from the fact that the variation of the bulk term alone can furnish the field equations (as explicitly demonstrated in Chap. 6 of [1]). Once the field equations and hence its solutions are obtained, which includes black hole solutions as well, one can evaluate the surface term on the horizon, which surprisingly reproduces the entropy of the horizon in the Euclidean sector. (In general it gives τTS where τ is the range of integration; we get S when $\tau = \beta$, the inverse temperature.) The fact that the surface term, which is not supposed to carry any information about the field equations, yields the entropy when integrated over a horizon is a direct hint that gravitational action principle contains information about horizon thermodynamics. An algebraic origin to this peculiar behavior lies in the fact that the bulk term L_{quad} and the surface term L_{sur} are not independent, but related to each other by the relation

$$\sqrt{-g}L_{\text{sur}} = - \left[\partial_c \left(g_{ab} \frac{\partial (\sqrt{-g}L_{\text{quad}})}{\partial (\partial_c g_{ab})} \right) \right]. \quad (2.6)$$

Since it relates a quantity on the surface with a quantity in the bulk, this relation has been termed “holographic” [17–19].

2.2.2 In Search of Alternative Variables in General Relativity

A natural question is whether g_{ab} is unique in providing us with a neat and clean variational principle for general relativity or are there other variables that does the same. It turns out that the Einstein-Hilbert action can be written in terms of a new variable $f^{ab} = \sqrt{-g}g^{ab}$ and we obtain the following structure [11]

$$\sqrt{-g}R = \sqrt{-g}L_{\text{quad}} - \partial_c \left[f^{ab} \frac{\partial (\sqrt{-g}L_{\text{quad}})}{\partial (\partial_c f^{ab})} \right] = \sqrt{-g}L_{\text{quad}} - \partial_c [f^{ab} N_{ab}^c] \quad (2.7)$$

where we have introduced the object N_{bc}^a to be the momentum conjugate to f^{ab} , since

$$N_{bc}^a \equiv \frac{\partial (\sqrt{-g}L_{\text{quad}})}{\partial (\partial_a f^{bc})} \quad (2.8)$$

$$= -\Gamma_{bc}^a + \frac{1}{2}(\Gamma_{bd}^d \delta_c^a + \Gamma_{cd}^d \delta_b^a) = [\mathcal{Q}_{be}^{ad} \Gamma_{cd}^e + \mathcal{Q}_{ce}^{ad} \Gamma_{bd}^e] \quad (2.9)$$

Note that it is symmetric in the lower two indices.

Next, the expressions for variation of the Einstein-Hilbert action in terms of the conjugate variables f^{ab} and N_{ab}^c becomes,

$$\delta(\sqrt{-g}R) = R_{ab} \delta f^{ab} + f^{ab} \delta R_{ab} = R_{ab} \delta f^{ab} - \partial_c [f^{ik} \delta N_{ik}^c], \quad (2.10)$$

In addition to simplifying the variation of the action, there are two main advantages of this conjugate pairs (f^{ab}, N_{ab}^c) . The first one is that many known expressions and formula simplify considerably as expressed in terms of these conjugate variables. Secondly and more importantly, the *variations of these variables on a horizon leads to a direct thermodynamical interpretation* (see [11] for details).

2.2.3 Einstein-Hilbert Action in Terms of the New Variables

Using these new pair of canonically conjugate variables (f^{ab}, N_{ab}^c) we will try to rewrite the bulk part of the Lagrangian. Substituting the connections in Eq. (2.5) by N_{ab}^c from Eq. (2.9) we obtain the bulk part of the Lagrangian to be:

$$\sqrt{-g}L_{\text{quad}} = g^{bd}N_{dj}^i N_{bl}^k \left[\delta_i^l \delta_k^j - \frac{1}{3} \delta_i^j \delta_k^l \right] = \frac{1}{2} N_{ab}^c \partial_c f^{ab} \quad (2.11)$$

with striking simplicity. Note that this exhibits the familiar ‘ $p\dot{q}/2$ ’ structure, which is a consequence of the fact that the Einstein-Hilbert Lagrangian is quadratic in $\dot{q}$. While for the surface term of the Einstein-Hilbert action we arrive at,

$$\sqrt{-g}L_{\text{sur}} = \partial_c(\sqrt{-g}V^c) = -\partial_c[f^{ab}N_{ab}^c]. \quad (2.12)$$

The above relation defines the object V^c , such that,

$$\sqrt{-g}V^c = -f^{ab}N_{ab}^c \quad (2.13)$$

Various other geometrical relations connecting curvature tensor components to the new set of conjugate variables (f^{ab} , N_{ab}^c) have been derived and explored in [11].

2.2.3.1 Hamilton’s Equations for General Relativity

In Lagrangian formulation of classical mechanics, variation of the action is carried out treating the dynamical variable q as independent, for the class of actions which do not depend on the higher time-derivatives of q . The momentum p on the other hand, is then defined as the partial derivative of the Lagrangian with respect to $\dot{q}$. In the Hamiltonian formulation one defines the Hamiltonian $H(q, p)$ through the following relation $L = p\dot{q} - H(q, p)$. In this case, fixing the variations of q at the boundary, yields the well-known Hamilton’s equations, which is referred to as modified Hamilton’s principle [20, 21].

