Shapley Value in a Priori Measuring of Intellectual Capital Flows

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Abstract. Analysis of transmission of intellectual capital as a specific types of information requires consequently different models. The graph presentation maybe in use but it needs more complicated structure including logical conditions and multi connections between the same nodes. In the process of evaluation of the role of each nodes of such graph the concepts of Shapley value and taxonomy dendrite were used. The obtained results let to evaluate the role of nodes not only as a separate element containing the intellectual capital but also as an element of much bigger structure.

Keywords: Group decisions · Shapley value · Taxonomy dendrite · Intellectual capital

1 Introduction

In this paper we deal with a specific types of information, i.e. intellectual capital by preparing a model with so called non-fast transmission. The model shares the basic assumption that each element of the system of non-fast transmission of knowledge takes part in it to some extent. The article presents a way of measuring this participation based on the Shapley value of cooperative game whose elements are the entities of transmission, or vertices of the graph.

The concept of non-fast transmission of information (Mercik 2017) allows to analyse the changes in the so-called intellectual capital (IC) of various participants involved in the movement of intellectual capital between the participants of this process. We assume that the IC transmission process takes place at various levels and with varying intensity. We intend to present the situation associated with the transmission of IC naturally in the form of a planar graph in which there is the possibility of multiple connections between adjacent vertices. Moreover, we assume that this graph also allows the mapping of the logical structure of transmission, in which continuation may require an additional condition to be met (e.g. the need to implement two of the three connections entering the node - the so-called OR condition). Similarly, there may be conditions on the output of a given vertex e.g. only one transmission may start from it (i.e. an EX-OR condition). So designed graph is calibrated using relevant transmission units and will serve to form a matrix of distances between all vertices. Further, the distance matrix will allow the creation of a taxonomic dendrite needed to determine the likelihood of execution of specific coalitions of vertices. Using these probabilities we

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can determine the Shapley values for cooperative games with pre-coalitions, which in turn will help to estimate the relative value of each node involved in the non-fast transmission of knowledge.

The article is set up as follows. First introduction. The next section outlines the way in which transmission of knowledge is modelled. The necessary preliminaries connected with the game theoretical language of modelling and ways of calculating power indices for a simple voting game together with taxonomy of object of a graph is presented in the following section. The next section presents assessment of a connection between vertices. This section describes a procedure to assessment of paths and importance of nodes in a given communication graph. After that, the next section presents transformation of a communication graph into taxonomy dendrite being an equivalent of it. The last section presents an example of simple graph with logical structure, its assessment, dendrite equivalent and evaluation of nodes in communication graph. Some conclusions and suggestions for future research are presented at the end of the paper.

2 Preliminaries

The model will use the following elements: a flat graph with a logical structure for the entry and exit of vertices and the possibility of multiple binary relationships between any two vertices; the definition of s-path, i.e. a path with the length s connecting any two vertices, the term coalition formed from the elements of the set of vertices[1] including a full coalition; and the Shapley value of payoffs for the coalition and the elements of the coalition in a cooperative game in which the players are vertices.

A graph G is an ordered pair $G = \langle N, U \rangle$ wherein each u edge corresponds to at least one pair of ordered vertices, $\langle x, y \rangle \in \mathrm{N} \times \mathrm{N}$ such that $\langle x, u, y \rangle \in \mathrm{N} \times \mathrm{U} \times \mathrm{N}$. A graph G has no so-called loops, by assumption. Because we assume that all the vertices of the graph are involved in the transmission of intellectual capital, all possible paths in the graph must be analysed. If $a_{ij} > 0$ there is connection between every two nodes i and j and G represents the structure of existing relations by incidence matrix $A(G) = \left[a_{ij}\right]_{nxm}$, clearly.

For modelling transmission of intellectual capital we assume that it is enough to know the set of nodes with the logical structure of connections and nodes, weights of each node, w_i, and connection $p^r_{i,j}$. Following (Mercik 2017) the assessment, $t_{i,j}$, of a connection between two vertices equals

$$t^r_{i,j} = \alpha_{i,j} w_{i,j} p^r_{i,j}, \tag{1}$$

where: $\alpha_{i,j}$ – is initially equal to 1, $r = 1, 2, \ldots$ (representing multi connections between any two nodes).

