A Stable Factor Approach of Input-Output-Based Sliding-Mode Control for Piezoelectric Actuators with Non-minimum Phase Property

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Abstract. This paper presents a new stable factor approach of input-output-based discrete-time sliding-mode control (IODSMC-SF), which dedicates to piezoelectric actuators (PEAs) with non-minimum phase (NMP) property. This control approach is developed based on a linear discrete-time input-output nominal model. A stable factor, which ensures stable and accurate motion control for PEAs with NMP nature, is designed, analyzed and introduced into the controller. One unique feature of the proposed controller lies in that it ensures stable and precision motion control for PEAs with NMP property. The construction of either a hysteresis model or a state observer is not needed. Moreover, the proposed controller releases the burden on parameter selection since only a stable factor is needed to stabilize the NMP system and this factor can be obtained by optimization algorithm. Experimental results with a piezoelectric actuator are presented to demonstrate the effectiveness of the proposed controller.

Keywords: Piezoelectric actuator · Precision motion control · Discrete-time Sliding-mode control · Non-minimum phase

1 Introduction

Piezoelectric actuators (PEAs) have been widely employed in many fields. However, the nonlinear effects existing in PEAs, including hysteresis nonlinearity and creep, can greatly degrade the motion accuracy [1]. Thus, for precise motion control of PEAs, it is essential to suppress these nonlinear effects to negligible levels.

As already be known, the discrete-time sliding-mode control (DSMC) is a very efficient feedback control approach featuring robustness in the presence of disturbance. The majority of existing DSMC have been realized based on system state feedback [2]. For actual application, it is difficult to measure all states of a system. Hence, state observers are always required for the DSMC implementation. Nonetheless, the state observer design clearly increases the burden of control design procedure. Furthermore, an inappropriately designed state observer affects the stability of the system. Thus, for practical implementation, it is desired to eliminate the use of state observers.

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Many efforts have been made to release the use of state observers in DSMC strategies [3–5]. Chan [3] proposed a discrete sliding-mode controller based on input-output model in the presence of modeling uncertainty and disturbance, but the reference inputs and disturbances of this method were assumed to be varying slowly. Janardhanan *et al.* [4] proposed a feedback sliding-mode controller based on multirate output feedback, where the system output is sampled at a rate faster than the control input. Sha *et al.* [5] designed an input-output-based adaptive sliding-mode controller, where only input and output data was needed. However, this controller was dedicated to a first-order model with long dead time. Yet, these restrictions make the control schemes not applicable to a piezoelectric actuation system which always possesses a high order model along with complicated nonlinearity. An input-output-based digital sliding-mode control (IODSMC) has been proposed by Xu [6] for piezoelectric micro/nano positioning systems, which successfully suppresses the unmolded hysteresis nonlinearity and disturbance in piezoelectric micro/nano positioning systems. However, these IODSMC controllers are only suitable for minimum phase systems. Many PEAs or piezoelectric micro/nano systems have non-minimum phase (NMP) nature and this greatly restricts the application of these methods.

In this paper, a stable factor approach of input-output-based discrete-time sliding-mode control (IODSMC-SF) for PEAs, which possess NMP nature preceded by hysteresis nonlinearity. This control scheme is developed based on a linear discrete-time input-output nominal model. A stable factor, which ensures stable and accurate motion control for PEAs with NMP nature, is designed, analyzed and introduced into the controller. The contribution of this controller can be summarized as follows:

(1) The proposed controller can realize stable and accurate motion tracking control for PEAs with NMP nature by overcoming the hysteresis nonlinearity and external disturbance.
(2) The proposed controller releases the burden on parameter selection. Unlike existing methods, only a stable factor is required to stabilize PEAs with NMP nature. This factor can be obtained by optimization algorithm. Moreover, the hysteresis model and state observer are not needed for the proposed controller.

The rest of this paper is organized as follows. The experiment setup of a PEA device and its model, which has NMP property, are presented in Sect. 2. The IODSMC-SF scheme is developed in Sect. 3. The stability analysis for NMP systems and optimized selection of the stable factor are presented. For experimental verification, in Sect. 4, the IODSMC-SF is implemented on the PEA device. Finally, Sect. 5 concludes this research.