Analogously there are two well-known variational principles in general relativity—one in which the variation of the gravitational action is carried out in terms of the variation of the metric. On the other hand, one can also consider the metric and the affine connection to be independent and hence varied separately, known as the Palatini variational principle [22]. We shall now outline a version of the Palatini variational principle in terms of the conjugate variables (f^{ab} , N_{ab}^c) for Einstein-Hilbert action following [11]. Using the variables (f^{ab} , N_{ab}^c), the gravitational Lagrangian density can be expressed as

$$\sqrt{-g}R = f^{ab}R_{ab} = f^{ab}(-\partial_c N_{ab}^c - N_{ad}^c N_{bc}^d + \frac{1}{3} N_{ac}^c N_{bd}^d). \quad (2.14)$$

The variation of $\sqrt{-g}R$ with respect to N_{ab}^c with fixed f^{ab} is given by

$$\delta(\sqrt{-g}R)|_{f^{ab}} = \left[\partial_c f^{ab} - 2f^{ad}N_{cd}^b + \frac{2}{3}f^{am}N_{dm}^d \delta_c^b \right] \delta N_{ab}^c - \partial_c(f^{ab}\delta N_{ab}^c) \quad (2.15)$$

Fixing N_{bc}^a at the boundary leads to the corresponding field equations, which can be obtained by equating the symmetrized coefficient of δN_{ab}^c to zero. These equations are

$$\partial_c f^{ab} = f^{ad} N_{cd}^b + f^{bd} N_{cd}^a - \frac{1}{3} f^{am} N_{dm}^d \delta_c^b - \frac{1}{3} f^{bm} N_{dm}^d \delta_c^a \quad (2.16)$$

yielding the desired relation between N_{ab}^c with f^{ab} and its derivative. The connection of the above relation with the standard result $\nabla_c g^{ab} = 0$ obtained from Palatini variational principle has been explored in [11].

At this stage it is possible to obtain an analogue of the Hamilton's equation $\dot{q} = \partial H / \partial p$. The analogy can be made more accurate by introducing a "Hamiltonian" as,

$$\mathcal{H}_g = f^{ab} (N_{ad}^c N_{bc}^d - \frac{1}{3} N_{ac}^c N_{bd}^d) \quad (2.17)$$

Comparing Eq. (2.17) with Eq. (2.11), we see that $\mathcal{H}_g = \sqrt{-g} L_{\text{quad}}$. Then, with the notional correspondence $f^{ab} \rightarrow q$ and $N_{ab}^c \rightarrow p$, one can immediately establish the following result:

$$\sqrt{-g} R \rightarrow -q \partial p - \mathcal{H}_g = \{p \partial q - \mathcal{H}_g\} - \partial(qp) = \sqrt{-g} (L_{\text{quad}} + L_{\text{sur}}) \quad (2.18)$$

Thus, the quadratic Lagrangian density used to derive the field equations can be reinterpreted as a Hamiltonian density. Equation (2.16) can then be rewritten as a Hamilton's field equation,

$$\partial_c f^{ab} = \frac{\partial \mathcal{H}_g}{\partial N_{ab}^c} \quad (2.19)$$

Proceeding by analogy with classical mechanics, one can also obtain the other Hamilton's equation of motion, viz., $\dot{p} = -\partial H / \partial q$. This can be achieved by considering the variation of the Einstein-Hilbert Lagrangian density with respect to f^{ab} , and can be expressed as,

$$\delta \left(\sqrt{-g} R \right) |_{N_{ab}^c} = R_{ab} \delta f^{ab} \quad (2.20)$$

Hence the field equations associated with variation of f^{ab} is equivalent to $R_{ab} = 0$, the vacuum field equations of Einstein. Referring back to Eq. (2.17), we see that this equation can also be re-expressed as

$$\partial_c N_{ab}^c = - \frac{\partial \mathcal{H}_g}{\partial f^{ab}} \quad (2.21)$$

yielding the second of the Hamilton's equations. However unlike the case in classical mechanics, where the momentum would be conserved in the absence of external forces, we observe that N_{ab}^c has the capability of driving its own change, thanks to the nonlinear nature of gravity.

2.2.3.2 Inclusion of Matter

The next natural step would be to consider the inclusion of the matter Lagrangian density $\sqrt{-g}L_m$, which can be accomplished by introducing a total Hamiltonian as

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_g - \sqrt{-g}L_m \quad (2.22)$$

If the first term $\mathcal{H}_g$ (incidentally this is equal to $\sqrt{-g}L_{\text{quad}}$) be considered to be kinetic, then it is natural to think of $\sqrt{-g}L_m$ as a potential term. We shall further assume that the matter Lagrangian density $\sqrt{-g}L_m$ under consideration depends only on f^{ab} and not on N_{ab}^c . In such a case, Eq. (2.16) retains its original form. Expressing everything in terms of the metric and its derivative it becomes clear that our assumption is equivalent to the fact that $\sqrt{-g}L_m$ does not depend on the derivatives of the metric. (This is similar in spirit to the case in classical mechanics when we consider velocity-independent potentials.)

