

Preface

Some mathematical problems of the dynamics of a two-dimensional viscous and ideal incompressible fluid on a rotating sphere are studied in this book. In this case, the fluid motion is completely governed by the barotropic vorticity equation (BVE). The analysis of vortex dynamics on a sphere is important because this approach is more natural for meteorological applications as compared to, e.g., using a β -plane approximation. Since the mid-twentieth century, this equation has played an important role in the study of hydrodynamic and meteorological processes. For example, in the case of an ideal fluid, a class of exact solutions of this equation represents meteorologically significant Rossby-Haurwitz waves, daily observed on the weather maps. In addition, the laws of conservation of enstrophy, total kinetic energy, and the average spectral number made it possible to study changes in the spectral distribution of the kinetic energy and the directions of the energy cascades as a result of nonlinear wave interactions. This equation was also chosen as the first approximate model of the atmosphere in attacking the problem of numerical weather prediction.

In this work, the turbulent viscosity term in the vorticity equation is taken in quite general form containing the derivative of real degree of the spherical Laplace operator. This required the use of orthogonal projections onto subspaces of spherical polynomials and the introduction of fractional derivatives of smooth functions on the sphere (multiplier operators) as well as a family of Hilbert spaces of functions with fractional derivatives of real degree. In addition, some embedding theorems were proved for Hilbert and Banach spaces of functions on the sphere.

In the case of a forced and viscous fluid, theorems on the existence and uniqueness of weak stationary and nonstationary BVE solutions are proved, and the asymptotic behavior of solutions with time and sufficient conditions for the global asymptotic stability of smooth and weak solutions are examined.

In the case of an unforced and ideal fluid, the stability of BVE solutions (Legendre polynomial flows, Rossby-Haurwitz waves, modons, and Wu-Verkley waves) is studied. For the Legendre polynomial flows and Rossby-Haurwitz waves, invariant sets of arbitrary perturbations are found. The nonlinear (Liapunov) instability of nonzonal Rossby-Haurwitz waves and dipole modons is also proved. Necessary

conditions for the normal mode instability are obtained for the Legendre polynomial flows, Rossby-Haurwitz waves, modons, and Wu-Verkley waves.

The method of normal modes for the linear stability study of stationary viscous and ideal flows on a sphere is analyzed. In particular, the spectrum of linearized operator and the block-diagonal structure of stability matrix are investigated. The problem of spectral approximation is considered for a viscous fluid, and the rate of convergence of the eigenvalues and eigenfunctions of discrete spectral problem to those of the original continuous spectral problem is estimated.

A general method that uses the energy and/or enstrophy norms is suggested for studying the geometric structure of growing perturbations to any steady flow on a sphere. The results of numerical stability study of the climatic January circulation of barotropic atmosphere show that this method correctly describes the two instability mechanisms, which were earlier opened by using the Eliassen-Palm flux diagnostics.

This book tries to build a bridge between basic concepts and concrete outcomes by a combination of theoretical, analytical, and numerical approaches. I hope this book will be a useful support tool for specialists developing mathematical methods for application to problems in physics, hydrodynamics, meteorology, and geophysics. The book contains a number of examples and can easily be used as a text for either an upper undergraduate or graduate course of the dynamics of incompressible fluid on a rotating sphere, theory of functions on a sphere, and flow stability.

Work on the book lasted for almost 30 years. During this period I had a chance to work in different institutions: Computing Center of Scientific and Technical Information, State Committee for Science and Technology, Council of Ministers of the USSR (Moscow); Institute of Numerical Mathematics, USSR Academy of Sciences (Moscow); Indian Institute of Tropical Meteorology (Pune), Indian Institute of Technology (New Delhi); Indian Institute of Science (Bangalore); and Centre of Atmospheric Sciences, Universidad Nacional Autónoma de México.

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