2 Experimental Setup and PEA Model

The experimental setup used in this study is a commercially available PEA system as shown in Fig. 1 The PEA system consists of an amplifier module (model: E505 from PI in Germany), a dSPACE1103, a piezoelectric actuator (model: P-843.30 from PI in Germany) with an inbuilt position sensor, a signal processing unit (model: E509 from PI in Germany), and a host PC. The dSPACE-DS1103 controller board equipped with

16-bit digital-to-analog converters and 16-bit analog-to-digital converters is adopted to generate control codes and obtain the displacement information. The sampling frequency of the dSPACE1103 is set to be 20 kHz in present work.

Fig. 1. Experimental platform.

Figure 2 shows the block diagram of the PEA model. The hysteresis nonlinearity, dynamic and creep are represented by blocks of H, G_M, and F, respectively. $u(t)$, $f(t)$, y_o(t), and $y(t)$ represent the input voltage, internal actuation force, output displacement without creep and the final displacement of PEA, respectively.

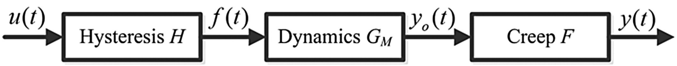

Fig. 2. PEA model.

In this paper, only the dynamic G_M is considered, while the hysteresis nonlinearity H and the creep F are taken as part of the disturbance. A generalized discrete-time dynamic model of G_M is established for PEAs as follow

$$x(k) = \sum_{i=1}^{n} a_i x(k-i) + \sum_{i=1}^{m} b_i u(k-i) + d(k) \tag{1}$$

where a_i and b_i are model coefficients. $x(k)$ and $u(k)$ represent the output position and input voltage of PEAs, respectively. In addition, $d(k)$ describes the lumped effect of unmodeled hysteresis nonlinear, residual dynamic, creep and external perturbations. If the plant Eq. (1) is NMP, then all or partial zeros of the plant are out of unit disk.

Experiments are performed to identify the parameters of Eq. (1), the plant transfer function can be obtained.

$$\begin{aligned} x(k) = 0.211x(k-1) + 0.153x(k-2) + 0.188x(k-3) - 0.009u(k-1) \\ + 0.064u(k-2) - 0.039u(k-3) \end{aligned} \tag{2}$$

The system has an unstable zero at 6.129, which is outside the unit disk, i.e., the PEA is a NMP system.

3 The Proposed Controller Design

3.1 Design of IODSMC-SF

The dynamic model Eq. (1) can be further expressed as

$$x(k+1) = A(z^{-1})x(k) + B(z^{-1})u(k) + d(k) \tag{3}$$

where $A(z^{-1})$ and $B(z^{-1})$ are polynomials in the unit-delay operator q^{-1} defined as

$$A(z^{-1}) = a_1 + a_2 z^{-1} + \cdots + a_n z^{-n+1}, \quad B(z^{-1}) = b_1 + b_2 z^{-1} + \cdots + b_m z^{-m+1} \tag{4}$$

The position error is represented as $e(k) = x(k) - r(k)$, where $r(k)$ is the desired position trajectory, an incremental type sliding function can be written as

$$s(k) = s(k-1) + F(z^{-1})[x(k) - r(k)] \tag{5}$$

where $F(z^{-1}) = 1 + f_1 z^{-1} + \ldots + f_n z^{-n}$. The controller is designed based on the reaching law $s(k + 1) = 0$. Substituting Eq. (3) into $s(k + 1) = 0$, the following deduction can be got

$$\begin{aligned} s(k+1) &= s(k) + F(z^{-1})[x(k+1) - r(k+1)] \\ &= s(k) + F(z^{-1})[A(z^{-1})x(k) + B(z^{-1})u(k) + d(k) - r(k+1)] \\ &= 0 \end{aligned} \tag{6}$$

The equivalent control $u_{eq}(k)$ is the solution to Eq. (6) by adding the stable factor $Q(z^{-1})$

$$u^{eq}(k) = G(z^{-1})[-F(z^{-1})A(z^{-1})x(k) + F(z^{-1})r(k+1) - s(k) - F(z^{-1})d(k)] \tag{7}$$

where $G(z^{-1}) = 1/(F(z^{-1})\, B(z^{-1}) + Q(z^{-1}))$.

From Eq. (7) we can see that the equivalent control only requires knowledge of measured output position, so the state observer is not needed. The unknown disturbance term $d(k)$ is obtained by its one-step delayed estimation $d(k - 1)$.