Let us now provide the usual definition of the matter energy-momentum tensor as

$$T_{ab} = -\frac{2}{\sqrt{-g}} \frac{\partial(\sqrt{-g}L_m)}{\partial g^{ab}}, \quad (2.23)$$

from which one can obtain the following equality:

$$\frac{\partial\sqrt{-g}L_m}{\partial f^{ab}} = -\frac{1}{2}\left[T_{ab} - \frac{1}{2}Tg_{ab}\right] \equiv -\frac{1}{2}\bar{T}_{ab}, \quad (2.24)$$

where a new object $\bar{T}_{ab}$ has been defined, which bears to T_{ab} the same relation as G_{ab} bears to R_{ab} .

When matter is included the second equation of Hamiltonian, i.e., Eq. (2.21) gets modified to read

$$\partial_c N_{ab}^c = -\frac{\partial\mathcal{H}_g}{\partial f^{ab}} + \frac{\partial(\sqrt{-g}L_m)}{\partial f^{ab}} = -N_{ad}^c N_{bc}^d + \frac{1}{3}N_{ac}^c N_{bd}^d - \frac{1}{2}\bar{T}_{ab} \quad (2.25)$$

which is equivalent to the usual Einstein's field equations, $2R_{ab} = \bar{T}_{ab}$ [11]. Finally, we should mention that the surface variation contains only variations of the conjugate momentum N_{ab}^c and not of f^{ab} . Thus, we need to fix only the “momenta” N_{ab}^c at the boundary in order to obtain the correct field equations.

2.2.4 Noether Current

In this section, we shall relate the Noether current—arising from a differential geometric identity—with the variations of N_{bc}^a and f^{ab} . The conservation of J^a using

differential geometric identity is straightforward and was first presented in [23], here we will briefly outline it.

Let us start with an arbitrary vector field v^a and decompose $\nabla_a v_b$ into symmetric and antisymmetric part as follows,

$$S_{ab} \equiv \nabla_a v_b + \nabla_b v_a; \quad J_{ab} \equiv \nabla_a v_b - \nabla_b v_a \quad (2.26)$$

It is obvious from the antisymmetry of J^{ab} , that one can construct a conserved current J^a out of it by defining $J^a = \nabla_b J^{ab}$. The only remaining task is to express the conserved current in terms of the newly defined N_{ab}^c variable. This can be achieved by considering Lie derivative of the connection, which has the following expression [1],

$$\mathfrak{L}_v \Gamma_{bc}^a = \nabla_b \nabla_c v^a + R_{cmb}^a v^m \quad (2.27)$$

From Eq. (2.9) one can relate Lie variation of connection to Lie variation of N_{ab}^c . This immediately leads to the following expression,

$$\mathfrak{L}_v N_{bc}^a = [Q_{be}^{ad} \mathfrak{L}_v \Gamma_{cd}^e + Q_{ce}^{ad} \mathfrak{L}_v \Gamma_{bd}^e] \quad (2.28)$$

where Q_{cd}^{ab} is defined in Eq. (2.2). Now one can consider contraction of the above object with inverse metric g^{bc} leading to the following expression,

$$\begin{aligned} g^{bc} \mathfrak{L}_v N_{bc}^a &= 2Q^{adc}_e (\mathfrak{L}_v \Gamma_{cd}^e) = (g^{ac} \delta_e^d - g^{dc} \delta_e^a) (\nabla_c \nabla_d v^e + R_{dmc}^e v^m) \\ &= \nabla_b J^{ab} - 2R_m^a v^m \end{aligned} \quad (2.29)$$

which immediately leads to the expression for Noether current

$$J^a(v) = 2R_b^a v^b + g^{cd} \mathfrak{L}_v N_{cd}^a \quad (2.30)$$

Thus, given a vector field v^a one can construct the Noether current by almost *trivial* differential geometric manipulations—which *happens* to be the Noether current arising from diffeomorphism invariance of Einstein-Hilbert Lagrangian. We will try to emphasize the physical significance of this result in later chapters.

2.2.5 Gravitational Momentum

Another structure we want to associate with an arbitrary vector field v^a is the gravitational four-momentum density $P^a(v)$, defined—in the context of general relativity—as

$$P^a(v) = -Rv^a - g^{ij} \mathfrak{L}_v N_{ij}^a \quad (2.31)$$

From Eq. (2.30) we can substitute for the Lie variation term in Eq. (2.31), which relates the gravitational momentum and the Noether current as

$$J^a(v) = -P^a(v) + 2G_b^a v^b \quad (2.32)$$

The physical meaning of the gravitational momentum can be understood from the following result. (This was motivated and discussed in detail in [16, 23]. We will not repeat the motivation and logic behind this definition here). Consider the special case in which v^a is the velocity of an arbitrary observer, who will attribute to the matter, with energy momentum tensor T_{ab} , the momentum density $\mathcal{M}^a = -T_b^a v^b$. We would expect the total momentum associated with matter plus gravitation to be conserved [16] in nature, for all observers. This condition requires:

