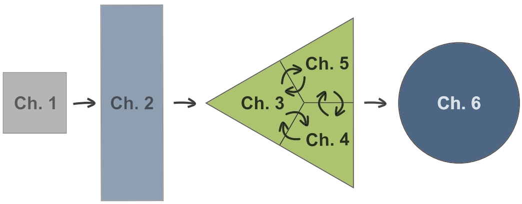
2 Problem Identification: Theoretical Concepts

This chapter contains a literature review and the theoretical background concepts relevant for a support project for engineering first-years in mathematics. This forms the basis for the following chapters covering the research approach (Chapter 3), the design and development of the project (Chapter 4), and the empirical analyses (Chapter 5), see orientation figure below.



Concerning transition to tertiary mathematics (section 2.1), the material presented here addresses transition processes in general (section 2.1.1), the learning of mathematics at tertiary level (section 2.1.2), and approaches to overcome the problems connected with this (section 2.1.3). The specifities of engineering mathematics (section 2.1.4) are explored as well as concrete projects at other universities (section 2.1.5) and the tangible conditions at RUB (section 2.1.6).

In reference to learning strategies (section 2.2), the assessment of learning behaviour with the help of questionnaires is considered (section 2.2.1), followed by a detailed account of the LIST questionnaire (section 2.2.2).

2.1 Transition to Tertiary Education in Mathematics

A human passing from kindergarten to primary and later to secondary school, and finally to some kind of education at the tertiary level, experiences transition more than once. From a general point of view, all these processes have three stages in common: letting go of what you have known and become

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accustomed to; getting to know new surroundings, new expectations, and new rules; and finally, getting used to these new concepts. How individuals experience their first transitions in life can influence the way they cope with the next one. This is reason to explore the characteristics of transition processes, in reference to mathematics education.

2.1.1 Characterising transition processes

This section describes the characteristics of transition processes in reference to to mathematics and summarises the specifities for Germany.

Early transition processes

The change from the elementary educational level, kindergarten, with its emphasis on daily routines and play, to primary school is characterised by a formalisation of learning (Grüßing, 2009). Whereas children learn basic mathematical concepts (like adding, taking away, sharing, distributing) in real-life situations at elementary level (in compliance with the ideas of Aebli, 1976), these processes are formalised in primary school, where arithmetic operations are abstracted from situational events (Clements & Sarama, 2007). The passage tends to be smooth, with primary teachers usually having a good idea of what children already know, and with constant reference of abstract operations to the real-life situation behind (Stern, 1997). This transition experience should ideally provide learners with positive feelings of success and independence.

The transition from primary to secondary school can contain obstacles (McGee, Ward, Gibbons, & Harlow, 2003), at least in Germany, where most of the federal states split children into two or three ability groups at the age of 10 or 12. Primary-secondary transition in mathematics is otherwise associated with assimilating to a new (presumably more homogeneous) group of peers, with more teachers for the different subjects, and with potentially higher demands for the achievement of cognitive goals (Reiss, 2009b; Maaz, Baumert, Gresch, & McElvany, 2010; Koch, 2006). Thus the start at a secondary school can be connected with fear of failure (Maaz, Gresch, McElvany, Jonkmann, & Baumert, 2010; Breen & Goldthorpe, 1997). With respect to mathematics, there is a consensus that mathematics at secondary level is more abstract, as it contains a growing number of abstract concepts, such as variables, functions, or infinity (Reiss, 2009a).

Specifities for Germany

The re-orientation that the teaching of mathematics (among other subjects) has undergone in Germany after PISA (*Programme for International Student Assessment*) and TIMSS (formerly *Third International Mathematics and Science Study*, now *Trends in International Mathematics and Science Study*) in the year 2000 (Baumert, 2001; Köller, Baumert, & Bos, 2001) aimed at improving lessons in general. The shock that German students did not rank on top in mathematics led to intensified efforts to help students to understand what mathematics is about (in contrast to just teaching them calculation routines).

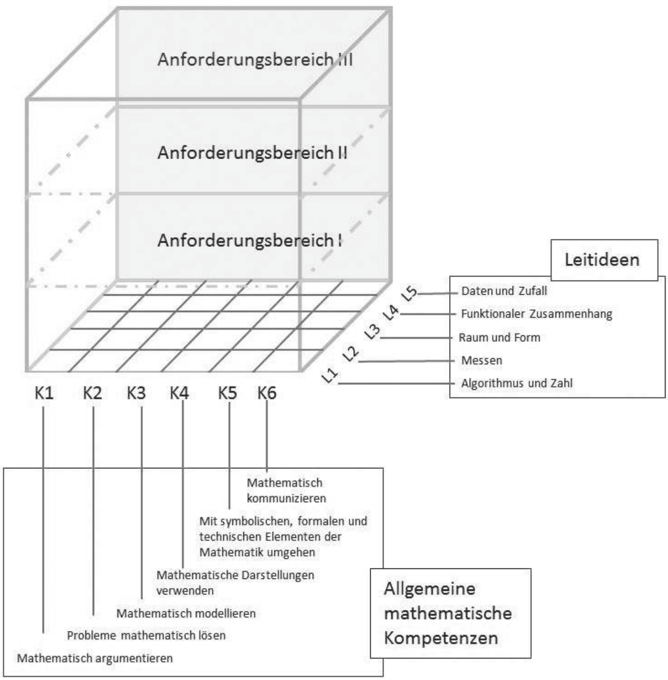

Figure 2.1: Competence Model of Education Standards for Mathematics at Higher Secondary Level, (KMK, 2012, p. 12)

This re-conceptualisation is based on a three-dimensional competence model comprising general mathematical competences (*allgemeine mathematische Kompetenzen*, content dimension), central ideas (*Leitideen*, action dimension), and challenge levels (*Anforderungsbereiche*, challenge dimension), see Figure 2.1.

This rethinking brought the change from stressing what was covered in class, input-orientation, to what students had truly mastered, output-orientation (Bildungsministerium für Bildung und Forschung, 2007), thus putting an emphasis on understanding, visualising, and connecting (Blum, Drüke-Noe, Hartung, & Köller, 2006). Ideally, students should experience mathematics at secondary level according to the three fundamental principles described by Winter (1996):

> (G1) To recognize and to understand phenomena in the world around us. (This principle recognizes the role of mathematics in acquiring important knowledge of our world.)
>
> (G2) To learn about and to understand mathematical issues represented in language, symbols, pictures, and formulas as intellectual creations, to recognize mathematics as a deductively ordered world of its own kind. (This principle recognizes mathematics as a rigorous science.)
>
> (G3) To acquire problem-solving (heuristic) skills for tasks that extend beyond the domain of mathematics. (This principle recognizes mathematics as a school of thought.)
>
> Winter (1996) – English translation and explanations according to H.-W. Henn (2003, p. 72).

In relation to the teaching of calculus, for instance, Danckwerts and Vogel (2006) discuss the tensions between those placing G1, real-world applications, in the centre of teaching calculus, and those preferring G2, concepts and interconnections. They summarise the criticism of established teaching approaches (Danckwerts & Vogel, 2006, p. 11) as the "tendency to neglect the fundamental principles G1 (applications) and G3 (heuristic strategies) in favour of the fundamental principle G2 (calculus as theory)"[1], and commiserate the reduction of G2 to calculation routines. Although their reflections refer to

[1] „Tendenz, die Grunderfahrung G1 (Anwendungen) und G3 (heuristisches Arbeiten) zugunsten der Grunderfahrung G2 (Analysis als Theorie) zu vernachlässigen", translation by author.

the teaching of calculus (which is the central issue in the advanced mathematics classroom), their descriptions fit other areas of teaching mathematics as well. In theory, Winter's triad of fundamental principles forms a perfect balance, as G3 (heuristic strategies) emphasises systematic testing, relating features to each other, and reasoning, and thus complements and deepens G1 and G2, which are regarded (respectively) as thought-out solutions for meaningful real-world problems and a developing network of interconnected concepts. In practise, however, there is severe doubt if these aspirations are being met for the majority of German students (Frey, Asseburg, Carstensen, Ehmke, & Blum, 2007; Frey, Asseburg, Ehmke, & Blum, 2008). Frey and colleagues present their findings of 15-year-olds' competencies (in the sense of a flexible ability to apply knowledge and skills to problems in a meaningful context). This (PISA) approach allows comparisons between countries and is detached from short-term rote learning, thus perfectly adequate in connection with the fundamental principles described above. The results place Germany in the middle range of OECD (Organisation for Economic Co-operation and Development, currently 34 member states, 21 from Europe) states. The findings show, among other results, that girls are significantly less competent in mathematics than boys. This point particularly implies need for further development in terms of teaching and instruction concepts, as does the fact that the dependency of educational success on social background is still remarkable, although it has slightly weakened in the three years since the previous study. Another important result includes that almost 20% of the teenagers have reached no more than the lowest competence level in mathematics, and that the variance of competence levels is comparably big. This last fact, in combination with the circumstance that the percentage of a generation that enrols for university courses has increased considerably in Germany in the last years from under 20% in the 1980s, to 34.4% in 2007, to 51.3% in 2013[2] supports the assumption that a notable percentage of students aspiring to university have not experienced mathematics as a field for intellectual involvement that requires active pursuit of understanding and cognitive processing (as Fischer, Heinze, and Wagner (2009) summarise in their paper), they rather see mathematics as an unconnected collection of cryptic formulas, unintelligible rules, presented via abstract formalism.

[2] See Bundesministerium für Bildung und Forschung (n.d., Tabelle 1.9.3).

Secondary-tertiary transition

In relation to other subjects, the gap between school and university mathematics seems extremely high and causes difficulties for students taking mathematics courses; Engelbrecht (2010) even describes first-year experiences at university as “traumatic” (p. 143). Although school mathematics itself is regarded as a tough, even polarising subject (H.-W. Henn & Kaiser, 2001), it is generally accepted that university mathematics is even tougher (Dreyfus, 1995; Zucker, 1996). The dramatic character is depicted in the word “abstraction shock”, used by some authors since there is a notable difference between mathematics taught at school and at university.

Clark and Lovric (2008) understand secondary-tertiary transition in mathematics as a rite of passage (a well-studied anthropological concept), an initiation into a new world. As such, they describe the three stages of transition from high school to university as follows:

> – separation (from high school); this stage takes place while students are still in high school, and includes anticipation of forthcoming university life;
>
> – liminal phase (from high school to university) includes the end of high school, the time between high school and university, and the start of the first year at university;
>
> – incorporation (into university) includes, roughly, first year at university (Clark & Lovric, 2008, p. 35)

In accordance with theories for rites of passage, Clark and Lovric realise the fact that secondary-tertiary transition is a “stressful, demanding, life-changing experience” (Clark & Lovric, 2008, p. 29), that it “involves both body and mind” (Clark & Lovric, 2009, p. 759), and that it needs a supportive environment, meaning “the totality of the contexts involved (social, psychological, cognitive etc.)” (Clark & Lovric, 2009, p. 759). This stresses the complexity of the transition process, particularly the notion that it is not only cognitive aspects that pose the obstacles – although studies often elaborate on cognitive difficulties and conceptual obstacles experienced by students in reference to specific content, such as aspects of functions (Attorps, Björk, Radic, & Viirmann, 2013; Viirmann, 2013; Winsløw, 2013), of linear algebra (Jaworski, Treffert-Thomas, & Bartsch, 2009; Hausberger, 2013), or proof (Hoffkamp, Schnieder, & Paravicini, 2013;

Selden & Selden, 2005; Moore, 1994). The way mathematics is communicated at university is also an area of intense research, cf. Artigue, Batanero, and Kent (2007).

There is a growing interest in investigating the various barriers that students encounter when starting a university course involving mathematics. The working groups on *University Mathematics Education* at the conferences of ERME (European Society for Research in Mathematics Education) discussing this subject reflect this trend: there were 21 papers in 2011, 23 in 2013, and 35 in 2015 submitted to and accepted by the group dedicated to the specifities of mathematical education at tertiary level, cf. CERME 7 (2011), CERME 8 (2013), and CERME 9 (2015).

