

# Chapter 1

## Basics of Semiconductor and Spin Physics

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This introductory chapter is mainly addressed to readers new to the field. In Sect. 1.1 a brief review of the historical roots of the current research is given. Section 1.2 describes various spin interactions. Section 1.3 is a mini text-book on semiconductor physics designed for beginners. A short overview of spin phenomena in semiconductors is given in Sect. 1.4. Finally, Sect. 1.5 presents the topics discussed in the chapters to follow.<sup>1</sup>

### 1.1 Historical Background

Maybe the first step towards today's activity was made by Robert Wood in 1923/24 when even the notion of electron spin was not yet introduced. In a charming paper [1] Wood and Ellett describe how the initially observed high degree of polarization of mercury vapor fluorescence (resonantly excited by polarized light) was found to diminish significantly in later experiments. "It was then observed that the apparatus was oriented in a different direction from that which obtained in earlier work, and on turning the table on which everything was mounted through ninety degrees, bringing the observation direction East and West, we at once obtained a much higher value of the polarization." In this way Wood and Ellet discovered what we now know as the

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<sup>1</sup>A special care is taken to indicate who has done what and when, since there exists much confusion on these matters in the current literature.

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Hanle effect, i.e. depolarization of luminescence by transverse magnetic field (the Earth's field in their case). It was Hanle [2] who carried out detailed studies of this phenomenon and provided the physical interpretation.

The subject did not receive much attention until 1949 when Brossel and Kastler [3] initiated profound studies of optical pumping in atoms, which were conducted by Kastler and his school in Paris in the 50 and 60s. (See Kastler's Nobel Prize award lecture [4].) The basic physical ideas and the experimental technique of today's "spintronic" research originate from these seminal papers: creation of a non-equilibrium distribution of atomic angular moments by optical excitation, manipulating this distribution by applying *dc* or *ac* fields, and detecting the result by studying the luminescence polarization. The relaxation times for the decay of atomic angular moments can be quite long, especially when hyperfine splitting due to the nuclear spin is involved.

A number of important applications have emerged from these studies, such as gyroscopes and hypersensitive magnetometers, but in my opinion, the knowledge obtained is even more valuable. The detailed understanding of various atomic processes and of many aspects of interaction between light and matter was pertinent to the future developments, e.g. for laser physics.

The first experiment on optical spin orientation of electrons in a semiconductor (Si) was done by Georges Lampel [5] in 1968, as a direct application of the ideas of optical pumping in atomic physics. The great difference, which has important consequences, is that now these are the free conduction band electrons (or holes) that get spin-polarized, rather than electrons bound in an atom. This pioneering work was followed by extensive experimental and theoretical studies mostly performed by small research groups at Ioffe Institute in St. Petersburg (Leningrad) and at Ecole Polytechnique in Paris in the 70s and early 80s. At the time this research met with almost total indifference by the rest of the physics community.

## 1.2 Spin Interactions

This Section serves to enumerate the possible types of spin interactions that can be encountered in a semiconductor.

The existence of an electron spin,  $s = 1/2$ , and the associated magnetic moment of the electron,  $\mu = e\hbar/2mc$ , has many consequences, some of which are very important and define the very structure of our world, while others are more subtle, but still quite interesting. Below is a list of these consequences in the order of decreasing importance.

### 1.2.1 The Pauli Principle

Because of  $s = 1/2$ , the electrons are fermions, and so no more than one electron per quantum state is allowed. Together with Coulomb law and Schroedinger equation,

It is this principle that is responsible for the structure of atoms, chemical properties, and the physics of condensed matter, biology included. It is interesting to speculate what our world would look like without the Pauli principle and whether any kind of life would be possible in such a world! Probably, only the properties of the high-temperature, fully ionized plasma would remain unchanged. Note that the Pauli exclusion principle is not related to any interaction: if we could switch off the Coulomb repulsion between electrons (but leave intact their attraction to the nuclei), no serious changes in atomic physics would occur, although some revision of the Periodic Table would be needed.

Other manifestations of the electronic spin are due to interactions, either electric (the Coulomb law) or magnetic (related to the electron magnetic moment  $\mu$ ).

### 1.2.2 Exchange Interaction

It is, in fact, the result of the electrostatic Coulomb interaction between electrons, which becomes spin-dependent because of the requirement that the wavefunction of a pair of electrons be anti-symmetric with respect to the interchange of electron coordinates and spins. If the electron spins are parallel, the coordinate part of the wavefunction should be antisymmetric:  $\psi_{\uparrow\uparrow}(\mathbf{r}_2, \mathbf{r}_1) = -\psi_{\uparrow\uparrow}(\mathbf{r}_1, \mathbf{r}_2)$ , which means that the probability that two electrons are very close to each other is small compared to the opposite case, when the spins are antiparallel and accordingly their coordinate wavefunction is symmetric. Electrons with parallel spins are then better separated in space, so that their repulsion is less and consequently the energy of electrostatic interaction for parallel spins is lower.

The exchange interaction is responsible for ferromagnetism. In semiconductors, it is normally not of major importance, except for magnetic semiconductors (like CdMnTe) and for the semiconductor-ferromagnet interface.

### 1.2.3 Spin-Orbit Interaction

If an observer moves with a velocity  $\mathbf{v}$  in an external electric field  $\mathbf{E}$ , he will see a magnetic field  $\mathbf{B} = (1/c)\mathbf{E} \times \mathbf{v}$ , where  $c$  is the velocity of light. This magnetic field acts on the electron magnetic moment. This is the physical origin of the spin-orbit interaction,<sup>2</sup> the role of which strongly increases for heavy atoms (with large  $Z$ ).

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<sup>2</sup>It is often stated that the origin of spin-orbit interaction is relativistic and quantum-mechanical. This is true in the sense that it can be derived from the relativistic Dirac equation by keeping terms on the order of  $1/c^2$ . However, the above formula  $\mathbf{B} = (1/c)\mathbf{E} \times \mathbf{v}$  is *not* relativistic: one does not need the theory of relativity to understand that, when moving with respect to a stationary charge, a current, and hence a magnetic field will be seen. Given, that the electron has a magnetic moment, the spin-orbit interaction follows directly. It is also not really quantum-mechanical: a classical object

The reason is that there is a certain probability for the outer electron to approach the nucleus and thus to see the very strong electric field produced by the unscreened nuclear charge  $+Ze$  at the center. Due to spin-orbit interaction, any electric field acts on the spin of a moving electron.

Being perpendicular both to  $\mathbf{E}$  and  $\mathbf{v}$ , in an atom the vector  $\mathbf{B}$  is normal to the plane of the orbit, thus it is parallel to the orbital angular momentum  $\mathbf{L}$ . The energy of the electron magnetic moment in this magnetic field is  $\pm\mu_B B$  depending on the orientation of the electron spin (and hence its magnetic moment) with respect to  $\mathbf{B}$  (or to  $\mathbf{L}$ ).<sup>3</sup>

Thus the spin-orbit interaction can be written as  $A\mathbf{L} \cdot \mathbf{S}$ , the constant  $A$  depending on the electron state in an atom. This interaction results in a splitting of atomic levels (the fine structure), which strongly increases for heavy atoms.<sup>4</sup>

In semiconductors, the spin-orbit interaction depends not only on the velocity of the electron (or its quasi-momentum), but also on the structure of the Bloch functions defining the motion on the atomic scale. Like in isolated atoms, it defines the values of the electron  $g$ -factors. More details can be found in [7].

Spin-orbit interaction is key to the subject of this book as it enables optical spin orientation and detection (the electrical field of the light wave does not interact directly with the electron spin). It is (in most cases) responsible for spin relaxation. And finally, it makes the transport and spin phenomena inter-dependent.

### 1.2.4 *Hyperfine Interaction with Nuclear Spins*

This is the magnetic interaction between the electron and nuclear spins, which may be quite important if the lattice nuclei in a semiconductor have non-zero spin (like in GaAs). If the nuclei get polarized, this interaction is equivalent to the existence of an effective nuclear magnetic field acting on electron spins. The effective field of 100% polarized nuclei in GaAs would be several Tesla!

Because the nuclear magnetic moment is so small (2000 times less than that of the electron) the equilibrium nuclear polarization at the (experimentally inaccessible) magnetic field of 100 T and a temperature of 1 K would be only about 1%. However,

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(Footnote 2 continued)

having a magnetic moment would experience the same interaction. The only place where quantum mechanics enters, is the value of the electron magnetic moment and, of course, the fact that the electron spin is 1/2.

<sup>3</sup>In fact the interaction energy derived in this simple-minded way should be cut into half (the “Thomas’s one half” [6]) if one takes properly into account that, because of the electron acceleration in the electric field of the nucleus, its moving frame is not inertial. This finding, made in 1926, resolved the factor of 2 discrepancy between the measured and previously calculated fine structure splittings.

<sup>4</sup>Interestingly, General Relativity predicts spin-orbit effects (on the order of  $(v/c)^2$ ) in the motion of planets. Thus the “spin” of the Earth should make a slow precession around its orbital angular momentum.

much higher degrees of polarization may be easily achieved through *dynamic nuclear polarization* due to hyperfine interaction with non-equilibrium electrons.

Experimentally, non-equilibrium nuclear polarization of several percent is easily achieved, recently values up to 50% were observed (see Chap. 12).

Similar to the spin-orbit interaction, the hyperfine interaction may be expressed in the form  $A \mathbf{I} \cdot \mathbf{S}$  (the Fermi contact interaction), where  $\mathbf{I}$  is the nuclear spin,  $\mathbf{S}$  is the electron spin, and the *hyperfine constant*  $A$  is proportional to  $|\psi(0)|^2$ , the square of the electron wave function at the location of the nucleus.

Like spin-orbit interaction, the hyperfine interaction strongly increases in atoms with large  $Z$ , and for the same reason. An  $s$ -electron in an outer shell has a certain probability to be at the center of the atom, where the nucleus is located, and the nearer it is to the center, the less is the nucleus shielded by the inner electrons. Thus the electron wavefunction of an  $s$ -electron will have a sharp spike in the vicinity of the nucleus. For example, for the In atom the value of  $|\psi(0)|^2$  is 6000 times larger than in the Hydrogen atom.

For  $p$ -states, and generally for states with  $l \neq 0$ , the Fermi interaction does not work, since  $\psi(0) = 0$ , and the electron and nuclear spins are coupled by the much weaker dipole-dipole interaction.

### 1.2.5 Magnetic Interaction

This is the direct dipole-dipole interaction between the magnetic moments of a pair of electrons. For two electrons located at neighboring sites in a crystal lattice this gives energy on the order of 1 K. This interaction is normally too weak to be of any importance in semiconductors.

## 1.3 Basics of Semiconductor Physics

A semiconductor is an insulator with a relatively small forbidden gap and shallow energy levels of electrons bound to impurities. The main feature of a semiconductor is its extreme sensitivity to impurities: a concentration of impurities like one per million of host atoms may determine the electrical conductivity and its temperature dependence.

### 1.3.1 Electron Energy Spectrum in a Crystal

The potential energy of an electron in a crystal is periodic in space. The most important consequence of this is that the energy spectrum consists of allowed and forbidden energy bands, and that the electron states can be characterized by its quasi-momentum

$\mathbf{p}$  (or quasi wave vector  $\mathbf{k} = \mathbf{p}/\hbar$ ). The energy in an allowed band is a periodic function of  $\mathbf{k}$ , so it may be considered only in a certain region of  $\mathbf{k}$ -space called the first Brillouin zone. The number of states in an allowed band is equal to twice the number of elementary cells in the crystal (the doubling is due to spin). Thus the energy spectrum is given by the dependence of energy on quasi-momentum,  $E(\mathbf{p})$ , for all the allowed bands.

