Reason vs. Rationality: From Rankings to Tournaments in Individual Choice

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Abstract. The standard assumption in decision theory, microeconomics and social choice is that individuals (consumers, voters) are endowed with preferences that can be expressed as complete and transitive binary relations over alternatives (bundles of goods, policies, candidates). While this may often be the case, we show by way of toy examples that incomplete and intransitive preference relations are not only conceivable, but make intuitive sense. We then suggest that fuzzy preference relations and solution concepts based on them are plausible in accommodating those features that give rise to intransitive and incomplete preferences. Tracing the history of those solutions leads to the works of Zermelo in 1920's.

1 Introduction

A basic concept in decision theory is that of rationality. While no decision theorist would maintain that all human behaviour is rational, most of them would probably argue that rational behaviour provides a useful benchmark for evaluating and explaining any kind of behaviour. In particular, if observed behaviour is found to agree with the dictates of rationality, no further explanation is typically needed for it. It is behaviours that exhibit deviations from rationality that require explanation. But what is then rational behaviour? The most precise definition – due to Savage (1954) – is based on a simple choice situation involving two alternatives, say, A and B. Suppose that the individual making the choice has a strict preference over these two so that he/she (hereinafter he) strictly prefers A to B. Choice behaviour is then called rational if it always, that is, with probability 1, results in A being chosen (see also Harsanyi 1977). Of course, we may encounter situations where the individual is physically prevented from choosing A or by making him believe that A is not really available or that by taking some new aspects of the situation into account, he does not prefer A to B or something similar. These kinds of considerations are, however, irrelevant since by suggesting that A is not available, the situation is no longer one involving a choice. Similarly, if the individual is led to believe that he is actually preferring

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B to A, the "original" preference no longer holds. So, we may argue that the definition holds at least as far as *preference-based rationality* is concerned. In this setting it is quite straight-forward and trivial to argue that rational behaviour aims at utility maximization since by assigning a larger utility value to A than to B, we guarantee that preferences coincide with utility maximization.

Things get more complicated when the alternative set is expanded. The standard way to proceed is to impose conditions on preference relations that guarantee that acting in accordance with preferences amounts to maximizing utilities. In other words, one looks for properties that the preferences have to possess in order for the choices made according to those preferences to be equivalent to utility maximization by the chooser. In fact, the theory of choice under certainty, risk and uncertainty focuses precisely on those conditions. The standard representation theorem (see e.g. Harsanyi 1977, 31) states that if the individual is endowed with a continuous, complete and transitive weak preference relation over the alternatives, then his choice behaviour – if it conforms with his preferences – can be represented as utility maximizing.[1] In the following sections we shall consider each one of these properties of preference relations in turn and discuss their plausibility. Our aim is to show that under relatively general circumstances each one of them can be questioned. We shall thereafter endeavour to show that fuzzy binary preference relations could provide a useful starting point for modelling reason-based behaviour and a more plausible benchmark than the traditional preference-based rationality.

2 Transitivity Assumption

It is common to assume that preferences are revealed by choices. This is, in fact, stated in the definition of preference. In the world of empirical observations it may, however, happen that a person may, for one reason or another, occasionally choose B even though his preference is for A over B. It would, then, be more plausible to translate the preference of A over B into a probability statement according to which the probability of A being chosen by the person is larger than the probability that B is chosen. Starting from this somewhat milder probability definition of preference, we shall now consider the transitivity property. May (1954) suggests that the appropriate definition of preference-based choice is one that – in addition to choice probability – includes the alternative set considered as well as the description of the experimental setting. In this framework the preference for A over B is expressed as the following probability statement:

$$p(A|A, B, E) > p(B|A, B, E)$$

Here E denotes the experimental setup.