[1] Because, naturally, not all paths can be implemented (this is usually not a complete graph) we are talking about the so-called pre-coalitions in the sense of (Owen 1977).

Definition. Path $s_{i,j}^r$ is a sequence of edges and vertices joining vertices i and j, $s_{i,j}^r = \{i, u_{ik}, \ldots, u_{lj}, \mathrm{j}\}$ for $\mathrm{k}, \mathrm{l} \neq \mathrm{i}, \mathrm{j}$, $\mathrm{k} \neq \mathrm{l}$, $r = 1, 2, \ldots$. Each path can be interpreted as a specific permutation of the vertices from the set of vertices that are on this path. This fact is used in the process of determining the Shapley value for the cooperative game formed by those vertices.

Definition (Mercik 2016)**.** Let i and j be two neighbouring vertices connected at least by one relationship $\{u_{i,j}^r, r = 1, 2, \ldots\}$. If their respective weights are w_i and w_j, $w_{ij} = w_i + w_j$. If i and j vertices are connected by at least one path, $\{s_{i,j}^r, r = 1, 2, \ldots\}$, the weight of the relationship equals for $k, l \in s_{ij}, k \neq i, l \neq j, w_{ij} = w_{ik} + \sum_{\substack{(k,l) \in s_{ij} \\ k \neq l}} w_{kl} + w_{lj} - \sum_{\substack{k \in s_{ij} \\ k \neq i,j}} w_k$. Moreover, $w_{ij} \geq w_i + w_j$ and can be different, for each path $s_{i,j}^r$. It is possible that for every $r = 1, 2, \ldots$ path $s_{i,j}^r$ has the same weight $w_{i,j}$.

Definition. For different q, weight w_{ij} meets the inequality $w_{ij} \geq \mathrm{q}$. The path $s_{i,j}^r$ is called a q-path and denoted by ${}^q s_{i,j}^r$. Changing parameter q allows for example to analyse propagation of information among nodes.

For any non-zero value $a_{ij} \in A(G)$ by replacing a_{ij} by w_{ij} one can obtain a weighted incidence matrix $A_w(\mathrm{G})$.

Formally the cooperative game on the set of vertices N is described by a set of vertices and the characteristic function v. The coalition is any subset of vertices ($cardN = 2^N$), including the grand coalition N made up of all the vertices. The characteristic function of the game (N, v) is the real function v defined for the set of all coalitions, interpreted so that for the coalition T size $v(T)$ it is the amount that the coalition T is able to achieve on its own.

With a fixed ordering, the contribution of a player to the game (N, v) is what he brings to the coalition composed of all the players preceding him in this ordering. This division is called the Shapley value (Shapley 1953) of the game v and is denoted by $\varphi(v)$. The components of This vector's components are Shapley values of individual players (vertices) - their contributions in the division given by the Shapley value. Formally, for any game (N, v) the Shapley value of the game $\varphi(v)$ is given by the formula $\varphi(v) = (\varphi_1(v), \ldots, \varphi_N(v))$,

$$\varphi_i(v) = E_\Pi(v(H_{\pi,i}) - v(H_{\pi,i} \backslash \{i\})) = \sum\nolimits_{\pi \in \Pi} \frac{v(H_{\pi,i}) - v(H_{\pi,i} \backslash \{i\})}{n!}, \tag{2}$$

where Π is the set of all permutations of the set N. E is the expected value, and with a fixed permutation $\pi \in \Pi$ by $H_{\pi,i}$ we denote the set of all those players who, in this permutation, occur not later than the player i (i.e. $H_{\pi,i} = \pi^{-1}$ for 1,2, ..., $\pi(i)$).

In the definition of Shapley value, each coalition has the same probability of realization. But this is not the case if we consider the vertices in the communication graph as the players, i.e. the arithmetic mean (expected value) has to be replaced with the weighted average using probabilities describing the likelihood of realisation of a given coalition (path). For this purpose we use the weight $w_{i,j}$ of a path connecting the vertices i and j in such a way that the greater the length of the path, the smaller is the chance for the realization of this path. The relationship between the weight of the path and the probability is inversely proportional. Thus, the probability assigned to the coalition is estimated by the expression

$$\frac{w_{i,j}^{min}}{w_{i,j}}, \tag{3}$$

where $w_{i,j}^{min}$ means the smallest of weights $w_{i,j}$ calculated for a given graph.