In insulators and pure semiconductors at zero temperature a certain number of the lowest allowed bands are completely filled with electrons (according to Pauli principle), while the higher bands are empty. In most cases only the upper filled band (valence band) and the first empty band (conduction band) are of interest. The conduction and valence bands are separated by a forbidden energy gap of width  $E_g$ . In semiconductors the value of  $E_g$  may vary from zero (so-called gapless semiconductors, like HgTe) to about 2 – 3 eV. For Si  $E_g \approx 1.1$  eV, for GaAs  $E_g \approx 1.5$  eV.

### 1.3.2 Effective Masses of Electrons and Holes

The important property of semiconductors is that the number of free carriers (electrons in the conduction band or holes in the valence band) is always small compared to the number of atoms. The carriers are produced either by thermal excitation, in which case one has an equal number of electrons and holes, or by doping (see Sect. 1.3.4). Whatever is the case, the carrier concentration never exceeds  $10^{20}\text{cm}^{-3}$  (normally much less than that), while the number of states per  $\text{cm}^3$  in a given band is on the order of  $10^{22}$ , which is also a typical electron concentration in a metal. This means that electrons occupy only a very small fraction of the conduction band where their energy is lowest (and holes occupy only a very small fraction of the valence band). Consequently, when dealing with a semiconductor, we should be mostly interested in the properties of the energy spectrum in the vicinity of the minimum of the function  $E(\mathbf{p})$  for the conduction band and in the vicinity of its maximum for the valence band. If these extrema correspond to the center of the Brillouin zone ( $\mathbf{p} = 0$ ), as it is the case for GaAs and many other materials, then for small  $\mathbf{p}$  the function  $E(\mathbf{p})$  should be parabolic:

$$E_c = \frac{p^2}{2m_c} \quad \text{for the conduction band,} \quad E_v = -\frac{p^2}{2m_v} \quad \text{for the valence band.}$$

Here  $m_c$  and  $m_v$  are the effective masses of electrons and holes respectively. The effective masses may differ considerably from the free electron mass  $m_0$ , for example in GaAs  $m_c = 0.067m_0$ . Generally, the extrema of  $E(\mathbf{p})$  do not necessarily occur at the center of the Brillouin zone, also the effective mass may be anisotropic, i.e. have different values for different directions in the crystal.

### 1.3.3 The Effective Mass Approximation

The effective masses were initially introduced just as convenient parameters to describe the curvature of the  $E(\mathbf{p})$  parabolic dependence in the vicinity of its minimum or maximum. However this concept has a more profound meaning. In many cases we are interested in what happens to an electron, or a hole, under the action of some external forces due, for example, to electric and magnetic fields, deformation of the crystal, etc.

It can be shown, that if the spatial variation of these forces is much slower than that of the periodic crystal potential and if the carrier energy remains small compared to the forbidden gap,  $E_g$ , we can forget about the existence of the periodic potential and consider our electrons (or holes) as free particles moving in this external field. The only difference is that they have an *effective mass*, not the free electron mass. Thus the classical motion of a conduction electron in an electric field  $\mathbf{E}$  and a magnetic field  $\mathbf{B}$  is described by the conventional Newton's law:  $m_c d^2\mathbf{r}/dt^2 = -e\mathbf{E} - (e/c)\mathbf{v} \times \mathbf{B}$ . In particular, the cyclotron frequency of an electron rotating in a magnetic field is determined by the effective mass  $m_c$ , and this gives a valuable method of determining the effective masses experimentally (the cyclotron resonance).

If quantum treatment is needed, one can use the Shrödinger equation for an electron in the external field with its effective mass, *forgetting* about the existence of the crystal periodic potential.

Clearly, the validity of the effective mass approximation simplifies enormously the understanding of various physical phenomena in semiconductors.

### 1.3.4 Role of Impurities

Consider a crystal of Germanium in which each atom is linked to its first neighbors by 4 tetrahedral bonds (Ge is an element of column IV of the Periodic Table, it has 4 electrons to form bonds). Replace one of the host atoms by an atom of As, which belongs to column V. Arsenic will give 4 of his valence electrons to participate in bonding, and give its remaining fifth electron to the conduction band of the crystal. Thus, Arsenic is a *donor* for Germanium. The extra electron can travel far away from the donor, which then has a positive charge. Alternatively, the electron may be bound by the positive charge of the donor forming a Hydrogen-like "atom".

If the binding energy is small compared to  $E_g$ , and if the effective Bohr radius  $a_B^*$  is large compared to the lattice constant, this bound state can be studied using the effective mass approximation described in the previous Section. This means that we can use the theory of the Hydrogen atom and simply replace in all final formulas the free electron mass  $m_0$  by the effective mass  $m_c$ . There is also another simple modification, which takes into account the static dielectric constant of the material,  $\epsilon$ . The Coulomb potential energy of two opposite charges in vacuum is  $-e^2/r$ , while inside a polarizable medium it should be replaced by  $-e^2/(\epsilon r)$ . The ionization energy

and the Bohr radius for the Hydrogen atom are, respectively:  $E_0 = m_0 e^4 / (2\hbar) = 13.6 \text{ eV}$ ,  $a_B = \hbar^2 / (m_0 e^2) \sim 10^{-8} \text{ cm}$ . To obtain the corresponding values for an electron bound to a donor in a semiconductor, we make the replacements:  $m_0 \rightarrow m_c$ ,  $e^2 \rightarrow e^2 / \epsilon$ .

Suppose, for example, that  $m_c = 0.1m_0$ , and  $\epsilon = 10$ , which are typical values for a semiconductor. Then our electron bound to a donor will have an ionization energy smaller by a factor of 1000 ( $E_0^* \sim 10 \text{ meV}$ ) and an effective Bohr radius larger by a factor of 100 ( $a_B^* \sim 10 \text{ nm}$ ) than the corresponding values for a hydrogen atom. This justifies the validity of the effective mass approximation. It is interesting, that within the electron orbit there are roughly  $10^5$  host atoms! The electron simply does not see these atoms, their only role being to change the free electron mass to  $m_c$ . Because of the small value of the binding energy  $E_0^*$ , the donor is very easily ionized at moderate temperatures.

Conversely, if we replace the Ge atom by a group III impurity, like Gallium, which has 3 valence electrons, it will take the fourth electron, needed to form the tetrahedral bonds, from the Ge valence band. Then the Ga *acceptor* will become a negatively charged center and a positively charged hole will appear in the valence band. Now the same story applies to the hole: it can either be free, or it may be bound to the negative acceptor forming a Hydrogen-like state. It is the effective mass of the hole,  $m_v$ , which will now define the ionization energy and the effective Bohr radius. Since in most cases  $m_v > m_c$ , the acceptor radius is normally smaller than the donor radius, and the ionization of acceptors occurs at higher temperatures. Some complications of this simple picture arise if the effective mass is anisotropic.

Semiconductors are always, either intentionally, or non-intentionally, *doped* by impurities and may be *n*-type or *p*-type depending on the dominant impurity type.

### 1.3.5 Excitons

An exciton in a semiconductor is a bound state of an electron and hole. It is again a Hydrogen-like system with properties similar to an electron bound to a donor impurity. The important difference is that an exciton as a whole can move inside the crystal. Another difference is that excitons practically never exist in conditions of equilibrium. Usually they are created by optical excitation. Excitons have a certain lifetime with respect to recombination, during which the bound electron-hole pair annihilates. They can be seen as an absorption line somewhat below  $E_g$ .

### 1.3.6 The Structure of the Valence Band. Light and Heavy Holes

The allowed bands in crystals may be thought of as originating from discrete atomic levels, which are split to form a band when isolated atoms become close to each other.



However atomic levels are generally degenerate, i.e. there may be several distinct states having the same energy. This degeneracy may have important consequences for the band energy spectrum of a crystal.

### Neglecting Spin-Orbit Interaction

We now restrict the discussion to cubic semiconductors and first do not consider spin effects. The  $\mathbf{p} = 0$  conduction band state is  $s$ -type ( $l = 0$ ), the corresponding valence band state is  $p$ -type ( $l = 1$ ) and is triply degenerate ( $m_l = 0, \pm 1$ ). Here  $l$  is the atomic orbital angular momentum, and  $m_l$  is its projection on an arbitrary axis. The problem is to construct an effective mass description of the valence band structure taking into account this threefold degeneracy. This may be done using symmetry considerations: we have a vector  $\mathbf{p}$  and a pseudo-vector of angular momentum  $\mathbf{L}$  (which is a set of  $3 \times 3$  matrices  $L_x, L_y$ , and  $L_z$ , corresponding to  $l = 1$ ,  $L_z$  is a diagonal matrix with eigenvalues 1, 0, and  $-1$ ), and a scalar Hamiltonian should be constructed, which must be quadratic in  $\mathbf{p}$ .

If we require invariance under rotations, the only possibility is the Luttinger Hamiltonian [8]:

$$H = Ap^2\mathcal{I} + B(\mathbf{p} \cdot \mathbf{L})^2, \quad (1.1)$$

where  $A$  and  $B$  are arbitrary constants,  $\mathcal{I}$  is a unit  $3 \times 3$  matrix.

Thus the Hamiltonian  $H$  is also a  $3 \times 3$  matrix, and the energy spectrum in the valence band should be found by diagonalizing this matrix. We can greatly simplify this procedure by noting that the choice of the axes  $x, y, z$  is arbitrary. Accordingly, we can choose the direction of the  $z$ -axis along the vector  $\mathbf{p}$  (naturally, the final result does not depend on how the axes are chosen). Then  $(\mathbf{p} \cdot \mathbf{L})^2 = p^2 L_z^2$ , so that  $H$  becomes diagonal with eigenvalues

$$E_h(p) = (A + B)p^2 \quad \text{for } L_z = \pm 1, \quad E_l(p) = Ap^2 \quad \text{for } L_z = 0.$$

Thus the valence band energy spectrum has two parabolic branches,  $E_h(p)$  and  $E_l(p)$ , the first one being two-fold degenerate. We can now introduce two effective masses,  $m_h$  and  $m_l$ , by the relations:  $A + B = 1/(2m_h)$  and  $A = 1/(2m_l)$  and say that we have two types of holes in the valence band, the *light* and *heavy* holes (usually  $B < 0$ , but  $A + B > 0$ ). The difference between these particles is that the heavy hole has a projection of its orbital momentum  $\mathbf{L}$  on the direction of  $\mathbf{p}$  (*helicity*) equal to  $\pm 1$ , while the light hole has a projection equal to 0.

### Effects of Spin-Orbit Interaction

If we now include spin but do not take into account the spin-orbit interaction, this will simply double all the states, both in the conduction band and in the valence band. However the spin orbit interaction essentially changes the energy spectrum of the valence band.

We start again with the atomic states from which the bands originate. The spin-orbit interaction results in an additional energy proportional to  $\mathbf{L} \cdot \mathbf{S}$  (see Sect. 1.2.3).

Because of this,  $\mathbf{L}$  and  $\mathbf{S}$  are no longer conserved separately, but only the total angular momentum  $\mathbf{J} = \mathbf{L} + \mathbf{S}$ .

The eigenvalues of  $J^2$  are  $j(j+1)$  with  $|l-s| \leq j \leq l+s$ . Thus the state with  $l=0$  (from which the conduction band is built) is not affected ( $j=s=1/2$ ), while the state with  $l=1$  (from which the valence band is built) is split into two states with  $j=3/2$  and  $j=1/2$ . In atomic physics this splitting leads to the fine structure of spectral lines.