Suppose now that A is preferred to B and B is preferred to C. I.e.

$$p(A|A, B, E) > p(B|A, B, E) \quad (1)$$

[1] The weak preference of A over B means that A is regarded as at least as desirable as B. Thus the weak preference relation is not asymmetric, while the strong one is.

$$p(B|B, C, E) > p(C|B, C, E) \tag{2}$$

Now, transitivity would require that Eqs. 1 and 2 imply that

$$p(A|A, C, E) > p(C|A, C, E) \tag{3}$$

It is, however, difficult to associate this implication with rationality, since the alternative sets considered are different in each Equation: in Eq. 1 it is $\{A, B\}$, in Eq. 2 it is $\{B, C\}$ and in Eq. 3 it is $\{A, C\}$. What May (1954, 2) argues is "that transitivity does not follow from this empirical [probabilistic] interpretation of preference, but must be established, if at all, by empirical observation." This point on which we completely agree leaves, however, open the possibility that transitivity would be normatively compelling (even if empirically contestable). Our position is stronger here: while we agree that there are circumstances where transitivity seems normatively plausible[2], there are others where it is not. Hence, defining rationality so that transitivity of preferences is a necessary part of it, is not acceptable in our view.

The reason is rather straight-forward. The grounds for preferring A over B might well be different from those used in ranking B ahead of C. Hence, it is purely contingent whether these or other grounds are used in preferring C to A or vice versa. Alternatively, the decision maker may use several criteria of "performance" of alternatives. Each of these may result in a complete and transitive relation over alternatives, but when forming the overall preference relation on the basis of these rankings, the decision maker may well end up with an intransitive relation. Consider a fictitious example.

Three universities A, B and C are being compared along three criteria: (i) research output (scholarly publications), (ii) teaching output (degrees), (iii) external impact (expert assignments, media visibility, R&D projects, etc.)

Publications	Teaching	External impact
A	B	C
B	C	A
C	A	B

Assuming that each criterion is of roughly equal importance, it is natural to form the overall preference relation between the universities on the basis of the majority rule: which one of any two universities is ranked higher than the other by a majority of criteria is preferred to the latter. In the present example this leads to a cycle: $A \succ B \succ C \succ A \succ \ldots$. Hence, intransitive individual preference relations can be made intelligible by multiple criterion setting and majority principle (cf. Fishburn 1970; Bar-Hillel and Margalit 1988).

[2] E.g. in preferences over monetary payoffs.

3 Completeness Assumption

The completeness of weak preference relation entails that for any pair (A, B) of alternatives either A is preferred to B or vice versa or both. Stated in another way, completeness means that it cannot be the case that A is not preferred to B and B is not preferred to A. In the following we show that there is nothing unnatural or irrational in situations where there are grounds for saying that neither A is preferred to B nor B is preferred to A. Perhaps the simplest way to show this is via a phenomenon known as Ostrogorski's paradox. It refers to the ambiguity in determining the popular preference among two alternatives (Daudt and Rae 1978). In the following we recast this paradox in an individual decision-making setting. The nominating individual is to make a choice between two alternatives A and B, e.g. applicants to the chair of economics in a university. Three types of merits are deemed of primary importance for this office, viz. research merits, teaching skills and ability to attract external funding to the university. The nominating individual received advice from three other individuals: one representing the peers (i.e. other economics professors), one representing the students of economics and one representing the university administration. The following table indicates the preferred applicant of each representative on each area of merit. Thus, e.g. applicant A has a preferable research record according to the peers than applicant B. Similarly, the representative of the administration deems B preferable in each merit area.[3]

Merit area	Research	Teaching	Funding potential	Row choice
Advisor 1	A	B	A	A
Advisor 2	A	A	B	A
Advisor 3	B	B	B	B
Column Choice	A	B	B	?

Suppose now that the nominating individual forms his preference in a neutral and anonymous manner, i.e. each merit area and each advisor is considered equally important. It would then appear natural that whichever applicant is deemed more suitable by more advisors than its competitor, is preferable in the respective merit area. Similarly, whichever candidate is more suitable than his competitor in more merit areas, is regarded as preferable by each advisor. Under these assumptions the nominating individual faces a quandary: if the aggregation of valuations is first done over columns – i.e. each advisor's overall preference is determined first – and then over rows – i.e. picking the applicant regarded more appropriate by the majority of advisors – the outcome is that B cannot be

[3] All preferences underlying the table are assumed to be strict. The composition of the advisory body may raise some eyebrows. So, instead of these particular categories of advisors, one may simply think of a body that consists of three peers.

preferred to A. If the aggregations is performed in the opposite order – first over rows and then over columns – the outcome is that A cannot be preferred to B. Hence, the preference relation over $\{A, B\}$ is not complete.