3 Estimating the Significance of a Communication Node in the Graph

The proposed estimation of significance of the vertex is a result of the implementation of the following steps:

Step 1. Marking all connections in the graph by assigning (formula 1) respective values $t_{i,j}^r$ to all $i \neq j; i,j = 1, \ldots, N$.

Step 2. Transformation of the graph shown in the matrix $A_w(G)$ into a dendrite showing the shortest connections in terms of length of path $s_{i,j}^r$. We propose to use this method known as Wroclaw taxonomy.

Step 3. On the basis of the dendrite the values v should be determined for the selected coalitions of vertices.

Step 4. For each permutation of the set of vertices {1, 2, ..., N} calculate the value $\varphi(v)$ and then, using the weights determined by (3) calculate the $\varphi_i(v)$ as a weighted average according to Eq. (2).

The resulting value in the fourth step will describe the importance of a given vertex of the graph of non-fast transmission of information. The value has all the good qualities of the Shapley value and can be easily interpreted.

Note that the calculations in step 4 have a considerable computational complexity, namely $O(n!)$. They can be reduced to the value $O(n2^{n-1})$ by replacing the analysis of all permutations with an analysis of all sets preceding a given vertex in permutations. It is also possible to use approximate algorithms, e.g. those specified in http://powerslave.val.utu.fi/.

4 Transformation of the Graph into Dendrite *T*

The Wroclaw Taxonomy Method (Florek et al. 1951) is based upon the idea of a spanning tree which is an undirected, acyclic and connected graph. The use of Wroclaw taxonomy guarantees that the connections (paths) in a dendrite achieved after te transformation of the communication graph are shortest (so-called Riker's postulate Riker 1962). Using this method we build a graph, whose nodes are all subsets of the set of the n analysed nodes – all the possible pre-coalitions. The edges of the graph are the distances between the respective vertices. The distances are found from the matrix D of distances between nodes (formula 1).

The set of the nodes of the obtained dendrite contains all possible pre-coalitions, whereas the set of edges contains the distances between them. The dendrite determines the shortest possible dendrite ordering (acyclic complete graph). Using this minimum ordering, we determine weights proportional to the probability of the formation of each coalition.

In order to generate partial orderings it is necessary to find among the coalitions the most probable coalition (a starting point of ordering). The choice of the most probable coalition is set arbitrarily. From among all coalitions we select the one that is composed of the smallest number of nodes (Riker 1962). If there is more than one such coalition, we choose the one that has the greatest value of w_i.

Assume that the selected most probable coalition is coalition K. Denote the corresponding node of the dendrite as W_k. The dendrite T has the property that for any of its nodes there will be a path connecting it with W_k. Thus, for any coalition P, it is possible to find the distance d between that coalition and coalition K. The distance is the sum of the lengths of all edges connecting W_p (the vertex corresponding to coalition P) with W_k. As the dendrite T determines the minimum ordering, the distance d between W_p and W_k is the minimum distance in terms of the similarity between the nodes (it is not the minimum distance in the ordinary sense).

For a given coalition P, the distance $d_{i,j}$ is related to the probability of the formation of that coalition. The greater the distance $d_{i,j}$, the smaller the probability of the formation of coalition P. The relation between the distance d and the probability of the formation of coalition P is inversely proportional. Coalition distance, d_{ij}, for $\{i, j\} \in E$, is defined as the length of the edge $\{i,j\}$ of the spanning tree. The dendrite T is therefore in general a nonlinear order over coalitions from the set of all coalitions. The sum of the lengths of edges of spanning tree T is called the length of the order and the best order is the one for which the length of the order is the least.

The described dendrite is therefore equivalent to communication graph. Its design ensures its uniqueness, and its character enables application of game theory approach and calculation of the power index for the games with precoalitions. Consequently, at a given size of, for example, intellectual capital contained in the individual vertices, we receive the assessment of their significance taking into account not only the size of the capital but also the complexity of the structure in which this intellectual capital moves.

