Routing Game on the Line: The Case of Multi-players

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Abstract. In this paper, we tackle the problem of a sequential routing game where multiple users coexist and competitively send their traffic to a destination over a line. The users arrive at time epoch with a given capacity. Then, they ship their demands over time on a shared resource. The state of players evolve according to whether they decide to transmit or not. The decision of each user is thus spatio-temporal control. We provide an explicit expression of the equilibrium of such systems and compare it to the global optimum case. In particular, we determine the expression of price of anarchy of such scheme and identify a Braess-type paradox in the context of sequential routing game.

Keywords: Sequential routing game · Nash equilibrium · Price of anarchy · Braess-type paradox

1 Introduction

We consider in this paper a spatio-temporal competitive routing game where the network is shared by several users in which each one having a non negligible cost while routes are chosen by the players so as to minimize the delay. Routing game is characterized by source-destination pair and demand function. It can be analyzed using the non-cooperative game theory. An appropriate solution concept in non-cooperative game theory is Nash equilibrium.

This concept has been long studied in the framework telecommunication networks. Such framework is used to model the flow configuration that results in networks where the routing decisions are made in a non-cooperative and distributed manner between the users.

The number of users can be infinite or finite. In the case of an infinite players each player is assumed to be atomless. Atomless means that the impact of routing choices of a single player on the utilities of other players is negligible.

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The resulting flow configuration corresponds to the Wardrop equilibrium [1]. This concept, has long been studied in the context of road traffic where there is an infinite of players (drivers) [2]. In the telecommunication community, Orda et al. [3] consider that the number of players is finite, where a player (typically corresponding to a service provider) takes the routing decisions for the whole class of users that it controls. It then decides on how to split the demand it controls between different possible routes. They establish existence and uniqueness of Nash equilibrium over large class of general cost functions. This approach also appeared in the road traffic literature (e.g. [4]), but was not much used there. Such a routing game may be handled by models similar to [5] in the special case of a topology of parallel links. An alternative class of routing games is the one in which a player has to route all the demand it controls through the same path. A special case of such framework is the "congestion games" introduced by Rosenthal in [6]. In [10], a load balancing network and Kameda type paradox has been studied, where losses occur on links in a way that may depend on the congestion, which by adding capacity, all players suffer larger loss rates. All the above works have been well studied in time-invariant networks in the last few years.

Our focus here is a spatio-temporal competitive routing where the network is shared by several users in which each one having a non negligible cost. The demand has to be split not only over space but also over time (see Fig. 1). As an example, assume that M players have each its own demand which should be shipped within a T days from a given source to a destination. Thus each player has to split its demand into that corresponding to each of the days of the week. At each day, the route corresponding to the daily demand of each player should be determined. Examples of such games in road traffic appear in [12].

An important property for an equilibrium is efficiency, (social optimality). It is well known that Nash equilibria in routing games are not efficient. Which can lead to the well known phenomena of Braess paradox where adding a link to the network, the cost to all users increases. For specific examples see [8,11].

Our starting point here is the work [7] in which the authors have already studied a routing game on a line where the decision of a user is spatio-temporal control. They addressed the case where only a single user arrives at time epoch. The game considered in [7] assumes that a single user ships its own traffic over a line. However, this assumption may be not always true. Indeed, several users can coexist and competitively need to ship their own resource over a line.

Without loss of generality, we extend their game problem into a general problem, where multiple users arrives on line. In particular, we consider an extreme scenario in which all traffic that arrives at a node could be shipped at the next node over a line. We show that even with this simple demand matrix, which is clearly biased in favor of choosing the direct path, we establish the counter-intuitive fact: "It is possible that not all players send their traffic through the direct path at equilibrium". Examples of such games in road traffic can be found in [8–11].

In the rest of the paper, we assume that M players have their own demands which should be shipped within a week from a given source to a destination. Thus each player has to split competitively its demand into that corresponding to each of the days of the week. At each day, the route corresponding to the daily demand of each player should be determined.

