

Preface

This book has grown out of an undergraduate course that I have taught at the Australian National University (ANU) for over 20 years. The course is one-semester long, which means that it runs for a total of 12 weeks. In each week I teach two lectures, where a lecture is defined as a 100-minute class with a 5-minute break in the middle. This lecture format is common in Europe but can be implemented almost everywhere; for example, at the ANU—where a normal teaching period is 50 minutes—I simply reserve two consecutive periods for every lecture. A 12-week semester can in principle accommodate 24 lectures of this kind but the course material only occupies 21, with the remaining time spent on discussing assignments, review, etc.

While I was transforming my lecture notes into a book, I decided to keep the splitting of the course material into 21 lectures. This approach is unusual as most authors would organise the content into chapters, with each chapter accommodating a particular topic. However, I find that dividing the material according to the way it is presented at the lectures has several advantages compared to the traditional topic-based book composition. Indeed, first of all, the lecture-based organisation ensures that the content is partitioned into (approximately) equal pieces, so that none of them stands out and looks intimidating to the students at least as far as the length is concerned. This issue becomes particularly important for those who wish to use the book for self-study and would like to keep a close eye on their overall progress. Secondly, the lecture-based format guarantees that the students get more training for the more advanced topics, which are spread over several lectures. Indeed, each lecture has its own unique set of exercises, and the students are strongly encouraged to do at least some of them before moving on. It is then automatic that the harder the topic, the more exercises one is expected to do to go through it. It should also be mentioned that some of the exercises for each lecture serve as a preparation for the following one. Thirdly, the lecture-based split-up gives clear teaching guidelines to the instructor, at the same time allowing for the possibility of re-arranging the material according to their own taste.

As the book covers a one-semester course, it is shorter than most complex analysis texts (approximately 200 pages). It is well-known that many students are intimi-

dated by long large books, so this shorter one—which contains exactly the material that needs to be learned in a one-semester course—is expected to have a broader appeal. Another feature of the book that the students may like is a reader-friendly conversational style of writing. For instance, there are plenty of fully worked-out examples and textual explanations of formal statements, with plain words systematically used in the formulations of theorems, propositions, etc. Furthermore, the reader is invited to participate in the exposition by filling in various details of formal arguments as indicated by parenthesised expressions, e.g., (check!), (explain!), (provide details!). The proofs of some of the statements are left as homework. In fact, doing weekly homework is strongly encouraged with plenty of exercises to choose from at the end of each lecture. Note that the exercises have a varying degree of difficulty to accommodate different cohorts of students and range from routine questions to rather hard problems.

Although the choice of topics covered in the book may appear to be standard, this is more than just a book on complex analysis since it discusses concepts that lie outside the scope of a typical complex analysis course, such as homotopy and algebraic properties of groups of conformal transformations. In fact, the exposition is non-standard in that the central result, from which most of the material follows, is Cauchy's Independence of Homotopy Theorem (in this regard, I was certainly influenced by A. Vitushkin from whom I took my first complex analysis course and who later became my PhD thesis adviser). Expositions based on the above theorem are hard to come by, and those that I am aware of do not satisfy me, often because of lack of rigour. At the same time, homotopy independence allows one to have a nice clean derivation of Cauchy's Integral Theorem and Cauchy's Integral Formula (see Lecture 10). This is one reason why I have decided to write my own book.

Another instance of a non-standard approach to exposition is the proof of the Fundamental Theorem of Algebra given in Lecture 1. The fact that this important result can be obtained so early in the book and in such an elementary way demonstrates the power of complex numbers and sets the tone for the entire course. The book concludes with a proof of another major milestone, the Riemann Mapping Theorem, which is rarely part of a one-semester undergraduate course. I stress that the exposition is almost entirely self-contained, with only a handful of statements included without proof.

Certainly, the nice extras incorporated in the book as described above come at a price: one has to possess a certain degree of mathematical maturity in order to understand and appreciate them. Namely, the students are required to have done a prerequisite course in real analysis and metric spaces, to which I refer for most facts mentioned without proof. If one does not assume such a course (as is the case at some universities in the US), the instructor may decide to exclude certain topics, e.g., the proof of the Riemann Mapping Theorem, and use the book for teaching a slightly lower-level course. The instructor will find that fitting even such a reduced amount of material in one semester requires a significant effort, so one should not be afraid of running out of content. Alternatively, although this book is primarily aimed at undergraduates, it can be used to teach a graduate course to students who have the right prerequisites.

Before proceeding, I would like to say another word about the exercises. Many of them are of my own making, but over the years I have also collected a good number of interesting unpublished problems informally communicated to me by various people, most of all by A. Vitushkin and E. Chirka. Some of the problems that I learned from them are included in the book, and I am grateful to these two mathematicians for their generous contributions.

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