Evolutionary Game Network Reconstruction by Memetic Algorithm with $l_{1/2}$ Regularization

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Abstract. Evolutionary Game (EG) theory is effective approach to understand and analyze the widespread cooperative behaviors among individuals. Reconstructing EG networks is fundamental to understand and control its collective dynamics. Most existing approaches extend this problem to the l_1-regularization optimization problem, leading to suboptimal solutions. In this paper, a memetic algorithm (MA) is proposed to address this network reconstruction problem with $l_{1/2}$ regularization. The problem-specific initialization operator and local search operator are integrated into MA to accelerate the convergence. We apply the method to evolutionary games taking place in synthetic and real networks, finding that our approach has competitive performance to eight state-of-the-art methods in terms of effectiveness and efficiency.

Keywords: Compressed sensing · Network reconstruction · Memetic algorithm · Evolutionary games · Sparse reconstruction

1 Introduction

An important class of collective dynamics is evolutionary games (EG) [8–10] in the human society. For example, through game theory, economists can analyze how people make choices about money. For the criminal gang, the police need to master the relationships between the members, namely, agent-to-agent networks. However, in the real life, it is difficult to directly access to this network, and maybe only the payoff and strategy of its members are available. Therefore, our goal is to reconstruct the agent-to-agent networks from these available information, namely, profit sequences.

There have been recent efforts in addressing EG network reconstruction problem which is converted into a sparse signal reconstruction problem that can be solved by exploiting l_1-minimization algorithms, such as the LASSO [1] and compressed sensing (CS) [2]. This problem also is solved by multiobjective evolutionary algorithm with l_1 regularization [29]. However, the l_1 regularization may generate inconsistent selections when coping with variable selection and often introduces extra bias in estimation. As a further modification, the $l_{1/2}$ regularization [11, 12] is naturally assured. Moreover, $l_{1/2}$ regularization can assuredly generate more sparse solutions than l_1 regularization.

The $l_{1/2}$ regularization is nonconvex, nonsmooth, and non-Lipschitz optimization problem. In general, it is different to solve. In this paper, we develop a memetic algorithm (MA) to cope with EG network reconstruction problem with $l_{1/2}$ regularization,

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termed as MAEGNet. MAs [4] are hybrids of global search procedures and local search procedures [5, 6]. They can explore better solutions around the best solution found so far. The proposed algorithm combines a genetic algorithm as the global search method and an iterative shrinkage-thresholding (IST) strategy [7] as the local search procedure. To achieve greater chance to increase the speed of convergence toward the optimal solutions, we employ a problem-specific initialization operator [29] for MA. To validate the performance of MAEGNet, EG models [8, 9] taking place on different model-based networks are used. The experimental results show that MAEGNet is able to effectively reconstruct EG networks. The systematic comparison with existing algorithms shows that MAEGNet matches or exceeds the other algorithms.

The remainder of this paper is organized as follows. Section 2 introduces the EG models and the network reconstruction problem. Section 3 gives an introduction to MAEGNet. Section 4 presents experimental data, and compares the performance of MAEGNet against eight state-of-the-art methods. Section 5 concludes the work in this paper.

2 Network Reconstruction Model

Evolutionary games (EG) model a common type of interactions in various complex, networked, natural and social systems. In an evolutionary game, at any time, one agent can select a certain strategy, such as cooperation or defection. The payoffs of the two agents in a game have four possibilities. For example, in the prisoner's-dilemma game (PDG) [10], the agents get rewards R or Pu if both choose to cooperate or defect, respectively. In the remaining two cases the defector's and cooperator's payoff are Te (temptation to defect) and S (sucker's payoff), respectively. The ranking of $Te > R > Pu > S$ and $2R > Te + S$ still holds. We use the same setting as in [8], $R = 1$, $Pu = S = 0$, and $Te = b$, where $b \in (1, 2)$ that keeps the essentials of the Prisoners' dilemma. At each round, all agents play game with their neighbors and then obtain payoffs. For agent i, its payoff is

$$Y_i = \sum_{j \in \Gamma_i} S_i^{\mathrm{T}} P S_j \tag{1}$$

where S_i and S_j denote the strategies of agents i and j at the time and the sum is over the neighbor-connection set Γ_i of i. After a round of game, an agent updates its strategy using the Fermi rule [11] which can maximize its payoff at the next round. Fermi rule is defined as follows

$$W\left(S_i \leftarrow S_j\right) = \frac{1}{1 + \exp\left[\left(Y_i - Y_j\right)/\kappa\right]} \tag{2}$$

where κ characterizes the stochastic uncertainties introduced to permit irrational choices. We use the same setting as in [29], b is set to 1.2, and $\kappa = 0.1$.