The challenge of comprehending (and influencing) how the learning of mathematics at university works is therefore often addressed by the use of cognitive development theories, as we will do in detail in the first part of section 2.1.2. But other aspects play a role, too; that is why we will explore non-cognitive aspects later in section 2.1.2, such as motivation, self-regulation, attribution theories, and particularly learning strategies in section 2.2.

2.1.2 Learning mathematics at university

Learning mathematics at tertiary level is a complex matter. For our project seizing on elements of *Design Research*, we are looking for theories that can be applied to higher mathematics, that describe learning processes adequately in their complexity, and that comprise cognitive as well as affective and motivational aspects. The last point is imperative as, apart from researching the conditions that support or hinder academic success in mathematics for engineering students, the focus lies on the interventions themselves, on approaching preconditions that are changeable.

In this chapter, we will subsequently look at how mathematics is taught and learned at university, and at the differences and commonalities between school mathematics and university mathematics. This leads to the cognitive stages essential for the development of abstract mathematical thought and concepts, to theories on advanced thinking and eventually to the non-cognitive concepts involved. Each section closes with a short retrospect, and finally chapter 2.1.2 ends with a conclusion of which theories are relevant for the research perspective at hand.

Mathematics at secondary and tertiary level

In accordance with the special position mathematics occupies both at school and at university, most researchers introduce a category to describe the difficulties stemming from the high level of abstraction and complexity typical of this subject. Various terms are used: de Guzmán, Hodgson, Robert, and Villani (1998, p. 747)[3] name this category “epistemological and cognitive”, Rach and Heinze (2013, p. 123) refer to it as “the aspect of subject content in mathematics and its character”[4], and Gueudet (2008, p. 237) calls it “individual difficulties”. The choice of terms here strictly distinguishes between individual lack of effort or perception, and “institutional practices” (Gueudet, 2008, p. 247) that entail “limited (...) and poorly connected” (Gueudet, 2008, p. 246) mathematical organisations unsuitable for the extensions needed at university level. Various work is dedicated to describing different facets of this category, e.g. the formality of mathematics, the stringent way of thinking, and the systematic structure. In this respect, the skills and knowledge shown by school leavers are often found wanting, like in Bruder et al. (2010), Hoyles, Newman, and Noss (2001), or Mündemann, Fröhlich, Ioffe, and Krebs (2016).

Rach (2014) gives a comprehensive overview that contrasts mathematical subject matter at secondary and tertiary level in five categories, see Table 2.1. She proceeds from the understanding that school mathematics and scientific mathematics have different goals: Whereas at school, the aim is to generally educate students and to enable them to solve text problems with the help of mathematics (in a world where concepts possess a concrete, or at minimum a symbolic representation), at university the aim is to introduce mathematical theories in a formal-axiomatic world[5]. Concerning proofs, apart from the fact that they are the central activity in university mathematics, Rach differentiates between dominant features and types on the one hand, and formalisation on the other. In each of these two subcategories, university mathematics is stricter,

[3] In this study, engineering students tended to see their passage from school to university as less difficult than students with mathematics major or aspiring teachers, although it was still “24 out of 118 (20%)” (de Guzmán et al., 1998, p. 749) who agreed or totally agreed to finding the transition to university mathematics difficult.

[4] “der Aspekt (1) des Lerninhalts Mathematik und dessen Charakter”, Rach and Heinze (2013, p. 123), translation by author.

[5] These terms refer to the theory of the three worlds of mathematics, cf. Gray and Tall (2001) and Tall (2004).

Table 2.1: Comparison of Characteristics of Mathematical Subject-Matter at Secondary and Tertiary Level, Rach (2014, p. 79), shortened and translated by the author

Category	School Mathematics	(Scientific) Mathematics, first-year course at university
Goal	general education, esp. solving text problems with mathematical knowledge	learning about scientific mathematics, esp. building mathematical theories
Mathematical thought processes	conceptual-embodied world / proceptual-symbolic world	formal-axiomatic world
Proofs: activity	one of many mathematical activities	central mathematical activity
Proofs: dominant feature and types	mostly explanation, partly verification; experimental or pre-formal proofs	more verification, additionally communication; formal-deductive proofs
Proofs: formalisation	low level of mathematical notation	high level of mathematical notation
Concept formation: objects and axiomatic system	objects refer to concrete real objects, embedded into contextual axiomatics	objects describe mental constructs, embedded into formal axiomatics
Concept formation: kind of description, level of abstraction	description via specific representatives	abstraction: description via characteristic features
Concept formation: level of formalisation	low level of formalisation	high level of mathematical formalisation

more formal, and more abstract (in the sense of further removed from examples and experiments).

While Rach contraposes school mathematics and university mathematics, Dreyfus (1991) sees the differences as more of a continuum, where generalizing ("To generalize is to derive or induce from particulars, to identify commonalities, to expand domains of validity", p. 35), synthesizing ("To synthesize means to combine or compose parts in such a way that they form a whole, an entity.", p. 35), and abstracting (closely connected to generalizing) are gaining more and more dominance. The notion of the learner experiencing different worlds of mathematics, as described by Gray and Tall (Gray & Tall, 2001; Tall, 2004), is incorporated here. They developed the theory that there are basically three different kinds of mathematical *objects*, each representing a mathematical world that has to explored and experienced in order to reach understanding:

> – those that arise through *empirical abstraction* (in the sense of Piaget[6]) by which is meant the study of *objects* to discover their properties
>
> – those that arise from what Piaget termed *pseudo-empirical abstraction* from focusing on *actions* (such as counting) that are symbolised and mentally compressed as *concepts* (such as number)
>
> – those that arise from the study of *properties*, and the logical deductions that follow from these, found in the modern formalist approach to mathematics.
>
> (Tall, 2004, p. 29)

Figure 2.2 illustrates the concept of the three worlds: There is an overlap between the first two worlds, the *embodied* and the *symbolic world*. Both belong to the two lower levels of practical and theoretical mathematics, whereas the second needs the first to build concepts through perception and action. The third *(axiomatic formal)* world is based on the two first and is the only one to be attributed to the highest level of formal mathematics – and thus to university mathematics, see Table 2.1, third row. The step from the embodied world to the symbolic world is regarded as smaller than the step to the axiomatic formal world, as the observation of objects is followed naturally by the handling of objects – but abstractly thinking about objects is further away.

[6] For more information on Piaget's theories on the development of cognitive abilities, see next section.

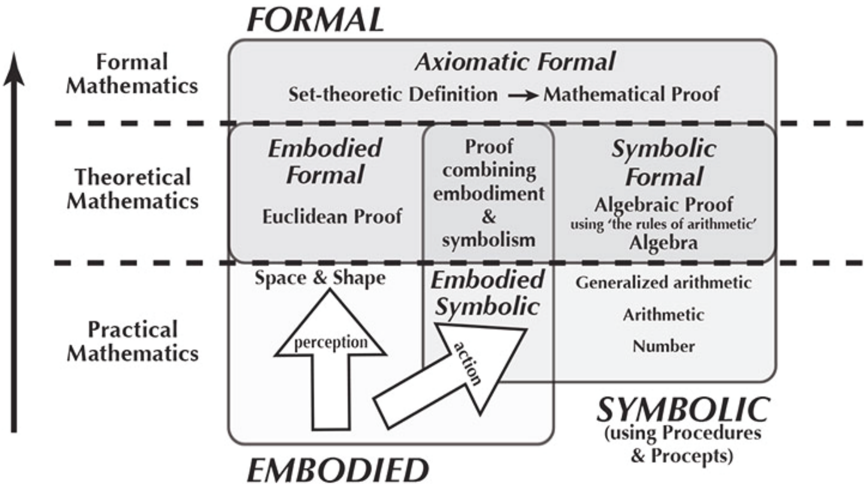

Figure 2.2: The Three Worlds of Mathematics, https://homepages.warwick.ac.uk/staff/David.Tall/themes/three-worlds.html

The question remains, however, of how the transition from one world to the next is to be accomplished. In university mathematics contexts, it is not unusual to present definitions and theorems in an axiomatic way, disregarding the demand "that students be given a feeling for the long history of magnificent failures" (von Glasersfeld, 1991, p. 179).

In sum, it has become clear that there are notable differences between school and university mathematics, in the areas of abstraction, formality, and concepts. These pose obstacles not easy to overcome.

Cognitive development

The development of understanding and cognitive abilities is of high importance for the learning of higher mathematics. In this section, we will look at the established theories of human behaviour and changes therein, i.e. learning. These theories aim at explaining cognitive processes via development models. When von Glasersfeld states, referring to his lifelong research focus, that

> Constructivism [...] is *one* possible way of thinking. It is a *model* – and models, no matter how useful they might prove, must never be claimed to be 'true'.
> (von Glasersfeld, 1991, p. 169)

this means that the theories on cognitive development do not come in categories of truth or deception, but appear in a continuum between viability and inutility for a given context. This is the perspective we will take when sorting the theoretical models for our purposes.

Early explanations of human behaviour include behaviorism (whose main representatives are Watson and Skinner[7]), a theory which focuses on simple models of stimulus and response and aims at changing behaviour patterns by using classical or operant conditioning. According to this way of thinking, an individual reacts in a predetermined way after experiencing a stimulus, which can be anything from a smell to a verbal utterance. The response can be influenced by combining different stimuli (classical conditioning) or by reinforcing wanted behaviour, sometimes in combination with punishing unwanted actions (operant conditioning). When applied to complex learning scenarios, behaviorism shows its limitation, true to Eisenberg's statement that "models of learning are either too specific or too general" (1991, p. 142): The teacher is seen as the centre of the learning situation, as he or she decides which behaviour is wrong and which is right, meaning ignoring or punishing the one and praising the other. This theory does not consider internal mental processes of building understanding, nor does it see the learner as taking an active part in the learning process.

In relation to learning mathematics at tertiary level, this approach is inappropriate in more than one way:

– There is no teacher / lecturer to take complete control of every individual student's learning progress when many hundred of them are affected.

– Mathematical learning at this level implies self-reliant work.

– The nature of mathematics at university (in the sense of the axiomatic formal world) requires understanding, not repeating.

[7] Watson (1913), Skinner (2002)

Therefore, behaviorism is inappropriate to describe a modification of human behaviour that stresses the mental processes and incorporates the idea of the learner taking an active part, of learning at a higher level.

Table 2.2: Piaget's Four Phases of Cognitive Development

Development Stage		Age
1	Sensorimotor Stage	0 to 2 years
2	Pre-operational Stage	2 to 7 years
3	Concrete Operational Stage	7 to 11 years
4	Formal Operations Stage	11 to 16 years and onwards

Epistemology, the theory of knowledge, is more suitable, it investigates how human beings develop knowledge and skills, apposite to mathematics at tertiary level. Piaget, one of the earliest representatives of non-behaviorism, saw four phases of cognitive development (Piaget, 1973), as shown in Table 2.2. At the core of Piaget's theory of cognitive development lies the notion that children build an understanding of the world as they experience it through perception and reflection. Whenever a child sees a discrepancy between its reconstruction of the world and a new perception, he or she will adapt their understanding to accommodate it. This has lead to the belief that children need a stimulative and inspiring environment in order to develop cognitively.

In the first stage, young children (before the age of having acquired language) experience the world through sensoric perceptions and discover how to use their motoric abilities. At the age of around two years, children usually discover language and thus reach the pre-operational stage where they play in the sense of pretending things to take a symbolic meaning (e.g. a box becomes a plane). At this stage, children are aware of objects without having them in front of them, but they find it difficult to occupy another's point of view. The later part of the pre-operational phase is characterised by aspiring after knowledge, particularly asking questions as to why things are the way they are. Some of Piaget's famous experiments for determining which stage a child has reached have a mathematical background as they refer to deciding between *more* and *less*, e.g. judging if an amount of liquid stays the same when poured into a narrower but higher container, if a row contains more blocks when they are spread more widely, or deducing the relation between two objects A and

C when $A > B$ and $B > C$ is given (transitivity). So, from a mathematical point of view, the pre-operational stage contains the beginning of mathematical reasoning, if only from acquiring misconceptions whose overcoming will later mark the transition to a new phase. The third stage, the concrete operational stage, is even more important in this respect: From the age of about seven years on, children develop inductive logical thinking (although the concept of transitivity may still pose problems), e.g. they understand conservation (that an amount of liquid stays the same no matter the shape of the container). They are more and more able to judge hypothetical situations without being exposed to them, and they start to use trial and error in a systematic way to solve concrete problems. In the formal operations stage, abstract concepts without relation to reality can be grasped, and symbols are used in a logical way. Trial and error is employed more systematically, which marks the beginning of problem-solving in a mathematical sense. Metacognition, the ability to reflect about thinking, is possible.