The paper is structured as follow: We present the system model with the assumptions considered in the next Section. Next we give the explicit expressions of the Nash equilibrium and the global optimum, respectively in Sects. 3 and 4. Section 5 presents some performance results including price of anarchy and Braess-type paradox. Finally we conclude the paper in Sect. 6.

2 The Model

Let $G = (\mathcal{N}, \mathcal{L}, \mathcal{I}, \mathcal{P})$ be a network routing game with $\mathcal{N}$ the set of nodes and $\mathcal{L}$ the set of links, $\mathcal{I}$ is the set of classes (e.g. players), and $P = (s_i, d^i, \phi_i)$ is a set that characterizes class i: s^i is the source, d^i is the destination and ϕ_i is the demand related to player i.

We describe the system with respect to the variables x_l^i which are restricted by the non-negativity constraints for each link l and player i: $x_l^i \geq 0$ and by the conservation constraints for each player i and each node v:

$$r_v^i + \sum_{j \in In(v)} x_j^i = \sum_{j \in Out(v)} x_j^i \tag{1}$$

where $r_v^i = \phi_i$ if v is the source node for player $i, r_v^i = \phi_i$ if v is its destination node, and $r_v^i = 0$ otherwise; $In(v)$ and $Out(v)$ are respectively all ingoing and outgoing links of node v.

A player i determines the routing decisions for all the traffic that corresponds to the corresponding class i. The cost of player i is assumed to be additive over links

$$J^i(\mathbf{x}) = \sum_l J_l^i(\mathbf{x}_l) \tag{2}$$

We shall assume that

(i) $K_l^i := \frac{\partial J_l^i(\mathbf{x})}{\partial x_l^i}$ exist and are continuous in x_l^i (for all i and l),

(ii) J_l^i are convex in x_l^i (for all i and l),

We shall also make the following assumptions for each link l and player i:

A1: J_l^i depends on $\mathbf{x}_l$ only through the total flow x_l and the flow of x_l^i of player i over the link.

A2: J_l^i is increasing in both arguments.

A3: Whenever J_l^i is finite, $K_l^i(x_l, x_l^i)$ is strictly increasing in both arguments.

We further restrict the cost to satisfy the following:
B1: For each link l there is a nonnegative cost density $T_l(x_l)$. T_l is a function of the total flow through the link and $J_l^i = x_l^i T_l(x_l)$.
B2: T_l is positive, strictly increasing and convex, and is continuously differentiable.

The Lagrangian with respect to the constraints on the conservation of flow is

$$L_i(\mathbf{x}, \lambda) = \sum_{l \in \mathcal{L}} J_l^i(x_l, x) + \sum_{v \in \mathcal{N}} \lambda_v^i \left(r_v^i + \sum_{j \in In(v)} x_j^i - \sum_{j \in Out(v)} x_j^i \right), \tag{3}$$

for each player i. Thus a vector $\mathbf{x}$ with nonnegative components satisfying (1) for all i and v is an equilibrium if and only if the following Karush-Kuhn-Tucker (KKT) condition holds. Below we shall use uv to denote the link defined by node pair u, v. There exist Lagrange multipliers λ_u^i for all nodes u and all players, i, such that for each pair of nodes u, v connected by a directed link (u, v),

$$K_{uv}^i(x_{uv}^i, x_{uv}) \geq \lambda_u^i - \lambda_v^i, \tag{4}$$

with equality if $x_{uv} > 0$.

Assume cost structure B. Then, the Lagrangian is given by

$$\begin{aligned} L_i(\mathbf{x}, \lambda) = &\textstyle\sum_{l \in \mathcal{L}} \left[T_l(x_l) + x_l^i \frac{\partial T_l(x_l)}{\partial x_l} \right] \\ &+ \textstyle\sum_{v \in \mathcal{N}} \lambda_v^i \left(r_v^i + \sum_{j \in In(v)} x_j^i - \sum_{j \in Out(v)} x_j^i \right), \end{aligned} \tag{5}$$

for each player i. Equation (4) can be written as

$$T_{uv}(x_{uv}) + x_{uv}^i \frac{\partial T_{uv}(x_{uv})}{\partial x_{uv}} \geq \lambda_u^i - \lambda_v^i. \tag{6}$$

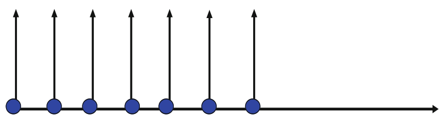

Fig. 1. Competitive routing on the line.