The symmetry properties of band states at  $\mathbf{p} = 0$  are completely similar to those of the corresponding atomic states. Thus for  $\mathbf{p} = 0$  we must have a four-fold degenerate state ( $j=3/2$ ,  $J_z = +3/2, +1/2, -1/2, -3/2$ ), which is separated by an energy distance  $\Delta$ , the *spin-orbit splitting*, from a doubly degenerate state ( $j=1/2$ ,  $J_z = +1/2, -1/2$ ). The conduction band remains doubly degenerate. The value of  $\Delta$  is small for materials with light atoms, like Si, and may be quite large (comparable to  $E_g$ ) in semiconductors composed of heavy atoms, like InSb (see Sect. 1.2.3). In GaAs  $\Delta \approx 0.3$  eV.

To see what happens to the  $j=3/2$  state for  $\mathbf{p} \neq 0$  for energies  $E(\mathbf{p}) \ll \Delta$  we construct the Luttinger Hamiltonian in a way quite similar to the procedure in the previous Section. The only difference is that the  $3 \times 3$  matrices  $L_x$ ,  $L_y$ , and  $L_z$ , corresponding to  $l=1$ , should now be replaced by  $4 \times 4$  matrices  $J_x$ ,  $J_y$ , and  $J_z$ , corresponding to  $j=3/2$ :

$$H = Ap^2\mathcal{I} + B(\mathbf{p} \cdot \mathbf{J})^2, \quad (1.2)$$

where now  $\mathcal{I}$  is a unit  $4 \times 4$  matrix, the matrix  $J_z$  is diagonal with eigenvalues  $3/2$ ,  $1/2$ ,  $-1/2$ , and  $-3/2$ .

Proceeding as above, we obtain the spectrum of the heavy and light holes, which is valid for energies much less than  $\Delta$ :

$$E_h(\mathbf{p}) = \left(A + \frac{9B}{4}\right)p^2 = \frac{p^2}{2m_h}, \quad (J_z = \pm 3/2) \text{ heavy hole band};$$

$$E_l(\mathbf{p}) = \left(A + \frac{B}{4}\right)p^2 = \frac{p^2}{2m_l}, \quad (J_z = \pm 1/2) \text{ light hole band}.$$

Both bands are doubly degenerate. Heavy holes have projection of the angular momentum  $\mathbf{J}$  on the direction of  $\mathbf{p}$  (or helicity) equal to  $\pm 3/2$ , while for light holes the helicity is  $\pm 1/2$ . Normally  $B < 0$ , but  $A + 9B/4 > 0$ , so that both masses are positive.

The combined description of all three bands (light, heavy, and split-off) on the energy scale  $\Delta \sim E(\mathbf{p}) \ll E_g$ , including effects of non-parabolicity, can be found in [9].

### Gapless Semiconductors

Interestingly, the signs of the expressions  $A + 9B/4$  and  $A + B/4$  may be opposite, which is the case of the so-called gapless semiconductors, like HgTe. In these

materials the light hole mass becomes negative, so that this band becomes a conduction band. The conduction band and the valence band (which now consists of heavy holes only) are degenerate at  $\mathbf{p} = 0$ , so that the energy gap is absent.

### Warping of the Iso-Energetic Surfaces

Also, it should be noted that the Luttinger Hamiltonian in (1.2) presents the so-called spherical approximation: it is invariant under arbitrary rotations. In a cubic crystal the symmetry is generally lower. Thus the true Luttinger Hamiltonian should have a more general form:

$$H = Ap^2\mathcal{I} + B(\mathbf{p} \cdot \mathbf{J})^2 + C(J_x^2 p_x^2 + J_y^2 p_y^2 + J_z^2 p_z^2), \quad (1.3)$$

where now the  $x, y, z$  axes are not arbitrary, they coincide with the crystallographic axes. The last term makes the iso-energetic surfaces of light and heavy holes anisotropic, so that the energy branches  $E_h(\mathbf{p})$  and  $E_l(\mathbf{p})$  will not have the simple parabolic form given above. (A similar term should be added to (1.1)).

### Oddities in the Behavior of Light and Heavy Holes

In the valence band the “spin” of light and heavy holes is tightly bound to their momentum, and this has many interesting consequences. If some external forces exist, the light and heavy hole states generally become mixed. A simple example is the reflection from an interface.

Suppose that a heavy hole is incident on an ideal flat potential wall. If the incidence is normal, nothing very interesting happens, except that the initial state with helicity  $+3/2$  (angular momentum  $\mathbf{J}$  parallel to  $\mathbf{p}$ ) will be transformed after reflection into a state with opposite helicity:  $-3/2$ . This can be explained by noting that while the initial momentum  $\mathbf{p}$  changes sign under reflection, the internal angular momentum remains unchanged.

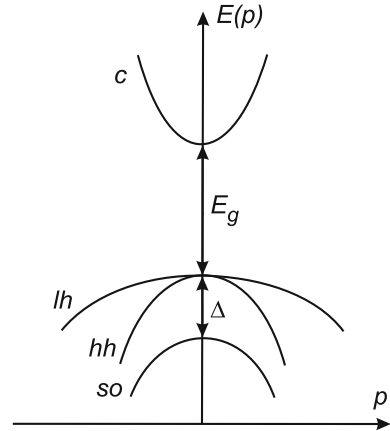
However for an arbitrary angle of incidence the same reasoning tells us that the reflected heavy hole will have a certain arbitrary angle between  $\mathbf{J}$  and  $\mathbf{p}$ . But such free states do not exist! This means that the incident heavy hole will be partly transformed into the light hole. (A similar phenomenon of transformation between ordinary and the extraordinary waves during reflection is known in optics of uniaxial crystals.)

One can re-consider all the textbook problems of Quantum Mechanics (potential well, tunnel effect, the Hydrogen problem, movement in magnetic field, etc.) for a particle, described by the Luttinger Hamiltonian, and these exercises reveal the rather bizarre physics of light and heavy holes in a semiconductor.

### 1.3.7 Band Structure of GaAs

The above considerations lead to the band structure presented in Fig. 1.1. Near the center of the Brillouin zone there is a simple isotropic conduction band, which is doubly degenerate in spin (for the moment we neglect the spin splitting, see Sect. 1.4.3).

**Fig. 1.1** Band structure of GaAs near the center of the Brillouin zone  $p = 0$ .  $c$  - conduction band,  $hh$  - heavy hole band,  $lh$  - light hole band,  $so$  - split-off band



The valence band, consists of the sub-bands of light and heavy holes, which are anisotropic (see Sect. 1.3.6), and the isotropic split-off band, all doubly degenerate.

### 1.3.8 Photo-Generation of Carriers and Luminescence

In the process of interband absorption of a photon with energy  $\hbar\omega > E_g$  in a semiconductor, an electron in the conduction band and a hole in the valence band are generated. During the process the (quasi)momentum is conserved, however the photon momentum  $\hbar k = 2\pi\hbar/\lambda$ , where  $\lambda$  is the photon wavelength, is very small (compared, for example, to the electron thermal momentum) and normally may be neglected.

In this approximation the optical transitions are *vertical*: to see what happens, we must simply apply a vertical arrow of length  $\hbar\omega$  to Fig. 1.1, so that the arrow touches one of the valence bands and the conduction band. The ends of the arrow will give us the initial energies of the generated electrons and holes. An electron may be created in company with a heavy hole, or a light hole; for  $\hbar\omega > E_g + \Delta$  the electron-hole pair can also involve a hole in the split-off band. Note, that for a given photon energy the initial electron energy will be different for these three processes.

The photoexcited carriers live some time  $\tau$  before recombination, which may be radiative (i.e. accompanied by photon emission, which results in luminescence), or non-radiative. In direct-band semiconductors, like GaAs, the recombination is predominantly radiative with a lifetime on the order of 1ns.

It is important to realize that this time is normally very long compared to the carriers thermalization time. Thermalization means energy relaxation of carriers in their respective bands due to phonon emission and absorption, which results in an equilibrium Boltzmann (or Fermi, depending on temperature and concentration) distribution function of electrons and holes. Thermal equilibrium *between* electrons and holes is established by recombination, on the time scale  $\tau$ .

Because the recombination time  $\tau$  is so long compared to the energy relaxation time, the luminescence is produced mostly by thermalized carriers and the emitted photons have energies close to the value of  $E_g$ , irrespective of the energy of exciting photons.<sup>5</sup>

It should be noted that semiconductors are normally either intentionally, or non-intentionally doped by impurities. In a  $p$ -type semiconductor at moderate excitation power the number of photo-generated holes is small compared to the number of equilibrium holes, so that the photo-created electron will recombine with these equilibrium holes, rather than with photo-generated ones.

### 1.3.9 Angular Momentum Conservation in Optical Transitions

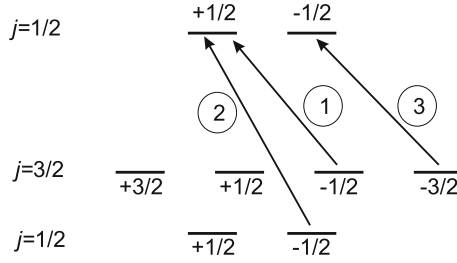
This Section is most important for our subject. Along with energy and momentum conservation, the conservation of the angular momentum is a fundamental law of Physics. Just like particles, electromagnetic waves have angular momentum. Photons of right or left polarized light have a projection of the angular momentum on the direction of their propagation (helicity) equal to  $+1$  or  $-1$ , respectively (in units of  $\hbar$ ). Linearly polarized photons are in a superposition of these two states.

When a circularly polarized photon is absorbed, this angular momentum is distributed between the photoexcited electron and hole according to the selection rules determined by the band structure of the semiconductor. Because of the complex nature of the valence band, this distribution depends on the value of the momentum of the created electron-hole pair ( $\mathbf{p}$  and  $-\mathbf{p}$ ). However, it can be shown that if we take the average over the directions of  $\mathbf{p}$ , the result is the same as in optical transitions between atomic states with  $j = 3/2, m_j = -3/2, -1/2, +1/2, +3/2$  (corresponding to bands of light and heavy holes) and  $j = 1/2, m_j = -1/2, +1/2$  (corresponding to the conduction band), see Sect. 1.4.2 below.

Possible transitions between these states, as well as between states in the split-off band and the conduction band, for absorption of a right circularly polarized photon with corresponding relative probabilities are presented in Fig. 1.2. Note, that if we add up *all* transitions, which is the correct thing to do if the photon energy sufficiently exceeds  $E_g + \Delta$  the two spin states in the conduction band will be populated equally. This demonstrates the role of spin-orbit interaction for optical spin pumping, see [9, 14] for the details of photon energy dependence of the spin polarization.

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<sup>5</sup>A small part of the excited electrons can emit photons *before* loosing their energy by thermalization. The studies of the spectrum and polarization properties of this so-called *hot luminescence* reveal interesting and unusual physics, see [10, 11].



**Fig. 1.2** Optical transitions between levels with  $j = 3/2$  and  $j = 1/2$  (the bands of light and heavy holes, and the split-off band) and the levels with  $j = 1/2$  (the conduction band) during an absorption of a right-polarized photon. The probability ratio for the three transitions is 3:2:1

### 1.3.10 Low Dimensional Semiconductor Structures

The development of semiconductor physics in the last two decades is mainly related to studies of artificially engineered low dimensional semiconductor structures, two-dimensional (quantum wells), one-dimensional (quantum wires), and zero-dimensional (quantum dots). By growing a structure consisting of a thin semiconductor layer, for example GaAs, surrounded by material with a larger band gap, for example a solid solution GaAlAs, one obtains a potential well for electrons (and for holes) with a typical width of 20 – 200 Å.