Although he saw close relations between biological and cognitive development, Piaget himself believed his theories could be applied to scientific knowledge as well – which Gray and Tall have done successfully (Gray & Tall, 1994; Tall, 2004) in their theory of the three worlds, see above. For the tertiary level, only the third and fourth stage are relevant, because, in summary, children begin to think logically in the concrete operational stage, but are in need of practical assistance and support. In the formal operational stage, they can think logically without practical help and can develop abstract thought[8]. University students have certainly passed the age of the concrete operational stage, but might profit from practical help when trying to understand complex formal concepts. Piaget stresses that the transition from one stage to the next forms a cycle of repeated actions, observations, and reflections, new actions with slightly different starting positions, new observations and consequently new reflections. He sees the learners as actively influencing the arrangements and observing the outcomes. This idea of learning by acting, observing, and reflecting corresponds with constructivism (see above) - as David Tall summarised as early as 1991: "The active participation in thinking is essential for the personal construction of meaningful concepts. Students

[8] It is an interesting issue to investigate if students of mathematics at university have actually reached the *Formal Operations Stage* – Leongson and Limjap (2003) found they had not.

need to be challenged to face the cognitive reconstruction explicitly, through conjecture and debate, through problem-solving [...]" (Tall, 1991b, p. 258f.). The same postulation of actively involving leaners in the learning process is highlighted by von Glasersfeld (1991, p. 175) who states that it "is an illusion that there is knowledge in textbooks or documents. [...] Texts *contain* neither meaning nor knowledge – they are a scaffolding on which readers can build their own interpretation".

Learning in this constructivist sense involves much social interaction, particularly between the teacher and the student. This needs knowledge and understanding for the point of view with which the student regards his or her environment, in summary "an almost infinitely flexible mind" (von Glasersfeld, 1991, p. 178). In detail, the implications of constructivism on a theory of learning are described by von Glasersfeld (1991, p. 177f.) as, among others:

> (3) If teachers want to modify a student's concepts and conceptual structures, they have to try and build up a model of the particular student's own thinking. Models of students' thinking can of course be generalised, but before assuming that a student fits the general pattern one should have some solid evidence that this is a viable assumption in the particular case. It should never be assumed that students' ways of thinking are simple or transparent. [...]
>
> (6) Successful thinking is far more important than "correct" answers. Successful thinking should be rewarded even if it was based on unacceptable premises.
>
> (7) To understand and appreciate students' thinking, the teacher must have an almost infinitely flexible mind (because students sometimes start from premises that seem inconceivable to teachers).

All in all, cognitive development, particularly in reference to higher mathematics, is a complex process requiring skilled teachers or lecturers, purposeful interaction, and time.

Advanced Mathematical Thinking

The necessity to inquire specifically into the processes and obstacles of advanced mathematical issues was particularly acknowledged in 1985, when a PME (International Group for the Psychology of Mathematics) Working Group on *Advanced Mathematical Thinking* was established. This resulted, among various other work, in a seminal volume by Tall (1991a) some years later, followed by a special journal issue by Dreyfus (1995) as an update. Much of the research in this area is dedicated to mathematics at tertiary level, but there is general agreement that advanced thinking should and must start at lower levels, as Selden and Selden (2005) assessed appropriately. Thus, in this context, *advanced* is not necessarily applied to *mathematics*, but to *thinking*.

A starting point is the fact that at university, mathematics is presented to the learners via the traditional deductive sequence of *definition – theorem – proof (– application)*, which does not reflect the nature of building mathematical knowledge, which happens "through trial and error, through partially correct (and partially wrong) statements, through intuitive formulations in which loose terms and imprecisions have intentionally been introduced, through drawings that try to visually present parts of the mathematical structures being thought about, through dynamic changes being made to these drawings, etc." (Dreyfus, 1991, p. 27). But as "the constructivist teacher should never present a solution as the only one" (von Glasersfeld, 1991, p. 179), other courses of action are imperative. To re-enact the process of finding connections and describing characteristics, Tall lists

> – the participation of the student in the process of mathematical thinking through an active process of "scientific debate", rather than passive receipt of preorganized theory,
>
> – the direct confrontation of the student with conflict which occurs in developing new theoretical constructs, to help them reflect on the problem and build a new, more coherent, cognitive structure.
>
> – the building up of appropriate intuitive foundations for the advanced mathematical concepts, through an approach which balances cognitive growth and an appreciation of logical development.
>
> – the use of visualization, particularly utilizing a computer, to give the student an overall view of concepts and enabling more versatile methods of handling the information,
>
> – the use of programming to cause the student to think through mathematical processes in a way which can be encapsulated by reflective abstraction.
>
> Tall (1991a, p. xiv-xv)

Much of this list is still up to date today, 25 years later, and can well suit as a guideline on which to model teaching concepts in mathematics, both at school and at university level. The suggested techniques are focused on activation of the learner, but the overall implication remains that the teaching concepts must originate from the teacher or lecturer, e.g. in not (or at least not exclusively) presenting content in a preorganised way that expects the learner to listen passively, or by setting exercises to force the learner to structure a mathematical process. This is in keeping with Piaget's theories on how knowledge is built in general. A selection of how mathematical knowledge is built specifically is presented in the following sections.

Procept Theory

The term *procept* was introduced by Gray and Tall (1991, 1994) in order to describe the twofold characteristic of mathematical symbols like $3 + 4$, which at the same time denotes the process of adding two numbers and the concept of the sum it results in. The authors define:

> An *elementary procept* is the amalgam of three components: a *process* which produces a mathematical *object*, and a *symbol* which is used to represent either process or object. [...]
> A *procept* consists of a collection of *elementary procepts* which have the same object.
> (Gray & Tall, 1994, p. 120)

Although many examples of procepts are taken from the area of elementary mathematics, the theory can be applied to more advanced concepts. Flexibility in the use of processes and representations for a given concept is the key to true understanding of mathematics at all levels. For example, the symbol $f(x) = x^2 + 4$ stands for the process of mapping x to $x^2 + 4$ and for the concept of the resulting quadratic function, which in turn has specific features and visual representations. A learner shows his or her expertise in flexibly switching between the process, the concept, its visualisations and the connected features of an object. Gray and Tall have incorporated their thoughts on this matter into the wider theory of the three worlds of mathematics, see above and Tall (2004). And the classification fits indeed: Studying an object in order to discover its properties (the first of the three worlds) corresponds to the object side of a procept, whereas handling and manipulating an object in order to further unveil its properties (the second world) focuses on the process aspect. When these explorations are conducted mentally and formally (i.e. a certain flexibility is reached in the use of the different aspects of a procept), the level of the third world is reached.

Procept theory allows the description and classification of mathematical comprehension problems, as, depending on the misconception shown, the lack of flexibility or variety can be diagnosed – and subsequently ameliorated. Like all theories on mathematical learning, it draws on stimulating exercises and the discussion of their solutions, thus stressing the importance of the teaching concept behind a university course and the design of its central tasks.

APOS Theory

The inventors of APOS theory, Dubinsky and McDonald (2001), view theory in general from a practical perspective, with regard to the help it can provide in mathematics education, substantiated in the six features as shown in Table 2.3. All these features refer to putting a theory in use in order to understand and improve teaching and learning processes. It has to be admitted, though, that this point of view is no exception: Most theorists develop an theory with the aim of predicting future outcomes, of explaining phenomena, or of helping to organise thinking and communication. But in the development of APOS, these guidelines were expressedly the starting point.

Table 2.3: Six Features for Models and Theories in Mathematics Education, according to Dubinsky and McDonald (2001, p. 275)

1	Support prediction
2	Be applicable to a broad range of phenomena
3	Serve as a tool for analyzing data
4	Have explanatory power
5	Help organize one's thinking about complex, interrelated phenomena
6	Provide a language for communication of ideas about learning that go beyond superficial descriptions

APOS theory was introduced by Dubinsky and McDonald (2001) in order to bring together the understanding of learning processes and the observations

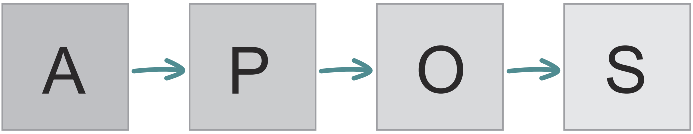

Figure 2.3: Stages of APOS Theory in Hierarchical Order

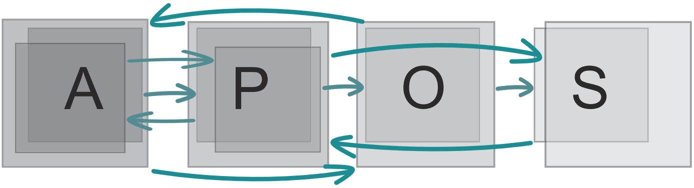

Figure 2.4: APOS Theory Accommodating More Sophisticated Sequences

made when teaching undergraduate students in mathematics. The concept of learning behind APOS theory is again constructivism (see above), meaning that individual learners have to (re-)build the constructs in order to comprehend them. This idea of learning comprises challenging problems, time to think and communications with others (an instructor or fellow learners). APOS has gained many supporters (Arnon et al., 2014), one reason for this is the stringent adherence to the six features from Table 2.3.

APOS theory focuses on the mental processes of an individual when learning mathematics. The hypothesis behind APOS theory is that individuals construct "mental *actions*, *processes*, and *objects*" and organize them in "*schemas* to make sense of the situations and solve the problem" (Dubinsky & McDonald, 2001, p. 276), implying the acronym APOS. Ideally, these four stages of building understanding are passed in linear order by the learners (Figure 2.3), but in reality, the learning process is often more sophisticated (Figure 2.4).

The elements of APOS theory are defined as follows: When an individual is in the act of comprehending a mathematical concept, first he or she starts with performing *actions* on mathematical objects, e.g. the algebraic transformations on a differential equation. At this first stage, these *actions* are perceived as "essentially external and as requiring [...] step-by-step instructions on how to perform the operation" (Dubinsky & McDonald, 2001, p. 276). With these accumulated *actions* in combination with reflections on them, the individual forms a *process* which in this context means "an internal mental construction [...] which the individual can think of as performing the same kind of action, but no longer with the need of external stimuli" (Dubinsky & McDonald, 2001, p. 276). In the example of differential equations, this would mean that the learner now combines and reverses *actions*, in the example above this might involve including other knowledge on integration or constructing new problems that can be solved by using the same *actions*, e.g. separation of variables or variation of parameters. An *object* describes the stage when the learner "becomes aware of the process as a totality" (Dubinsky & McDonald, 2001, p. 277) and does not have to think about single steps, but can view the transformations as a whole and conjecture of how variations in the initial equation affect the outcome. The *schema* finally denominates the individual learner's compilation of *actions*, *processes* and *objects* as well as the network that connects them, in the form of general principles and in the form of concrete relations. In the example above this would comprise a universal ability to solve (certain kinds of) differential equations. APOS' origin in Piaget's phases of cognitive development is obvious; the advantage of APOS is the specifity for mathematics, which Piaget's theory does not feature. As such, APOS is a generally accepted theory to classify cognitive mathematical processes, although there is criticism (Tall, 1999) that it may not apply very well to very advanced or very formal concepts.

So, apart from the conceptual details, APOS theory sees learning as an activity of the learner, and the teacher / lecturer / tutor as supporting this activity by establishing a general set-up that facilitates movement from one stage to another.