3 The Transient Equilibrium for Multi-user Sequential Game

Consider a sequential game: Assume at each day i, there are M new players that arrive. Player i comes with a demand of ϕ_i each competing for the link. This demand has to be shipped within 2 days. The decision of each player influences the cost of the others. This is a scheduling game: how much should player i send upon arrival and how much should it delay to the next day? If the game is already on for a long time, we can expect to have a symmetric solution (this will be made exact below). This means that at each day, the same amount will be sent. And since each day an amount of ϕ_i arrives, then the amount that will have to leave each day is also ϕ_i. An optimal solution will therefore be to send immediately all the arriving demand. We shall see however that at equilibrium, each player sends some of its traffic in the next day. We solve this problem by viewing it as an equivalent routing problem. The cost per unit of packet sent each day is $f(x)$. The cost for waiting another day is d.

Formally, let the demand of each player $k = 1, 2, \cdots, M$ arriving at day i be $\phi_{i,k} > 0$ that has to be shipped to destination within a period of 2 days. Let $x_j^{i,k}$ denotes the amount of flow sent by user k on day j knowing that user k arrived at day i. The total flow on day i is then denoted as $x_i = \sum_{k=1}^{M} x_i^{i,k} + x_i^{i-1,k}$. Let the vector $\mathbf{x}^{i,k} = (x_i^{i,k}, x_{i+1}^{i,k})$ denote the amount of flow sent by user k arriving at day i. The vector $\mathbf{x}^{i,k}$ is said to be feasible if $x_i^{i,k} + x_{i+1}^{i,k} = \phi_{i,k}$. For a given flow configuration of users $(\mathbf{x}^{i,1}, \mathbf{x}^{i,2}, \cdots, \mathbf{x}^{i,M})$, user k pays a congestion cost of $f(x_i)$ and delay cost of d per unit of its flow on day i. The objective of user l arriving at day i is to minimize his cost given by

$$J^{i,l} = x_i^{i,l} f(x_i) + x_{i+1}^{i,l}(f(x_{i+1}) + d) \tag{7}$$

By differentiating the cost function with respect to $x_i^{i,l}$ and setting the derivative equal to zero, we get:

$$x_i^{i,l} = \frac{d + f(x_{i+1}) - f(x_i) + \phi_{i,l} g(x_{i+1})}{g(x_i) + g(x_{i+1})} \tag{8}$$

so that

$$x_{i+1}^{i,l} = \phi_{i,l} - x_i^{i,l} = \frac{-d - f(x_{i+1}) + f(x_i) + \phi_{i,l} g(x_i)}{g(x_i) + g(x_{i+1})} \tag{9}$$

and also

$$x_{i+1}^{i+1,l} = \frac{d + f(x_{i+2}) - f(x_{i+1}) + \phi_{i,l} g(x_{i+2})}{g(x_{i+1}) + g(x_{i+2})} \tag{10}$$

Taking the sum, we obtain

$$\begin{aligned} x_{i+1} = \sum\nolimits_{k=1}^{M} & \frac{d + f(x_{i+2}) - f(x_{i+1}) + \phi_{i,k} g(x_{i+2})}{g(x_{i+1}) + g(x_{i+2})} \\ & + \frac{-d - f(x_{i+1}) + f(x_i) + \phi_{i,k} g(x_i)}{g(x_i) + g(x_{i+1})} \end{aligned} \tag{11}$$

3.1 The Steady State

In the steady state, we have $x_i = \sum_{k=1}^{M} \phi_{i,k}, \forall i$. Hence, for every user l we have from Eq. (8)

$$x_i^{i,l} = \frac{\phi_{i,l}}{2} + \frac{d}{2g(\sum_{k=1}^{M} \phi_{i,k})} \tag{12}$$

