

## Chapter 2

# Light as an Electromagnetic Wave

### 2.1 The Development of Classical Electrodynamics

From the late 1700s through the 1800s there was intense activity and progress in defining the properties of electrostatic and electromagnetic forces, resulting in what is known as *classical electrodynamics*. These investigations would prove to be valuable prerequisites in later descriptions of the nature of light. Some of the scientists and their contributions are summarized below.

**Georg Simon Ohm** (1787–1854) stated that for conducting materials, the ratio of the current density and the electric field is a constant, being independent of the electric field producing the current. That is, the current density  $J$  and the electric field  $E$  are related by  $J = \sigma E$ , where the constant of proportionality  $\sigma$  is the *conductivity*. This empirical relationship is known as *Ohm's law*.

**Charles Augustin de Coulomb** (1736–1806) described the electrostatic force of attraction or repulsion between point charges, where the unlike charges attract and like charges repel. This was formulated around 1783 as *Coulomb's law*, where the electrostatic force  $F$  has the units of Newtons [N].

$$F = \left( \frac{1}{4\pi\epsilon_0} \right) \frac{q_1 q_2}{r^2}, \quad (2.1)$$

where

$\left( \frac{1}{4\pi\epsilon_0} \right)$  = Coulomb's constant =  $9 \times 10^9 \text{ N m}^2/\text{C}^2$ ,  
 $q_1$  and  $q_2$  are the values of the charges in units of **C**,  
 $r$  is the distance between the charges in meters,  
 $\epsilon_0$  is the permittivity of free-space =  $8.8542 \times 10^{-12} \text{ C}^2/\text{N m}^2$ .

The unit of charge **C** is defined in terms of one of the smallest known charges  $e$ , that of a single electron, where one unit of **C** corresponds to the charge carried by  $6.3 \times 10^{18}$  electrons.

**Carl Friedrich Gauss** (1777–1855) described basic properties of magnetic flux using the magnetic field vector  $\mathbf{B}$ . This was formalized as *Gauss' law of magnetism*. It can be expressed in an integral or differential form, these forms being mathematically equivalent. We present the integral form here.

$$\int_S \mathbf{B} \cdot d\mathbf{a} = 0, \quad (2.2)$$

where  $\mathbf{B}$  is the magnetic vector,  $d\mathbf{a}$  is a vector perpendicular to an infinitesimal area of a closed surface  $S$ , and  $\mathbf{B} \cdot d\mathbf{a}$  is the scalar product of these vectors (product of the magnitude of the vectors and the cosine of their included angle).  $\int_S \mathbf{B} \cdot d\mathbf{a}$  is the net magnetic flux  $\Phi_{mag}$  through this closed surface. The net magnetic flux is zero since isolated magnetic poles do not exist.

Gauss also developed a law for electric flux that is often simply called *Gauss' law*. It relates the net flux  $\Phi$  through a closed surface  $S$  to the electric field vector  $\mathbf{E}$  and the enclosed charge.

$$\Phi = \int_S \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\epsilon_0}, \quad (2.3)$$

where  $d\mathbf{a}$  is a vector perpendicular to an infinitesimal area of a closed surface  $S$ ,  $\mathbf{E} \cdot d\mathbf{a}$  is the scalar product of these vectors,  $q_{enc}$  is the net charge inside the closed gaussian surface  $S$ , and  $\epsilon_0$  is the permittivity of free-space.

**Hans Christian Oersted** (1777–1851), around 1819 was the first to discover that a compass needle was deflected when placed in the proximity of a current-carrying wire. He concluded that electric currents produce magnetic fields, and thus established a link between electricity and magnetism, called *electromagnetism*.

**André-Marie Ampère** (1775–1833), in 1826 formulated his circuital laws relating electric current to the magnetic field over a closed path around a steady current source. One form of *Ampère's law* states that for a symmetric conductor (eg. a straight wire) carrying a steady electric current with a closed path surrounding the conductor, the magnetic field  $\mathbf{B}$  on the path and the enclosed electric current  $I_{enc}$  are related by

$$\int_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}, \quad (2.4)$$

where  $\int_C$  is the line integral of the closed path,  $\mathbf{B}$  is the magnetic field vector tangent to the path, vector  $d\mathbf{l}$  is an infinitesimally small section of the path,  $\mathbf{B} \cdot d\mathbf{l}$  is the scalar product of  $\mathbf{B}$  and  $d\mathbf{l}$ ,  $\mu_0$  is the permeability of free-space, and  $I_{enc}$  is the total steady current through the conductor enclosed by the path.