Concept Image and Concept Definition

Teaching mathematics in the traditional, deductive way involves starting from definitions of the concepts that are to be covered. Most of the time, these definitions are formal, and desirably, according to Vinner (1991, p. 65f.), they should be "minimal" (not containing parts that can be inferred from others), "elegant" (which is a question of taste, undoubtedly, but most mathematicians will consider short, condensed definitions as more elegant than longer, elaborated ones), and (more or less obviously) "arbitrary". Even when a definition does not stand at the beginning of the process of introducing a concept, but at the end of a phase of exploring objects and their characteristics, the formal definition is expected to be the reference to turn to when arguing for or against certain properties. Tall and Vinner (1981) found that this was often not the case, though, because the mental image formed in the mind of the learner takes precedence over the formal definition. Thus, intuitive thinking via the concept image prompts the answers, without reference to the concept definition. Particularly with concepts such as functions, continuity, and limits, Tall and Vinner (1981) as well as Eisenberg (1991) showed that even good students have problems forming concept images that will hold in non-standard situations.

RBC Theory / Abstraction In Context

On the quest of not only a theoretical framework, but of what can be observed to determine the level of understanding a student has reached, and what can be done to foster (deep) understanding, Dreyfus (2012) developed a theory that is based on “three observable epistemic actions: Recognizing, Building-with and Constructing” (p. 1) and describes how abstract mathematical concepts can be learned in classroom situations. It postulates the idea of the learner as actively and responsibly engaging in the learning process in an inquiry-encouraging environment. Based on the works by Freudenthal (1991), Davydov (1990) and thus Vygotsky (1978), the three components are described as follows, complemented by a fourth, consolidation:

> Recognizing refers to the learner realizing that a specific previous knowledge construct is relevant in the situation at hand. Building-with comprises the combination of recognized constructs, in order to achieve a localized goal such as the actualization of a strategy, a justification or the solution of a problem. The model suggests constructing as the central epistemic action of mathematical abstraction. Constructing consists of assembling and integrating previous constructs by vertical mathematization to produce a new construct. It refers to the first time the new construct is expressed or used by the learner. This definition of constructing does not imply that the learner has acquired the new construct once and forever; the learner may not even be fully aware of the new construct, and the learner’s construct is often fragile and context dependent. Constructing does not refer to the construct becoming freely and flexibly available to the learner. Becoming freely and flexibly available pertains to consolidation.

> Consolidation is a never-ending process through which students become aware of their constructs, the use of the constructs becomes more immediate and self-evident, the students' confidence in using the construct increases, and the students demonstrate more and more flexibility in using the construct.
>
> (Dreyfus, 2012, p. 3f.)

The ensuing theory is consequently abbreviated RBC+C, and often focuses on the highly relevant C-processes, which can be further differentiated into "different modes of thinking: numerical (C1), algebraic (C2), analytic (C3), and visual (C4) (Dreyfus, 2012, p. 9). Together with his co-authors Hershkowitz and Schwarz, Dreyfus described in detail this theoretical-methodological framework whose long name is "dynamically nested epistemic actions model of abstraction in context" (see Hershkowitz, Schwarz, & Dreyfus, 2001; Schwarz, Dreyfus, & Hershkowitz, 2009). Due to the broad observational basis, the framework covers numerous aspects of the process of forming abstract mathematical concepts. First, it is characteristic for abstract mathematical learning processes that the learner feels the need for a new construct, this implies an active learner and a (teacher- or lecturer-created) challenging learning situation. Second, there is often a moment of enlightenment (or aha) in which the learner accepts a new concept into his or her existing network of mathematical concepts. Third, newly-accepted concepts are not necessarily completely correct. RBC+C allows for studying partially correct or wrong concepts and for identifying the nature of the misconception. Fourth, social interaction plays a major role in the forming of new abstract concepts as "Abstraction often takes place in interacting groups of students" (Dreyfus, 2012, p. 12). Fifth, the use of technology to help test new concepts for their validity, is another facet. All these aspects are covered in the RBC+C framework (also called AiC, short for abstraction in context). As AiC was developed observing students interacting, i.e. used as a methodological approach, the researchers were well aware of the dual nature of the framework as a theory and a methodology, and many have found this fact an asset to their own research (see Bikner-Ahsbahs et al., 2010; Wood, Williams, & McNeal, 2006).

The universal applicability as well as the emphasis on learner activities and how they should be guided makes this framework a promising candidate for the purpose we have in mind for the project of supporting engineering students in mathematics. It also suits the overall concept of *Design Research* in its two-footed balance of theory and method, see section 3.1.

In recapitulation, all models presented for explaining progress in mathematical thinking, have the common feature that they require learners and teachers or lecturers to interact, that learners have to adapt new concepts to already acquired ones, and that this process is often not strictly linear, but rather characterised by misconceptions, inconsistencies, and intellectual contention.

Non-cognitive aspects

Apart from the agreement on cognitive difficulties, the research approaches differ slightly when describing obstacles in the transition from secondary to tertiary education in mathematics in regard to aspects not related to cognition: de Guzmán et al. (1998) call the remaining categories “sociological and cultural” and “didactical” (p. 749), Gueudet (2008) chooses the labels “social” and “institutional” (p. 237), and Rach and Heinze (2013) classify them as referring to institutional frame conditions of university education and individual requirements respectively determinants of learning mathematics at university[9]. Inverted, they mean roughly the same, i.e. what Rach and Heinze see as characteristics of the individual are seen as features of society by de Guzmán et al. and Gueudet. All researchers identify the learning conditions at tertiary level as a problem area, although they place themselves at different positions of the continuum between the extremes *researcher* and *university teacher*, between theory and practise.

An interesting perspective is presented by Gueudet (2008), who focuses on the interrelations of difficulties observed, possible views of transition and the resulting didactical actions. Thus, when describing difficulties, she combines the observation of subject-specific difficulties (like abstract thinking when conducting proofs) with social problems (like regarding training for university lecturers as unnecessary) and institutional obstacles (like deficient textbooks). This comprehensive view yields a more precise picture of the mechanisms involved. The cause of the difficulties experienced varies, depending if the focus is on the students’ or the university teachers’ point of view, as (de Guzmán et

[9] „Aspekte [. . .] der institutionellen Rahmenbedingungen der Hochschulbildung sowie (3) der individuellen Voraussetzungen bzw. Determinanten des Mathematiklernens an der Hochschule“, p. 123.

Table 2.4: Causes of Difficulties in the Learning of Mathematics as Seen by Students and University Teachers, cf. de Guzmán et al. (1998)

Students	University Teachers
way teachers present mathematics at the university level	lack of interest in mathematics
changes in the mathematical ways of thinking	lack of prerequisite knowledge
lack of appropriate tools to learn mathematics	deficient learning style, e.g. lack of organisation or autonomy

al., 1998) have found, see Table 2.4. University teachers tend to attribute the difficulties exclusively to the students, while students mostly divide the causes for their difficulties between their instructors and the complexity of mathematics itself (the lack of tools playing only a minor role in the survey).

Gueudet stresses the fact that the way mathematics is taught is a relevant factor in itself, coming to the conclusion that the "difficulties of the students are not individual difficulties, but consequences of [...] institutional practices" (Gueudet, 2008, p. 247)[10]. In this, she would probably agree with Tall's statement that it is "no longer viable, if indeed it ever was, to lay the burden of failure of our students on their supposed stupidity, when now the reasons behind their difficulties may be seen to be in part to be due to the epistemological nature of mathematics and in part to misconceptions by mathematicians of how students learn" (Tall, 1991b, p. 251f.). These positions stress the responsibility of educational institutions, of teachers, lecturers and researchers in education, to create learning environments and to foster learning techniques and conditions that support learners of mathematics and show understanding for their difficulties. This categorisation suits the purpose of a project drawing on *Design Research*, as it implies interventions[11].

[10] Here Gueudet refers to the practise of promoting only one technique to solve narrow types of tasks.

[11] In fact, (de Guzmán et al., 1998) suggest various measures to counter the difficulties observed, among them a contact desk where students can get answers for their mathematical questions.

Social aspects

As stated above, interaction is the key to building mathematical constructs; and interaction can only take place in a social context. Although the interaction between the teacher / instructor and the learner is the focus of various educational research, from the sheer quantity, there is much more interaction between individual learners – particularly at the university level, where the ratio between lecturer and students is much smaller than between teacher and pupils in a school environment. Social aspects must be considered in detail, both in their potential for creating understanding while discussing and explaining concepts, and in their ability to help learners' motivation and perseverance in a peer environment.

Yackel and Cobb (1996) have researched the sociomathematical mechanisms behind the process of becoming autonomous in mathematics in detail. They argue that sociomathematical norms play an important role and that "the teacher's role as a representative of the mathematical community" (p. 458) is central: In a classroom situation, explanations and argumentations based on the generally accepted norms are the key to building mathematical knowledge and understanding. The teacher as the authority that decides whether an argument holds or not is imperative for keeping the learning processes on track. This perspective stresses the importance of interaction between teacher and students.

At the university level, many students regularly meet in groups to work on tasks, often outside of organised learning arrangements. These group sessions can be very fruitful, as von Glasersfeld (1991, p. 176) explains, elaborating on social instruction.

> I take it this refers to "group learning", and there is a lot to be said about this.
>
> (1) Students who work at a problem together with other students have to verbalize how they see the problem and what they want to do about it. This is one way of generating reflection, which requires awareness of what one is thinking and doing. This, in turn, provides occasions for active abstraction (repeating, writing down, and learning by heart what a teacher says, does none of this).
>
> (2) Explaining something to a peer usually leads to seeing things more clearly and often to spotting inconsistencies in one's own thoughts.

And when a small group explains its “solution” (irrespective of whether it happens to be viable or not) to the whole class, this provides a wonderful opportunity for learning [...].

(3) Knowing that those you work with have no ready-made answer increases everyone’s courage to try and find one.

(4) If one of the group finds an answer, this more often than not generates motivation to try a new problem.

(5) To have an inconsistency or “error” explained by a peer is far less painful than have the teacher tell you that you are wrong. Etc., etc.

To use the social aspects of learning therefore seems a sensible thing to do. As normally social interactions between students take place in an unorganised way, there is the possibility to make them more purposeful.

And there is another, wider, perspective to the social aspects of learning mathematics. It is not restricted to the relationship and interactions observable in classroom interactions, but incorporates aspects regarding society and the positions of power therein. Skovsmose, Valero, and Christensen (2009) and Halai (2014) regard mathematical knowledge as a way to gain empowerment in a political sense – as power and influence are often connected to technological and therefore mathematical skills. Alrø, Ravn, Valero, and Skovsmose (2010, p. 15) summarise that the “use of knowledge is to be seen as political, i.e., mathematical knowledge can be used in order to influence society”. Although this aspect is not central to our project and research focus, it is not unrelated: As engineering is a career often aspired to by descendants from non-academic families (Bathke, Schreiber, & Sommer, 2000; Becker, Haunberger, & Schubert, 2010)[12], supporting first-year engineering students in mathematics truly contributes to their participation in society.

Affective aspects

Affect has gained a growing role in the research on mathematics education in recent years. This is mirrored in the many publications and activities on this topic, for an early overview see McLeod (1992), for more recent developments

[12] The total image is diverse, though, as there are different ways to gain entrance to university, various types of higher education institutions, and several kinds of engineering courses, see Schindler (2012).

Hannula, Evans, Philippou, and Zan (2004), Leder, Pehkonen, and Törner (2002), Leder and Grootenboer (2005), Roesken and Casper (2011), and Zan, Brown, Evans, and Hannula (2006). In particular, the theory of dual processes in cognitive psychology has been adapted to mathematics education, and the role of affective variables has been pointed out in this context (e.g. Evans, 2007). These perspectives provide novel views on learning processes and have done much to reach a deeper understanding of the obstacles involved. It is now generally accepted that “affect plays a significant role in mathematics learning and instruction” (McLeod, 1992, p. 575), in a more pronounced way than in other subjects, i.e. learning is considerably influenced by the affective aspects accompanying the learning process. Goldin (2002, p. 60) even ventures to say that “When individuals are doing mathematics, the affective system is not merely auxiliary to cognition - it is central”. Liston and O’Donoghue (2008, p. 10) found that “Enjoyment of maths and mathematical self-concept, were the strongest affective predictors of exam results together with the level of maths previously studied”. On the other hand, negative affective factors (Nardi & Steward, 2002) are feared to be omnipotent at least for less able students (Larcombe, 1985). When these are combined with deficient mathematical skills, which are steadily on the increase, as some researchers[13] report, the impact on university education can be severe.