We assume that only the profit sequences of all agents and their strategies in each round are available. The key to reconstruct agent-to-agent networks lies in the relationships between agents' payoffs and strategies. The interactions among agents in the network can be characterized by an $N \times N$ adjacency matrix X with elements $x_{ij} = 1$ if agents i and j are connected, and $x_{ij} = 0$ otherwise. The payoff of agent i can be expressed by

$$Y_i(t) = \sum_{\substack{j=1 \\ i \neq j}}^{N} x_{ij} S_i^{\mathrm{T}}(t) P S_j(t) \tag{3}$$

where x_{ij} ($j = 1, 2, \ldots, N$) represents a possible connection between agent i agent j; $x_{ij} S_i^T(t)\ PS_j(t)$ ($l = 1, 2, \ldots, N$) stands for the possible payoff of agent i from the game with agent j; and $t = 1, 2, \ldots, m$ is the number of rounds that all agents play the game with their neighbors. Equation (3) can be simplified as $Y_i = A_i \times X_i$, where

$$Y_i = (Y_i(1), Y_i(2), \cdots, Y_i(m))^{\mathrm{T}} \tag{4}$$

$$X_i = \left(x_{i1}, \cdots, x_{i,i-1}, x_{i,i+1}, \cdots x_{iN}\right)^{\mathrm{T}} \tag{5}$$

$$A_i = \begin{pmatrix} D_{i1}(1) & \cdots & D_{i,i-1}(1) & D_{i,i+1}(1) & \cdots & D_{iN}(1) \\ D_{i1}(2) & \cdots & D_{i,i-1}(1) & D_{i,i+1}(2) & \cdots & D_{iN}(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ D_{i1}(m) & \cdots & D_{i,i-1}(m) & D_{i,i+1}(m) & \cdots & D_{iN}(m) \end{pmatrix} \tag{6}$$

where $D_{x,y}(t) = S_x^T(t)\ PS_y(t)$. Y_i can be obtained directly from the payoff data and A_i can be calculated from the strategy data.

Our goal is to reconstruct X_i from Y_i and A_i. To solve this network reconstruction problem, the following tradeoff form is developed:

$$\min_{X_i} \left(\frac{1}{2} \|A_i X_i - Y_i\|_2^2 + \lambda \|X_i\|_{1/2}^{1/2} \right) \tag{7}$$

where λ is a constant that controls the tradeoff between the reconstruction error and the sparseness of network. Being different from L_0 and L_1 regularization of R^N, $L_{1/2}$ can assuredly generate sparser solutions than L_1 regularization [11, 12]. Thus, in this paper, we optimize (7) using MA and can obtain the solution X_i. In a similar fashion, the neighbor-connection vectors of all other agents can be predicted, yielding the network adjacency matrix $X = (X_1, X_2, \cdots, X_N)$.

3 MAEGNet

To solve Eq. (7), we employ the proposed problem-specific memetic algorithm, termed as MAEGNet. The whole framework of MAEGNet is shown in Algorithm 1. For each agent, the steps from line 3 to line 13 are implemented. In line 3, the data for Eq. (7) are assigned. In lines 3 to 12, MA is employed to dealing with the problem (Eq. (7)). In line 4, MAEGNet completes the population initialization task according to Sect. 3.1. In line 5, the individual with the minimum fitness is selected as the best individual. Inline 7, MAEGNet uses the tournament selection method to select *pool* parental individuals for mating. Then in line 8, the BLX-α crossover [13] and the non-uniform mutation operation [14] are employed on the chosen parental individuals $P^t_{Selection}$. In line 9, MAEGNet performs IST local search for P^t_{GO}. In line 10, the current population is refreshed by taking the best *pop* individuals from $P^t_{Local} \cup P^t$. In line 11, we update the best individual and λ. When stopping criterion satisfies, MA stops and outputs $X_i^{temp} \leftarrow P^{t+1}_{best}$ (line 12). When $i > N$, MAEGNet stops and outputs X.