It should be observed that there is nothing arbitrary or irrational in the above example. The use of expert information (advisors) or other evaluation criteria in assessing applicants would seem quite natural way to proceed. Also, the task to be performed by the successful applicant often has several aspects (merit areas) to it. Similarly, the use of majority principle in determining the "winners" of aggregation is quite reasonable, certainly not counterintuitive.

4 Continuity Assumption

Continuity axiom states that both the inferior and superior sets for any given alternative are closed (Harsanyi 1977, 31). To elaborate this a little, consider a set $\mathcal{X}$ of alternatives and an element x in it. Let now $x_1, x_2, \ldots$, a sequence of alternatives converging to x_0, have the property that for each x_i in the sequence, $x \succ_j x_i$. In other words, individual j prefers x to each element of the sequence. Then, continuity requires that $x \succ_j x_0$ as well. Similarly, continuity requires that the sequence has the property that if $x_i \succ_j x$ for each x_i in the sequence, then $x_0 \succ_j x$ as well. The above pertains to infinite sequences. In finite ones, continuity requires that small changes in the alternatives are accompanied with small changes in their desirability.

Let us now see how continuity assumption translates into multiple-criterion settings. We shall take advantage of Baigent (1987) fundamental result in social choice theory. This result has subsequently been augmented, modified and generalized by Eckert and Lane (2002), Baigent and Eckert (2004), as well as by Baigent and Klamler (2004). We shall, however, largely make use of the early version (Baigent 1987). It states the following.

Theorem 1. *Anonymity and respect for unanimity of a social choice function cannot be reconciled with proximity preservation.*

Proximity preservation is a property defined for social choice functions. It amounts to the requirement that choices made in profiles more close to each other ought to be closer to each other than those made in profiles less close to each other. Profiles – it will be recalled – are n-tuples of preference rankings over the set of alternatives (n being the number of individuals). What this requirement intuitively means is that if we make a small modification in the preference rankings, the change in the outcome of the social choice function should be smaller than if we make a larger modification. Anonymity, in turn, requires that relabelling of the individuals does not change the choice outcomes. In multi-criterion setting anonymity means that permuting the criteria does not change the outcome of evaluation. Respect for unanimity is satisfied whenever the choice function agrees with a preference ranking held by all individuals, i.e. if $x \succ_j y$ for all individuals j, then this will also be the social ranking between x and y. In multi-criterion environment this amounts to the requirement that if all criteria suggest the same ranking of alternatives, then this ranking should also be the outcome.

To illustrate the incompatibility exhibited by Baigent's theorem, let us turn again to the fictitious example of nominating the chair of economics. Suppose that there are two applicants A and B. Moreover, only two criteria are being used by the nominating authority: research merits (R) and teaching record (T).[4] To simplify things, assume that only strict preferences are possible, i.e. each criterion produces a strict ranking of the applicants. Four different configurations of rankings (i.e. profiles) $(S_1, \ldots, S_4)$ are now possible:

S_1		S_2		S_3		S_4	
R	T	R	T	R	T	R	T
A	A	B	B	B	A	A	B
B	B	A	A	A	B	B	A

Let us denote the rankings in various configurations S_{mi} where m is the number of the configuration and i the criterion. We consider two types of metrics: one that is defined on pairs of rankings and one defined on configurations. The former is denoted by d_r an the latter by d_S. The number of criteria is N. The metrics are related as follows:

$$d_S(S_m, S_j) = \sum_{i \in N} d_r(S_{mi}, S_{ji}).$$

In other words, the distance between two configurations is the sum of distances between the pairs of rankings of the first, second, *etc.* criterion.