In this context, the term affect (McLeod, 1992; Lester, Garofalo, & Kroll, 1989) indicates a broad range of phenomena, and can be differentiated into beliefs, attitudes and emotions (here ordered by decreasing stability over time, and by decrease in cognition), see Table 2.5[14], in which McLeod summarises constructs by Mandler (1984) and Snow and Farr (1987). All three major categories impact on learning outcomes and are therefore worth looking at. The next paragraph briefly summarises descriptions of the three main categories of affect.

The first category, beliefs, is further differentiated into beliefs about mathematics, beliefs about self, beliefs about mathematics teaching, and beliefs about the social context (McLeod, 1992; Mandler, 1984; Snow & Farr, 1987; Roesken, Hannula, & Pehkonen, 2011). These facets influence the way learners approach a task, the tools they employ to solve it, the perseverance they

[13] For example Berger and Schwenk (2006), G. Henn and Polaczek (2007), or Knospe (2013).
[14] Later, DeBellis and Goldin (1997) added a fourth category, values.

Table 2.5: The Affective Domain in Mathematics Education, (McLeod, 1992, p. 578)

Category	Examples
Beliefs	
About Mathematics	Mathematics is based on rules
About self	I am able to solve problems
About mathematics teaching	Teaching is telling
About the social context	Learning is competitive
Attitudes	Dislike of geometric proof Enjoyment of problem-solving Preference for discovery learning
Emotions	Joy (or frustration) in solving non-routine problems Aesthetic responses to mathematics

show while doing so, the expectations they confront their teaches or lecturers with, and the meaning they perceive behind it. Beliefs are largely cognitive and tend to be stable over long periods of time – but are changeable in principle. Attitudes are less stable over time and less cognitive than beliefs (Hannula, Evans, et al., 2004). They may change more frequently and more easily. Emotions are the most unstable and less cognitive construct of the three. They may differ from situation to situation, and often for (at first sight) incomprehensible reasons. What these affective aspects have in common is that they influence achievement (Ma & Kishor, 1997; Juter, 2005; Kloosterman, 1988) via metacognition or social aspects (among others).

Mathematical beliefs offer a depth of views on the subject central to our studies. Students in their first year at university are on the one hand experienced pupils, they have passed trough more than a dozen years of mathematics instruction at school. On the other hand, they have hardly any experience in certain areas of mathematics – and thereby, certain beliefs about mathematics might be limited. Goldin (2002) gives an overview to illustrate the width of types of mathematical beliefs.

- Beliefs about the physical world, and about the correspondence of mathematics to the physical world (e.g., number, measurement);

- Specific beliefs, including misconceptions, about mathematical facts, rules, equations, theorems, etc. (e.g., the law of exponents, the quadratic formula, the idea that "multiplication always makes larger");
- Beliefs about mathematical validity, or how mathematical truths are established;
- Beliefs about effective mathematical reasoning methods and strategies or heuristics;
- Beliefs about the nature of mathematics, including the foundations, metaphysics, or philosophy of mathematics;
- Beliefs about mathematics as a social phenomenon;
- Beliefs about aesthetics, beauty, meaningfulness, or power in mathematics;
- Beliefs about individual people who do mathematics, or famous mathematicians, their traits and characteristics;
- Beliefs about mathematical ability, how it manifests itself or can be assessed;
- Beliefs about the learning of mathematics, the teaching of mathematics, and the psychology of doing mathematics;
- Beliefs about oneself in relation to mathematics, including one's ability, emotions, history, integrity, motivations, self-concept, stature in the eyes of others, etc.

(Goldin, 2002, p. 67f.)

Especially the last three types of beliefs are relevant when encountering university mathematics. They refer both to mathematics and to the self, thus bridging the eventual gap between what learners aspire after and what they perceive themselves to be able to. Creating a balance would go a long way to help first-year students on their way to become successful graduates.

There are other psychological concepts that have an influence on the learning of mathematics but do not necessarily come under the heading of affect, e.g. confidence, self-concept, self-efficacy, mathematics anxiety, causal attributions, effort and ability attributions, learned helplessness, and motivation, which McLeod (1992) terms "mini-theories about parts of the affective domain" (p. 583).

It would mean going beyond the scope of this chapter to elaborate on all these concepts in detail. It is obvious, though, that the above-mentioned categories of affective domains (see Table 2.5) can be applied. For example, confidence, self-concept, and self-efficacy can be classified as beliefs about self, mathematics anxiety as an emotion, and attributions can be classified as attitudes. Still, taking a closer look at these concepts yields more insight into affective influences on learning. The idea that confidence in oneself promotes learning, for example, has been widely tested and accepted (Schoenfeld, 1992). The attitude learners have towards mathematics helps achievement, and not vice versa, found Ma and Kishor (1997). This is closely connected to gender issues (Leder, 1995; Pehkonen, 1997; Hannula, Maijala, & Pehkonen, 2004), as boys tend to show more confidence and a more positive attitude towards mathematics. The statement that there is a "tendency to believe that learning mathematics is a question more of ability than effort" (McLeod, 1992, p. 575) can be understood in this context that success or failure in mathematics is more often attributed to stable (uncontrollable) than to unstable (controllable, thus influenceable) issues. This impacts on teaching concepts which postulate that success is possible, i.e. influenceable.

For our purposes, beliefs about self, which cover a range of views individuals have on how they will perform and for what reasons, are of special interest. This includes motivation (for more detail on motivation, see next section) and attribution theories, which are associated with Weiner (e.g. 1994) and have been widely researched (e.g. Bempechat, Nakkula, Wu, & Ginsburg, 1996), gender differences included (Forgasz, 1995; Hyde, Fennema, Ryan, Frost, & Hopp, 1990). The reasons an individual specifies for success or failure come in three dimensions. They can be internal or external, stable or temporary, and controllable or uncontrollable (this last dimension was added later), see Table 2.6. For example, if a student fails a mathematics test, he or she can attribute it to an external, stable cause: the task was too difficult. Or he or she can attribute failure to an internal unstable cause: I did not put enough effort into it. Attribution typically varies if applied to success or failure, and gender-specific attributions were often observed, albeit with some variation, depending on other factors like achievement or cultural background (Freislich & Bowen-James, 2001; Fennema & Leder, 1990). Males tend to attribute failure to external stable causes, and success to internal unstable causes. Females tend to do the opposite, resulting in a negative self-image, as success is attributed

Table 2.6: Attribution Model, according to Weiner (1994),
columns = locus of causality, rows = stability dimension

	Internal	External
Stable	Ability	Task difficulty
Temporary	Effort	Luck

externally. The third dimension of controllability is an important addition, as it potentially assigns the learner an active part in the learning process.

Having explored literature on cognitive and non-cognitive theories with application to the transition to tertiary mathematics, the choice by Liston and O'Donoghue (2008)[15] was first considered as a starting point for our project for supporting first-year engineering students in mathematics. To keep the concept manageable, three affective aspects were addressed as a basis for the conceptualisation:

- beliefs (about oneself, about mathematical ability, and about the learning of mathematics, cf. the list by Goldin on page 32),
- attitude towards mathematics, and
- approaches to learning.

The last, approaches to learning, seemed the most easily accessible, it is therefore hypothesised that it may have the potential to influence the other, more hidden variables, beliefs and attitude towards mathematics. The supposition is that this will in turn influence mathematical achievement.

Self-regulation and motivation in stressful situations

For many students in a university course including mathematics, the first months are a potentially stressful time. They encounter new people, novel situations and unique challenges. Seminal research on stress has been conducted by Bandura (1977), who explored the influence of performance accomplishments,

[15] Liston and O'Donoghue (2008) identify five affective factors to "impact strongly on the transition to university mathematics": attitude, beliefs, mathematics self-concept, one's conception of mathematics, and approaches to learning (p. 2).

verbal persuasion, and emotions. As learning mathematics at tertiary level may not always run smoothly, the way an individual emotionally copes in perceived crisis situations is relevant, too. This has been researched in depth by (Lazarus, 1991), see Figure 2.5. Transferred to the learning of mathematics, a learner would encounter a potentially stressful environment, e.g. a task in an important examination. He or she would then primarily appraise the situation, as either positive (he or she can solve the task and interprets it as a success), dangerous (he or she anticipates failure and due consequences), or irrelevant (he or she does not care about the outcome, maybe because the mark does not count). Depending on a secondary, more rational, appraisal, in which the learner evaluates his or her resources (in reference to learning mathematics, his or her prior knowledge, heuristic skills, help available etc.), according to Lazarus (1991), stress does or does not develop. The coping (in whatever way) can focus on the problem (tackle the task, increase resources etc.), or on the emotion (avoid feeling of failure by external attributing). This results in a reappraisal of the situation which can be labelled learning - although maybe not in the desired sense of increasing mathematical skills. Beside the details of Lazarus' theory, the implications for university mathematics are clear: Learners of mathematics at university will be more successful if they have adequate resources available, and if they focus on the problem. Having lived through (and coped with) stressful situations can furthermore help to cope with future challenges. This is in keeping with theories on self-regulation and their impact on the learning process (Fox & Riconscente, 2008).

Concepts of learning styles, of metacognition, of self and affect come together in the theory of self-regulation (Vohs & Baumeister, 2011; Boekaerts, 1999; B. Schmitz & Wiese, 2006; Zimmerman, 2000; Winne, 1996; Livingston, 2003). Self-regulation is a concept that has come into focus in recent years because in a technology-based world it is becoming more and more important to adapt to new issues and to learn in all stages of one's educational and professional life, often without direct or individual support (cf. Köller & Schiefele, 2003). For an individual, to be able to regulate his or her own learning behaviour according to the needs of the situation, is a precondition for personal and social success. What is more, "being able to regulate your own learning is viewed by educational psychologists and policy makers alike as the key to successful learning in school and beyond" (Boekaerts, 1999, p. 446), and thus as the lever by which to open educational and social possibilities for everyone. The

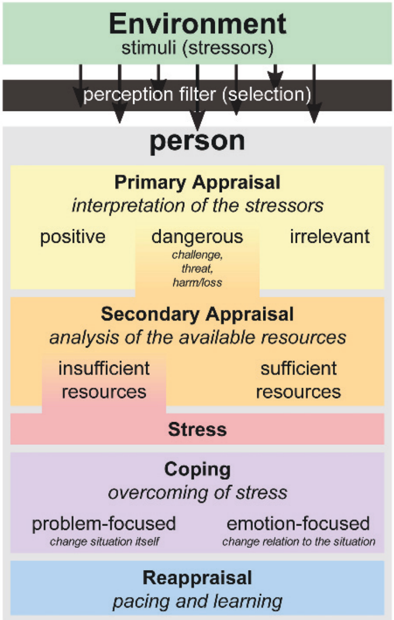

Figure 2.5: Transactional Model of Stress and Coping, according to Lazarus (1991), graphic by Philipp Guttmann, https://commons.wikimedia.org/wiki/File:Transactional_Model_of_Stress_and_Coping_-_Richard_Lazarus.svg, licensed under the Creative Commons Attribution-Share Alike 4.0 International license.

connection between mastering self-regulation and academic success has been observed many times, e.g. by Pintrich and de Groot (1990). Findings reveal that students' cognitive reflection, as a metacognitive variable, their beliefs about mathematics, and their self-efficacy, are all correlated positively and significantly with mathematical achievement (Gómez-Chacón, García-Madruga, Vila, Elosúa, & Rodríguez, 2014). There is also evidence that metacognition impacts positively on learning strategies which in turn influences achievement (Glasmachers, Griese, Kallweit, & Roesken, 2011).

Definitions of self-regulated learning behaviour differ, though, due to the background of the individual researcher. Zimmerman (1990) describes the common understanding that self-regulated learners are “metacognitively, motivationally, and behaviorally active participants in their own learning” (p. 4). Winne (1996) outlines self-regulated learning, similarly, “as metacognitively governed behavior wherein learners adaptively regulate their use of cognitive tactics and strategies in tasks” (p. 327). According to Hannula, Evans, et al. (2004, p. 10),

> Self-regulation processes represent the central combining feature of self-system processes with affect. In addition to self-appraisals and self-judgments, these metalevel mental processes involve students’ self-directive constructions, self-control and self-regulatory actions.