3.1 Initialization Operator for MA

In this paper, we employ a new initialization operator to initialize population [29]. The L_1-minimization algorithm LASSO [3] solves the following problem:

$$\min_{X_i}\left(\frac{1}{2}\|A_iX_i - Y_i\|_2^2 + \lambda\|X_i\|_1\right) \tag{8}$$

In LASSO, different choices for λ in the above equation may result in different optimal solutions. Moreover, we will obtain a set of solutions by using LASSO with different value of λ. Then, we briefly present this operator as follows. In order to generate *pop* individuals, we need to set *pop* different value of $\lambda_i \in [0.00001,10]$ randomly, $i = 1, 2, 3,\ldots, pop$, and then solve Eq. (8) using the LASSO to generate *pop* solutions.

3.2 Shrinkage-Thresholding Local Search

Iterative shrinkage-thresholding algorithm (ISTA) [7] is one of L_1-minimization algorithms for solving linear inverse problem Eq. (8). Basically, we can use the following equation to update the next individual.

$$X_i^{k+1} = \Xi_\alpha\left(g\left(X_i^k\right)\right) \tag{9}$$

where $\alpha = \lambda/L$ and Ξ_α: $R^n \rightarrow R^n$ is the shrinkage operator defined by:

$$\Xi_\alpha(x) = \text{sgn}(x)(|x| - \alpha)_+ \tag{10}$$

and $g\left(X_i^t\right)$ stands for

$$g\left(X_i^t\right) = X_i^t - \frac{1}{c}\nabla f\left(X_i^t\right) \tag{11}$$

Algorithm 1 Framework of MAEGNet

Input:

maximum generation: *maxgen*;
population size: *pop*;
mating pool size: *pool*;
tournament size: *tour*;
crossoverprobability: *pc*;
mutation probability: *pm*;
balance factor in Eq. (7): λ;
beta: β;
lambda bars: λ_{bar};
thepayoff data: Y;
the strategy data: A;

Output:

Agent-to-agent network X.

1: Agent $i \leftarrow 1$;
2: **while** ($i < N$) **do**
3: Obtain Y_i from the payoff data and calculate A_i from the strategy data;
4: Population initialization: $P^0 = \left\{P_1^0, P_2^0, \cdots, P_{pop}^0\right\}^{\mathrm{T}}$;
5: Obtain the individualwith the minimum fitness: $P_{best}^0 = P_i^0$; Generation $t \leftarrow 0$;
6: **while** ($t < maxgen$) **do**
7: Select parental chromosomes: $P_{Selection}^t = Selection\left(P^t, pool, tour\right)$;
8: Perform genetic operators on: $P_{GO}^t = GeneticOperation\left(P_{Selection}^t, pc, pm\right)$;
9: Perform local search: $P_{Local}^t = LocalSerach\left(P_{GO}^t, \lambda, A_i, Y_i\right)$;
10: Update population: $P^{t+1} = UpdatePopulation\left(P_{Local}^t, P^t\right)$;
11: Update the best individual P_{best}^{t+1} and $\lambda \leftarrow max\ (\beta\lambda, \lambda_{bar})$; $t \leftarrow t+1$;
12: **end while**
13: $X_i \leftarrow P_{best}^{t+1}$; $i \leftarrow i+1$;
14: **end while**

where $f(X_i) = \|A_i X_i - Y_i\|_2^2$ and the parameter c is chosen by optimizing the so-called Barizilai-Borwein equation:

$$c = \frac{\left(X_i^k - X_i^{k-1}\right)^T \left(\nabla f\left(X_i^k\right) - \nabla f\left(X_i^{k-1}\right)\right)}{\left(X_i^k - X_i^{k-1}\right)^T \left(X_i^k - X_i^{k-1}\right)} \tag{12}$$

To obtain better solutions efficiently, we employ a shrinkage-thresholding strategy improved from ISTA to refine the individuals of P_{GO}. In this paper, the regularizing coefficient λ is generated by a decreasing sequence of $\{\lambda_k\}$. To obtain X_i^k and X_i^{k-1}, we first sort P_{GO} and split P_{GO} into two equal subsets $\{P_{GOL}, P_{GOR}\}$, where P_{GOL} is better than P_{GOR}. We then calculate c and obtain X_i^{k+1} using Eq. (9).