5 Example

Analyse the example in Fig. 1. In this graph we have four nodes. We also assume that we are dealing with a directed graph, i.e. the graph with the lower number from a pair of connected vertices is the source-vertex. Let the weights of vertices, w_i, are respectively: 2, 2, 4, 3. Figure 1 presents these values in the upper part of the vertex.

Suppose also that the vertices of the graph have different logical conditions on inputs and outputs. Figure 1 the value in the lower part of the vertex on the left denotes the number of connections which must be completed before the signal goes to another vertex. Similarly, the value on the right side describes the number of connections

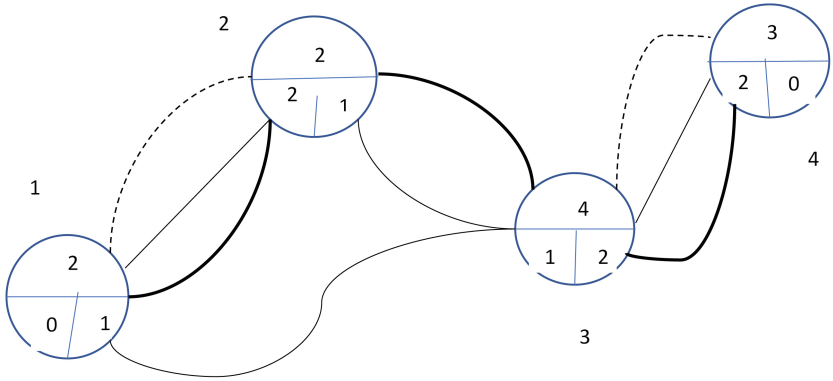

Fig. 1. Various connections (represented by the different kinds of arrows: the dashed line - type a; solid line - type b; the solid bold line - type c) between four objects with different logical structure of entry and output of each node.

simultaneously activated from a given vertex. E.g. the input value of 0 means that the given vertex is the source, and the input value of 2 means that the start of transmission from a given vertex requires the completion of two different transmissions leading to it.

Calculations of the significance of the vertex start with determining the weights of connections. For the graph in Fig. 1 we receive: $w_{1,2} = 4; w_{1,3} = 6; w_{2,3} = 6; w_{3,4} = 7.$

$t^r_{i,j}(1)$ is weight of direct connections between each pair of vertices, where different types of a, b or c of relationships is marked by $r = 1, 2, 3$. If there is a connection between two vertices, but it is not direct communication we calculate the value of the path connecting them. Of course, for some vertices there are mixed connections, i.e. direct and path connections ($\alpha_{i,j} = 1$).

Table 1. Distances between every pair of nodes from the example

		Nodes			
		1	2	3	4
Nodes	1	0	4	12	33
	2		0	12	33
	3			0	14

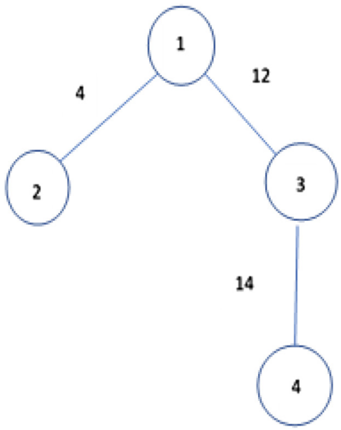

Fig. 2. Dendrite spinned over nodes from the example

Table 2. Permutations of grand coalition, the probability of their realization and contributions made by the individual elements (vertices) to the value of the grand coalition.