3.2 The Case of Linear Cost

Similarly to the previous sections, let $f(x) = ax$. Then, Eq. (8) gives

$$-x_{i+2} + (\frac{2}{M} + 2)x_{i+1} - x_i = (\frac{2}{M})\sum_{k=1}^{M} \phi_k \tag{13}$$

The solution of this difference equation has the form

$$x_i = c_1(r_1)^i + c_2(r_2)^i + \sum_{k=1}^{M} \phi_k \tag{14}$$

where r_1 and r_2 are the solution of the characteristic equation

$$-r^2 + (\frac{2}{M} + 2)r - 1 = 0 \tag{15}$$

They are thus given by

$$r_{1,2} = 1 + \frac{1}{M}(1 \pm \sqrt{1+2M}) \tag{16}$$

Similarly, using the condition $x_0 = 0$ we conclude that

$$x_i = \sum_{k=1}^{M} \phi_k [1 - (1 + \frac{1}{M}(1 - \sqrt{1+2M}))^i] \tag{17}$$

4 The Global Optimum for a Semi Infinite Line

Let us now compute the global cost for the multi-user case at day i.

$$J^i(\mathbf{x}) = \sum_{i=1}^{M} J^{i,l}(\mathbf{x}) \tag{18}$$

Deriving the sum with respect to $x_i^{i,l}$, we have

$$\begin{aligned}\frac{\partial J^l(\mathbf{x})}{\partial x_i^{i,l}} &= f(x_i) + x_i^{i,l} g(x_i) - (\phi_{i,l} - x_i^{i,l}) g(x_{i+1}) - f(x_{i+1}) - d \\ &\quad + (\phi_{i-1,l} - x_{i-1}^{i-1,l}) g(x_i) - x_{i+1}^{i+1,l} g(x_{i+1}) \end{aligned} \tag{19}$$

Equating the above equation to zero gives

$$x_i^{i,l} = \frac{d + f(x_{i+1}) - f(x_i) + \phi_{i,l} g(x_{i+1})}{g(x_i) + g(x_{i+1})} - \frac{\phi_{i-1,l} g(x_i) + x_{i+1}^{i+1,l} g(x_{i+1}) + x_{i-1}^{i-1,l} g(x_i)}{g(x_i) + g(x_{i+1})} \tag{20}$$

so that

$$\phi_i - x_i^{i,l} = \frac{-d - f(x_{i+1}) + f(x_i) + \phi_{i,l} g(x_i)}{g(x_i) + g(x_{i+1})} + \frac{\phi_{i-1,l} g(x_i) - x_{i+1}^{i+1,l} g(x_{i+1}) - x_{i-1}^{i-1,l} g(x_i)}{g(x_i) + g(x_{i+1})} \tag{21}$$

and also

$$x_{i+1}^{i+1} = \frac{d + f(x_{i+2}) - f(x_{i+1}) + \phi_{i+1,l}g(x_{i+2})}{g(x_{i+1}) + g(x_{i+2})} - \frac{\phi_{i,l}g(x_{i+1}) + x_{i+2}^{i+2,l}g(x_{i+2}) + x_i^{i,l}g(x_{i+1})}{g(x_{i+1}) + g(x_{i+2})} \tag{22}$$

Taking the sum, we obtain

$$\begin{aligned} x_{i+1} = & \sum_{k=1}^{M} \frac{-d - f(x_{i+1}) + f(x_i) + \phi_{i,l}g(x_i)}{g(x_i) + g(x_{i+1})} + \frac{\phi_{i-1,l}g(x_i) - x_{i+1}^{i+1,l}g(x_{i+1}) - x_{i-1}^{i-1,l}g(x_i)}{g(x_i) + g(x_{i+1})} \\ & + \sum_{k=1}^{M} \frac{d + f(x_{i+2}) - f(x_{i+1}) + \phi_{i+1,l}g(x_{i+2})}{g(x_{i+1}) + g(x_{i+2})} \\ & - \frac{\phi_{i,l}g(x_{i+1}) + x_{i+2}^{i+2,l}g(x_{i+2}) + x_i^{i,l}g(x_{i+1})}{g(x_{i+1}) + g(x_{i+2})} \end{aligned} \tag{23}$$