$$\begin{aligned} 0 &= \nabla_a (P^a + \mathcal{M}^a) = \nabla_a (-J^a + 2G_b^a v^b - T_b^a v^b) \\ &= \nabla_a (2G_b^a v^b - T_b^a v^b) = (2G_b^a - T_b^a) \nabla_a v^b \equiv \mathcal{S}^{ab} \nabla_a v_b \end{aligned} \quad (2.33)$$

where $\mathcal{S}^{ab} \equiv (2G^{ab} - T^{ab})$ is a symmetric tensor and in the last line we have used Bianchi identity and the fact that J^a and T_b^a are conserved. The above relation should hold for any normalised time-like vector field v^a , which requires $\mathcal{S}^{ab} = 0$, i.e., $G_{ab} = 8\pi T_{ab}$, which are the field equation for gravity. (This result should be obvious from the fact that $\nabla_a v_b$ can be chosen to be arbitrary at any event even for normalised timelike vector fields. A more formal proof, suggested by S. Date, goes as follows: Choose first v^a to be a normalized *geodesic* velocity field with $v^a v_a = -1$ and $v^a \nabla_a v^b = 0$. Then the most general $\mathcal{S}^{ab}$ which satisfies $\mathcal{S}^{ab} \nabla_a v^b = 0$ has the form $\mathcal{S}^{ab} = \alpha(X^a v^b + X^b v^a) + \beta v^a v^b$ with two arbitrary functions α and β and an arbitrary vector X^a which can be chosen without loss of generality to be purely spatial, i.e., $v_a X^a = 0$. Choose next the velocity field to be $u_a = -N \nabla_a t$. Using the form of $\mathcal{S}^{ab} \nabla_a u_b = 0$ leads to $\alpha = \beta = 0$. This immediately gives $\mathcal{S}^{ab} = 0$.) The gravitational momentum introduced above will find quiet a few interesting applications latter on.

2.3 Lanczos-Lovelock Gravity: A Brief Introduction

One key aspect for general relativity is that, it follows directly from the geometric properties of the Riemann curvature tensor—in particular the Bianchi identity plays a crucial role. Generalizing the curvature tensor by including higher curvature terms, if one imposes the criteria that the trace of its Bianchi derivative vanishes, leads to a divergence free second rank tensor. This tensor also uniquely leads to the Lanczos-Lovelock Lagrangian and agrees with the one obtained by variation of the Lanczos-Lovelock action [24]. In this section we will describe several algebraic and thermodynamic results related to Lanczos-Lovelock Lagrangian, which will be of

extensive use in later chapters. The importance of Lanczos-Lovelock gravity is two-fold. It generalizes Einstein-Hilbert action by introducing higher curvature terms, which can arise naturally in strong gravity regime (incidentally, the first correction to Einstein-Hilbert action, known as Gauss-Bonnet action was obtained earlier from a string theoretic point of view [25]). Secondly, several thermodynamic features related to horizons in Einstein-Hilbert action carry over to Lanczos-Lovelock theories of gravity. In emergent gravity paradigm thermodynamics of horizons appear as a key input and hence there is a natural extension of this paradigm to Lanczos-Lovelock gravity. We will be mainly concerned about the thermodynamic aspects of Lanczos-Lovelock gravity in later chapters of this thesis.

2.3.1 How Does Lanczos-Lovelock Gravity Comes About?

In the first part of this chapter, we have been working with Einstein-Hilbert action in four-dimensional spacetime. A natural question to ask in that context is, why spacetime has four dimensions? Even though there is no satisfactory answer to that question, most of the high-energy physics community believes in existence of extra dimension, exploiting which one can answer some subtle questions, which cannot be answered in the standard four-dimensional models. One such problem has to do with the large difference between scale of weak interaction to that of strong interaction, known in the literature as the hierarchy problem. It turns out that extra dimensional models can provide a *natural* solution to this fine-tuning problem [26–28].

Another natural query being—what about gravity in these higher-dimensional scenarios? One can assume that gravitational action is still given by the Einstein-Hilbert Lagrangian, however there is one caveat. The effect of these extra dimensions can only be felt at very high energies and one can ask at such high energies how legitimate it is to use the Einstein-Hilbert Lagrangian. There is a general consensus in the community that at such high energies the Einstein-Hilbert Lagrangian would be supplemented by higher curvature terms. In principle there can be a huge number of such higher curvature corrections gravitational action can have, e.g., $f(R)$, $g(R_{ab}R^{ab})$ and so on. Thus we need some physical guiding principles to find out the most natural candidate.

2.3.1.1 A General Lagrangian

Before going into the guiding principles, let us start with an action, which is an arbitrary function of the curvature tensor R^a_{bcd} and the metric g^{ab} in a D -dimensional spacetime. We will work with a single timelike coordinate, i.e., the total spacetime dimensions can be written as $1 + (D - 1)$. With imposition of all these conditions, the action functional for gravity turns out to be,

$$A = \int_{\mathcal{V}} d^D x \sqrt{-g} L(g^{ab}, R^a_{bcd}) \quad (2.34)$$

As we have already mentioned, the Lagrangian depends both on the curvature and the metric but not on the derivatives of the curvature. Given this Lagrangian, one can construct another fourth rank tensor P^{abcd} , obtained by taking derivative of L with respect to curvature tensor R_{abcd} . This tensor, which will turn out to be quite significant for our later purpose, can be written as