**Michael Faraday** (1791–1867) was the first to describe electromagnetic induction, where a changing magnetic field induces a current in a nearby conductor, or produces an electric field. This was discovered independently by **Joseph Henry** (1797–1878). It was formalized in *Faraday's law of induction*, where a changing magnetic field produces a proportional electromagnetic force. He also rotated the plane of polarization of linearly polarized light with a magnetic field oriented in the direction of the light travel. This is called the *Faraday effect*, and established a connection with light and electromagnetism.

**Wilhelm Eduard Weber** (1804–1891) was a colleague of Gauss in Germany where he worked with him in studies of magnetism. He invented the electrody-namometer, which measured electric current and voltage by the interaction of two magnetic coils. Weber formalized an electrical law which integrated Ampère's law with the laws of induction and Coulomb's law. He also collaborated in a break-through experiment in electromagnetism, as described in the next section.

## 2.2 The Experiment of Weber and Kohlrausch

In 1855 [Wilhelm Eduard Weber](#) and [Rudolf Kohlrausch](#) performed an experiment that established a fundamental relationship between electricity, magnetism, and light [1]. The measured quantity was the ratio of the absolute unit of electrostatic charge to the absolute unit of magnetic charge, and remarkably had units of velocity, although no actual velocities were involved in the experiment. They measured this constant ratio by an experiment that involved charging a Leyden jar capacitor and measuring the electrostatic force. Then the [Leyden jar](#) was discharged and the magnetic force from the discharge current was measured. The ratio of the forces had a value  $3.107 \times 10^8$  m/s, which was close to known measurements of the speed of light. Weber had previously named this constant ratio the letter “*c*” in an 1846 paper, although this ratio was not associated with the speed of light. This measured force ratio corresponds to the value of the later well-known expression

$$c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}.$$

## 2.3 Maxwell and Electromagnetism

### 2.3.1 Maxwell's Electromagnetic Equations

During the period 1864–1865 James Clerk Maxwell developed a remarkable set of equations describing the properties of electromagnetic radiation [2]. There is a large volume of literature describing the development and application of these equations.

In our case the concern is primarily on their influence in the development of the physics of light in free-space. Maxwell had originally formulated 20 quaternion equations describing electric and magnetic phenomenon. These were later combined into four equations that we now refer to as *Maxwell's equations*. We state them here in their integral form with a brief explanation.

The first Maxwell equation is Gauss' law for electric flux.

The First Equation:

$$\int_S \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\epsilon_0} \quad (2.5)$$

The second Maxwell equation is Gauss' law of magnetism.

The Second Equation:

$$\int_S \mathbf{B} \cdot d\mathbf{a} = 0 \quad (2.6)$$

The third of Maxwell's equations is Faraday's law of induction. The mathematical form is

The Third Equation:

$$\int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_{mag}}{dt} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a} \quad (2.7)$$

This equation states that the line integral of the scalar product of the electric field vector  $\mathbf{E}$  with an infinitesimal length vector  $d\mathbf{l}$  around a closed path equals the time rate of change of the magnetic flux  $\Phi_{mag}$  through that path.

The fourth of Maxwell's equations is an extension of Ampère's law that adds the effect of time-varying electric flux, and is often called the *Ampère-Maxwell law*.

The Fourth Equation:

$$\int_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc} + \epsilon_0 \mu_0 \frac{d}{dt} \int_S \mathbf{E} \cdot d\mathbf{a} \quad (2.8)$$

Here  $I_{enc}$  is the current flowing through a closed curve  $C$ , and  $\int_S \mathbf{E} \cdot d\mathbf{a}$  is the electric flux  $\Phi_{elec}$  through the surface  $S$  bounded by the curve  $C$ . The  $\mu_0 I_{enc}$  term in

Eq. (2.8) is called the *displacement current*. Using these four Maxwell equations, a complete description of electromagnetic phenomena can be obtained.

### 2.3.2 The Electromagnetic Wave Equations

The crowning achievement of Maxwell was the synthesis of the wave equations for electric and magnetic fields in 1864 using the four electromagnetic equations. The wave equations can be derived by converting the integral forms of Maxwell's equations to the differential form, and in free-space are given by

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (2.9)$$

and

$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \quad (2.10)$$

where  $\nabla^2$  is the Laplacian operator, which represents the second spatial derivative of the vector following it.