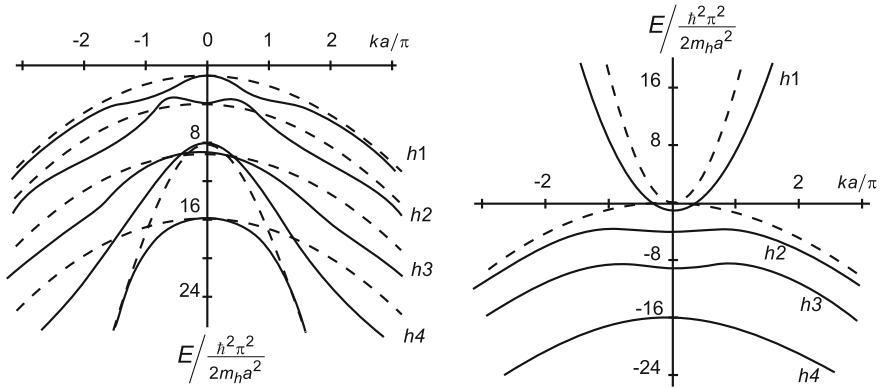
Thus the first problem in Quantum Mechanics courses, a particle in a one-dimensional rectangular potential well, which since 1926 was tackled by generations of students as the simplest training exercise, has finally become relevant to some reality!

#### Energy Spectrum of Electrons and Holes in a Quantum Well

The motion in the direction perpendicular to the layer (the *growth direction*,  $z$ , is quantized in accordance with textbooks, while the motion in the plane of the layer  $xy$  is unrestrained. Thus the energy spectrum of an electron in a quantum well consists of two-dimensional *subbands*:  $E_n(\mathbf{p}) = E_n^0 + p^2/(2m)$ , where  $E_n^0$  are the energy levels for the one-dimensional motion in the  $z$  direction,  $\mathbf{p}$  is the two-dimensional (quasi)momentum in the  $xy$  plane, and  $m$  is the electron effective mass.

In most cases the electron concentration in the well is such that only the lowest subband is occupied. The motion of such electrons is purely two-dimensional (2D). One important consequence is that in an applied magnetic field perpendicular to the 2D plane the spectrum becomes discrete: it consists of Landau levels. A magnetic field parallel to the 2D plane has no effect on the orbital motion of electrons, however it has the usual influence on their spins.

For the case of holes in a quantum well, the problem is not so simple. It was tackled in 1982 by Dyakonov and Khaetskii [12]. For  $\mathbf{p} = 0$  one has two independent ladders of levels for heavy and light holes, given (for an infinite well) by the textbook formula  $E_n^0 = (\pi n \hbar)^2 / (2ma^2)$ , where  $m$  is the respective effective mass,  $a$  is the well width,



**Fig. 1.3** The energy spectrum  $E(k)$  of holes (left) and of carriers in a gapless semiconductor (right) in an infinite rectangular quantum well within the spherical approximation [12]. Dashed lines represent the spectrum that would exist if the two type of carriers were independent particles

and  $n = 1, 2, 3, \dots$ . However, for  $\mathbf{p} \neq 0$  the spectrum is determined by the mutual transformations of light and heavy holes during reflections from the potential walls, see Sect. 1.3.6.

Figure 1.3 shows the spectrum of holes and of carriers in a gapless semiconductor in an infinite quantum well calculated in [12] within the spherical approximation, (1.2)<sup>6</sup>.

Especially interesting is the case of a gapless semiconductor. In a quantum well, a gap will obviously appear due to quantization of the transverse motion. Naively, one would expect this gap to be  $E_{e1}^0 - E_{h1}^0 = (1/2)(\pi\hbar/a)^2(1/m_e - 1/m_h)$ , i.e. mostly determined by the small electron mass. In fact, this is not true, because the  $h1$  subband, originating from the first hole level at  $p = 0$ , becomes *electronic* (see Fig. 1.3). Thus the gap is  $\approx m_e/m_h \sim 1/10$  times smaller than expected.

For  $pa/\hbar \gg 1$  the first electronic subband  $h1$  corresponds to *surface states* localized near the well boundaries. Such states should exist also near the surface of a bulk gapless semiconductor [13].

In fact, it is not even necessary to have a sandwich structure to obtain 2D electrons. A simple interface between two different materials plus an electric field of ionized donors gives the same effect, except that now the quantum well is not rectangular, but more like triangular, and that its shape depends on the electron concentration.

The heterostructure design allows to accomplish what was impossible in bulk semiconductors: a spatial separation of the electrons and the donors, from which they originate. The technique of *delta doping* provides a 2D electron gas with previously unimaginable mobilities on the order of  $10^7$  V/cm<sup>2</sup>·s.

<sup>6</sup>More accurately, one should use the Hamiltonian in (1.3), which takes care of the warping of iso-energetic surfaces. In fact, the energy spectrum depends on the growth direction, and on the orientation of the vector  $\mathbf{p}$  in the  $xy$  plane with respect to the crystal axes. However the general properties of the spectrum are the same.

## Quantum Dots

Quantum dots are zero-dimensional structures, a sort of large artificial atoms. Under certain growth conditions, *self-assembled* quantum dots appear spontaneously. Typically, they have the form of a flat cake with a height  $\sim 30 \text{ \AA}$  and a base diameter of  $\sim 300 \text{ \AA}$ . They are embedded in a different material, so that there is a large potential barrier at the interface.

Normally, samples contain an ensemble of many quantum dots with varying parameters, however special techniques allow to deal with individual dots. Like in an atom, the energy spectrum is discrete. A quantum dot may contain a few electrons or holes.

## 1.4 Overview of Spin Physics in Semiconductors

The basic ideas concerning spin phenomena in semiconductors were developed both theoretically and experimentally more than 30 years ago. Some of these ideas have been rediscovered only recently. A review of non-equilibrium spin physics in bulk semiconductors can be found in [14], as well as in other chapters of the Optical Orientation book.

### 1.4.1 Spin Splittings of Energy Bands in Three and Two Dimensions

Spin-orbit interaction results in a spin splitting of energy bands in bulk semiconductors as well as in two-dimensional semiconductor structures. While these splittings are small, they are key to understanding spin relaxation and a variety of other interesting spin phenomena in optics and transport that are described in this book.

The additional spin-dependent term in the electron Hamiltonian can be generally presented as

$$V_{so} = \hbar \mathbf{\Omega}(\mathbf{p}) \cdot \mathbf{S}, \quad (1.4)$$

which can be viewed as the energy of a spin in an effective magnetic field. Here  $\mathbf{\Omega}(\mathbf{p})$  is a vector depending on orientation of the electron momentum  $\mathbf{p}$ .

### Bulk Zincblende Semiconductors

For non-centrosymmetric semiconductors like GaAs (but not Si or Ge, which are centrosymmetric) the spin splitting of the electron energy spectrum was discovered theoretically in 1955 by Dresselhaus [15], who has shown that the vector  $\mathbf{\Omega}(\mathbf{p})$  in (1.4) is cubic in momentum and given by:



$$\Omega_x \sim p_x(p_y^2 - p_z^2), \quad \Omega_y \sim p_y(p_z^2 - p_x^2), \quad \Omega_z \sim p_z(p_x^2 - p_y^2). \quad (1.5)$$

Here the indices  $x, y, z$  denote vector components along the crystal axes.

### Bulk Semiconductors With Wurtzite Structure

Using Dresselhaus's method, in 1959 Rashba and Sheka [16] derived the spin splitting of the conduction band in uniaxial crystals with the wurzite structure and found that now (to the first order in  $\mathbf{p}$ ) the vector  $\boldsymbol{\Omega}(\mathbf{p})$  is a *linear* function of  $\mathbf{p}$ :

$$\Omega_x \sim p_y, \quad \Omega_y \sim -p_x, \quad \Omega_z = 0, \quad (1.6)$$

where  $z$  is the wurzite  $C$  axis. Thus the spin-dependent term in (1.4) is:

$$V_{so} = \alpha(\mathbf{S} \times \mathbf{p})_z, \quad (1.7)$$

where  $\alpha$  is a constant.<sup>7</sup> At the same time a similar result was obtained by Glasser [17] and Casella [18].

This correction to the energy spectrum shifts the extremum of the energy surface away from the point  $\mathbf{p} = 0$ , so that a *loop of extrema* is formed in the dependence  $E(\mathbf{p})$  (see also Casella's article [19]). Starting with [20], the consequences of this for the electron spin resonance and other phenomena were studied in a series of papers, see Rashba's review [21].

### Bulk Zincblende Semiconductors Under Strain

An experimental and theoretical study of optical spin orientation in strained crystals of GaAs, GaSb, and InP was performed in 1986 by Dyakonov et al. [22]. Strain induces additional terms to  $V_{so}$  in (1.4).

From symmetry considerations, these terms can be obtained by replacing any pair  $p_i p_j$  in (1.5) by the component of the strain tensor  $\epsilon_{ij}$ . This procedure gives two possible forms of the vector  $\boldsymbol{\Omega}(\mathbf{p})$ :

$$\hbar\Omega_x^{(1)} = v^{(1)}(\epsilon_{xy}p_y - \epsilon_{xz}p_z), \quad \hbar\Omega_x^{(2)} = v^{(2)}(\epsilon_{yy} - \epsilon_{zz})p_x, \quad (1.8)$$

where  $v^{(1)}$  and  $v^{(2)}$  are some coefficients with the dimension of velocity. The remaining components of these vectors are obtained by cyclic permutations of the indices  $x, y, z$  (here  $x, y, z$  denote the principal crystal axes).

If the strain axis coincides with the  $\langle 100 \rangle$  direction, then  $\boldsymbol{\Omega}^{(1)} = 0$  and the strain-induced splitting should be entirely determined by the second term in (1.8). Very careful measurements done by Marushchak and Titkov [22] showed that in this case there is no noticeable effect, while for other directions the effects of strain are large. Thus the essential contribution comes from the first term only. As G.E. Pikus has

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<sup>7</sup>In most of the current literature (as well as in some chapters of this book) this expression is called "Rashba term" or "Rashba interaction". Rather, it should be referred to as Rashba-Sheka term. There is no good reason for ignoring one of the authors, it is unfair and incorrect.

explained us at the time, the splitting related to  $\mathbf{\Omega}^{(1)}$  appears as a result of interaction between the conduction band and the nearest valence band  $\Gamma_{15}$  while the term  $\mathbf{\Omega}^{(2)}$  is due to much weaker interaction with the higher conduction band  $\Gamma_{12}$  [23].

The effective magnetic field directed along the vector  $\mathbf{\Omega}^{(1)}$  lies in a plane perpendicular to the vector  $\mathbf{N}$ , such that  $N_x = \epsilon_{yz}$ ,  $N_y = \epsilon_{zx}$ ,  $N_z = \epsilon_{xy}$ . Indeed, one can see that  $\mathbf{N} \cdot \mathbf{\Omega}^{(1)} = 0$ .

The experimental results on spin relaxation for different directions of the applied strain [22] were consistently interpreted as resulting from the first term in (1.8) and the parameter  $v^{(1)}$  was measured for all the studied materials. Thus for GaSb  $v^{(1)} = 3 \cdot 10^8$  cm/s.

