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Dirk Draheim

Generalized Jeffrey Conditionalization

A Frequentist Semantics of
Partial Conditionalization



Springer

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Foreword

In my view, mathematics of the 21st century can be characterized by the attempt at *automating* mathematical reasoning. By Gödel, we know that this will never be completely possible: The higher we go in the sophistication of automation the more remote is the horizon of where we would like to go next. However, it is possible and exciting to conquer higher and higher levels of automation.

The 20th century was the century of *formalizing* mathematical reasoning, which is the first step towards *automating* mathematical reasoning. As a side-product of mathematical formalization, the notion of “universal computer” – which in essence is a mathematical and not an engineering concept – was invented. The enormous impact of this notion on all aspects of science, technology, economy, and society as a whole, by now, is understood by everybody. The impact of *automating* mathematical reasoning (mathematical invention and mathematical verification) will generate bigger and bigger waves of understanding the world and of societal transformation. The waves will include such theoretical areas like, for example, the build-up of web-accessible global and comprehensive mathematical knowledge bases and such practical effects like, for example, deriving hidden knowledge from social media messages.

The level of formalization is not equally high in all areas of mathematics. In this book, Dirk Draheim lays the ground for the formalization of an important part of mathematics that also has high relevance to modern data science: probabilistic reasoning. He clarifies the frequentist semantics of the fundamental notion of partial conditional probability and reveals the subtle differences and the relation between this frequentist and the established Bayesian view. This is the first time that the many results that are due to earlier publications in this area are brought into a coherent form. The concepts can be made operational in today’s standard programming paradigms. Thus, the foundational results are immediately available also for practical probabilistic modeling, which is of course of high relevance in current data science and artificial intelligence.

I wish this book wide distribution both in the research community and in the business world.

Research Institute for Symbolic Computation
Johannes Kepler University
Linz/Hagenberg, October 2017

Bruno Buchberger

Preface

Statistics is the language of science; however, the semantics of probabilistic reasoning is still a matter of discourse. In this book, I provide a frequentist semantics for conditionalization on partially known events. The resulting frequentist partial (F.P.) conditionalization generalizes Jeffrey conditionalization from partitions to arbitrary collections of events. Furthermore, the postulate of Jeffrey's probability kinematics, which is rooted in Ramsey's subjectivism, turns out to be a consequence in our frequentist semantics.

I think the book appeals to researchers that are involved in any kind of knowledge processing systems. F.P. conditionalization is a straightforward, fundamental concept that fits our intuition. Furthermore, it creates a clear link from the Kolmogorov system of probability to one of the important Bayesian frameworks. This way, I think it is interesting for anybody who investigates semantics of reasoning systems. The list of these mutually overlapping theories, methods and tools includes, without preference, multivariate data analysis, Bayesian frameworks, fuzzy logic, many-valued logics, conditional logic, Nilsson probabilistic logic, probabilistic model checking and also current efforts in unifying probability theory and logics such as the current rational programming.

Tallinn, August 2017

Dirk Draheim

Contents

1	Introduction	1
1.1	From Conditional Probability to Partial Conditionalization	2
1.2	Background, Motivation, Perspectives and Outlook	5
1.3	Chapter Overview	8
2	F.P. Conditionalization	11
2.1	On Modeling Repeated Experiments	12
2.2	Defining Frequentist Partial Conditionalization	20
2.3	Projective F.P. Conditionalizations	23
2.4	Cutting Repetitions	24
2.5	Conditional Events	27
3	F.P. Semantics of Jeffrey Conditionalization	33
3.1	The Case of Basic Jeffrey Generalization	33
3.2	The Case of Jeffrey Conditionalization in General	36
4	Properties of F.P. Conditionalization	41
4.1	Computing F.P. Conditionalization	43
4.2	Conditional Segmentation of F.P. Conditionalizations	48
4.3	Independence and F.P. Conditionalization	50
4.4	Updating with Particular Probability Values	55
4.5	Conditional Probabilities A Posteriori	57
4.6	F.P. Expectations	61
5	Probability Kinematics and F.P. Semantics	65
5.1	Jeffrey's Postulate and Jeffrey's Rule	67
5.2	Commutative Jeffrey Chaining	69
5.3	Jeffrey Desirabilities A Posteriori	73
5.4	The Jeffrey-Donkin Correspondence	75
	Bibliographic Notes	81

Basic Formulary and Notation	87
Technical Lemmas and Proofs	89
References	95
Index	103

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