Coalition	v(1) = 2	v(2) = 2	v(3) = 4	v(4) = 3	Length of connection	Probability of coalition	Weighted values of elements in a given permutation of coalition N			
							{1}	{2}	{3}	{4}
1-2-3-4	2	10	0	35	34	0.882353	1.764706	8.823529	0	30.88235
1-2-4-3	2	10	35	0	48	0.625	1.25	6.25	21.875	0
1-3-2-4	2	0	0	43	59	0.508475	1.016949	0	0	21.86441
1-3-4-2	2	4	0	31	56	0.535714	1.071429	2.142857	0	16.60714
1-4-2-3	2	0	43	0	72	0.416667	0.833333	0	17.91667	0
1-4-3-2	2	4	31	0	56	0.535714	1.071429	2.142857	16.60714	0
2-1-3-4	10	2	0	35	30	1	10	2	0	35
2-1-4-3	10	2	35	0	44	0.681818	6.818182	1.363636	23.86364	0
2-3-1-4	0	2	0	43	54	0.555556	0	1.111111	0	23.88889
2-3-4-1	43	2	0	0	56	0.535714	23.03571	1.071429	0	0
2-4-1-3	0	2	43	0	68	0.441176	0	0.882353	18.97059	0
2-4-3-1	43	2	0	0	56	0.535714	23.03571	1.071429	0	0
3-1-2-4	0	0	4	41	46	0.652174	0	0	2.608696	26.73913
3-1-4-2	0	16	4	29	68	0.441176	0	7.058824	1.764706	12.79412
3-2-1-4	0	0	4	41	46	0.652174	0	0	2.608696	26.73913
3-2-4-1	41	0	4	0	72	0.416667	17.08333	0	1.666667	0
3-4-1-2	29	16	4	0	44	0.681818	19.77273	10.90909	2.727273	0
3-4-2-1	41	0	4	0	48	0.625	25.625	0	2.5	0
4-1-2-3	0	0	42	3	46	0.652174	0	0	27.3913	1.956522
4-1-3-2	0	15	30	3	54	0.555556	0	8.333333	16.66667	1.666667
4-2-1-3	0	0	42	3	46	0.652174	0	0	27.3913	1.956522
4-2-3-1	42	0	0	3	58	0.517241	21.72414	0	0	1.551724
4-3-1-2	30	15	0	3	30	1	30	15	0	3
4-3-2-1	42	0	0	3	34	0.882353	37.05882	0	0	2.647059
Shapley value for element of coalition N = {1.2.3.4}							0.324677	0.100063	0.270942	0.304318

Source: own calculations.

Assuming respectively that $p^r_{i,j}$, for a, b, c are 1, 2 and 3 one may find the following direct attributes: $t^1_{1,2} = 4; t^2_{1,2} = 8; t^3_{1,2} = 12; t^1_{1,3} = 12; t^1_{2,3} = 12; t^2_{2,3} = 18; t^1_{3,4} = 14;$ $t^2_{3,4} = 21; t^3_{3,4} = 7.$

Calculated parameters lead to matrix of taxonomy distances and taxonomy dendrite respectively (Table 1 and Fig. 2).

The achieved dendrite describing the taxonomical valuation allows to determine the pre-coalitions and the likelihood of their realization. As we can see, the following winning coalitions described by the value of the function v exist: $v(1,2) = max\left(t^r_{1,2}\right) = 12$, $v(1,3,4) = \max\left(t^r_{1,3,4}\right) = \max\left(t^r_{1,3}\right) + max\left(t^r_{3,4}\right) = 12 + 21 = 33$, $v(1,2,3,4) = 45$. The values thus obtained allow for the calculation of the contribution individual vertices make to all permutations of vertices. Distances in Table 1 also allow to determine the probability of a given permutation. Table 2 shows the calculated values.

From Table 2 one may evaluate elements of the graph shown in Fig. 1. As shown, the output proportion of knowledge between the nodes 0.18182; 0.18182; 0.36364; 0.27273 was adjusted by taking into account the structure and nature of connections into the proportion corresponding to the Shapley values: 0.324677; 0.100063; 0.270942; 0.304318. In our opinion, the latter proportions better correspond to the estimated role of each vertex in the flow of intellectual capital (knowledge) through this graph.

6 Conclusions

The proposed a priori evaluation of the elements of graph representing the transmission of intellectual capital via Shapley value may also be used for any other kind of information transmission, not only the slow one.

The very essential logical structure of all transmissions was up to now evidently omitted. This novel approach eliminate this gap.

In the future, the problem of summing such graphs into bigger unit should be analysed. It seems that connection between two graphs by single or multi connections looks relatively easy. The sum of two games where at least one player is common for both of them needs more attention. Some ideas how to solve the problem can be found in Malawski (2017).

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