4 Experiments

Our experiments fall into three parts. Section 4.1 introduces performance measures to evaluate the performance of MAEGNet and parameters settings for MAEGNet. Section 4.2 compares the performance of MAEGNet against eight state-of-the-art methods on synthetic networks. Section 4.3 shows the experiments on six real social networks.

4.1 Experimental Setup

To quantify the performance of our reconstruction method, two standard measurement indices are introduced, namely, the area under the receiver operating characteristic curve (AUROC) and the area under the precision-recall curve (AUPR) [16].

In practice, the learned x_{ij} is real number. Thus, a cut-off CO can be used to distinguish the relationship between node i and node j. in this paper, we set $CO = 0.2$. If nodes i and j are connected, $x_{ij} \geq CO$; otherwise, $x_{ij} < CO$.

Numerical simulation of EG is described as follows. Initially, a fraction of agents is set to choose the strategy of cooperation and the remaining agents are set to choose the strategy of defection. Nodal states are updated in parallel. For agent i of degree $\langle k \rangle$, at round t, the payoff of this agent is calculated using Eq. (2). To maximize the payoff of agent i, its strategy is updated using Eq. (3). A Monte Carlo round t is referred to the situation where all the states at $t + 1$ have been updated according to their states at t.

For MAEGNet, we have chosen a reasonable set of values and have not made any effort in finding the best parameter settings. We leave this task for a future work. The parameters of MAEGNet are showed in Table 1. Six real social networks are employed, including football [30], polbooks [31], dolphin [32], ZK [33], lesmis [15], neuralnet [17].

Table 1. The parameter settings of MAEGNet.

Parameter	Meaning	Value
maxgen	The maximum generation	1000
pop	Population size	100
pm	Mutation rate	0.2
pool	Size of the mating pool	50
tour	Tournament size	2
pc	Crossover rate	0.5
β	Beta	0.98

4.2 Analysis of MAEGNet on Synthetic Networks

In this section, we study the effect of the parameters, such as $\langle k \rangle$, on MAEGNet. We simulate evolutionary games on different model-based networks, including Erdős-Rényi random networks (ER) [17], Barabási-Albert scale-free networks (BA) [18], Newman-Watts small-world networks (NW) [19], and Watts-Strogatz small-world networks (WS) [20]. The experiments are conducted on network size $N = 100$. $\langle k \rangle = 18$. Note that we can obtain common conclusion for other values of N and $\langle k \rangle$. N_M is the total data length M divided by network size N. Here, N_M is increased from 0.2 to 0.8 in steps of 0.1. Rewriting probability of small-world networks is 0.3. Each data point is obtained by averaging over 30 independent runs. The performance of MAEGNet is compared with those of LASSO [1], OMP [21], basis pursuit (BP) [22], homotopy method [23, 24], fast iterative soft-thresholding algorithm (FISTA) [25], LARS [26], primal augmented lagrangian methods (PALM) [27], and L1LS [28] in terms of AUROC and AUPR, which are reported in Figs. 1 and 2, respectively. In order to fairly compare with other methods, we set λ = (100, 10, 1, 0.1, 0.01, 0.001, 0.0001, 0.00001). For all algorithm, we select the best results in terms of AUPR and AUROC. For these methods, we have identical parameter settings as