## Two-Dimensional Semiconductor Structures

### *Asymmetric 2D structures*

In 1974 Ohkawa and Uemura [24] derived for the first time the spin splitting for a two-dimensional semiconductor structure. They considered an inversion layer of a zincblende semiconductor. Such a structure is asymmetric in the growth direction and can be characterised by a unit vector  $\mathbf{n}$  perpendicular to the 2D plane.

Their result can be presented as

$$V_{so} = \alpha(\mathbf{S} \times \mathbf{p}) \cdot \mathbf{n}, \quad (1.9)$$

and the coefficient  $\alpha$  was expressed through the gradient of the energy corresponding to the bottom of the conduction band in the inversion layer.

A similar result was independently obtained in 1979 by Vasko [25] for an asymmetric quantum well.

The expression in (1.9) is formally identical to the Rashba-Sheka spin splitting obtained for bulk crystals with wurtzite structure, (1.7), although their physical origin and the value of the constant  $\alpha$  are quite different. Because of this, in a vast number of publications the corresponding term is labelled as “Rashba interaction” (see also footnote 6), and the names of Ohkawa, Uemura, and Vasko, who were the first to establish this interaction for two dimensions, are only rarely cited. Clearly, this is not correct: it should be noted that this similarity was by no means obvious, actually it was a controversial subject for a long time (see e.g. [26, 27]), and the derivation is far from being trivial.

In 1984 Bychkov and Rashba [28, 29] have studied the consequences of this spin splitting for electron spin resonance and Shubnikov-de Haas oscillations in two-dimensions.<sup>8</sup>

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<sup>8</sup>In many publications [28, 29] are erroneously cited as the origin of the “Rashba term” in 2D. Meanwhile, in [28] (but not in the better known [29]) the authors correctly attribute this result to Vasko [25], although do not mention the work of Ohkawa and Uemura [24].

### Symmetric Quantum Wells

Another reason for a linear in momentum spin splitting in *both* symmetric and asymmetric quantum wells was identified in 1986 by Dyakonov and Kachorovskii [30]. Since the projection of momentum perpendicular to the 2D plane is quantized and fixed, and since it is normally much greater than the in-plane projections, one should replace any pair of  $p_i p_j$  in (1.5) by their quantum-mechanical average values in the lowest subband (somewhat similar to the case of bulk zincblende semiconductors under strain, see above).

The result crucially depends on the growth direction. For the simplest case when the growth direction ( $z$ -axis) is  $\langle 001 \rangle$ , the average values of  $p_z$  and  $p_z^2$  are equal to 0 and  $\langle p_z^2 \rangle$  respectively (for a deep rectangular well of width  $a$ ,  $\langle p_z^2 \rangle = (\pi\hbar/a)^2$ ). These considerations give:

$$\Omega_x \sim -p_x, \quad \Omega_y \sim p_y, \quad \Omega_z = 0. \quad (1.10)$$

Thus the effective magnetic field (or the vector  $\Omega$ ) is linear in  $\mathbf{p}$  and lies in the 2D plane. Combining the results of (1.9, 1.10) we can present the spin orbit interaction in an asymmetric quantum well with the  $\langle 001 \rangle$  growth direction as:

$$V_{so} = \alpha(S_x p_y - S_y p_x) + \beta(S_x p_x - S_y p_y), \quad (1.11)$$

where the indices  $x$  and  $y$  denote the in-plane vector components. Here the first term was derived by Ohkawa and Uemura [24] and Vasko [25] and the second one - by Dyakonov and Kachorovski [30].<sup>9</sup>

An interesting case is when the growth direction corresponds to  $\langle 110 \rangle$ . If we now take *this* direction as the  $z$  axis, and take  $x$  and  $y$  axes along the in-plane  $\langle 1\bar{1}0 \rangle$  and  $\langle 001 \rangle$  directions respectively, in the same manner as above we obtain [30]:

$$\Omega_x = 0, \quad \Omega_y = 0, \quad \Omega_z \sim p_x, \quad (1.12)$$

so that we have an unusual situation when the effective magnetic field is always parallel or anti-parallel to the direction perpendicular to the 2D plane, while its sign and value depend on the electron momentum projection on the in-plane  $x$  axis! This leads to an interesting phenomenon of *persistent spin helix* [31, 32] to be described below in Sect. 1.4.3.

Another amusing consequence of (1.11) is the resulting deviation from circular cyclotron motion of an electron in magnetic field, leading to the appearance of

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<sup>9</sup>It has become a bad habit to label the two terms as “Rashba” and “Dresselhaus” terms, without any good reason or justification: the results given by (1.9 - 1.11), for 2D structures were *not known* prior to the publications in [24, 25, 30] and neither Rashba, nor Dresselhaus have introduced any linear in  $\mathbf{p}$  terms for the 2D case (Rashba and Sheka have derived such terms in bulk wurtzite-type crystals, Dresselhaus had no linear terms at all). The normal practice is to give credit to those who have *in fact* obtained previously unknown results.

higher impair harmonics of cyclotron resonance with an amplitude proportional to  $\alpha\beta$  (except for the particular case when  $\alpha = \beta$ ) [33].

### 1.4.2 Optical Spin Orientation and Detection

To date, the most efficient way of creating non-equilibrium spin orientation in a semiconductor is provided by interband absorption of circularly polarized light.

It can be seen from Fig. 1.2 that for  $E_g < \hbar\omega < E_g + \Delta$  absorption produces an average electron spin along the direction of excitation equal to  $(-1/2)(3/4) + (+1/2)(1/4) = -1/4$  and an average hole spin equal to  $+5/4$ , with a sum  $+1$ , equal to the angular momentum of the absorbed right circularly polarized photon. Thus in a  $p$ -type semiconductor the degree of spin polarization of the photo-excited electrons will be  $-50\%$ , the minus sign indicating that the spin orientation is opposite to the angular momentum of incident photons.

If our electron immediately recombines with its partner hole, a 100% circularly polarized photon will be emitted. However in a  $p$ -type semiconductor electrons will predominantly recombine with the majority holes, which are not polarized. Then the same selection rules show that the circular polarization of luminescence should be  $\mathcal{P}_0 = 25\%$ , if the holes are not polarized, and if no electron spin relaxation occurs during the electron lifetime  $\tau$ , i.e. if  $\tau_s \gg \tau$ , where  $\tau_s$  is the electron spin relaxation time. Generally, the degree  $\mathcal{P}$  of circular polarization of the luminescence excited by circularly polarized light is less than  $\mathcal{P}_0$ :

$$\mathcal{P} = \frac{\mathcal{P}_0}{1 + \tau/\tau_s}. \quad (1.13)$$

In an optical spin orientation experiment a semiconductor (usually  $p$ -type) is excited by circularly polarized light with  $\hbar\omega > E_g$ . The circular polarization of the luminescence is analyzed, which gives a direct measure of the electron spin polarization. Actually, the degree of circular polarization is simply equal to the average electron spin. Thus various spin interactions can be studied by simple experimental means. The electron spin polarization will be measured provided the spin relaxation time  $\tau_s$  is not very short compared to the recombination time  $\tau$ , a condition, which often can be achieved even at room temperature.

### 1.4.3 Spin Relaxation

Spin relaxation, i.e. disappearance of initial non-equilibrium spin polarization, is the central issue for all spin phenomena. Spin relaxation can be generally understood as a result of the action of fluctuating in time magnetic fields. In most cases, these are

not real magnetic fields, but rather “effective” magnetic fields originating from the spin-orbit, or, sometimes, exchange interactions, see Sect. 1.2.

### Generalities

A randomly fluctuating magnetic field is characterized by two important parameters: its amplitude (or, more precisely, its rms value), and its correlation time,  $\tau_c$ , i.e. the time during which the field may be roughly considered as constant. Instead of the amplitude, it is convenient to use the rms value of the spin precession frequency in this random field,  $\omega$ .

Thus we have the following physical picture of spin relaxation: the spin makes a precession around the (random) direction of the effective magnetic field with a typical frequency  $\omega$  and during a typical time  $\tau_c$ . After a time  $\tau_c$  the direction and the absolute value of the field change randomly, and the spin starts its precession around the new direction of the field. After a certain number of such steps the initial spin direction will be completely forgotten.

How this happens, depends on the value of the dimensionless parameter  $\omega\tau_c$ , which is the typical angle of spin precession during the correlation time. Two limiting cases may be considered:

(a)  $\omega\tau_c \ll 1$  (*Most Frequent Case*)

The precession angle is small, so that the spin vector experiences a slow angular diffusion. During a time  $t$ , the number of random steps is  $t/\tau_c$ , for each step the squared precession angle is  $(\omega\tau_c)^2$ . These steps are not correlated, so that the total squared angle after a time  $t$  is  $(\omega\tau_c)^2(t/\tau_c)$ . The spin relaxation time may be defined as the time at which this angle becomes of the order of 1. Hence:

$$\frac{1}{\tau_s} \sim \omega^2 \tau_c \quad (1.14)$$

This is essentially a *classical* formula (the Planck constant does not enter), although certainly it can be also derived quantum-mechanically. Note, that in this case  $\tau_s \gg \tau_c$ .

(b)  $\omega\tau_c \gg 1$

This means that during the correlation time the spin will make many rotations around the direction of the magnetic field. During the time on the order of  $1/\omega$  the spin projection transverse to the random magnetic field is (on the average) completely destroyed, while its projection along the direction of the field is conserved. At this stage the spin projection on its initial direction will diminish 3 times. [Let the random magnetic field have an angle  $\theta$  with the initial spin direction. After many rotations the projection of the spin on the initial direction will diminish as  $(\cos \theta)^2$ . In three dimensions, the average of this value over the possible orientations of the random field yields  $1/3$ .]

After time  $\tau_c$  the magnetic field changes its direction, and the initial spin polarization will finally disappear. Thus in the case  $\omega\tau_c \gg 1$  the time decay of spin

polarization is not exponential, and the process has two distinct stages, the first one has a duration  $1/\omega$ , and the second one has a duration  $\tau_c$ . The overall result is:  $\tau_s \sim \tau_c$ .

This consideration is quite general and applies to any mechanism of spin relaxation. We have only to understand the values of the relevant parameters  $\omega$  and  $\tau_c$  for a given mechanism.

### Spin Relaxation Mechanisms

There are several possible mechanisms providing the fluctuating magnetic fields responsible for spin relaxation.

#### *Elliott-Yafet Mechanism* [34, 35]

The electrical field, accompanying lattice vibrations, or the electric field of charged impurities is transformed to an effective magnetic field through spin-orbit interaction. Thus momentum relaxation should be accompanied by spin relaxation.

For phonons, the correlation time is on the order of the inverse frequency of a typical thermal phonon. Spin relaxation by phonons is normally rather weak, especially at low temperatures.

For scattering by impurities, the direction and the value of the random magnetic field depends on the geometry of the individual collision (the impact parameter). This random field can not be characterized by a single correlation time, since it exists only *during* the brief act of collision and is zero between collisions. In each act of scattering the electron spin rotates by some small angle  $\phi$ . These rotations are uncorrelated for consequent collisions, so the average square of spin rotation angle during time  $t$  is on the order of  $\langle \phi^2 \rangle (t/\tau_p)$ , where  $\tau_p$  is the time between collisions and  $\langle \phi^2 \rangle$  is the average of  $\phi^2$  over the scattering geometry.

Thus  $1/\tau_s \sim \langle (\phi)^2 \rangle / \tau_p$ . The relaxation rate is obviously proportional to the impurity concentration.