$$P^{abcd} = \left(\frac{\partial L}{\partial R_{abcd}} \right)_{g_{ij}} \quad (2.35)$$

having all the algebraic symmetry properties of the curvature tensor. Note that for Einstein-Hilbert action the tensor P^{abcd} exactly coincides with the tensor Q^{abcd} defined in Eq. (2.2). An analogue of the Ricci tensor in general relativity can also be constructed by the definition

$$\mathcal{R}^{ab} \equiv P^{aijk} R^b_{ijk}. \quad (2.36)$$

It is a straightforward exercise to show that for Einstein-Hilbert action $\mathcal{R}_{ab}$ is indeed the Ricci tensor. The field equations for this gravity theory can be obtained by varying the action in Eq. (2.34) with respect to the metric g^{ab} , leading to the following result:

$$\begin{aligned} \delta A &= \delta \int_{\mathcal{V}} d^D x \sqrt{-g} L \\ &= \int_{\mathcal{V}} d^D x \sqrt{-g} E_{ab} \delta g^{ab} + \int_{\mathcal{V}} d^D x \sqrt{-g} \nabla_j \delta v^j \end{aligned} \quad (2.37)$$

Alike the case for Einstein-Hilbert action one obtains a bulk variation term, leading to field equations and a surface variation term. The term E_{ab} in the bulk variation is the field equation term and δv^a corresponds to the boundary term. They are given by the following expressions [1]

$$\begin{aligned} E_{ab} &\equiv \frac{1}{\sqrt{-g}} \left(\frac{\partial \sqrt{-g} L}{\partial g^{ab}} \right)_{R_{abcd}} - 2 \nabla^m \nabla^n \left(\frac{\partial L}{\partial R^{amnb}} \right) \\ &= \mathcal{R}_{ab} - \frac{1}{2} g_{ab} L - 2 \nabla^m \nabla^n P_{amnb} \end{aligned} \quad (2.38)$$

$$\delta v^j = 2 P^{ibjd} \nabla_b \delta g_{di} - 2 \delta g_{di} \nabla_c P^{ijcd} \quad (2.39)$$

Note that for a general Lagrangian, P^{abcd} , being derivative of the Lagrangian with respect to R_{abcd} involves second order derivative of the metric through curvature tensor. Hence the term $\nabla^m \nabla^n P_{amnb}$ in the field equations contains fourth order derivatives of the metric. It is well known that field equations having more than second order derivatives of the metric (to be precise, time derivative) in general is

plagued with various instabilities, e.g., Ostrogradski instability, existence of tachyonic modes etc.

2.3.1.2 The Lanczos-Lovelock Lagrangian

So far we have been discussing the most general Lagrangian constructed out of metric and curvature. However as we have just witnessed such Lagrangians in general have field equations, which involve fourth order derivatives of the metric leading to instabilities. To avoid all these instabilities and to obtain a physically well motivated theory we will work with *only* those gravitational theories, for which field equations contain at *most* second derivatives of the metric.

Therefore, we need to impose certain restrictions on the form of the Lagrangian, presented in Eq. (2.34) in order to yield second order field equations. From the general field equations in Eq. (2.38) to obtain second order field equations, we must impose an extra condition on P^{abcd} , such that

$$\nabla_a P^{abcd} = 0. \quad (2.40)$$

Thus the problem of finding an action functional leading to a second order field equation reduces to finding scalar functions of curvature and metric such that Eq. (2.40) is satisfied. Such a scalar indeed exists and is unique; it is given [1, 10, 18, 29, 30] by the Lanczos-Lovelock Lagrangian in D dimensions, as

$$L = \sum_m c_m L_m = \sum_m \frac{c_m}{m} \frac{\partial L_m}{\partial R_{abcd}} R_{abcd} = \sum_m \frac{c_m}{m} P_{(m)}^{abcd} R_{abcd} \quad (2.41)$$

This Lagrangian L_m being a homogeneous function of R_{abcd} of order m can also be written as $L_m = Q_{(m)}^{abcd} R_{abcd}$, which can be used to identify $P_{(m)}^{abcd} = m Q_{(m)}^{abcd}$. This relation can also be obtained using explicit expression for P_{cd}^{ab} in terms of the curvature tensor and L_m as

$$P_{cd(m)}^{ab} = \frac{\partial L_m}{\partial R_{ab}^{cd}} = m \delta_{cdc_2d_2 \dots c_m d_m}^{aba_2b_2 \dots a_m b_m} R_{a_2b_2}^{c_2d_2} \dots R_{a_m b_m}^{c_m d_m} \equiv m Q_{cd(m)}^{ab} \quad (2.42)$$