4.1 The Case of Linear Cost

Let $f(x) = ax$ and assume that $\phi_{i,k} = \phi_k, \forall i, k$ does not depend on i. Then

$$-\frac{M}{2}x_{i+2} + (M+1)x_{i+1} - \frac{M}{2}x_i = \sum_{k=1}^{M} \phi_k + \theta \tag{24}$$

where $\theta = \frac{1}{2}\sum_{k=1}^{M} x_{i+2}^{i+2,k} - x_{i+1}^{i+1,k} + x_i^{i,k} - x_{i-1}^{i-1,k}$. Noting $a_i = \sum_{k=1}^{M} x_i^{i,k}$, we have

$$x_{i+1} = \sum_{k=1}^{M} \phi_k - a_i + a_{i+1} \tag{25}$$

$$x_{i+2} = \sum_{k=1}^{M} \phi_k - a_{i+1} + a_{i+2} \tag{26}$$

From (25) and (26), we obtain

$$\begin{aligned} \theta &= \tfrac{1}{2}[a_{i+2} - a_{i+1} + -a_i - a_{i+1}] \\ &= \tfrac{1}{2}[x_{i+2} - x_{i+1}] \end{aligned} \tag{27}$$

We then have the following recursive function

$$-(M+1)x_{i+2} + (2M+3)x_{i+1} - Mx_i = 2\sum_{k=1}^{M} \phi_k \tag{28}$$

The solution of this difference equation has the form

$$x_i = c_1(r_1)^i + c_2(r_2)^i + \sum_{k=1}^{M} \phi_k \tag{29}$$

where r_1 and r_2 are the solution of the characteristic equation $-(M+1)r^2 + (2M+3)r - M = 0$. They are thus given by

$$r_{1,2} = 1 + \frac{1 \pm \sqrt{9+8M}}{2(M+1)} \tag{30}$$

Using the condition $x_0 = 0$, we conclude that, for every user l, we have

$$x_i = \sum_{k=1}^{M} \phi_k [1 - (1 + \frac{1 - \sqrt{9+8M}}{2(M+1)})^i] \tag{31}$$

5 Performance Analysis

5.1 Price of Anarchy

At the steady state, the cost for player i is

$$\begin{aligned} J^{i,l}(d) &= x_i^i f(\textstyle\sum_{k=1}^{M} \phi_{i,k}) + x_{i+1}^i (f(\textstyle\sum_{k=1}^{M} \phi_{i,k}) + d) \\ &= (x_i^i + x_{i+1}^i) f(\textstyle\sum_{k=1}^{M} \phi_{i,k}) + d x_{i+1}^i \\ &= \phi_{i,l} f(\textstyle\sum_{k=1}^{M} \phi_{i,k}) + d(\frac{\phi_{i,l}}{2} - \frac{d}{2g(\sum_{k=1}^{M} \phi_{i,k})}) \end{aligned} \tag{32}$$

Since $x_{i+1}^i \geq 0$, we have $d \leq \phi_{i,l} g(\sum_{k=1}^{M} \phi_{i,k})$. $J^{i,l}(d)$ is concave in d with a maximum at $d = \frac{\phi i,lg(\sum_{k=1}^{M} \phi_{i,k})}{2}$. Interestingly, we observe that at $d = \phi_{i,l} g(\sum_{k=1}^{M} \phi_{i,k})$, the cost function $J^{i,l}(d)$ is minimized and equal to $\phi_{i,l} f(\sum_{k=1}^{M} \phi_k)$ which is equal to the cost at $d = 0$.