The derivation of these wave equations is explained in several texts, such as that of Fleisch [3]. Since both  $\mathbf{E}$  and  $\mathbf{B}$  are vector quantities, each wave equation actually contains three equations, one for each of the  $x$ ,  $y$ , and  $z$  spatial directions.

The three-dimensional wave equation for a vector  $\mathbf{U}$  has the form  $\frac{\partial^2 \mathbf{U}}{\partial t^2} = v^2 \nabla^2 \mathbf{U}$ , where  $v$  is the propagating velocity. The propagating velocity  $v$  in the wave equations for  $\mathbf{E}$  and  $\mathbf{B}$  must then be  $v = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$ . When previously known values of  $\mu_0$  and  $\epsilon_0$  were inserted in this equation, a value of  $v$  was obtained that was very close to Foucault's measured speed of light in 1850,  $c \approx 2.98 \times 10^8$  m/s.

Without any direct experimental confirmation, Maxwell proposed that light was an electromagnetic wave that propagated through space at velocity  $c$ . We anticipate this later confirmation and state that

$$c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}. \quad (2.11)$$

Another electromagnetic wave relationship that follows from Maxwell's equations is

$$c = \frac{E}{B}, \quad (2.12)$$

where  $E$  and  $B$  are the magnitudes of the  $\mathbf{E}$  and  $\mathbf{B}$  vectors at any instant of time.

## 2.4 The Experiments of Hertz

Around 1886 Heinrich Hertz was the first to experimentally produce the electromagnetic waves proposed by Maxwell. He built a transmitter of electromagnetic waves using an induction coil connected to two small spherical electrodes separated by a small gap. The receiver was a loop of wire with two small spheres separated by another small gap, with the transmitter and receiver separated by several meters. When the oscillation frequency of the receiver was adjusted to match that of the transmitter, a small high-frequency spark appeared at the receiver. These propagated waves through space became known as *radio waves*.

The continuous propagation of electromagnetic waves in free-space can be described as a supportive interaction between varying electric and magnetic fields. An initial change in the electric field induces a change in the magnetic field from Maxwell's third equation (Faraday's law), Eq. (2.7). In turn, the changing magnetic field induces a change in the electric field from Maxwell's fourth equation (Ampère-Maxwell law), Eq. (2.8), and the process continues as a sustainable continuous wave having a velocity  $c$ .

In 1888 Hertz experimentally measured the frequency  $f$  and wavelength  $\lambda$  of radio waves in order to calculate their velocity  $v$  from the relationship  $v = \lambda f$ . The frequency of the wave was first measured. He then performed the more difficult measurement of the wavelength by wave interference effects using a reflected wave from a boundary, and measuring the nodal points. In one method he transmitted guided waves on a two-wire coaxial line [4]. His calculated value for  $v$  was remarkably close to the speed of light  $c$ . Hertz concluded that light was indeed an electromagnetic wave, thereby affirming Maxwell's theory and the electromagnetic nature of light. Hertz was also the first to observe the photoelectric effect (Sect. 4.1). Ironically, the photoelectric effect later provided evidence for the particle nature of light.

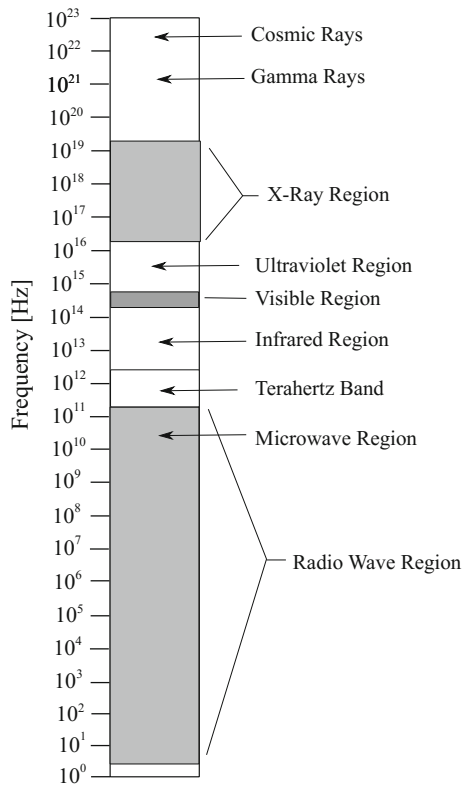
## 2.5 The Discovery of X-Rays

In 1895 Wilhelm Roentgen was studying the high-speed movement of electrons in an evacuated glass vessel. The impact of the electrons on the glass envelope produced a type of radiation that penetrated solid materials and produced distinct shadows of interior structures, such as human bones, on developed photographic film. Because of the unknown nature of this type of radiation, it was referred to as an *X-ray*. X-rays were positively identified as another type of electromagnetic radiation when in 1912 Max von Laue demonstrated interference effects of X-rays and measured their wavelength.