Since there exists unmodeled hysteresis nonlinearity, modeling uncertainty, and external disturbances, the switching control $u_{sw}(k)$ is needed here.

$$K_s \text{sat}\{s(k)\} = \begin{cases} K_s \text{sign}\{s(k)\}, & \text{if } |s(k)| > \varepsilon \\ K_s s(k)/\varepsilon, & \text{if } |s(k)| \le \varepsilon \end{cases} \tag{8}$$

where K_s is a positive control gain and sat$\{s(k)\}$ is the saturation function. The positive constant ε, which ensures that $s(k)$ is always bounded by $\pm\,\varepsilon$, represents the boundary layer thickness. The overall control action can be obtained

$$u(k) = (F(z^{-1})B(z^{-1}) + Q(z^{-1}))^{-1}[-F(z^{-1})A(z^{-1})x(k) + F(z^{-1})r(k+1) - s(k) - F(z^{-1})d(k-1)] - K_s \mathrm{sat}\{s(k)\} \tag{9}$$

3.2 Stability Analysis

The stability of IODSMC-SF is analyzed in this section. Substituting the equivalent control $u_{eq}(k)$ Eqs. (7) into (3), the control law gives rise to the closed-loop response

$$x(k+1) = A(z^{-1})x(k) + \frac{B(z^{-1})}{H(z^{-1})}[-F(z^{-1})A(z^{-1})x(k) + F(z^{-1})r(k+1) - s(k) - F(z^{-1})d(k-1)] + d(k) \tag{10}$$

where $H(z^{-1}) = F(z^{-1})B(z^{-1}) + Q(z^{-1})$. The closed-loop response can be simplified as

$$\left[z - A(z^{-1}) + \frac{B(z^{-1})F(z^{-1})A(z^{-1})}{H(z^{-1})}\right]x(k) = \frac{B(z^{-1})}{H(z^{-1})}[F(z^{-1})r(k+1) - s(k)] + d(k) - \frac{B(z^{-1})F(z^{-1})}{H(z^{-1})}d(k-1) \tag{11}$$

The closed-loop system behavior is governed by the roots of polynomial in Eq. (12). The system will be stable if the eigenvalues of characteristic equation locate inside the unit disk in the z-plane.

$$z - A(z^{-1}) + \frac{B(z^{-1})F(z^{-1})A(z^{-1})}{H(z^{-1})} = 0 \tag{12}$$

It can be concluded that the parameters $Q(z^{-1})$ and $F(z^{-1})$ are both included in the roots of the characteristic equation. So, by properly adjusting the parameters of $Q(z^{-1})$ and $F(z^{-1})$, the eigenvalues of the system could locate inside the unit disk in the z-plane, even if the system has NMP property.

3.3 Parameter Optimization

As shown in Eq. (7), the equivalent control $u_{eq}(k)$ has an additional control parameter: the stable factor $Q(z^{-1})$. A minimization optimization is designed to release the burden on factor selection.

Selection of the fitness function (or objective function) is the key point of the optimization. We can see that if $G(z^{-1})$ has suitable fitted amplitude- and phase-frequency characteristics with objective output transfer function $G_o(z^{-1}) = 1/(F(z^{-1})B(z^{-1}))$ in the desired frequency range, $u_{eq}(k)$ can remain the tracking accuracy. $G(z^{-1})$ is defined as

$$G(z^{-1}) = \frac{k(z - z_1)(z - z_2)\ldots(z - z_m)}{(p - p_1)(p - p_2)\ldots(p - p_n)} \tag{13}$$

where k, z_i, and p_i are gain, the zeros and poles of $G(z^{-1})$. In this work, the number of zeros and poles are determined by trials via experiments. So in this work, the fitness function is chosen as

$$E(w) = \sum_{k=0}^{n} [G_o(jw_k) - G(jw_k)] \tag{14}$$

where w_k denotes the total number of the frequency data. When $G(z^{-1})$ is close to the experimental data of $G_o(z^{-1})$, the identified result is thought to be a good result. Therefore, the minimize optimization problem with constraints can be formulated as follows

$$\min E(w) = \sum_{k=0}^{n} [G_{FB}(jw_k) - G_{FB+Q}(jw_k)] \tag{15}$$

s.t.

$$\begin{cases} |z_i| < 1, |p_i| < 1 \\ |\lambda_i| < 1 \end{cases} \tag{16}$$

where λ_i are the solutions of the characteristic equation Eq. (12).

