

Preface

Neighborhood models generalize the well-known relational models, or Kripke models, for modal logic. Although the idea underlying neighborhood models is implicit in the seminal work of McKinsey and Tarski (1944), they were first formally defined by Dana Scott (1970) and Richard Montague (1970). The original motivation for generalizing relational models was to provide semantics for the so-called *classical* systems of modal logic:

The qualification “classical” has not yet been given an established meaning in connection with modal logic....Clearly one would like to reserve the label “classical” for a category of modal logics which—if possible—is large enough to contain all or most of the systems which for historical or theoretical reasons have come to be regarded as important, and which also posses a high degree of naturalness and homogeneity.

(Segerberg 1971, pg. 1)

This quote is from Krister Segerberg’s dissertation, *An Essay in Classical Modal Logic* (1971), that included early results about neighborhood models and the classical systems of modal logic, also called non-normal modal logics, that correspond to them. A few years later, Brian Chellas incorporated these and other salient results in part III of his textbook *Modal Logic: An Introduction* (1980). Nevertheless, in the apparent absence of applications, neighborhood models and non-normal modal logics were studied mainly in view of their intrinsic mathematical interest. A notable exception is David Lewis’s seminal book on conditional logic (Lewis 1969). The so-called *Lewis sphere models* are one of the earliest and most interesting applications of neighborhood models.

Interest in neighborhood models increased steadily over the past 30 years. Neighborhood models form an interesting and rich class of mathematical structures that can be fruitfully studied using modal logic. This is most evident in the extensive literature on the topological interpretation of modal logic. In this book, I highlight additional motivations for studying neighborhood models:

- Neighborhood models naturally show up when studying game theory.
- Neighborhood models can be used to represent the evidence and beliefs of a rational (and not so rational) agent.
- Neighborhood models offer an interesting new perspective on the Barcan and Converse Barcan formulas in the first-order modal logic.

Finally, one can learn a great deal about normal systems of modal logic by looking at how these systems behave in a more general semantics.

This book will quickly familiarize the reader with the general theory of neighborhood semantics for modal logic. I explain how neighborhood models fit within the large family of semantic frameworks for modal logic. In addition, I explain both the pitfalls and potential uses of neighborhood models. This book is designed to be a supplemental text for a course on modal logic, logic in AI, or philosophical logic (either at the advanced undergraduate or graduate level), or as a source for researchers interested in using neighborhood structures in their work. One of the constraints on writing a book for this series is that the length must be kept short. This means that I had to make some hard choices about which topics to leave out. Some topics are only briefly discussed, such as proof theory for non-normal modal logics and the topological interpretation of modal logic. Other topics, such as coalgebraic models for modal logic, are not discussed in any detail.

The book is divided into three chapters. Chapter 1 motivates our study by discussing applications of non-normal modal logics and different interpretations of neighborhood models. Chapter 2 is focused on the core logical theory. This includes questions about axiomatizations, definability, decidability, and relationships with other logical systems. Chapter 3 surveys different extensions to the basic logical framework, including the first-order quantifiers, dynamic modalities, and multi-agent modalities. Finally, there is a short Appendix providing some background on relational semantics for modal logic. Exercises are included throughout the text. There is a website for this book:

<http://pacuit.org/modal/neighborhoods>.

The website includes an extended appendix, solutions to some of the exercises, lecture slides and videos on topics discussed in this book, and links to relevant readings and tutorials on modal logic.

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