Preface

Although impulsive systems were defined in the 1960s by Millman and Mishkis [94, 95], the theory of impulsive differential equations started its rapid development in the 1980s and continues to develop today. The development of the theory of impulsive differential equations gives an opportunity for some real-world processes and phenomena to be modeled more accurately. Impulsive equations are used for modeling in many different areas of science and technology (see, e.g., [46, 106]).

In the literature there are two popular types of impulses:

- *Instantaneous impulses*—the duration of these changes is relatively short compared to the overall duration of the whole process. The model is given by impulsive differential equations (see, e.g., monographs [59, 79, 104], and the cited therein bibliography).
- *Non-instantaneous impulses*—an impulsive action, which starts at an arbitrary fixed point and remains active on a finite time interval. E. Hernandez and D. O'Regan [56] introduced this new class of abstract differential equations where the impulses are not instantaneous, and they investigated the existence of mild and classical solutions.

In this book the impulses start abruptly at some points, and their actions continue on given finite intervals. As a motivation for the study of these systems, we consider the following simplified situation concerning the hemodynamical equilibrium of a person. In the case of a decompensation (e.g., high or low levels of glucose), one can prescribe some intravenous drugs (insulin). Since the introduction of the drugs in the bloodstream and the consequent absorption for the body are gradual and continuous processes, we can interpret the situation as an impulsive action which starts abruptly and stays active on a finite time interval. The model of this situation is the so-called non-instantaneous impulsive differential equation.

This book is the first published book devoted to the theory of differential equations with non-instantaneous impulses. A wide class of differential equations with non-instantaneous impulses are investigated, and these include:

- Ordinary differential equations with non-instantaneous impulses (scalar and n-dimensional case)
- Fractional differential equations with non-instantaneous impulses (with Caputo fractional derivatives of order $q \in (0, 1)$)
- Ordinary differential equations with non-instantaneous impulses occurring at random moments (with exponential, Erlang, or gamma distribution)

In Chapter 1 a systematic development of the theory of differential equations with non-instantaneous impulses is presented. In Section 1.2 some existence results are presented. In Section 1.3 stability theory is studied using modifications of Lyapunov's method. Classical continuous Lyapunov functions are commonly used for the qualitative investigation of different types of differential equations without impulses (see, e.g., the books [31, 70, 135]). Since the solutions of non-instantaneous impulsive equations are piecewise continuous functions, it is necessary to use appropriately defined piecewise continuous analogues of classical Lyapunov functions. It is noted that many authors apply piecewise continuous Lyapunov functions to study the stability of solutions of instantaneous impulsive equations (see the monographs [29, 79] and the cited therein bibliography). In Section 1.4 the monotone—iterative technique is applied to initial value problems for non-instantaneous impulsive equations. The main characteristic of our approximate method is the combination of the method of lower and upper solutions and an appropriate monotone method. These techniques are applied successfully to different types of differential equations without impulses (see, e.g., the book [74] and the cited therein references) and differential equations with instantaneous impulses (see, e.g., the book [59], and the cited therein references).

Chapter 2 is devoted to Caputo fractional differential equations with non-instantaneous impulses. In Section 2.1 the concepts of the presence of non-instantaneous impulses in Caputo fractional differential equations are given. Some existence results are presented. The basic stability theory to nonlinear fractional differential equations with non-instantaneous impulses by Lyapunov functions is developed. Several sufficient conditions for various types of stability for Caputo fractional derivatives are obtained. Also approximate methods for solving the initial value problem for fractional equations are developed.

In Chapter 3 non-instantaneous impulses starting at a random time and acting on an interval with initially given fixed length are studied. The p-exponential stability is defined and several sufficient conditions are given. We investigate both ordinary differential equations and Caputo fractional differential equations with random non-instantaneous impulses. The cases of exponentially, Erlang, and gamma distributed moments of the occurrence of impulses are studied.

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