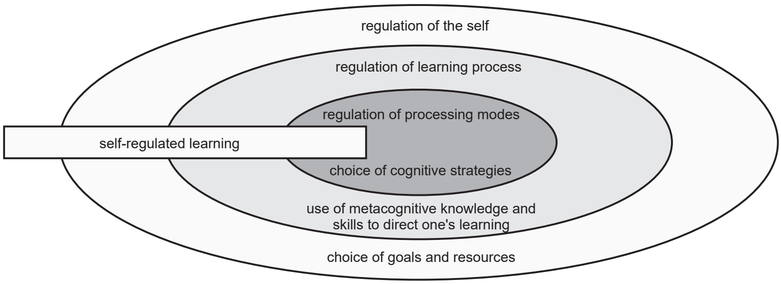

Figure 2.6: The Three-Layered Model of Self-Regulated Learning (Boekaerts, 1999, p. 449)

A broader and more comprehensive perspective is taken by Boekaerts et al. (Boekaerts, 1999, 1997; Boekaerts, Pintrich, & Zeidner, 2000) who describe self-regulated learning in a three-layered model, see Figure 2.6. They take three schools of thought into account, referring to “(1) research on learning styles, (2) research on metacognition and regulation styles, and (3) theories of the self, including goal-directed behavior” (Boekaerts, 1999, p. 445). Each research focus is represented by a layer in the model, which allows for concentrating on single aspects as well as exploring interrelations and the big picture. The hierarchy implied by introducing layers is justified by ordering the aspects of self-regulated learning from concrete to abstract, from inner to outer layer. The inner layer refers to processing modes, i.e. choices of cognitive strategies. Examplified, processing new information would mean underlining important passages in a text, summarising them, finding examples, or connecting them to previously learned facts. Boekaerts stresses the fact that a learner has to be aware (in this context: has to know) that there are alternatives before actively making a choice. The middle layer in Boekarts' model is titled *regulation of the learning process*, meaning "use of metacognitive knowledge and skills to direct one's learning" (Boekaerts, 1999, Figure 1, here reproduced in Figure 2.6). More abstract than the first layer, this layer relates to thinking about the learning process, and comparing it to other, alternative learning processes, with the aim of increasing efficiency. Operationalised, the skills needed here are orienting, planning, executing, monitoring, and correcting (Brown, 1987; Weinstein & Mayer, 1986; Wild, 1994).

The fact that there is yet another layer in the model, can be seen in connection to the psychological origin of the authors. They are aware of the fact that not only knowledge about the learning process is needed in order to successfully accomplish learning (particularly complicated academic material), but that regulation of the self is guided by "the students' involvement in and commitment to self-chosen goals" (Boekaerts, 1999, p. 451), and by the resources they have available.

This leads us to theories on motivation and self-determination to understand what prompts people to do something. According to Deci and Ryan (1990), an individual's motivation is guided by the expectation if his / her basic psychological needs will be met. These comprise competence (the need to experience control over what is regarded as important), autonomy (the need to perceive oneself as actively making decisions), and relatedness (the need to feel connected to others). The authors (Deci & Ryan, 1990; Ryan & Deci, 2000; Deci & Ryan, 2000) also elaborate on different forms of motivation (from amotivation over four different grades of extrinsic motivation to intrinsic motivation, see Figure 2.7) which they connect to the degree to which these basic needs are fulfilled. Ideally, all three needs are satisfied to an individually acceptable degree, which leads to volition (expressly wanting to reach the learning aims), motivation (feeling able and eager to reach the learning aims) and engagement (in actions that lead to reaching the learning aims), see Deci and Ryan (1990). The presence of these attitudes then results in increased effort and performance. If an individual does not expect that participation in an activity will fulfil his or her basic psychological needs, he or she might probably not even endeavour to engage in the activity. The awareness of these circumstances has even found its way into human resource management and helps to motivate employees through catering for their need to experience social support and leeway for their actions (Nerdinger, 2014).

Behavior	Nonself-determined					Self-determined
Type of Motivation	Amotivation	Extrinsic Motivation				Intrinsic Motivation
Type of Regulation	Non-regulation	External Regulation	Introjected Regulation	Identified Regulation	Integrated Regulation	Intrinsic Regulation
Locus of Causality	Impersonal	External	Somewhat External	Somewhat Internal	Internal	Internal

Figure 2.7: The Self-Determination Continuum (Deci & Ryan, 2000, p. 237)

More specifically, Heckhausen has explored the mechanisms of achievement motivation (Heckhausen, 1977, 1989; Heckhausen & Heckhausen, 2010). He describes four phases: "(1) the initial *situation* as appraised, (2) the person's own *action*, (3) the *outcome* of the action or of the situation, and (4) the *consequences* of the outcome, with their various incentives determining the value of the outcome" (Heckhausen, 1977, p. 286f.). The crucial point is in what an individual anticipates will be the probable outcome of his or her action. The incentives that ideally lead to evaluating the outcome as valuable enough to perform an action are

> affective states people expect to experience after or while performing a certain behavior [...] In a learning context an incentive may be, for example, the anticipated pride (affective goal state) regarding a good performance in an exam. This anticipation is likely to lead to revising for this exam (goal-directed behavior). Depending on the positive (e.g., pride after passing the exam) or negative (e.g., disappointment after failing the exam) value of the anticipated affective state, people develop approach or avoidance tendencies towards the exam situation.
> (Schüler & Engeser, 2009, p. 340)

The distinction between an outcome and the consequences of an outcome is necessary (Heckhausen, 1977, p. 286) because an outcome (e.g. an 80% score in an exam) can have different consequences for different individuals (e.g. feelings of pride or shame, depending on the expectations), and because an outcome can bring about more than one consequence (e.g. concerning self-evaluation, other-evaluation and superordinate goals).

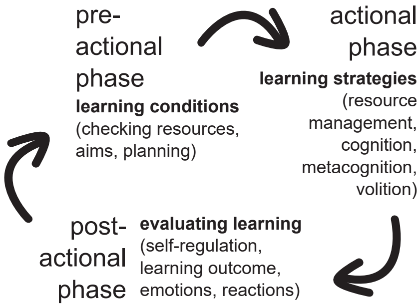

Figure 2.8: The Cycle of Self-Regulation

The consequences experienced in a learning situation can in turn influence the following learning experiences, when self-regulation is viewed more as a cyclic process, as described by B. Schmitz and colleagues (Landmann & Schmitz, 2007b; B. Schmitz, 2003; B. Schmitz & Wiese, 2006), based on the work of Zimmerman (e.g. Zimmerman, 2000, 1990), and involves “goal-setting, monitoring and regulation” (B. Schmitz, 2003, p. 221) in a three-step-cycle, see Figure 2.8.

According to Landmann and Schmitz (2007b), the process of self-regulation itself is divided into three parts: the pre-actional, actional, and post-actional phases. The authors elaborate that before a learning activity is started (during the initial appraisal of the situation, according to Heckhausen), the individual has certain resources at his or her disposal, aims at certain goals to reach with the learning activity, and plans what to do exactly, influencing how the actional phase, the immediate learning activity, is approached. As depicted in this theory (B. Schmitz & Wiese, 2006; B. Schmitz, 2003), the actional phase is characterised by the individual’s strategies, meaning resources (e.g. works of reference), cognition (knowledge and skills), metacognition, and volition. This influences the post-actional phase (in Heckhausen’s terms, the outcome and its consequences), where the individual assesses his or her learning, the way they can control the learning process, by the results that are reached, by emotions and the reactions to these emotions. According to this line of thinking, if the learning activity produces ample satisfying results, there will probably be positive emotions, and the individual will continue the activity until the end without big difficulty. But if the learning activity turns out to be a frustrating experience that cannot be remedied by employing available resources, emotions may falter, and the individual may give up the learning activity (Schmidt et al., 2011). One outcome may be the realisation that further help is needed (consult another textbook) or that an easier or shorter task may lead to a more satisfying experience, which in turn will influence the next pre-actional phase, in which the individual reviews his / her resources and learning aims and plans the next learning activity.

Viewing self-regulation as a (cyclic) process is no contradiction of the three-layered model described above, however. Rather, the view of the process raises the awareness of how interdependent the concept is – whereas the view of the layers accounts for its complexity. This results in the insight that success or failure of self-regulation in individuals depend on different features, among them, learning strategies, learning styles, and personality (Cervone, Mor, Orom, Shadel, & Scott, 2011; Zimmerman, 1989).

Coping with stress, maintaining motivation, and regulating the self via metacognition are the key challenges when understanding learning in academic contexts. They are multi-faceted processes and involve many variables. All these constructs bear the common feature of thinking about one's own thoughts and perceptions, i.e. of having reached Piaget's formal operations stage, see Table 2.2. To tackle the problem of improving learning mathematics at university, it is essential

– to point to resources sufficient enough to overcome the challenge (cf. Lazarus),
– to offer a choice of alternatives in organising the learning process (following Boekaerts),
– to emphasise that the decision which alternatives to employ is an autonomous one (as understood by Deci and Ryan, (1990)),
– to discuss the possible outcomes of different actions (according to Heckhausen), and
– to convey in the learner feelings of control over possible outcomes (termed competence by Deci and Ryan, (1990)).

2.1.3 Approaches to overcome transition problems

The attempts to overcome the obstacle of transition from secondary to tertiary education in mathematics vary according to the underlying hypothesis about what forms the core of the problem, the perspective of the initiators, and the means at their disposal. A number of predictors for success in university mathematics courses have been identified: Among these are the mathematics mark reached in school (Rach & Heinze, 2013; Rach, Siebert, & Heinze, 2016), mathematical competence (G. Henn & Polaczek, 2007), the use of learning activities offered by the university, the teaching quality in terms of cognitive activation or discussion of alternative solutions, deep learning strategies, and motivation for / interest in mathematics, see Griese, Glasmachers, Kallweit, and Roesken (2011); Gómez-Chacón et al. (2014), and, for an empirically backed overview, Blömeke (2016), and the meta-analysis by Trapmann, Hell, Weigand, and Schuler (2007). Other researchers are looking for reasons for failure and find early and constant use of calculators (Weinhold, 2014) or lowered demands in school curricula (G. Henn & Polaczek, 2007). This list can be complemented with the reasons students give when asked why they decided to drop out of a university mathematics course (Heublein & Barthelmes, 2010): they most often report difficulties to meet the standards (33%), or a lack of motivation (25%). In a survey among students from universities and universities of applied sciences, Bargel (2015, p. 28, Figure 6) found that the top requests for improving study conditions were more courses in small groups (2013: 25%), a stronger practical relevance (2013: 24%) and fixed tutorials (2013: 20%). In 2007, however, it was a different list. Then, bridging courses came first (39%), before more courses in small groups (36%) and a stronger practical relevance (29%). This development takes account for the fact that bridging courses have become a common feature.

It is important to note that there are many projects around aiming at overcoming transition problems in mathematics (Dunn, Lo, Mulvenon, & Sutcliffe, 2012; Hoppenbrock, Biehler, Hochmuth, & Rück, 2016)[16]. This in itself is an indication of how huge this problem is, how many people are affected, and of how urgently a solution is needed. The obstacle exists all around the world, as the fact shows that similar approaches can be found all over Germany, in Europe, and in many other countries.

[16] A list (under certain criteria) is compiled in section 2.1.5 later in this work.

For reasons of completeness, at first some categories of possible interventions are described that are not directly connected with support projects for first-year students. To paint the whole picture, these arrangements often complete and complement a university's or a district's catalogue of interventions to help young people on their way to participate in society, according to their capability and inclination.