4 Experimental Studies

4.1 Parameter Selection

First, sliding surface parameter $F(z^{-1}) = 1 - 0.75z^{-1} + 0.0425z^{-2}$ is chosen by trials for the IODSMC-SF controller. In the present research, a fourth-order model is employed to make a tradeoff between the model accuracy and complexity in order to demonstrate the effectiveness of the proposed scheme. The stable factor $Q(z^{-1})$ and $G(z^{-1})$ can be obtained by a minimization optimization, which is shown as below

$$G(z^{-1}) = \frac{(z + 0.1183)^4}{0.0213(z - 0.8322)(z - 0.263)(z^2 + 1.214z + 0.3729)} \tag{17}$$

Spectral analysis is conducted to obtain the frequency responses, as depicted in Fig. 3. It is obvious that the identified output transfer function $G(z^{-1})$ (dot red curve) is a suitable fitting of $G_o(z^{-1})$ behavior up to 100 Hz. So the IODSMC-SF controller can preserve the tracking accuracy in the frequency range 1–100 Hz.

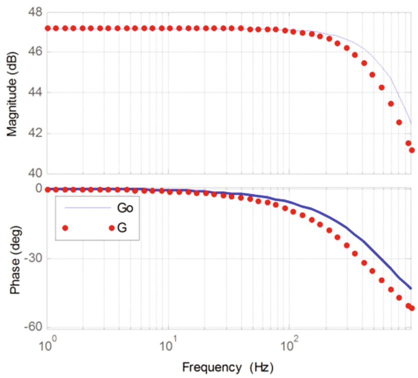

Fig. 3. Frequency response of $G_o(z^{-1})$ and $G(z^{-1})$.

The eigenvalues of the closed-loop system can be got by substituting the optimized stable factor into the constraint Eq. (12). The solutions are shown in Table 1, where all the solutions locate inside the unit disk in the z-plane.

Table 1. Solutions of the constraint equation.

	Solutions	
0.0627	$-0.1659 -0.1259i$	$0.2537 -0.6120i$
$-0.0666 +0.0265i$	$-0.1659 +0.1259i$	$0.2537 +0.6120i$
$-0.0666 -0.0265i$	$0.7363 +0.0473i$	$0.7363 -0.0473i$
$-0.6961 +0.6257i$	$-0.6961 -0.6257i$	

For comparison, the input-output-based discrete-time sliding-mode control (IODSMC) [6], and a traditional PID controller are also implemented on the model Eq. (2) to demonstrate the effectiveness of the proposed method.

Moreover, there are two control parameters both incorporated in the IODSMC-SF and IODSMC controllers: switching gain K_s and the boundary layer thickness ε. The parameters can be tuned by trials via experiments. The PID gains K_p, K_i, and K_d are also tuned by trials via experiments. Control parameters of IODSMC-SF and PID are listed in Table 2.

Table 2. Parameters of IODSMC-SF and PID.

Controller	Parameters	Value
IODSMCSF	K_s	5
	ε	10
PID	K_p	2.2
	K_i	200
	K_d	0.00001

4.2 Tracking Performance Experiment

Here, the reference trajectories are sinusoidal signals with amplitudes of 9 μm and frequencies of 10 Hz and 100 Hz, respectively. It can be found that no matter how to adjust control parameters, IODSMC cannot be stable for both sinusoidal reference inputs. The tracking errors of the IODSMC-SF controller and PID are illustrated in Fig. 4(b) and (d). The root mean square (RMS) tracking errors are listed in Table 3. Compared with the popular PID controller, the tracking errors of the IODSMC-SF are much smaller. Moreover, as the frequency increases, the performance improvement is more obvious. The reason why PID produces such a worse result is mainly attributed to its band-width limit and the inherent hysteresis nonlinearity effect of PEA.