#### *Dyakonov-Perel Mechanism* [9, 36]

The physical idea was put forward in 1969 by Gershenzon et al. [37] in analogy to the so-called *motional narrowing* of nuclear magnetic resonance. The mechanism is related to the spin-orbit splitting of the conduction band caused by the absence of inversion symmetry, see Sect. 1.4.1.

For a given  $\mathbf{p}$ ,  $\Omega(\mathbf{p})$  is the spin precession frequency in this field. The effective magnetic field changes in time because the direction of  $\mathbf{p}$  varies due to electron collisions. Thus the correlation time is on the order of the momentum relaxation time,  $\tau_p$ , and if  $\Omega\tau_p$  is small, which is normally the case, we get:

$$\frac{1}{\tau_s} \sim \Omega^2 \tau_p. \quad (1.15)$$

In contrast to the Elliott-Yafet mechanism, now the spin rotates not during, but *between* the collisions. Accordingly, the relaxation rate *increases* when the impurity concentration decreases (i.e. when  $\tau_p$  becomes longer). It happens that this mechanism is often the dominant one, both in bulk  $A^{III}B^V$  and  $A^{II}B^{VI}$  semiconductors, like GaAs, as well as in 2D structures.

*Bir-Aronov-Pikus Mechanism [38]*

This is a mechanism of spin relaxation of non-equilibrium electrons in  $p$ -type semiconductors due to the exchange interaction between the electron and hole spins (or, expressing it otherwise, exchange interaction between an electron in the conduction band and all the electrons in the valence band). This spin relaxation rate, being proportional to the number of holes, may become the dominant one in heavily  $p$ -doped semiconductors.

*Relaxation via Hyperfine Interaction with Nuclear Spins*

The electron spin interacts with the spins of the lattice nuclei (see Sect. 1.4.6 below), which are normally in a disordered state. Thus the nuclei provide a random effective magnetic field, acting on the electron spin. The corresponding relaxation rate is rather weak, but may become important for localized electrons, when other mechanisms, associated with electron motion, do not work.

*Spin Relaxation of Holes in the Valence Band*

The origin of this relaxation is in the splitting of the valence band into subbands of light and heavy holes. In this case,  $\hbar\Omega(\mathbf{p})$  is equal to the energy difference between light and heavy holes for a given  $\mathbf{p}$  and the correlation time is again  $\tau_p$ . However, in contrast to the situation for electrons in the conduction band, we have now the opposite limiting case:  $\Omega(\mathbf{p})\tau_p \gg 1$ . So, the hole spin relaxation time is on the order of  $\tau_p$ , which is very short. One can say that the hole “spin”  $\mathbf{J}$  is rigidly fixed with respect to its momentum  $\mathbf{p}$ , and because of this, momentum relaxation leads automatically to spin relaxation.

For this reason, normally it is virtually impossible to maintain an appreciable non-equilibrium polarization of bulk holes. However, Hilton and Tang [39] have managed to observe the spin relaxation (on the femtosecond time scale) of both light and heavy holes in undoped bulk GaAs. The general theory of the relaxation of spin, as well as helicity and other correlations between  $\mathbf{J}$  and  $\mathbf{p}$ , for holes in the valence band was given by Dyakonov and Khaetskii [40].

**Influence of Magnetic Field on Spin Relaxation**

In the presence of an external magnetic field  $\mathbf{B}$ , the spins perform a regular precession with a frequency  $\Omega = g\mu B/\hbar$ , and one should distinguish between relaxation of the spin component along  $\mathbf{B}$  and the relaxation, or *dephasing*, of the perpendicular components. In the magnetic resonance literature it is customary to denote the corresponding longitudinal and transverse times as  $T_1$  and  $T_2$  respectively.

To understand what happens, it is useful to go to a frame rotating around  $\mathbf{B}$  with the spin precession frequency  $\Omega$ . In the absence of random fields, the spin vector would remain constant in the rotating frame. Relaxation is due to random fields in the rotating frame, and obviously these fields now rotate around  $\mathbf{B}$  with the same frequency  $\Omega$ .

Thus random fields directed along  $\mathbf{B}$  are the same as in the rest frame, and cause the same relaxation of the perpendicular spin components with a characteristic time  $T_2 \sim$

$\tau_s$ . However the perpendicular components of the random field, which are responsible for the relaxation of the spin component along  $\mathbf{B}$ , do rotate. The importance of this rotation is determined by the parameter  $\Omega\tau_c$ , the angle of rotation of the random field during the correlation time.

If  $\Omega\tau_c \ll 1$ , then rotation is of no importance, since the random field will anyway change its direction after a time  $\tau_c$ . However, for  $\Omega\tau_c \gg 1$  the rotating random field will effectively average out during the correlation time, resulting in a decrease of the longitudinal spin relaxation rate.

A simple calculation gives:

$$\frac{1}{T_1} = \frac{1}{\tau_s} \frac{1}{1 + (\Omega\tau_c)^2} = \frac{\omega^2\tau_c}{1 + (\Omega\tau_c)^2}. \quad (1.16)$$

Interestingly, with increasing magnetic field the longitudinal spin relaxation rate changes from being proportional to  $\tau_c$  to becoming proportional to  $1/\tau_c$ .

Again, the classical formula (1.16) can be derived quantum mechanically. From the quantum point of view the longitudinal relaxation is due to flips of the spin projection on  $\mathbf{B}$ , which requires an energy  $g\mu B$ . Since the energy spectrum of the random field has a width  $\hbar/\tau_c$  the process becomes ineffective when  $g\mu B \gg \hbar/\tau_c$ , or equivalently, when  $\Omega\tau_c \gg 1$ .

Ivchenko [41] has calculated the influence of magnetic field on the Dyakonov-Perel spin relaxation. The result coincides with (1.16) with  $\tau_c = \tau_p$ , except that the spin precession frequency  $\Omega$  is replaced by the (greater) electron cyclotron frequency,  $\omega_c$ . The reason is that for this case the variation of the vector  $\boldsymbol{\Omega}(\mathbf{p})$  is primarily due to the rotation of the electron momentum  $\mathbf{p}$  in magnetic field.

### Spin Relaxation of Two-Dimensional Electrons and Holes

Usually the Dyakonov-Perel mechanism is the dominant one. The relaxation rate is governed by the linear momentum dependence of the effective magnetic field, or the vector  $\boldsymbol{\Omega}(\mathbf{p})$ , see Sect. 1.4.1.

The spin relaxation is generally anisotropic and depends on the growth direction [30]. As seen from (1.11), for quantum wells grown in the  $\langle 001 \rangle$  direction the effective magnetic field lies in the 2D plane. As a consequence, the spin component perpendicular to the plane decays 2 times faster than the spin in-plane components.<sup>10</sup>

An interesting case is when the growth direction is  $\langle 110 \rangle$ . (1.12) says that now the random effective magnetic field is always perpendicular to the 2D plane! Its value and sign depend only on the projection of electron momentum on the  $\langle 1\bar{1}0 \rangle$  direction. This means that the relaxation times for both in-plane components of the spin are equal, however the normal to the plane spin component does not relax at all.<sup>11</sup>

<sup>10</sup>The reason is that the  $z$  projection of the spin is rotated by both  $x$  and  $y$  components of the random field, while the  $x$  spin projection is influenced by the  $y$  component only, since the  $z$  component of the random field is zero.

<sup>11</sup>In fact, the normal spin component *will* slowly decay because of the small cubic in  $\mathbf{p}$  Dresselhaus terms, which were neglected in deriving (1.12). Experimentally, a  $\sim 20$  times suppression of the relaxation rate for the  $z$  spin component in  $\langle 110 \rangle$  quantum wells was observed by Ohno et al. [42].



When both the Ohkawa-Uemura-Vasko and the Dyakonov-Kachorovskii types of interactions coexist and are of the same order of magnitude ( $\alpha \sim \beta$  in (1.11)), a specific anisotropy of relaxation in the  $xy$  plane arises due to a kind of interference between the two terms. This was demonstrated in 1999 by Averkiev and Golub [43].

The spin structure of holes in a quantum well is also completely different from that in the bulk. More details on spin-orbit interaction in two dimensional systems can be found in Winkler's book [44].

### Persistent Spin Helix

A most unusual and unexpected phenomenon was predicted in 2006 by Bernevig et al. [31]. As we have seen, momentum-dependent spin splitting  $V_{so} = \hbar \mathbf{\Omega}(\mathbf{p}) \cdot \mathbf{S}$  is equivalent to an effective magnetic field acting on the electron spins,  $\mathbf{\Omega}(\mathbf{p})$  being the spin precession frequency for a given  $\mathbf{p}$ . Since the electron momentum  $\mathbf{p}$  rapidly changes because of collisions, this normally will destroy spin polarization via the Dyakonov-Perel mechanism.

However, in [31] it was shown that there exist special situations when certain *periodic* spin structures are immune to this type of relaxation and can live much longer than the spin polarization that is uniform in space.<sup>12</sup>

Such a special situation arises for a symmetric quantum well grown in the  $\langle 110 \rangle$  direction, see (1.12) derived by Dyakonov and Kachorovskii [30].

The in-plane spin projections rotate around the  $z$ -axis, perpendicular to the plane, with a frequency proportional to the electron momentum, or velocity, along the  $x$ -axis:  $\Omega = \alpha m v_x$ , where  $\alpha$  is a constant. The time needed to rotate the spin by  $360^\circ$  is  $2\pi/(\alpha m v_x)$  and the distance travelled by the electron in the  $x$ -direction during this time is  $L = 2\pi/\alpha m$ .

This distance does not depend on the value of the electron velocity, nor on its direction. Moreover, it remains the same even if the velocity changes in a random way due to collisions! This means that a periodic in the  $x$ -direction (and uniform in the  $y$ -direction) polarization pattern with a period  $L$  is *stable*.

The prediction in [31] was experimentally confirmed in 2009 by Koralek et al. [32] for another special situation where  $\mathbf{\Omega}(\mathbf{p})$  is given by (1.11) with  $\alpha = \beta$ .

In this case, instead of  $x$  and  $y$ , one can introduce new in-plane coordinates  $\xi = (x + y)/\sqrt{2}$  and  $\eta = (x - y)/\sqrt{2}$ , so that  $V_{so} = 2\alpha p_\xi S_\eta$ , and the only non-zero component of the vector  $\mathbf{\Omega}$  is  $\Omega_\eta = 2\alpha p_\xi$ . This is similar to the previous case where the non-zero component of  $\mathbf{\Omega}$  was  $\Omega_z = \alpha p_x$ , except that now the spin is rotated not in  $(xy)$  plane, but in the  $(\xi z)$  plane, and the observation of the  $S_z$  component by optical means is easier.

In fact, the observed spin helix slowly decays because of the small cubic in  $\mathbf{p}$  terms in  $V_{so}$  [32], see also the footnote 11.

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<sup>12</sup>Bernevig et al. [31] have explained their result as a consequence of the exact SU(2) symmetry in special cases and used the Keldysh formalism. Here a more pedestrian qualitative explanation is given.

### 1.4.4 Hanle Effect

Depolarization of luminescence by a transverse magnetic field (first discovered by Wood and Ellett, as described in Sect. 1.1 is effectively employed in experiments on spin orientation in semiconductors.

The reason for this effect is the precession of electron spins around the direction of the magnetic field. Under continuous illumination, this precession leads to the decrease of the average projection of the electron spin on the direction of observation, which defines the degree of circular polarization of the luminescence. Thus the degree of polarization decreases as a function of the transverse magnetic field. Measuring this dependence under steady state conditions makes it possible to determine both the spin relaxation time and the recombination time.