The price of anarchy (PoA) is then given by

$$\begin{aligned} PoA &= \frac{J^{i,l}(d)}{\phi_{i,l} f(\sum_{k=1}^{M} \phi_{i,k})} \\ &= 1 + \frac{d(\frac{\phi_{i,l}}{2} - \frac{d}{2g(\sum_{k=1}^{M} \phi_{i,k})})}{\phi_{i,l} f(\sum_{k=1}^{M} \phi_{i,k})} \end{aligned} \tag{33}$$

For the numerical results we set the value of $\phi_{i,l}$ to 1 and a =1.

Notice that the PoA is equal to 1 for $d = 0$ and $d = \phi_{i,l} g(\sum_{k=1}^{M} \phi_{i,k})$. We depict in Fig. 2 the price of anarchy as function of delay for different number of users. As already said, we notice that the PoA is equal to 1 for $d = 0$, and $d = 1$. We also remark that the PoA increases as the number of users increases which leads to the interaction of users and hence induces more cost to ship their demand over the line.

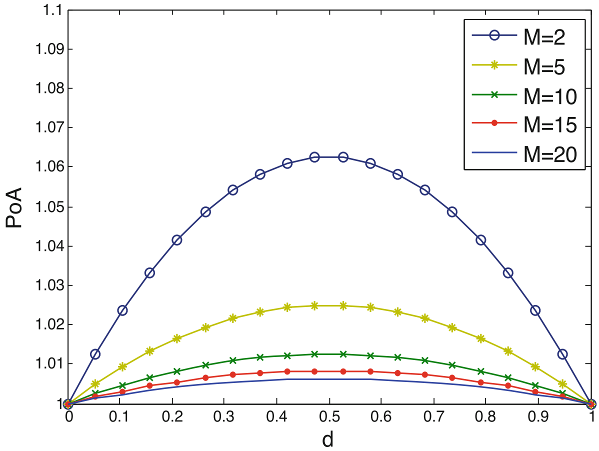

Fig. 2. Price of anarchy as function of cost

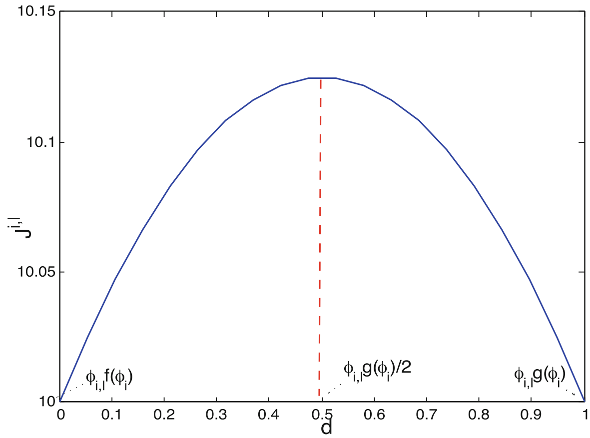

Fig. 3. The Braess-type paradox for the sequential game.

5.2 Braess-Type Paradox

At the steady state, the cost for player l at day i is:

$$\begin{aligned} J^{i,l}(d) &= x_i^i f(\textstyle\sum_{k=1}^M \phi_{i,k}) + x_{i+1}^i (f(\sum_{k=1}^M \phi_{i,k}) + d) \\ &= (x_i^i + x_{i+1}^i) f(\textstyle\sum_{k=1}^M \phi_{i,k}) + d x_{i+1}^i \\ &= \phi_{i,l} f(\textstyle\sum_{k=1}^M \phi_{i,k}) + d(\frac{\phi_{i,l}}{2} - \frac{d}{2g(\sum_{k=1}^M \phi_{i,k})}). \end{aligned} \tag{34}$$

It is clear that the cost for player i is non-negative for

$$d \in [\frac{\phi_{i,l} g(\sum_{k=1}^M \phi_{i,k}}{2}, \phi_{i,l} g(\sum_{k=1}^M \phi_{i,k})].$$

