

Preface

This book consists of a long manuscript devoted to the study of diverse aspects of braid representations of knots and links. We focus predominantly on 3, and partially on 4 strands, but do try to generalize some results to higher braid groups.

Here is a brief summary of the content. We show that 3-braid links with given (non-zero) Alexander or Jones polynomial are finitely many and can be effectively determined. We classify among closed 3-braids strongly quasi-positive and fibered ones and show that 3-braid links have a unique incompressible Seifert surface. We also classify the positive braid words with Morton-Franks-Williams bound 3 and show that closed positive braids of braid index 3 are closed positive 3-braids. For closed braids on more strings, we study the alternating links occurring. In particular we classify those of braid index 4 and show that their Morton-Franks-Williams inequality is exact. We settle the strong form of Jones' exponent sum conjecture for (arbitrary) braid representations of 3-braid links. Finally, we use the Burau representation to obtain new braid index criteria, including an efficient 4-braid test.

The manuscript was mainly written between 2004 and 2006, with the latest important additions made around 2009. In my previous book [St5], I attempted to extend a similarly minded research manuscript by a broadly accessible and at times entertaining introductory part and finished with an outlook exposition (citing further-going work, mostly of myself). Here I decided, perhaps somewhat extremely, to preserve as much as possible the style of a research monograph and the original state (and level and scope) of the material. Thus editing was limited to formalities like relayout, update of references, and addition of an index. Apart from correcting obvious typos I found, I have also retained the language style, including a few instances of perhaps perceivably peculiar writing, for example, the "ß" of Gauß. (While this letter has meanwhile become obsolete even in German, it is generally agreed that the so-called orthography reform has, as a whole, made German writing only more peculiar. . .)

I had several reasons to refrain from substantial alterations. One was that there were no changes recommended by the referee. Moreover, the publisher requested a space limitation (in the spirit of calling the volumes in the series to be "brief").

In particular, I saw no problem that all subsequent related study be also left entirely (and tacitly) to its own separate place.

In result, again (and even more so than with my previous book), the work primarily concentrates on providing complete proofs for a certain number of theorems and is not aimed to serve well as a textbook on knot/braid theory or as an overview of the topic. I realize that this makes the exposition somewhat less self-contained than some readers would desire, but I hope there are enough references given for the needed basics.

There is one noteworthy update that must be mentioned. D. Emmes in “An expression for the Homflypt polynomial and some applications,” *Topology and its Applications* 160 (2013) 2069–2087, has fully answered affirmatively the question posed and partially settled in Section 4.2: does the Jones polynomial of a 3-braid link determine its skein polynomial? Apart for this important point (and drawing attention to the reference to Y. Ni given in the introduction), I am not (and have not been made) aware of errors found, proofs simplified, results improved, or questions answered.

Several people must be thanked for realizing this project. Crucially, I owe gratitude to M. Hirasawa, M. Ishiwata, and K. Murasugi for providing some missing pieces of the proofs, given in the appendix. I also like to thank the referee, R. Amzad, and the technical team at Springer for their support.

Gwangju, Korea (Republic of)
August 2016

Alexander Stoimenow

<http://www.springer.com/978-3-319-68148-1>

Properties of Closed 3-Braids and Braid
Representations of Links

Stoimenow, A.

2017, X, 110 p. 89 illus., Softcover

ISBN: 978-3-319-68148-1