Take now two configurations, S_1 and S_3, from the above list and express their distance using metric d_S as follows:

$$d_S(S_1, S_3) = d_r(S_{11}, S_{31}) + d_r(S_{12}, S_{32}).$$

Since, $S_{12} = S_{32} = \text{A} \succ \text{B}$, and hence the latter summand equals zero, this reduces to:

$$d_S(S_1, S_3) = d_r(S_{11}, S_{31}) = d_r((A \succ B), (B \succ A)).$$

Taking now the distance between S_3 and S_4, we get:

$$d_S(S_3, S_4) = d_r(S_{31}, S_{41}) + d_r(S_{32}, S_{42}).$$

Both summands are equal since by definition:

$$d_r((B \succ A), (A \succ B)) =$$
$$d_r((A \succ B), (B \succ A)).$$

Thus,

[4] The argument is a slight modification of Baigent's (1987, 163) illustration.

$$d_S(S_3, S_4) = 2 \times d_r((A \succ B), (B \succ A)).$$

In terms of d_S, then, S_3 is closer to S_1 than to S_4. This makes sense intuitively.

We now turn to procedures used in aggregating the information on criterion-wise rankings into an overall evaluation or choice. Let us denote the aggregation procedure by g. We make two intuitively plausible restrictions on choice procedures, *viz.* that they are anonymous and respect unanimity. In our example, anonymity requires that whatever is the choice in S_3 is also the choice in S_4 since these two profiles can be reduced to each other by relabelling the criteria. Unanimity, in turn, requires that $g(S_1) = A$, while $g(S_2) = B$. Therefore, either $g(S_3) \neq g(S_1)$ or $g(S_3) \neq g(S_2)$. Assume the former. It then follows that $d_r(g(S_3), g(S_1)) > 0$. Recalling the implication of anonymity, we now have:

$$d_r(g(S_3), g(S_1)) > 0 = d_r(g(S_3), g(S_4)).$$

In other words, even though S_3 is closer to S_1 than to S_4, the choice made in S_3 is closer to - indeed identical with - that made in S_4. This argument rests on the assumption that $g(S_3) \neq g(S_1)$. Similar argument can, however, easily be made for the alternative assumption, *viz.* that $g(S_3) \neq g(S_2)$.

The example shows that small mistakes or errors in criterion measurements are not necessarily accompanied with small changes in evaluation outcomes. Indeed, if the true criterion rankings are those of S_3, then a mistaken report on criterion 1's leads to profile S_1, while mistakes on both criteria lead to S_4. Yet, the outcome ensuing from S_1 is further away from the outcome resulting from S_3 than the outcome that would have resulted had more – indeed both – criteria been erroneously measured whereupon S_4 would have emerged. This shows that measurement mistakes do make a difference. It should be emphasized that the violation of proximity preservation occurs in a wide variety of aggregation systems, viz. those that satisfy anonymity and unanimity. This result is not dependent on any particular metric with respect to which the distances between profiles and outcomes are measured. Expressed in another way the result states that in nearly all reasonable aggregation systems it is possible that a small number of measurement errors has greater impact on evaluation outcomes than a big number of errors.

The theorem – when interpreted in the multiple-criterion choice context – does not challenge completeness or transitivity of individual preferences, but calls into question the continuity of preferences, i.e. their representation by smooth utility functions.

5 Invoking Reasons

The upshot of the preceding is that all assumptions underlying the utility maximization theory can be questioned, not only from the descriptive accuracy but also from the normative point of view. The deviations from the assumptions described above are not unreasonable or irrational. In fact, it can be argued that they are just the opposite, viz. based on reasons for having opinions (cf. Dietrich and List 2013). Incompleteness of preference relations as exhibited by Ostrogorski's paradox is a result of a systematic comparison of alternatives using a set of criteria and a set of aspects or dimensions or purposes ("functions") that the alternatives are associated with. There is a reason for the incompleteness: simple majority rule gives different results when row-column aggregation or column-row aggregation is resorted to. The simple majority rule is not the sole culprit: the paradox can occur with super-majority rules as well. The point is that one can build a plausible argument for the incompleteness under some circumstances.

The same goes for intransitivity. The argument is, however, somewhat different in invoking reasons for having a given binary preference: the reason for preferring A to B may differ from the one for putting B ahead of C and this, in turn, may differ from the basis for preferring A to C or vice versa. As May (1954) pointed out, the basic sets from which choices are made are different in each of these three cases.