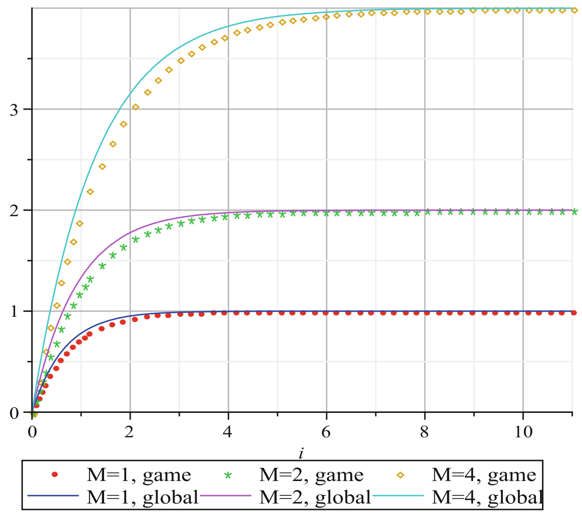

Fig. 4. Dynamics of the total flow sent over the horizon where $\phi_k = 1 \forall k = 1, \ldots, M$.

In this region, $J^{i,l}$ decreases as d increases, which is a Braess type paradox (see Fig. 3). This paradox was obtained in the context of a load balancing network in [11]. We depict in Fig. 4 the variation of the Nash equilibrium in Eq. (17) and the global optimum in Eq. (31) over the horizon for $\phi_k = 1; \forall k = 1, \ldots, M$. It is clearly illustrated that the Nash equilibrium over a semi infinite line converges to the global optimum from day $i = 4$.

6 Conclusion

We have considered a sequential routing game networks where several users coexist and competitively send their traffic to a destination on a line over a period of two days. Under some assumptions, we have obtained the explicit solutions for the Nash equilibrium and the global optimum and derive the price of anarchy. Finally, we have showed that under a semi infinite line, the Nash equilibrium converges to the global optimum and, have identified a Braess-type paradox behavior in the context of sequential games.

References

1. Wardrop, J.G.: Some theoretical aspects of road traffic research communication networks. Proc. Inst. Civ. Eng. Part 2 **1**, 325–378 (1952)
2. Patriksson, M.: The Traffic Assignment Problem: Models and Methods. VSP, Utrecht (1994)
3. Orda, A., Rom, R., Shimkin, N.: Competitive routing in multi-user environments. IEEE/ACM Trans. Netw. **1**, 510–521 (1993)

4. Haurie, A., Marcotte, P.: On the relationship between Nash-Cournot and Wardrop equilibria. Networks **15**, 295–308 (1985)
5. Park, K., Sitharam, M., Chen, S.: Quality of service provision in noncooperative networks with diverse user requirements. Decis. Support Syst. **28**(1–2), 101–122 (2000)
6. Rosenthal, R.W.: A class of games possessing pure strategy Nash equilibria. Int. J. Game Theory **2**, 65–67 (1973)
7. Haddad, M., Altman, E., Gaillard, J.: Sequential routing game on the line: transmit or relay? In: Proceeding of Communications and Information Technology (ICCIT), June 2012
8. Hanawal, M.K., Altman, E., El-Azouzi, R., Prabhu, B.J.: Spatio-temporal control for dynamic routing games. In: Proceedings of Game Theory for Networks (GameNets), Shanghai, China, April 2011
9. Altman, E., Basar, T., Jimenez, T., Shimkin, N.: Competitive routing in networks with polynomial cost. IEEE Trans. Autom. Control **47**, 92–96 (2002)
10. Altman, E., Kuri, J., El-Azouzi, R.: A routing game in networks with lossy links. In: 7th International Conference on NETwork Games COntrol and OPtimization (NETGCOOP), Trento, Italy, October 2014
11. Kameda, H., Pourtallier, O.: Paradoxes in distributed decisions on optimal load balancing for networks of homogeneous computers. J. ACM **49**(3), 407–433 (2002)
12. Wie, B.W., Friesz, T., Tobin, R.: Dynamic user optimal traffic assignment on congested multidestination networks. Transp. Res. **24B**(6), 431–442 (1990)

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