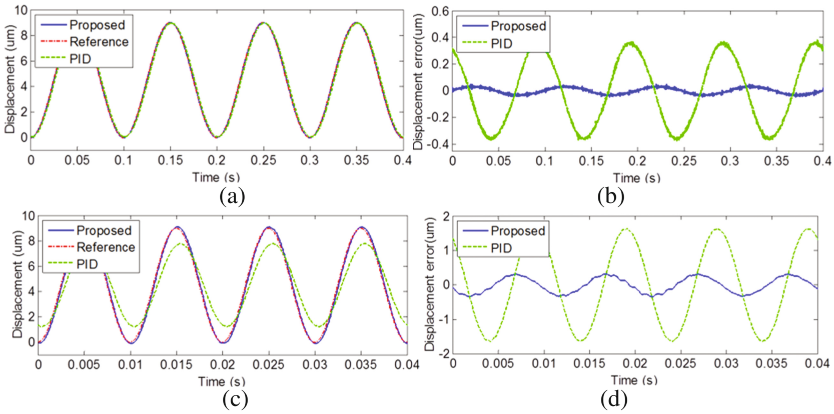

Fig. 4. (a) Sinusoidal tracking results at 10 Hz. (b) Tracking error at 10 Hz. (c) Sinusoidal tracking results at 100 Hz. (d) Tracking error at 100 Hz.

Table 3. Tracking error RMS (%) of IODSMC-SF and PID.

Input frequency	IODSMC-SF	PID
10 Hz	0.19%	2.82%
100 Hz	2.13%	12.88%

4.3 Robust Performance Experiment

The robustness of the proposed IODSMC-SF and PID is examined by internal and external disturbance. As shown in Sect. 4.2, since no hysteresis model-based feed-forward controller is used, the hysteresis nonlinearity is taken as part of the internal disturbance to the feedback controller. Note that smaller tracking error of the proposed method is achieved under the influence of hysteresis nonlinearity, the proposed method shows better robustness than PID.

Step load disturbance is added into the control system to evaluate the robustness of IODSMC-SF and PID with external disturbance. The desired reference is a sinusoidal

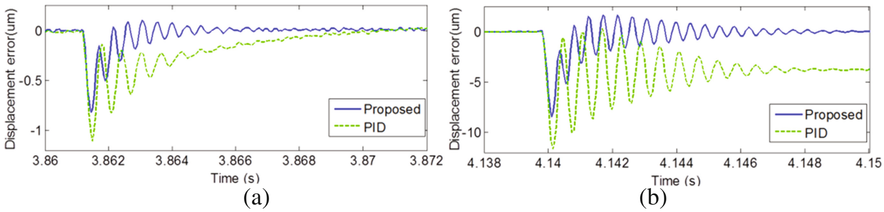

Fig. 5. Tracking error with external disturbance: (a) 5 V step load. (b) 50 V step load.

trajectory with 9 μm in amplitude and 5 Hz in frequency. The disturbance is added in the 5th second with voltages of 5 V and 50 V, respectively. The tracking errors of two situations are exhibited in Fig. 5.

As shown in Fig. 5(a), it can be concluded that both controllers are capable of precisely tracking the predefined trajectory with the 5 V step load disturbance. On the other hand, the IODSMC-SF and PID produce the settling time of 5 ms and 10 ms, respectively. As compared with PID, the IODSMC-SF renders a more rapid transient response with smaller tracking error. As shown in Fig. 5(b), it is notable that the tracking error of PID cannot converges to zero within a ±0.2 μm band and leaves large tracking error, whereas the tracking error of the proposed controller converges to zero within a ±0.2 μm band in 11 ms. The reason why PID produces such a worse result in the second test is mainly attributed to the large disturbance of 50 V step load. The experimental results demonstrate the robustness of IODSMC-SF under the influence of the internal and external disturbance. The proposed controller shows better performance than PID.

5 Conclusion

Simplification of the SMC controller design in front to hysteresis nonlinearity is a challenging task in many applications requiring precision motion control. If the controlled object is NMP, the problem becomes even more difficult. In this proper, a stable factor approach of input-output-based discrete-time sliding-mode control, which dedicates to precision motion tracking control of piezoelectric actuation systems with NMP property, was presented. The stability condition of the proposed control algorithm was analyzed, and based on the stability condition, the stable factor can be obtained by an optimization process, which released the burden on parameter selection. To illustrate the effectiveness of the proposed controller, experiments were performed and the results were compared with the IODSMC and PID. It has been shown that the proposed controller is stable for NMP systems, and shows superior performance than PID.

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