This relation will be used extensively later. Like the Einstein-Hilbert Lagrangian, one can decompose the Lanczos-Lovelock Lagrangian in a quadratic part and a surface contribution. This decomposition can be written as,

$$\begin{aligned} \sqrt{-g} L &= \sqrt{-g} Q_a^{bcd} R_{bcd}^a \\ &= 2\sqrt{-g} Q_a^{bcd} \Gamma_{dk}^a \Gamma_{bc}^k + \partial_c [2\sqrt{-g} Q_a^{bcd} \Gamma_{ba}^c] \end{aligned} \quad (2.43)$$

$$= \sqrt{-g} L_{\text{quad}} + \sqrt{-g} L_{\text{sur}} \quad (2.44)$$

where for m th order Lanczos-Lovelock Lagrangian $Q_a^{bcd} = Q_a^{bcd}_{(m)}$, while for general Lagrangian $Q_a^{bcd} = \sum c_m Q_a^{bcd}_{(m)}$. Further note that due to complete antisymmetry of the determinant tensor in a D dimensional space-time we have the following restriction $2m \leq D$, otherwise the determinant tensor would vanish identically. Lanczos-Lovelock models at dimension $D = 2m$ are known as critical dimensions for a given Lanczos-Lovelock term. In these critical dimensions the variation of the action functional reduces to a pure surface term [31, 32]. Further, the tensor $\mathcal{R}_{ab}$ defined in Eq. (2.36) in the context of Lanczos-Lovelock gravity is indeed symmetric; but the result is nontrivial to prove (for this result and other properties of these tensors, see [33]). So far we have been dealing with geometrical properties of the Lovelock action, but for the rest of the thesis some thermodynamic constructs would be useful. We will now briefly introduce these thermodynamic ingredients for Lanczos-Lovelock gravity.

2.3.2 Noether Current and Entropy for Lanczos-Lovelock Gravity

The Noether current for Lanczos-Lovelock gravity can also be derived manipulating several differential geometric identity. It coincides with the one derived from diffeomorphism invariance of the action and has the following expression [1, 29, 33, 34]:

$$J^a(\xi) \equiv (2E^{ab}\xi_b + L\xi^a + \delta_\xi v^a) \quad (2.45)$$

From the property of the Noether current $\nabla_a J^a = 0$, we can define an antisymmetric tensor referred to as *Noether Potential* by the condition, $J^a = \nabla_b J^{ab}$. Using Eq. (2.38) we can substitute for the boundary term, leading to an explicit form for both the Noether current and Potential. These general expressions can be found in [1]. In the context of Lanczos-Lovelock theories, where $\nabla_a P^{abcd} = 0$, they are given by

$$J^{ab} = 2P^{abcd}\nabla_c \xi_d \quad (2.46)$$

$$J^a = 2P^{abcd}\nabla_b \nabla_c \xi_d = 2\mathcal{R}^{ab}\xi_b + 2P_k^{ija}\xi_{ij}\Gamma_{ij}^k \quad (2.47)$$

with Γ_{bc}^a being the metric compatible connection.

The Noether current has a direct thermodynamic interpretation as well. For Lanczos-Lovelock theories of gravity, assuming the zeroth law, one can derive the first law [35], from which one can identify the entropy associated with Lanczos-Lovelock gravity. Also the Noether charge computed from the Noether current corresponding to diffeomorphism symmetry will play the role of entropy when evaluated on null surfaces. Further this entropy is referred to as the Wald entropy associated with null surfaces in all Lanczos-Lovelock models. The corresponding entropy density (which, integrated over the horizon gives the entropy) is given by [35–44]

$$s = -\frac{1}{8}\sqrt{q}P^{abcd}\mu_{ab}\mu_{cd} \quad (2.48)$$

where $\mu_{ab} = u_a r_b - u_b r_a$ stands for the bi-normal, with the following normalization conditions, $u_a u^a = -1$ and $r_a r^a = 1$. Since the tensor P^{abcd} appears directly in the expression for Wald entropy, it is sometimes called the entropy tensor as well. The best way to get an intuitive feel for this result is to consider Einstein-Hilbert action, which can be obtained by considering $P^{abcd} = (1/2)(g^{ac}g^{bd} - g^{ad}g^{bc})$. For this case one will immediately obtain the entropy density to be $s = \sqrt{q}/4$, the standard Bekenstein entropy. We will use this expression for entropy density quite frequently in later chapters.

2.4 Construction of Gaussian Null Coordinates

We will now elaborate on the construction of a suitable system of coordinates associated with an arbitrary null surface. This construction will be used extensively in later chapters. Let us consider the four-dimensional spacetime $V^4 = M^3 \times R$, where M^3 is a compact three-dimensional manifold. We will consider spacetimes to be time orientable with null embedded hypersurfaces, which are diffeomorphic to M^3 with closed null generators. We take $\mathcal{N}$ to be such a null hypersurface with null generator [45, 46]. On this null surface $\mathcal{N}$ we can introduce spacelike two surface with coordinates (x_1, x_2) defined on them. The null geodesics generating the null hypersurface $\mathcal{N}$ goes out of this spacelike two surface. Thus we can use these null generators to define coordinates on the null hypersurface. The intersection point of these null geodesics with the spacelike two surface can be determined by the coordinates (x_1, x_2) , which then evolves along the null geodesic, which is parametrized by u , and label each point on the null hypersurface as (u, x_1, x_2) . The above system of coordinates readily identify three basis vectors: (a) the tangent to the null geodesics, $\ell = \partial/\partial u$, and (b) basis vectors on the two surface $\mathbf{e}_A = \partial/\partial x_A$.