The eventual failure on continuity rests on yet another consideration. By Baigent's theorem any rule that is anonymous (does not discriminate for or against alternatives) and respects unanimity (in agreeing with the ranking that is identical on all criteria) can lead to discontinuities. One could argue that any reasonable rule is prone to discontinuous utility representations.

To reiterate: the grounds for deviating from the assumptions of utility maximization are normative, not just descriptive. In other words, it makes perfect sense to have preferences that deviate from the assumptions. The question now arises: are there alternatives to these assumptions that could be used in analyzing individual choice behavior? In what follows we shall argue that there are and, moreover, these alternatives provide adequate foundations for institutional design.

6 Dealing with Incomplete, Intransitive and Discontinuous Preferences

The most natural way of handling intransitive preference relations is to start from complete relations and look for methods to aggregate them. This approach has a long history. The most important of the early pioneers is Ernst Zermelo (1929). The starting point is the concept of tournament, i.e. a complete and asymmetric relation. With a finite (and small) number k of alternatives this can conveniently be represented as a $k \times k$ matrix where the element a_{ij} on the ith row and jth column equals 1 whenever ith alternative is preferred to the jth one. Otherwise, the element equals 0.

Given an individual preference tournament we might be interested in forming a ranking that would preserve the essential features of the tournament, while at the same time augmenting it so that a complete and transitive relation emerges. The latter might be necessary e.g. for aggregating individual preference information to end up with a social ranking or choice. By the fundamental result of

Edward Szpilrajn (1930) every partial order – that is a asymmetric and transitive relation – has a linear extension. In other words, if the individual gives a preference relation that is asymmetric (strict preferences only) and transitive, but not complete (not all pairs of alternatives are comparable), then preference rankings can be constructed that preserve those aspects provided by the individual. The problem is that the resulting rankings are rarely unique. In fact, if x and y are two non-comparable alternatives in the relation given by the individual, there are rankings in which $x \succ y$ and rankings in which $y \succ x$ (Dushnik and Miller 1941). Thus, there seems to be no general way of extending a partial order into a unique linear one.

However, it can be argued that tournaments put less structure into individual preferences than partial orders.[5] After all, they are complete and asymmetric, not necessarily transitive. Over past decades many ways of translating tournaments into rankings have been suggested. The usual way – called scoring method by Rubinstein (1980) – is the straight-forward summing of row entries in the tournament matrix whereby one ends up with $s_i = \sum_j a_{ij}$ for each alternative i. The ranking over the alternatives is then determined by the order of scores. The resulting ranking is, of course, weak since several alternatives may receive the same score.

The scoring method may, however, lead to an outcome ranking where a higher rank is given to an alternative that is deemed inferior to one or several of the lower ranked ones. Several methods to avoid this problem has been suggested. Thus, for example, Goddard's (1983) proposal is to choose those rankings that minimize the number of times a binary preference between any two alternatives is upset (i.e. reversed) in the outcome ranking.[6] Upon closer inspection this proposal turns out to be similar to Kemeny's (1959) rule. Viewed as a social choice function this rule has a host of desirable properties (see, e.g. Nurmi 2012, 257). It is, however, intended for finding the "closest" social ranking for any given set of individual rankings over several alternatives. A function that – given a set of individual preference tournaments – looks for the collective one that is closest to the individual tournaments in a specific sense is – regrettably nowadays largely forgotten – Slater's (1961) rule. It seems identical to the rule that Goddard advocates. It works, as was already stated, on the basis of individual tournaments, i.e. complete and asymmetric relations. It then generates all $k!$ complete and transitive relations (strict rankings) that can be obtained from the k alternatives and converts them into tournament matrices. Each of these generated matrices is then a candidate for the collective preference tournament (i.e. the winning tournament). The winning tournament has the distinction that it is closest to the individual tournaments in the sense that it requires the minimum number of changes from 0 to 1 or vice versa in individual opinions to be unanimously adopted.