To start out, it seems advisable to provide more information on university courses at school level. There are initiatives that aim at improving the information about university courses involving mathematics that are available to students[17]. Thus, their prospects are expected to be more realistic. Despite an ambitious school curriculum, it is hard to impart the concept of mathematics as a complex science at high school level. Nevertheless, multifarious facts are available, online and otherwise, that describe university science and engineering courses and that aim at conveying the idea of hard work, long hours, and deep thinking. In addition to this, universities offer open days where students can partake in lectures and lab work. This first-hand experience is supposed to not only attract excellent students, but also to inform average ones.

Entrance tests for certain university courses are another suggestion. Some universities use tests as a means to choose their students. In Germany, this is not usually an option (for STEM subjects[18]), but universities can take an applicant's school mark in mathematics as a further criterion for awarding places to study (in combination with the average mark in the school-leaving examination, beside other criteria such as waiting time). In other countries, this is completely different: Depending on their status and ranking, universities can either choose from among the very best of a generation[19], or contend themselves with whoever is able to pay the college tuition fees.

[17] For example www.studifinder.de for North Rhine-Westphalia, which combines the search for suitable courses with tests and customised preparation courses, for Germany see also https://studieren.de, http://www.hochschulkompass.de/, http://www.studienwahl.de and http://www.wege-ins-studium.de.

[18] For medicine, there is a tradition of entrance tests in Germany.

[19] The US-American Ivy League universities are famous for accepting below 10% of applicants, with tuition fees ranging around 35,000$ a year, see https://www.studential.com/applying/studying-abroad/USA/ivy-league-universities.

Third, some universities try to reform and improve their courses, e.g. the mathematics department at Ruhr-Universität Bochum[20]. They enhance first year lectures according to didactic demands: Lecturers and teaching assistants are coached to openly show interest and respond to newcomers' problems, to visualise their explanations and give illustrating examples, to warn off typical misconceptions and to be aware of characteristic mistakes. The accompanying tutorials are also an objective for innovations – and Szczyrba and Wiemer (2011) believe they can be a motor for good teaching. It is almost standard procedure today to try out technical support for lectures in different ways: tasks can be set online, lectures are audio- or video-recorded, digital versions of scripts are uploaded, e-mail support is offered, and much more. Modern software even allows for individual and constructive feedback, a huge step forward from the true/false single or multiple choice tasks from before, like the STACK plugin for the learning platform moodle[21]. More information on technological support can be found later in this section.

Formal relief for the start at university is another approach to support average or weak first-years. Several universities have taken the fact into account that problems mostly occur at the beginning of a university course, and that, once these initial transition problems have been overcome, most students' courses indeed run rather smoothly. Logically, they try to make the start easier by allowing for (more) unsuccessful attempts in exams, by enabling students to reduce the workload in their first year, or by giving them more time to adapt to the new learning conditions. For example, Ruhr-Universität Bochum has devised a model where mathematics students concentrate on linear algebra and take calculus later, in order to take pressure from students who normally would have to attend both lectures in their first year. So far, there is no systematic evaluation if this idea has indeed raised the number of graduates.

As mentioned above, the use of modern technology offers a whole new range of support devices. It makes sense, therefore, that many of the foregoing projects make use of modern technology in the form of online support, e.g. by using learning management systems, software that allows users to upload and share files and links, with comments, tasks and assessment tools. In education,

[20] http://www.ruhr-uni-bochum.de/imperia/md/content/mathematik/service-zentrum/szma_schulung.pdf

[21] https://moodle.org/plugins/qtype_stack

moodle is one of the most popular. It is free, open source, and features countless plug-ins for different purposes. But also in the commercial sector, there is hardly any popular sharing site that does not serve those with mathematical questions. For example, www.youtube.com gives you 5,600,000 hits for “math help” (in November 2013), the top ones sporting more than 2,000,000 clicks, while some German mathematics help videos on www.youtube.de accomplish around 145,000 clicks. The current generation of students has grown up with the perception that the answer to any possible question, however specific it may be, is usually just one google search away. But even before the Internet was available everywhere and all the time, there was software to support visualisation and thus understanding. The availability of computers in general and dynamic geometry software and graphing software in particular have backed this development, even though they may not have conquered classrooms as far and wide as might be desirable. The chances offered by these devices were discerned decades ago, as McLeod revealed when expressing the view that “technology can play an important role in changing beliefs about mathematics and possibly even in improving attitudes toward mathematics” (1992, p. 588).

As early as (1998), de Guzmán et al. suggested offering learning centres or help desks to students in need of mathematical support. An intervention like this is usually part of almost every support project (e.g. M. Schmitz & Grünberg, 2016). Ideally, students can get their questions on current homework assignments or the lecture answered in an informal surrounding. Often, these help desks or learning centres are manned by more advanced students, so that the atmosphere is more relaxed than it could ever be in a professor’s office hour. Apart from face-to-face explanations, often there are additional learning materials available, e.g. summaries of basic domains, example exercises, a collections of test papers from the past, explanatory videos, or Internet access. Like many other universities (for a large-scale Irish survey on the impact of support centres, see Nì Fhloinn, Fitzmaurice, Mac an Bhaird, & O’Sullivan, 2014), Ruhr-Universität Bochum has help desks both for service mathematics lectures (established 2007[22]) and for mathematics majors (established 2012[23]). Other universities have more resources and sport award-winning facilities,

[22] http://www.ruhr-uni-bochum.de/helpdesk-mathematik/

[23] http://www.ruhr-uni-bochum.de/ffm/Lehrstuehle/stochastik/lernzentrum.html

like Loughborough University with its Mathematics Learning Support Centre[24] (Jaworski, 2008).

Preliminary or bridging courses offered between school and university or at the beginning of a university course are among the most popular measures a university can take to support students. Many universities offer voluntary bridging courses: Before lectures start, new students are invited to attend a course, usually lasting a few weeks, where what is considered general school knowledge in mathematics is revisited, mostly in the form of a lecture, sometimes in combination with tutorials and homework, usually in groups of less than 30 students. As many as 48% of engineering students report that there were bridging courses at their university, and that they took part (Bargel, 2015, p. 38). Although this seems promising, details like how to address an appropriate target group are not always solved. Those students who enrol early enough to feel addressed by bridging courses may be above average. What is more, bridging courses require students to visit university regularly *before* their regular time there has started, and they normally do not warrant any credit points. Both reasons result in not reaching the target group the bridging course was initially intended for, namely average (or even below average) students with an incomplete mastery of school mathematics. Although there is a unique consensus that first-year students of mathematics or of subjects related to mathematics often lack basic skills in high school mathematics, such as rearranging terms, solving equations, fractions, powers, and roots, the efficacy of bridging courses has not yet been universally observed, but positive effects are there (Heiss & Embacher, 2016; Greefrath & Hoever, 2016). For example, Kürten, Greefrath, Harth, and Pott-Langemeyer (2014) found significant differences between students who had attended a bridging course and those who had not only in selected subgroups of their sample. Despite the statistics, students attending bridging courses often report to feel their time well-spent; according to Bargel (2015, p. 48) the large majority judge bridging courses as useful (65%) or partly useful (31%).

The idea to specifically review the skills and concepts that have not been mastered (e.g. modes of thought, see Hoffkamp, Paravicini, & Schnieder, 2016), and at the time when they are needed, expands the notion of bridging courses into courses accompanying students during their first months at university. This

[24] http://www.lboro.ac.uk/departments/mlsc/

kind of course can also incorporate some of the ideas described above, like the use of learning platforms or other technology (like Ellis, Goodyear, Rafael, & Prosser, 2008, who explore approaches to learning and understanding in a blended-learning environment), as well as reviewed didactical concepts with purposely trained and carefully chosen staff. It needs thoughtful planning, though, as it is meant to complement and not to replace the regular courses at university.

Table 2.7: Description Categories for Objectives, according to WiGeMath, translation by author

Learning objective		
	Knowledge-related objectives	
		Basic mathematical knowledge and skills
		Academic mathematical knowledge and skills
		Terminology
	Action-related objectives	
		Mathematical methods
		Learning strategies
		Learning behaviour (e.g. study time)
	Attitude-related objectives	
		Beliefs
		Affective features
		Practical relevance
		Mathematical enculturation
System-related objectives		
	Reviewing skills and abilities	
	Introducing academic methods and procedures	
	Imparting (mathematic-related) learning strategies	
Quality of objective		
	Clarity resp. concreteness of objectives	
	Transparency and propagation of objectives	

A conceptualisation to describe a framework for the reconstruction of supportive interventions is currently being developed in the course of the

WiGeMath project[25] at Leibniz Universität Hannover and Universität Paderborn, which researches conditions of effect and success of support interventions for mathematics-related learning (*Wirkung und Gelingensbedingungen von Unterstützungsmaßnahmen für mathematikbezogenes Lernen in der Studieneingangsphase*). According to presentations at the kickoff workshop in autumn 2015 and at a conference in spring 2016 (Colberg et al., 08.03.2016), a first version of the framework categorises support interventions in regard to their objectives, the interventions and their characteristics, and general parameters. The categorisation of the objectives is presented in Table 2.7.

The second group of categories, which refers to the interventions and their characteristics, comprises structural characteristics (format, duration, temporal structure and appointed times), didactical elements (didactic principles and guidelines, exercises, interactive and social forms, and summative vs. formative tests), and characteristics of the teachers or teaching teams (number, status, role, as well as subject-specific, subject-didactic or university-didactic qualification), see Colberg et al. (08.03.2016). The last group of conditions covers general parameters, like number of students and their characteristics (age, gender etc.), mathematical foreknowledge, genesis and development of the intervention, curricular embedding (optional or mandatory), organisational characteristics (e.g. selective measures), rooms (e.g. flexibility of furniture), financial conditions, and characteristics of the teaching and learning culture (epistemological aspects of the subject-specific culture, interaction between teachers / lecturers and students, students' and lecturers' expectations). The WiGeMath framework is still in the stage of being tested and will be adapted to improve the fit to the interventions that are explored in detail. At this point, however, the impression is that all bases are covered, and that this framework will serve not only to categorise support projects, but also to enable evaluating their impact and success, as planned. For the research purposes at hand, the framework provides a reference frame to classify the interventions in the project MP^2-Math/Plus.

Furthermore, the possible factors influencing effect and success of support projects listed in the WiGeMath framework can serve to check our hypothesis for completeness and detail. WiGeMath have identified the influence factors as presented in Table 2.8, with the main categories person-related

[25] http://www.hochschulforschung-bmbf.de/de/1901.php

Table 2.8: Possible Impact and Effect Variables for the Evaluation of Mathematics-Related Support Interventions, according to WiGeMath, translation by author

Person-related factors	
	(Socio)demographic objectives
	Cognitive / Metacognitive characteristics
	Motivational characteristics
Environment-related factors	
	General conditions
	Structural characteristics
Teaching and Learning activity	
	Learning process
	Didactical elements
	Teacher / Lecturer
Impact variables	
	Cognitive impact
	Motivational and affective impact

factors, environment-related factors, teaching and learning activity, and impact variables.

All these projects and initiatives aim at bringing about change in the form of a better kind of learning in mathematics: be prompter, offer more specific help, make the learners more content and more motivated, or help them understand the concepts on a deeper level. The research project at hand will be classified according to the categories and framework presented in this section.

2.1.4 Specifities of engineering mathematics

The Mathematics Working Group of the European Society for Engineering Education (SEFI) has the aim to foster exchange and to compile documents offering orientation (Alpers, 2016). Their consensus on competencies, curricula, and assessment is a qualified basis for mathematical education for aspiring engineers. SEFI's understanding of mathematical competence is based on Niss (2003), who states

> *Mathematical competence* then means the ability to understand, judge, do, and use mathematics in a variety of intra- and extramathematical contexts and situations in which mathematics plays or could play a role. Necessary, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills [...] (p. 6f.).

A. Entwistle and Entwistle describe the importance and implications of understanding mathematics

> not as a cognitive process, but as an experience. It involves a feeling of satisfaction as sets of information and ideas are brought together into a coherent whole. It also creates a feeling of confidence that the understanding reached can be used to construct explanations or to solve problems in novel contexts. The development of understanding may also have a social component through the discussions which lead to the negotiation of shared meaning (p. 18).