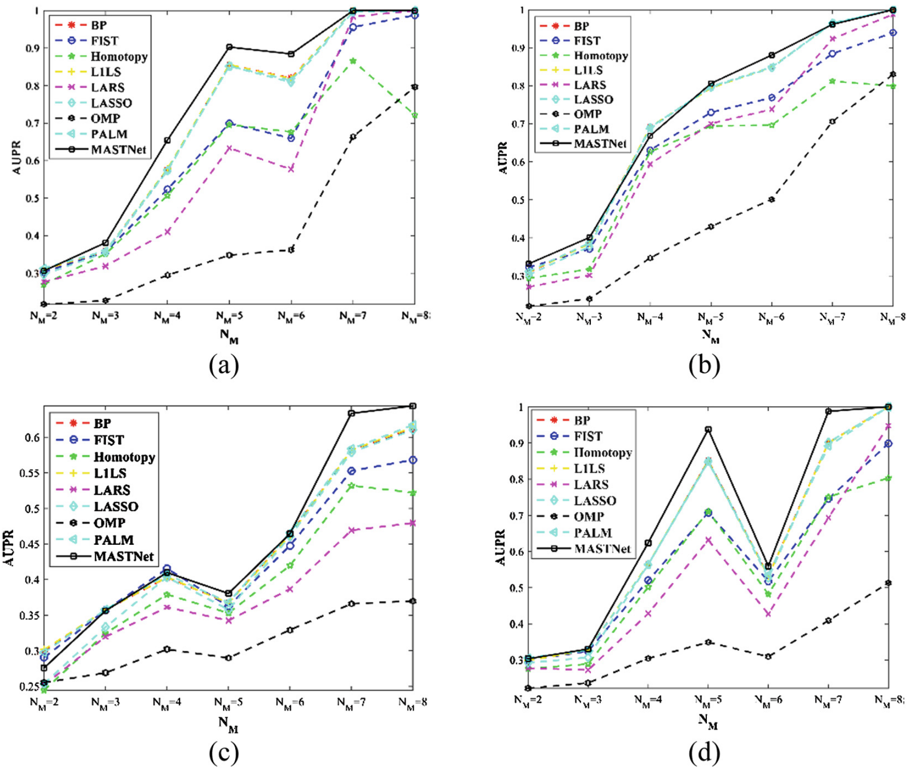

Fig. 1. AUPR as a function of the relative data length N_M of time series for (a) ER networks, (b) BA networks, (c) NW networks, and (d) WS networks. Here, $\langle k \rangle = 18$.

suggested in the original codes. The code of these methods can be obtained from https://people.eecs.berkeley.edu/∼yang/software/l1benchmark/ or http://sparselab.stanford.edu/.

The results demonstrate that the length of data sequences has an important effect on the performance of MAEGNet, even for small value of N_M, most links can be identified, as reflected by the high values of AUPR and AUROC. As seen, in terms of average AUPR and AUROC, MAEGNet almost outperforms all the others. Note that there are the curves descending rapidly for WS networks in terms of AUPR and AUROC. The reason is that the average degree $\langle k \rangle$ of WS networks with a certain value of N_M become greater than the general. Furthermore, the experimental results show that although MAEGNet cannot perform better than all methods in some cases, it just performs slightly worse than the best performer.

4.3 Application to Real-World Networks

We also test MAEGNet on several empirical networks. The experiments are conducted on EG dynamic with six real-world networks. Here, N_M is increased from 0.3 to 0.6 in