Having fixed the coordinates on the null surface $\mathcal{N}$ we now move out of the surface using another set of null generators with tangent k^a satisfying the following constraints: (a) $k_a k^a = 0$, (b) $e_A^a k_a = 0$ and finally $\ell_a k^a = -1$. These null geodesics are taken to intersect the null surface at coordinates (u, x_1, x_2) and then move out with affine parameter r , such that any point in the neighborhood of the null surface can be characterized by four coordinates (u, r, x_1, x_2) . In this coordinate system, the null surface is given by the condition $r = 0$. This defines a coordinate system $\{u, r, x^A\}$ over the global manifold V^4 . This system of coordinates are formed in a manner analogous with Gaussian Normal Coordinate and hence is referred to as *Gaussian Null Coordinates (GNC)*.

Having set the full coordinate map near the null surface we now proceed to determine the metric elements in that region. Note that $\ell_a \ell^a = 0$ leads to $g_{uu} = 0$ on the null surface since $\ell = \partial/\partial u$. We also note that the basis vectors e_A^a have to lie on the null surface implying $\ell_a e_A^a = 0$ on $\mathcal{N}$, which leads to $g_{uA} = 0$. Also the metric on the two-surface is given by $g_{AB} = e_A^a e_B^b g_{ab}$, which we denote by q_{AB} . We also need the criteria that q_{AB} is positive definite with finite determinant ensuring invertibility

and non-degeneracy of the two metric. Thus the following metric components gets fixed to be:

$$\begin{aligned} g_{uu}|_{r=0} &= g_{uA}|_{r=0} = 0; \\ g_{AB} &= q_{AB} \end{aligned} \quad (2.49)$$

Let us now proceed to determine the other components of the metric. For this, we will use the vector $\mathbf{k} = -\partial/\partial r$ such that from $k^a k_a = 0$ we get $g_{rr} = 0$ throughout the spacetime manifold. Also from the criteria that the null geodesics are affinely parametrized by r we readily obtain $\partial_r g_{r\alpha} = 0$, where $\alpha = (u, x_1, x_2)$. Again, from the conditions $\ell^a k_a = -1$, we readily get $g_{ru} = 1$ and from $k_a e_A^a = 0$ we get $g_{rA} = 0$. From the criteria derived earlier showing $\partial_r g_{r\alpha} = 0$ we can conclude that the above two-metric coefficients are valid everywhere. Thus within the global region V^4 we can have smooth functions α and β_A such that, $\alpha|_{r=0} = (\partial g_{uu}/\partial r)|_{r=0}$ and $\beta_A|_{r=0} = (\partial g_{uA}/\partial r)|_{r=0}$. With these two identifications we have the following expression for the line element as

$$ds^2 = g_{ab} dx^a dx^b = -2r\alpha du^2 + 2dudr - 2r\beta_A dudx^A + q_{AB} dx^A dx^B \quad (2.50)$$

where q_{AB} is the two-dimensional metric representing the metric on the null surface. Note that the construction presented above is completely general; it can be applied in the neighborhood of any null hypersurface and, in particular, to the event horizon of a black hole and can also be generalized to D spacetime dimensions, with q_{AB} being the $(D - 2)$ -dimensional metric.

We will now introduce the time development vector ξ^a appropriate for this coordinate system as the one with the components $\xi^a = \delta_0^a$ in the GNC; that is:

$$\xi^a = (1, 0, 0, 0); \quad \xi_a = (-2r\alpha, 1, -r\beta_A) \quad (2.51)$$

It can be easily shown that ξ^a will be identical to the timelike Killing vector corresponding to the Rindler time coordinate if we rewrite the standard Rindler metric in the GNC form. Therefore, we can think of ξ^a as a natural generalization of the time development vector corresponding to the Rindler-like observers in the GNC; of course, it will not be a Killing vector in general. Since $\xi^2 = -2r\alpha$, we see that, in the $r \rightarrow 0$ limit, ξ^a becomes null. Given ξ^a , we can construct the four-velocity u_a for a comoving observer by dividing ξ^a by its norm $\sqrt{2r\alpha}$ obtaining:

$$u^i = \left(\frac{1}{\sqrt{2r\alpha}}, 0, 0, 0 \right); \quad u_i = \left(-\sqrt{2r\alpha}, \frac{1}{\sqrt{2r\alpha}}, \frac{-r\beta_A}{\sqrt{2r\alpha}} \right) \quad (2.52)$$

The form of u^i shows that the comoving observers can also be thought of as observers with $(r, x^A) = \text{constant}$. This proves to be convenient for probing the properties of the null surface.

This four-velocity, has the four-acceleration $a^i = u^j \nabla_j u^i$. The magnitude of the acceleration $\sqrt{a_i a^i}$, multiplied by the redshift factor $\sqrt{2r\alpha}$, has a finite result in the null limit, (i.e., $r \rightarrow 0$ limit):

$$Na|_{r \rightarrow 0} = \left(\sqrt{2r\alpha} \right) a|_{r \rightarrow 0} = \alpha - \frac{\partial_u \alpha}{2\alpha} \quad (2.53)$$

When the acceleration α varies slowly in time (i.e., $\partial_u \alpha / \alpha^2 \ll 1$) the second term is negligible and $Na \rightarrow \alpha$. The redshifted Unruh-Davies [47, 48] temperature associated with the $r = 0$ surface, as measured by $(r, x^A) = \text{constant}$ observer is given [23] by Eq. (2.53). We will call this temperature as the “acceleration temperature”.