[5] Admittedly, this claim rests on a specific intuitive concept of structure.

[6] Goddard is not the first one to suggest this method. For earlier discussions, see Kendall (1955) and Brunk (1960).

The principle of Slater's rule can, of course, be used in individual decision making as well. To wit, given an individual preference tournament one generates the tournaments corresponding to all $k!$ preference rankings involving the same number of alternatives. One then determines whether the individual tournament coincides with one of them. If it does, then this gives us the ranking we are looking for. Otherwise one determines which of the generated tournaments is closest to the individual's. The closest one indicates the ranking. It may happen that there are several equally close tournaments and thus there may be several "solutions".

Zermelo's (1929) approach to tournaments is based on observations of chess playing contests which often take the form of a tournament.[7] Each player plays against every other player several times. The outcome of each game is either a victory of one player or a tie. We assume that the games are independent binomial trials so that the probability of player i beating player j is p_{ij}. Zermelo then introduces the concept *Spielstärke*, playing strength, denoted by V_i, that determines the winning probability as follows:

$$p_{ij} = \frac{V_i}{V_i + V_j}.$$

The order of the V_i values is the ranking of the players in terms of playing strength. Apparently player i is ranked no lower than player j if and only if $p_{ij} \geq 1/2$, i.e. players with greater strength defeat contestants with smaller strength more often than not. Now, given the matrix A of results, i.e. a $k \times k$ matrix of 0's and 1's denoting losses and victories of the alternatives represented by the rows, Zermelo defines maximum likelihood estimates, denoted by v_i, for the playing strengths of players. Consider any k vector of strengths v. One can associate with it the probability that the observed matrix A is the result of the tournament when the strengths are distributed according to v. The probability is the following:

$$p(v) = \prod_{i,j}(\frac{v_i}{v_i + v_j}).$$

and this is what is to be maximized. Conditions under which a unique maximizing vector of strengths can be found are discussed by Zermelo and found to be rather general. A particularly noteworthy property of the Zermelo rankings is that they always coincide with the rankings in terms of scores defined above. So, were one interested in rankings only, the easy way to find them is simply to compute the scores. However, the v_i values give us more information about the players than just their order of strength; it also reveals how much stronger player i is when compared with player j.

[7] The differences between Zermelo's and Goddard's approaches are cogently analyzed by Stob (1985). Much of what is said in this and the next paragraph is based on Stob's brief note.

Leaving aside now the game context and looking at Zermelo's method from the point of view of fuzzy systems, it is not difficult to envision a new interpretation whereby the outcome matrix expresses the individual's choice between pairs of alternatives. The values V_i and their estimates v_i can be viewed as values of *desirability* of alternatives. A ranking based on desirability of alternatives is certainly a worthy goal of inquiry and Zermelo's approach gives us plausible way to achieve it.[8]

The above remarks pertain to situations where we are given an individual preference tournament and, for one reason or another, are looking for a ranking that would best approximate it. It is, however, quite easy to envision situations where no ranking at all is required, but rather choice of a subset of "best alternatives". These kinds of situations have been dealt with elsewhere (see Aizerman and Aleskerov 1995; Nurmi and Kacprzyk 1991; Kacprzyk and Nurmi 2000; Kacprzyk et al. 2008, 2009).

7 Concluding Remarks

We have attempted to show above that there are quite plausible reasons for individuals to deviate from the behavior dictated by preference-based utility maximization theory. Indeed, behavior based on reasons would seem to be particularly prone to these kinds of deviations. Rankings being the basic concept underlying the maximization theory, our main conclusion is that alternatives to ranking assumption already exist. One of these, individual preference tournament, has been discussed at some length above. Of particular interest is the re-discovery of Zermelo's approach to tournaments since it provides a natural link between directly observable pairwise choices and the underlying fuzzy notion of desirability. It thus provides a method for estimating fuzzy preference degrees for observational data.

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[8] We shall here ignore the ties in pairwise comparisons. These can certainly be dealt with in fuzzy systems theory. Also the tournament literature referred to here is capable of handling them. Ties are typically considered as half-victories, i.e. given a value 1/2 in the tournament matrices.

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