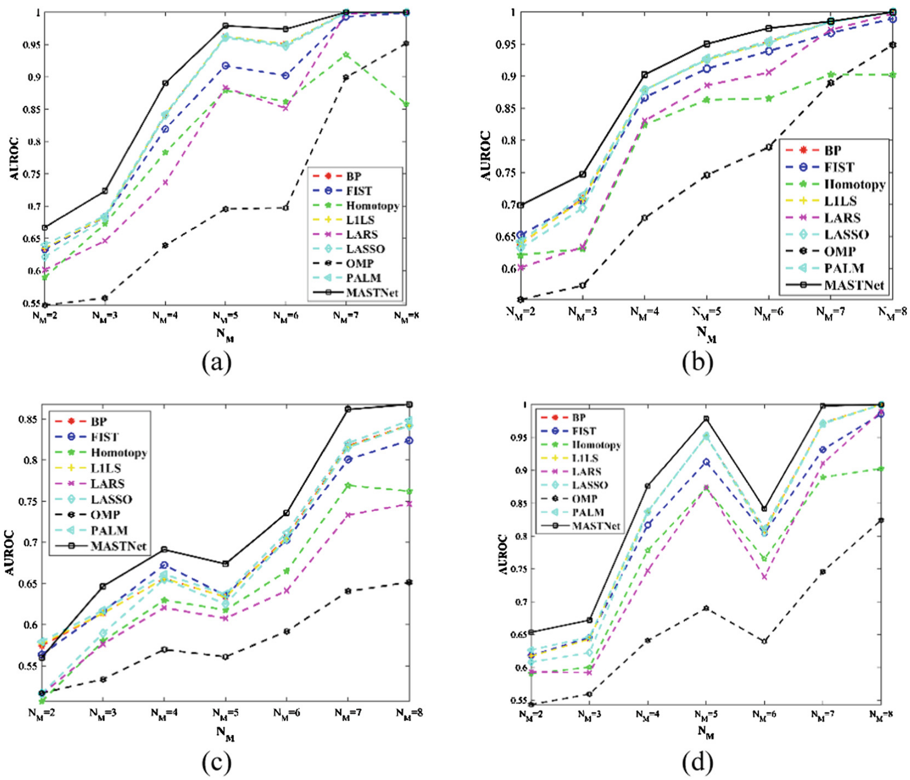

Fig. 2. AUROC as a function of the relative data length N_M of time series for (a) ER networks, (b) BA networks, (c) NW networks, and (d) WS networks. Here, $\langle k \rangle = 18$.

steps of 0.1. Each data point is obtained by averaging over 30 independent realizations. The parameters of these approaches are similar to Sect. 4.2. The results are reported in Tables 2 and 3.

As seen, on the cases with $N_M \geq 0.4$, the performance of MAEGNet matches or exceeds that of the remaining methods in terms of AUPR and AUROC. On the football dataset with $N_M = 0.3$, AUPR of MAEGNet outperforms that of LARS, LASSO and OMP, but is worse than that of BP, FIST, L1LS, and PALM. On this case, AUROC of MAEGNet has a similar performance to AUPR of MAEGNet. On the neuralnet dataset with $N_M = 0.3$, AUPR of MAEGNet outperforms that of FIST, LARS, LASSO and OMP, but is worse than that of BP, L1LS, LASSO, and PALM. On the ZK dataset with $N_M = 0.3$, AUROC of MAEGNet outperforms that of LARS, LASSO and OMP, but is worse than that of BP, FIST, L1LS, and PALM. On each of polbooks, dolphin, ZK, and lesmis datasets, AUPR of MAEGNet performs better than the remaining methods. On each of polbooks, dolphin, lesmis, and neuralnet datasets, the performance of MAEGNet performs better than the remaining methods in terms of AUROC.

Table 2. The comparison of MAEGNet against other methods in terms of AUPR.

N_M	Dataset	BP	FIST	L1LS	LARS	LASSO	OMP	PALM	MAEGNet
0.3	football	**0.239**	0.234	**0.240**	0.170	0.187	0.139	0.238	0.223
	polbooks	0.681	0.617	0.682	0.551	0.659	0.204	0.679	**0.723**
	dolphin	0.885	0.770	0.890	0.767	0.881	0.358	0.882	**0.902**
	ZK	0.825	0.809	0.825	0.808	0.789	0.719	0.825	**0.830**
	lesmis	0.607	0.492	0.609	0.405	0.531	0.273	0.605	**0.620**
	neuralnet	**0.218**	0.192	**0.216**	0.132	0.207	0.072	0.210	0.201
0.4	football	0.492	0.449	0.491	0.351	0.476	0.194	0.495	**0.546**
	polbooks	0.746	0.672	0.747	0.631	0.748	0.301	0.740	**0.778**
	dolphin	**1.000**	0.982	**1.000**	**1.000**	**1.000**	0.824	**1.000**	**1.000**
	ZK	**1.000**	0.987	**1.000**	**1.000**	0.973	0.841	**1.000**	**1.000**
	lesmis	0.922	0.827	0.919	0.817	0.860	0.512	0.922	**0.979**
	neuralnet	0.306	0.228	0.303	0.185	0.304	0.110	0.295	**0.354**
0.5	football	0.656	0.543	0.654	0.436	0.654	0.212	0.644	**0.764**
	polbooks	0.967	0.865	0.970	0.872	0.976	0.510	0.965	**0.984**
	dolphin	**0.976**	**0.976**	**0.976**	**0.976**	**0.976**	**0.976**	**0.976**	**0.976**
	ZK	**1.000**	0.987	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
	lesmis	**0.998**	0.901	**0.998**	0.958	**0.999**	0.634	**0.998**	**0.997**
	neuralnet	0.393	0.339	0.390	0.215	0.367	0.150	0.374	**0.429**
0.6	football	0.977	0.867	0.976	0.878	0.982	0.454	0.974	**1.000**
	polbooks	0.961	0.862	0.961	0.900	0.960	0.497	0.954	**0.996**
	dolphin	**0.973**	0.967	**0.973**	**0.973**	0.971	0.947	**0.973**	**0.973**
	ZK	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
	lesmis	**0.997**	0.953	0.995	0.982	0.988	0.884	**0.996**	**0.995**
	neuralnet	0.565	0.479	0.567	0.351	0.555	0.212	0.548	**0.619**