We will next introduce the relevant null vectors associated with the GNC. Given the four-velocity u_a and four-acceleration a_i we can construct two null vectors $\bar{\ell}^i$ and $\bar{k}^i$ as:

$$\bar{\ell}^a = \frac{\sqrt{2r\alpha}}{2} (u^a + r^a); \quad \bar{k}^a = \frac{1}{\sqrt{2r\alpha}} (u^a - r^a) \quad (2.54)$$

where r_i is the unit vector in the direction of the acceleration, i.e., $r_i = a_i / a$. These two vectors $\bar{\ell}^i$ and $\bar{k}^i$ satisfy: $\bar{\ell}^2 = 0$, $\bar{k}^2 = 0$ and $\bar{\ell}^a \bar{k}_a = -1$ and we have the following components on the null surface:

$$\bar{\ell}_i \Big|_{r \rightarrow 0} = (0, 1, 0, 0) \quad (2.55)$$

$$\bar{k}_i \Big|_{r \rightarrow 0} = \left(-1, \frac{q_{AB} a^A a^B}{4r\alpha a^2}, -\frac{a_A}{\sqrt{2r\alpha} a} \right) \quad (2.56)$$

Since we are essentially interested only in the $r \rightarrow 0$ limit, it is more convenient to work with a simpler vector field $\ell_i \equiv \nabla_i r$ *everywhere*, which reduces to this $\bar{\ell}_i$ on the null surface and defines the natural null normal to the $r = 0$ surface as a limiting case. Similarly, we can introduce another vector k_a in place of $\bar{k}_a$ to simplify the computations. Using the non-uniqueness in the definition of $\bar{k}_a$, we can change it to another vector k_a such that,

$$\bar{k}_a = k_a + A \ell_a + B_A e_a^A \quad (2.57)$$

where e_a^A are basis vectors on the null surface and $\ell_a = \bar{\ell}_a$. From the property $\ell_a e_a^A = 0$ and $\ell^2 = 0$ we get $\ell_a k^a = -1$, since $\ell_a \bar{k}^a = -1$. The condition $k^2 = 0$, leads to a condition between A and B_A as: $2A = q_{CD} B^C B^D$. Choosing $A = (q_{AB} a^A a^B / 4r\alpha a^2)$ and $B_A = -(a_A / \sqrt{2r\alpha} a)$ leads to the simple form $k_a = (-1, 0, 0, 0)$. Thus the vector $\ell_a = \nabla_a r$ and the auxiliary $k_a = -\nabla_a u$ have the following components in the GNC:

$$\ell_a = (0, 1, 0, 0), \quad \ell^a = (1, 2r\alpha + r^2\beta^2, r\beta^A) \quad (2.58a)$$

$$k_a = (-1, 0, 0, 0), \quad k^a = (0, -1, 0, 0) \quad (2.58b)$$

Along with these two vectors we also have the vector ξ^a , which is the time development vector introduced earlier. Thus, through the GNC, we have introduced three vectors ℓ_a, k_a and ξ_a . The bi-normal associated with the null surface can be obtained in terms of ℓ_a and k_a as $\epsilon_{ab} = \ell_a k_b - \ell_b k_a$. It turns out that, for the $r = 0$ surface the non-affinity parameter, κ corresponding to the null normal ℓ^a is obtained from $\ell^b \nabla_b \ell^a = \kappa \ell^a$. Evaluation of which for the null normal in Eq. (2.58a) yields the non-affinity parameter to be: $\kappa = \alpha$. While the vector $k_a = -\nabla_a u$, is tangent to the ingoing null geodesic and is affinely parametrized, with affine parameter r . These null vectors will be used extensively as we go along the thesis.

2.5 Looking to the Future

In the previous sections we have discussed in detail, various properties of Einstein-Hilbert and Lanczos-Lovelock gravity. The obvious question to ask at this stage, whether, just like Einstein-Hilbert action, for Lanczos-Lovelock gravity as well, there exist some geometrical variables that can have nice thermodynamic features. We will try to answer this question in the later chapters. We will first introduce a new set of conjugate variables from differential geometric perspective, which can be used to describe Lanczos-Lovelock gravity in the next chapter. After identifying the correct geometric variables we will proceed to understand their thermodynamic importance as well and shall illustrate that variations of these variables are related to either variation of temperature or entropy associated with null surfaces in the subsequent chapters.

Also, we reiterate that in a recent work [23] it has been shown that evolution of spacetime can be interpreted as the difference between surface and bulk degrees of freedom, but in the context of Einstein-Hilbert action. In later chapters we will show that the same works for Lanczos-Lovelock gravity as well. Further, the notion of gravitational momentum have been defined for general relativity, whose thermodynamic significance as well as generalization to Lanczos-Lovelock gravity will be an interesting problem to look upon. All of these discussions will form an integral part of this thesis and will be dealt with greater detail in later chapters.

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