This effect is due to the precession of electron spins in a magnetic field  $\mathbf{B}$  with the Larmor frequency  $\Omega$ . This precession, along with spin pumping, spin relaxation, and recombination is described by the following simple equation of motion of the average spin vector  $\mathbf{S}$ :

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\Omega} \times \mathbf{S} - \frac{\mathbf{S}}{\tau_s} - \frac{\mathbf{S} - \mathbf{S}_0}{\tau}, \quad (1.17)$$

where the first term on the rhs describes spin precession in a magnetic field ( $\Omega = g\mu B/\hbar$ ), the second term describes spin relaxation, and the third one describes generation of spin by optical excitation ( $\mathbf{S}_0/\tau$ ) and recombination ( $-\mathbf{S}/\tau$ ). The vector  $\mathbf{S}_0$  is directed along the exciting light beam, its absolute value is equal to the initial average spin of photo-created electrons.

In the stationary state ( $d\mathbf{S}/dt = 0$ ) and in the absence of a magnetic field, one finds:

$$S_z(0) = \frac{S_0}{1 + \tau/\tau_s}, \quad (1.18)$$

where  $S_z(0)$  is the projection of the spin on the direction of  $\mathbf{S}_0$  ( $z$ -axis). Since  $S_z(0)$  is equal to the degree of polarization of the luminescence (Sect. 1.4.2), this formula is equivalent to the expression for  $\mathcal{P}$  in (1.13). In the presence of magnetic field transverse to  $\mathbf{S}_0$  we obtain:

$$S_z(B) = \frac{S_z(0)}{1 + (\Omega\tau^*)^2}, \quad \frac{1}{\tau^*} = \frac{1}{\tau} + \frac{1}{\tau_s}. \quad (1.19)$$

The effective time  $\tau^*$  defines the width of the depolarization curve. Thus the spin projection  $S_z$  (and hence the degree of circular polarization of the luminescence) decreases as a function of the transverse magnetic field. Combining the measurements of the zero-field value  $\mathcal{P} = S_z(0)$  and of the magnetic field dependence in the Hanle effect, we can find the two essential parameters: the electron lifetime,  $\tau$ , and the spin relaxation time,  $\tau_s$ , under steady-state conditions.

If polarized electrons are created by a short pulse, time-resolved measurements reveal, very impressively, the damped spin precession around the direction of magnetic field [45], which follows from (1.17) for a given initial spin value.

### 1.4.5 Interconnections Between Spin and Charge

Because of spin-orbit interaction, charge and spin are interconnected: there is a number of phenomena where an electrical current produces spin current and/or homogeneous spin polarization and *vice versa*. Generally, an electric current induces a transverse spin current leading to spin accumulation at the boundaries, or a uniform spin polarization, or both effects simultaneously. Inverse effects exist too.

Phenomenologically, all these effects can be derived from pure symmetry considerations, according to the general principle: *everything, that is not forbidden by symmetry or conservation laws, will happen*. The microscopic theory should provide the physical mechanism of the phenomenon under consideration, as well as the values of observable quantities.

#### *Electric Current Producing Spin Current*

Spin accumulation at the lateral boundaries of a current-carrying sample (the Spin Hall Effect) was predicted in 1971 by Dyakonov and Perel [46, 47], together with the Inverse Spin Hall Effect [46], which was observed for the first time in 1984 by Bakun et al. [48]. The name “Spin Hall Effect” was given by Hirsch [49] who re-predicted this phenomenon 28 years later.

The first experimental observations of the (direct) Spin Hall Effect were reported in 2004 by Kato et al. [50] and in 2005 by Wunderlich et al. [51].

Related phenomena that have recently attracted much attention are the Spin Hall Magnetoresistance [52] and the *swapping* effect - a mutual transformation of spin currents introduced in 2009 by Lifshits and Dyakonov [53].

#### *Electric Current Producing Homogeneous Spin Polarization*

In gyrotropic systems (like Te crystals, or III-V quantum wells) the electric current can also generate *homogeneous* spin polarization.<sup>13</sup> This effect was predicted in 1978 by Ivchenko and Pikus [54] for bulk Te crystals and observed in 1979 by Vorob'ev et al. [55]. In 2D structures the mechanism of spin orientation by current is different from that in Tellurium (it is caused by the asymmetry of spin relaxation). In two dimensions this effect was treated theoretically in 1979 by Vasko and Prima [56], in 1985 by Levitov et al. [57], in 1990 by Edelstein [58], and in 1991 by Aronov and Lyanda-Geller [59]. Experimentally it was observed for the first time in 2004 for quantum wells by Silov et al. [60] and by Ganichev et al. [61], and also for strained bulk materials by Kato et al. [62].

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<sup>13</sup>I thank S. Ganichev and E. Ivchenko for an illuminating discussion of this issue.

Inversely, spin polarization in such systems can induce electric current as shown theoretically in 1989 by Ivchenko et al. [63]. The first experimental demonstration of this effect, named the *spin-galvanic effect*, was reported in 2002 by Ganichev et al. [64].

Furthermore, the interconnection between the spin polarization, spin current, and charge current is particularly pronounced in the application of polarized radiation giving rise to photogalvanic phenomena. The *circular photogalvanic effect* was independently predicted in 1978 by Ivchenko and Pikus [54] and by Belinicher [65]. It was first observed for Te crystals in 1978 by Asnin et al. [66] and for quantum wells in 2001 by Ganichev et al. [67].

This kind of effects have been re-discovered theoretically several times.<sup>14</sup> Hence, a large variety of different names exist labelling one and the same thing: appearance of non-equilibrium spin polarization induced by *dc* electric current in gyrotropic media with linear in  $\mathbf{p}$  spin splitting of energy bands, and *vice versa*.<sup>15</sup>

A more elaborate description can be found in Chaps. 8, 9, and 11.

#### 1.4.6 Interaction Between the Electron and Nuclear Spin Systems

The non-equilibrium spin-oriented electrons can easily transmit their polarization to the lattice nuclei, thus creating an *effective nuclear magnetic field*  $\mathbf{B}_N$ . This field will, in turn, influence the spin of electrons (but not their orbital motion!). For example, it can strongly influence the electron polarization via the Hanle effect [68]. Thus the spin-oriented electrons and the polarized lattice nuclei form a strongly coupled system, in which spectacular non-linear phenomena, like self-sustained slow oscillations and hysteresis are observed by simply looking at the circular polarization of the luminescence [14, 69]. Optical detection of the nuclear magnetic resonance in a semiconductor was demonstrated for the first time by Ekimov and Safarov [70]. Recently, spectacular new results were obtained for two-dimensional structures, see [71–73] and references therein.

The physics of these phenomena are governed by three basic interactions:

##### (a) Hyperfine Interaction

The interaction has the form  $A \mathbf{I} \cdot \mathbf{S}$ , where  $\mathbf{I}$  is the nuclear spin and  $\mathbf{S}$  is the electron spin. If the electrons are in equilibrium, this interaction provides a mechanism for nuclear spin relaxation. If the electron spin system is out of equilibrium, it leads

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<sup>14</sup>Probably, because announcing a new effect is more exciting than digging into literature to find out that it is already known.

<sup>15</sup>Some of the labels used are: current-induced spin polarization (CISP), inverse spin-galvanic effect (ISGE), current-induced spin accumulation (CISA), magneto-electric effect (MEE), kinetic magneto-electric effect (KMEE), Edelstein effect (EE), inverse Edelstein effect (IEE), and even Rashba-Edelstein effect (REE).

to dynamic nuclear polarization. These processes are very slow compared to the characteristic electron time scale.

On the other hand, if the nuclei are polarized, this interaction is equivalent to the existence of a nuclear magnetic field acting on the electron spins:  $\mathbf{B}_N = A\langle\mathbf{I}\rangle/(g\mu)$ , where  $\langle\mathbf{I}\rangle$  is the average value of the nuclear spin. The field of 100% polarized nuclei in GaAs would be about 6 T! Experimentally, nuclear polarization of several percent is easily achieved.

The time of build-up of nuclear polarization due to interaction with electrons is given by the general formula (1.14), where  $\omega$  should be understood as the precession frequency of the nuclear spin in the effective electron magnetic field due to hyperfine interaction, and the correlation time  $\tau_c$  depends on the electron state. For mobile electrons this time is extremely short:  $\tau_c \sim \hbar/E$ , where  $E$  is the electron energy. As first pointed out by Bloembergen [76], nuclear polarization (or depolarization) by electrons is much more effective when the electrons are localized, for example, bound to donors, or confined in a quantum dot. In this case  $\tau_c$  is generally much longer than for mobile carriers. It is defined by the shortest of processes like recombination, hopping to another donor site, thermal ionization, or spin relaxation.<sup>16</sup>

#### (b) Dipole-Dipole Interaction Between Nuclear Spins

This interaction can be characterized by the local magnetic field,  $B_L$ , on the order of several Gauss, which is created at a given nuclear site by the neighboring nuclei.<sup>17</sup> The precession period of a nuclear spin in the local field, on the order of  $T_2 \sim 10^{-4}$  s, gives a typical intrinsic time scale for the nuclear spin system. During this time, thermal equilibrium within this system is established, with a *nuclear spin temperature*  $\Theta_N$ , which may be very different from the crystal temperature  $T$ , for example, something like  $10^{-6}$  K.

Since the times characterizing the interaction of the nuclear spin system with the outside world (electrons, or lattice) is much greater than  $T_2$ , the nuclear spin system can be considered as always being in a state of internal thermal equilibrium with a nuclear spin temperature defined by the energy exchange with the electrons and/or the lattice. Accordingly, the nuclear polarization is always given by the *thermodynamic* formula:  $\mathcal{P}_N \sim \mu_N B / (k\Theta_N)$ , where  $\mu_N$  is the nuclear magnetic moment. The most important concept of the nuclear spin temperature was introduced by Redfield [74], see also [75].

<sup>16</sup>The concept of a mean nuclear magnetic field is well justified for electrons in semiconductors, either moving freely in the conduction band or localized on a shallow donor. Thus, for a shallow donor in GaAs the Bohr radius is around 10 nm, so that about  $10^5$  lattice nuclei are simultaneously seen by a localized electron.

<sup>17</sup>As it was pointed out in Sect. 1.2, the magnetic dipole-dipole interaction between electron spins can be usually neglected. Given that a similar interaction between nuclear spins is about a million times smaller, it may seem strange that this interaction may be of any importance. The answer comes when we consider the extremely long time scale in the nuclear spin system (seconds or more) compared to the characteristic times for the electron spin system (nanoseconds or less).

The dipole-dipole interaction is also responsible for the nuclear spin diffusion [76] – a process that tends to make the nuclear polarization uniform in space. The nuclear spin diffusion coefficient can be estimated as  $D_N \sim a_0^2/T_2 \sim 10^{-12} \text{ cm}^2/\text{s}$ , where  $a_0$  is the distance between the neighboring nuclei. Thus it takes about 1 s to spread out the nuclear polarization on a distance of 100 Å, and several hours for a distance of 1 μm.

(c) *Zeeman Interaction of Electron and Nuclear Spins*

The energy of a nuclear magnetic moment in an external magnetic field is roughly 2000 times smaller than that for the electron. However, it becomes important in magnetic fields exceeding the local field  $B_L \sim 3 \text{ G}$ . Accordingly, the behavior of the nuclear spin system in small fields, less than  $B_L$ , is quite different than in larger fields. At zero magnetic field the nuclear spins can not be polarized (the Zeeman energy is zero, while  $\Theta_N$  remains finite, see the thermodynamic formula above).