Table 3. The comparison of MAEGNet against other methods in terms of AUROC.

N_M	Dataset	BP	FIST	L1LS	LARS	LASSO	OMP	PALM	MAEGNet
0.3	football	0.660	0.658	0.660	0.593	0.610	0.562	**0.666**	0.658
	polbooks	0.899	0.887	0.899	0.851	0.890	0.652	0.903	**0.935**
	dolphin	**0.960**	0.937	**0.960**	0.930	0.951	0.784	**0.961**	**0.960**
	ZK	**0.984**	0.952	**0.984**	0.922	0.914	0.901	**0.984**	0.940
	lesmis	0.897	0.855	0.897	0.792	0.848	0.700	0.897	**0.908**
	neuralnet	0.702	0.688	0.702	0.661	0.698	0.583	0.705	**0.749**
0.4	football	0.826	0.807	0.825	0.756	0.818	0.632	0.834	**0.864**
	polbooks	0.937	0.923	0.938	0.907	0.939	0.740	0.938	**0.958**
	dolphin	**1.000**	**0.999**	**1.000**	**1.000**	**1.000**	0.971	**1.000**	**1.000**
	ZK	**1.000**	0.991	**1.000**	**1.000**	0.992	0.969	**1.000**	**1.000**
	lesmis	0.980	0.952	0.978	0.954	0.968	0.860	0.980	**0.998**
	neuralnet	0.779	0.768	0.780	0.730	0.780	0.634	0.781	**0.828**
0.5	football	0.903	0.870	0.902	0.826	0.901	0.662	0.904	**0.944**
	polbooks	0.994	0.976	0.995	0.973	0.995	0.855	0.995	**0.998**
	dolphin	**0.984**	**0.984**	**0.984**	**0.984**	**0.984**	**0.984**	**0.984**	**0.984**
	ZK	**1.000**	**0.999**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
	lesmis	**1.000**	0.983	**1.000**	0.992	0.996	0.974	**1.000**	**1.000**
	neuralnet	0.828	0.807	0.828	0.773	0.822	0.673	0.830	**0.869**
0.6	football	0.994	0.976	0.994	0.977	0.996	0.842	0.993	**1.000**
	polbooks	0.992	0.967	0.992	0.975	0.992	0.860	0.992	**0.999**
	dolphin	**0.983**	**0.982**	**0.983**	**0.983**	0.981	0.979	**0.983**	**0.983**
	ZK	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
	lesmis	**0.999**	0.994	**0.999**	0.992	0.996	0.974	**1.000**	**1.000**
	neuralnet	0.896	0.875	0.896	0.852	0.894	0.729	0.897	**0.925**

5 Conclusions

In this paper, we have developed an efficient MA to reconstruct EG networks from profit sequences. It is noteworthy that the proposed approach is quite flexible and not limited to the networked systems discussed here, such as gene regulatory networks, transportation networks, and communications networks. Being different from l_1 minimization methods, we solve EG network reconstruction problem with $l_{1/2}$ regularization. Then, a problem-specific memetic algorithm incorporated both LASSO initialization and shrinkage-thresholding local search has been proposed to optimize this problem. The experiments on synthetic and real networks illustrate that the proposed MAEGNet achieve good performance in terms of accuracy.

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