

Preface to the German Edition

The main topic of this book is an elementary introduction to the construction of the number systems encountered by mathematics students in their first semesters of study. Beginning with the natural numbers, we successively construct, along with the requisite algebraic machinery, all the number fields containing the natural numbers, including the real numbers, complex numbers, and Hamiltonian quaternions. Our experience has shown us that time is frequently lacking in introductory mathematics courses for a well-founded construction of number systems; this book represents a contribution toward filling that gap.

The construction of number systems also represents an important component in the professional education of mathematics teachers. For this reason, this book offers a self-contained and compact construction of the number systems that are of relevance to different grade levels from a mathematical perspective with a view toward aspects of pedagogical content knowledge.

This book arose from a course in elementary abstract algebra and number theory given a number of times at the Humboldt University of Berlin. Parts of the first-named author's book *Zahlen für Einsteiger: Elemente der Algebra und Zahlentheorie* (Vieweg Verlag, Wiesbaden, 2008) have been revised and expanded for inclusion in this newly conceived book on the construction of number systems. Numerous exercises with extensive solutions facilitate the reader's engagement with the subject.

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