Also, as the magnetic field increases, the time of polarization will increase according to (1.16), where  $\Omega$  is the electron spin precession frequency. Quantum mechanically, this increase is the result of the strong mismatch between the electron and nuclear Zeeman energies. Because of this mismatch the electron-nucleus flip-flop transitions would violate energy conservation. They can occur, however because of the energy uncertainty  $\Delta E \sim \hbar/\tau_c$ .

The interplay of these interactions under various experimental conditions accounts for the extremely rich and interesting experimental findings in this domain, see Chap. 12.

### Overhauser Effect

An extraordinary effect was predicted more than 60 years ago by Overhauser [78]. In the simplest form, it can be described as follows.

Suppose that there is a single species of nuclei with spin  $I = 1/2$ , interacting with electrons via the hyperfine interaction  $V = A\mathbf{I} \cdot \mathbf{S}$ , which conserves the total spin  $\mathbf{I} + \mathbf{S}$ . In a magnetic field  $\mathbf{B}$  the nuclear and electron energies are  $\mu_n \mathbf{B} \cdot \mathbf{I}$  and  $g\mu \mathbf{B} \cdot \mathbf{S}$  respectively, and the latter is  $\sim 2000$  times greater than the former, because such is the ratio of the electron ( $\mu$ ) and the nuclear ( $\mu_n$ ) magnetic moments (here  $g$  is the electron g-factor).

Let  $N_+$ ,  $N_-$  and  $n_+$ ,  $n_-$  be the populations of the spin up (+) and spin down (−) states for the nuclei and electrons respectively. The balance equations for  $N_+$  and  $N_-$  are:

$$\frac{dN_+}{dt} = -W_1 N_+ n_- + W_2 N_- n_+, \quad \frac{dN_-}{dt} = W_1 N_+ n_- - W_2 N_- n_+, \quad (1.20)$$

where  $W_1$  and  $W_2$  are the probabilities of transitions  $(+-) \rightarrow (-+)$  and  $(-+) \rightarrow (+-)$ . Here the first and second signs designate the spin states of the nucleus and of the electron respectively.

Under stationary conditions we have:

$$W_1 N_+ n_- = W_2 N_- n_+, \quad (1.21)$$

In thermodynamic equilibrium, the populations  $N_+$ ,  $N_-$  and  $n_+$ ,  $n_-$  are related by the respective Boltzman factors:

$$N_+/N_- = \exp(2\mu_n B/T), \quad n_+/n_- = \exp(-g\mu B/T), \quad (1.22)$$

where  $T$  is the temperature in units of energy (it is assumed that the electrons are non-degenerate). Hence the following general relation between the transition probabilities in (1.20, 1.21) should hold:  $W_1/W_2 = \exp(-g\mu B/T)$ , where the very small nuclear Zeeman energy is neglected.

Suppose now that the electron spin populations are *forced to be equal* by some external agent, so that  $n_+ = n_-$ . Then, it follows immediately from (1.21, 1.22) that  $N_+/N_- = W_2/W_1 = \exp(g\mu B/T)$  or, in other words, the nuclear polarization  $P_N = (N_+ - N_-)/(N_+ + N_-)$  is given by the formula:

$$P_N = \tanh(g\mu B/2T), \quad (1.23)$$

which means that in these non-equilibrium conditions the nuclear polarization becomes very high, equal to the eventual *equilibrium electron polarization* (but with an opposite sign). This is the essence of the Overhauser effect.

Originally, Overhauser proposed to maintain  $n_+ = n_-$  by saturation of the electron paramagnetic resonance in metals. In semiconductors, one can excite electrons by unpolarized or linearly polarized light. In the presence of magnetic field the photocreated electrons will have a tendency towards thermodynamic equilibrium, resulting in electron spin polarization following from (1.22). The characteristic time for this process is the electron spin relaxation time  $\tau_s$ .

If the recombination time,  $\tau$ , is much shorter than  $\tau_s$ , the photocreated electrons will not have enough time to achieve this equilibrium value, the steady state electron spin polarization will be negligible, and the condition  $n_+ = n_-$  will be satisfied with a high degree of accuracy.

The Overhauser effect was demonstrated experimentally in 1953 by Carver and Slichter [79], see also ([80], [81]), and was extensively studied ever since, with important applications for chemistry [82].

### Nuclear Self-Polarization

As a certain development of Overhauser's idea, in 1972 Dyakonov and Perel [83] have predicted that below a certain critical temperature, a large *spontaneous* nuclear polarization should appear in a small applied magnetic field, provided that the *electron spins are maintained in an unpolarized state*.

If the nuclei are polarized then, for any spin effect, the nuclear field  $\mathbf{B}_N$  should be added to the external magnetic field,  $\mathbf{B}$ . In particular, this should be done in

(1.23) describing the dynamic nuclear polarization in the Overhauser effect. The self-polarization is due to Overhauser effect in the effective magnetic field  $\mathbf{B}_N$  created by the *polarized nuclei themselves*.

Consider the situation when the external field  $\mathbf{B}$  is very small, so that  $g\mu B \ll T$ . Then, using (1.23), we obtain the following self-consistent equation for the nuclear polarization:

$$P_N = \tanh(g\mu B_N/2T), \quad (1.24)$$

where the nuclear magnetic field is given by  $\mathbf{B}_N = A\langle \mathbf{I} \rangle / (g\mu) = AP_N / (2g\mu)$ . Thus (1.24) may be rewritten as

$$P_N = \tanh\left(P_N \frac{T_c}{T}\right), \quad (1.25)$$

with the critical temperature,  $T_c = A/4$  (for the case, when  $I = 1/2$ ).

For  $T > T_c$  this equation has a single solution  $P_N = 0$ . However, for  $T < T_c$  there are also two nonzero solutions (with opposite signs) describing nuclear self-polarization, and by using (1.20) it can be shown that these states are stable, while the state with  $P_N = 0$  is unstable. The dependence  $P_N(T)$  has the character of a second-kind phase transition.

The generalisation for the case when there are several species of nuclei with different hyperfine constants,  $A$ , and spins,  $I$ , is given by: [14, 83, 84]

$$T_c = \Sigma AI(I + 1)/3, \quad (1.26)$$

where the sum is over all the nuclei in the unit cell, and the abundance of different isotopes should be taken into account. Calculations give  $T_c = 9K$  for InSb and  $T_c = 1.3K$  for GaAs.

The phenomenon of nuclear self-polarization has never been observed so far and remains a challenge.<sup>18</sup> An unsuccessful attempt was made in 1998 by Farah et al. [85] where optical spin orientation in InP was studied. While spectacular nuclear effects were observed, no spontaneous nuclear polarization at low magnetic field was found.

However, a very similar phenomenon was independently predicted in 1987 by Fortson and Heckel [86] for a gas and experimentally observed in 1996 in an optically pumped Cesium vapor by Klipstein et al. [87].

Conditions for observation of nuclear self-polarization under optical spin orientation in semiconductors were studied in detail by Dyakonov in 1984 [84].

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<sup>18</sup>It should be noticed that obtaining a high degree of nuclear polarization in solids is a very desirable goal for experimental nuclear and particle physics.



## 1.5 Overview of the Book Content

Within the scope of this introductory chapter it is only possible to briefly outline the main directions of the current research.

*Time-Resolved Optical Techniques.* The innovative time resolved optical techniques, based on Faraday or Kerr polarization rotation, were developed by Awschalom's group in Santa Barbara [88] and by Harley's group in Southampton [89]. These techniques opened a new era in experimental spin physics. They have allowed the visualization of spin dynamics on the sub-picosecond time scale and study of the intimate details of various spin processes in a semiconductor. This book presents several subjects, where most of the experimental results are obtained by using these optical techniques.

*Spin Dynamics in Quantum Wells and Quantum Dots.* The spin dynamics of carriers in quantum wells is discussed in Chap. 2. Exciton spin dynamics and the fine structure of neutral and charged excitons are presented in Chaps. 3 (quantum wells) and 4 (quantum dots). The interplay between carrier exchange and confinement leads to quite a number of interesting and subtle effects, that are now well understood. These chapters show how many important parameters, like spin splittings and relaxation times, can be accurately determined.

*Spin Noise Spectroscopy.* Chapter 5 gives a general introduction to experimental time-resolved techniques. It also presents a quite new way of research in spin physics, where the methods of noise spectroscopy, known in other domains, are applied to the spin system in a semiconductor. Unlike other techniques, this allows the study of spin dynamics without perturbing the system by an external excitation.

*Coherent Spin Dynamics in Quantum Dots.* This topic is covered in Chap. 6. It contains extraordinary interesting and surprising new results on "mode-locking" of spin coherence in an ensemble of quantum dots excited by a periodic sequence of laser pulses and, in particular, on spin precession "focusing" induced by the hyperfine interaction with the nuclear spins.

*Spin Properties of Confined Electrons in Silicon.* Spin-related studies in Silicon were somewhat neglected in recent years, because it practically does not give photoluminescence, has a weak spin-orbit interaction, and contains few nuclear spins. However, Chap. 7 demonstrates interesting new spin physics in Si-based quantum wells and quantum dots, studied mostly by the Electron Spin Resonance, which may have extremely small linewidths.

*Interconnections Between Spin and Charge.* Chapter 8 is devoted to the coupling between the spin and charge currents due to spin-orbit interaction and the Spin Hall Effect, which was predicted almost half a century ago [46], but observed for the first time only in 2004 [50] causing a widespread interest. A related subject is treated in Chap. 9 describing spin-related photocurrents and circular photo-galvanic effect in two-dimensional structures. There are a variety of interesting experiments, which reveal subtle physics.

*Spin Injection.* Spin injection from a ferromagnet to a normal metal, originally proposed by Aronov [90], and spin detection using a ferromagnet, originally proposed by Silsbee [91], was first observed by Johnson and Silsbee [92]. Injection through a ferromagnet/semiconductor junction has been investigated in many recent works. Chapter 10 describes these and related phenomena, which have some promising applications.

*Spin Orbit-Torques.* Chapter 11 is devoted to spin-orbit torques and their interaction with the magnetization in magnetically ordered materials. During recent years this field of research is continuously expanding. The possibility of switching magnetic domains or dynamically exciting magnetization in ferromagnets by injection of spin currents (via spin Hall effect or inverse spin-galvanic effect) can open new ways of information processing. Potentially, this could become the most important practical application of spin physics.

*Nuclear Spin Effects in Optics and Electron Transport.* Chapter 12 discusses electron-nuclear spin systems formed by the hyperfine interaction in quantum wells and quantum dots. Nuclear spin polarization results in spectacular optical effects, including unusual magnetic resonances and hysteretic behavior which are fairly well understood, see the review by Fleisher and Merkulov [69].

Chapter 13 describes some astonishing manifestations of nuclear spins in low temperature magneto-transport in two dimensions, first observed by Dobers *et al.* [93]. Strong changes of the magnetoresistance in the Quantum Hall Effect regime are observed and shown to be caused by the dynamic nuclear spin polarization. Such studies yield unique insights into the properties of fragile quantum Hall states, which only exist at ultra-low temperatures and in the highest mobility samples. Some of the experimental results still remain to be understood.

*Spin Dynamics in Diluted Magnetic Semiconductors.* Mn doped III-V and II-VI systems, both bulk and two-dimensional, have attracted intense interest. The giant Zeeman splitting due to exchange interaction with Mn, combination of ferromagnetic and semiconductor properties, and the possibility of making a junction between a ferromagnetic and a normal semiconductor have been the focus of numerous studies. The basic physics, the magnetic and optical properties are reviewed in Chap. 14.

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