## 2.6 The Electromagnetic Spectrum

At this point it is convenient to join together the radio waves of Hertz, the visible light perceived by the human eye, and the X-rays of Roentgen, as types of electromagnetic radiation whose primary difference is their wavelength  $\lambda$  and frequency  $f$ . They all propagate at a common speed, which is the speed of light in vacuum  $c = \lambda f$ . These are usually presented as regions of a general *electromagnetic spectrum*, as illustrated in Fig. 2.1. The visible light spectrum is the smallest of these three frequency ranges, having a frequency spread between about  $4 \times 10^{14}$  and  $7.9 \times 10^{14}$  Hz, where one Hz (Hertz)  $\equiv$  one cycle/s. The radio wave portion of the spectrum is the broadest, running from about 3 to  $3 \times 10^{11}$  Hz. This includes the microwave region. The X-ray region of the spectrum covers the frequency range from about  $3 \times 10^{16}$  to  $3 \times 10^{19}$  Hz. The combined ultraviolet, visible, and infrared region is often called the *optical region*. The terahertz radiation region from about  $3 \times 10^{11}$  to  $3 \times 10^{12}$  Hz, is useful for communication and security screening. Other important high-frequency regions of the electromagnetic spectrum are identified as cosmic radiation and gamma radiation.

**Fig. 2.1** Frequency regions of the electromagnetic spectrum

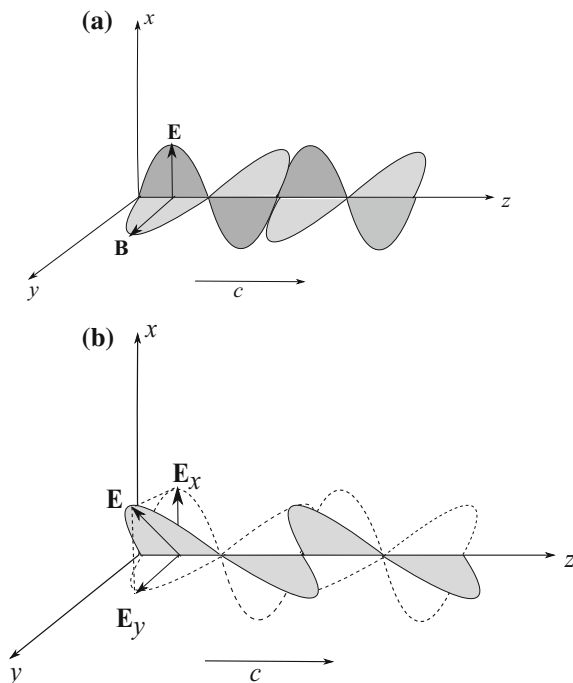


It is important to emphasize that all types of radiation in the electromagnetic spectrum are considered as types of light having similar physical properties. For example, we can speak of *visible light*, *X-ray light*, *gamma ray light*, and *radio wave light*. Thus certain properties of one type of light can be determined or verified from experiments that are specifically designed for, or more easily performed on another type of light. This technique has proved to be extremely useful in the scientific investigation of light.

## 2.7 Polarization of Light

Considering light as a transverse electromagnetic wave, the  $\mathbf{E}$  and  $\mathbf{B}$  vectors are perpendicular to each other and are perpendicular to the wave direction, as shown in Fig. 2.2a. Polarization of light is described by consideration of the motion of the  $\mathbf{E}$  vector. In the splitting of light rays in a calcite birefringent crystal, the split rays are polarized, with each ray linearly polarized in orthogonal planes to each other. Figure 2.2b illustrates a linear polarized light wave, where the  $\mathbf{E}$  vector remains in a plane tilted with respect to the propagation axis  $z$ . The wave theory of light provides a convincing explanation of the polarization of light.

**Fig. 2.2** **a** Illustration of a transverse electromagnetic wave. **b** Illustration of a linear polarized light wave



## References

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