This justifies the central position understanding has in the learning of mathematics. After a first version of a curriculum document for engineering mathematics from 1992, which according to Alpers (2016) was little more than a list of the subject-specific content[26], the more recent framework document (Alpers et al., 2013) has accomplished the step to a competence-based description of learning outcomes. These competencies comprise thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modelling mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating in, with, and about mathematics, and making use of aids and tools[27] (Alpers et al., 2013, p. 13f.) – meant to overlap. In conformity with Niss' above statement, the SEFI framework (Alpers et al., 2013) contains a list of subject-specific factual knowledge and skills as well, ordered from core level 0 resp. 1 to level 2 and 3. The level descriptions include:

> Core Zero [...] comprises such essential material that only minor omissions are acceptable. [...] Core level 1 comprises the knowledge and skills which are necessary in order to underpin the general Engineering Science that is assumed to be essential for most engineering graduates. [...]

[26] „Das Curriculum-Dokument [...] bestand zum größten Teil aus aus einer Liste von zu behandelnden Themen" (Alpers, 2016, p. 645).

[27] The analogy to the competencies in the German competence model of education standards (see section 2.1.1) stems from Niss' participation in the PISA studies.

> Level 2 comprises specialist or advanced knowledge and skills which are considered essential for individual engineering disciplines. [...] Level 3 comprises highly specialist knowledge and skills which are associated with advanced levels of study and incorporates synoptic mathematical theory and its integration with real-life engineering examples (p. 20f.).

It is notable, though, that even core level 0 (which covers more than six pages listing learning objectives completing the phrase “As a result of learning this material you should be able to”, p. 23–29) has a considerable number of items starting with “understand” (e.g. “understand the role of the arbitrary constant” in the section on indefinite integration, p. 26) and consistently addresses the learner, and not the teacher / lecturer. This highlights the change to describing learning outcome (and not input) and the personal responsibility of the learner, true to Alpers’ belief that “If we cannot communicate to the learner what we expect them to learn, we cannot demand or expect that they learn it” (Alpers et al., 2013, p. 69).

In a similar frame of mind, the advisory board of teachING-learnING.eu has formulated eight theses on next generation engineering[28] that back this position. Particularly the first three theses draw a progressive picture of the future of engineering education:

> Next generation engineering education will prepare graduates to solve open problems and well as competing requirements. This is based on mastery of problem definition and analysis skills, domain specific knowledge in required areas, interaction collaboration, and self management skills as well as self guided initiative and creativity. [...]
> Next generation engineering education will be aware of the fact, that especially the first year of study is crucial for the personal development and thus requires personal coaching of the students by experienced mentors [...].
> Next generation engineering education will move from teacher centered (passive) to student centered (active) learning. [...] [29]

The question of how to assess these kinds of competencies arises naturally, as traditional written examinations do not seem appropriate. Students tend to

[28] http://www.teaching-learning.eu/fileadmin/documents/News/Theses_AdvBoard_2011_news.pdf
[29] http://www.teaching-learning.eu/fileadmin/documents/News/Theses_AdvBoard_2011_news.pdf

think and work with a strong orientation towards the examination, “What you get is what you assess” (Alpers, 2016, p. 651). In his *Mathematics Curriculum for a Practice-oriented Study Course in Mechanical Engineering*, Alpers (2014) gives concrete examples of how assessment can follow a competence-based curriculum, for example by splitting assessment into a classical written part assessing the “capability of performing short mathematical reasoning along lines encountered before or performing standard problem solving routines” (Alpers, 2014, p. 64), and a second part including “larger assignments, project work documentations and oral presentations” (p. 65)[30]. The SEFI framework finds reasons why a student should only pass if he or she has mastered every task from the first part, neglecting “minor ‘numerical’ errors” (Alpers et al., 2013, p. 70). Other research is dedicated to “Conceptions of Understanding in Engineering Mathematics” (Khiat, 2010), to exploring theoretical frameworks for mathematical competencies for engineering students (Hochmuth, Roesken-Winter, & Jaworski, 2013; Noss & Kent, 2002), or to developing measurement instruments for them (Neumann et al., 2015). In more or less complex applications, future engineers are expected to use mathematical models and techniques to solve open practical problems which can be described not only by the number of competencies involved, but also by the degree of coverage (“the extent to which the person masters the characteristic aspects“), the radius of action (“contexts and situations in which a person can activate” a competency), and the technical level (indicating “how conceptually and technically advanced the entities and tools are with which the person can activate the competence”) needed (Niss, 2003, p. 10).

In contradiction to this mode of thought, Dreyfus (1991) finds that “what most students learn in their mathematics courses is, to carry out a large number of standardized procedures, cast in precisely defied [sic (defined)] formalisms, for obtaining answers to clearly delimited classes of exercise questions. They thus acquire the capability to perform, albeit much slower, the kind of operation which a computer can perform” (p. 28). This hints at the perception that there is a tendency to focus strongly on procedure, be it because procedural tasks are easier to grade, or because there is a general consensus that procedural knowledge is at the basis of mathematical competence. In contrast to students

[30] Alpers et al. (2013) are well aware of the problem of resources, however.

majoring in mathematics, engineering students do not have to conduct formal proofs, but to calculate demands on material properties, statics, and solidity.

But, as demonstrated in section 2.1.2, the universally accepted way of learning mathematics is to (re)construct each piece of knowledge by studying objects in order to abstract their properties, by building concepts, and by logical deductions from these concepts (Tall, 2004). It also involves discovering discrepancies and accommodating previous mental concepts accordingly (Piaget, 1973), which implies bringing discrepancies to light and discussing them. This requires active participation of the learner, guided and challenged by a teacher or lecturer (Tall, 1991b). In keeping with this, engineering curricula contain much more than just technical and procedural requirements (albeit no formal proofs), as the examples from Neumann et al. (2015) and Alpers et al. (2013) show. The question remains if university mathematics courses focus on the entirety of competences, or on what is sometimes perceived as the foundation of mathematics, namely basic skills and techniques. This particularly applies to the support projects for first-years, which will be elaborated upon in the next section.

2.1.5 Projects at other universities

As presented on in section 2.1.3, there are many ways in which to deal with the problem that students tend to experience difficulties in mathematics when starting a course at university. These considerations have led a huge number of universities to initiate various projects in order to overcome the obstacle. To attempt to compile a comprehensive list of who does what in this field would be presumptuous, so the elaborations in this section are restricted under the following criteria.

– The initiative caters for mathematics for (not necessarily exclusively) engineering students.

– It concentrates on the first semester or first year at university.

– It exchanges and discusses ideas in an organised way with other initiatives working in the same field.

– The intervention is connected with research in mathematics education.

– It is located at a German university.

This implies that there is still variety in the kind of university (private or state university, university of applied sciences, technical college etc.), in the specific engineering course (e.g. machine engineering, civil engineering, environmental engineering, industrial engineering), and in the particular focus of the initiative (e.g. target groups, size of project, specific learning activity).

According to their homepage[31], the competence centre for university didactics in mathematics (*Kompetenzzentrum für Hochschuldidaktik Mathematik*), khdm, is a common institution of the universities at Kassel, Lüneburg and Paderborn, and has been active since 2010. Their central fields of work include

– investigating the teaching and learning of mathematical modes of thought and practise,

– investigating students' attitudes and learning behaviour,

– developing and investigating innovations from the areas of teaching and learning methods and digital media, and

– revising and innovating existing university curricula in relation to content under the aspect of competence and recipient orientation.

khdm has a working group on engineering mathematics (AG Ing-Math[32]), concentrating on mathematics for first-year engineering students. Their central projects[33] deal with modelling in machine engineering and with situational acquisition of mathematical knowledge in electrical engineering. Other associated projects research requirements in mathematical problem-solving in electrical engineering (part of the bigger project KoM@ING[34]), produce mathematical video lessons for students of electrical engineering (LEMMA[35]), and develop multimedia material for virtual preliminary courses for STEM subjects (VEMINT[36]).

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31 www.khdm.de/

32 www.khdm.de/ag-ing-math/

33 https://www.khdm.de/fileadmin/khdm/Kolloquien_und_Oberseminare/Poster/Ing_110117_A4.pdf

34 www.kom-at-ing.de/

35 https://www.khdm.de/ag-ing-math/lemma-lehrinnovationen-zur-mathematikausbildung-in-der-elektrotechnik/

36 www.vemint.de

In connection and collaboration with khdm, the above-mentioned project WiGeMath was founded. It aims at a conceptualisation to describe a framework for the reconstruction of supportive interventions. To reach this aim, they have brought together several representatives active in support projects in mathematics. In order to test the framework, some of the projects will be chosen to collect more data. The result is expected to deliver a theoretical framework and insights into what is relevant for support projects to be effective and successful. Thus, WiGeMath does not itself design support projects, but takes a general perspective. The WiGeMath project partners, supported by funds from the Qualitätspakt Lehre[37], were chosen from among the several initiatives contacted, many partaking in the kickoff meeting in 2015.

- RWTH Aachen, with 42,000 students the biggest university for technological courses in Germany: RWTH Aachen has offered preliminary courses in mathematics to their students for decades and has accumulated expertise and experience in teaching first-year STEM students in mathematics.
- MINT-Kolleg Baden-Württemberg[38], a joint college of KIT (Karlsruher Institut für Technologie) and Universität Stuttgart: The MINT-Kolleg is dedicated to improving the subject-specific preconditions in the transition from school to university. It offers preliminary courses, a help desk, information for those interested in studying a STEM subject, and tests.
- Ruhr-Universität Bochum: With more than 14,000 students attending courses in STEM subjects (about half of them enrolled in engineering courses[39]), the department of mathematics at RUB offers preliminary courses, a special help desk for service mathematics, and the support project MP^2-Math/Plus/Practice.
- Technische Universität Darmstadt: Its 26,000 students[40] mainly study STEM subjects; it offers individual information and activities for prospective students as well as bridging courses. There is research of the

[37] http://www.qualitaetspakt-lehre.de/
[38] http://www.mint-kolleg.de/
[39] http://www.ruhr-uni-bochum.de/universitaet/fakten/menschlich/index.html
[40] http://www.tu-darmstadt.de/universitaet/selbstverstaendnis/zahlenundfakten/index.de.jsp

transition from secondary to tertiary education in mathematics, particularly diagnostic tests (Schaub & Bruder, 2015).

- Technische Universität Dortmund: The majority of its around 30,000 students attend courses in STEM subjects[41]. TU Dortmund offers three different preliminary courses in mathematics[42] which especially aim (a) at students of physics, computer science (and others), (b) at students majoring in mathematics, and (c) at students of engineering and chemistry (and others). TU Dortmund also tests didactical concepts for motivating students who attend service lectures in mathematics.
- Universität Hamburg: Students can choose between more than 150 different courses (though none in engineering) and has over 40,000 students (more than 13,000 in science and mathematics[43]). There is one preliminary mathematics course that STEM students are advised to attend, including an online assessment; both are optional. The research groups for didactics are part of the faculty of education.
- Leibniz Universität Hannover: Of the more than 25,000 students, about a third are enrolled in STEM subjects[44], engineering courses are popular. The university is home of a research project that researches competences in mathematics-related engineering application tasks for first-year students (*Kompetenzen bei mathematikhaltigen ingenieurwissenschafltichen Anwendungsaufgaben in der Studieneingangsphase*[45]), and a project to explorate the development of interest in mathematics during the first year at university (*Interessenentwicklung im ersten Studienjahr*[46]) which are both connected to khdm.
- Universität Kassel: The university has 24,000 students, about a third are enrolled in STEM subjects. It offers a learning centre as well as prelimi-

[41] http://www.tu-dortmund.de/uni/Uni/Zahlen__Daten__Fakten/Statistik/Publikationen/Studierendenstatistik/StuSta_SoSe15_web.pdf
[42] http://www.mathematik.tu-dortmund.de/sites/vorkurs-mathematik-2015
[43] www.uni-hamburg.de/beschaeftigtenportal/services/statistik/download/up-pvv-u-2012w.pdf+&cd=2&hl=de&ct=clnk&gl=de
[44] https://www.uni-hannover.de/fileadmin/luh/content/strat_controlling/statistiken/studierendenstatistik/studierendenstatistik_wisem_2015_2016.pdf
[45] https://www.khdm.de/ag-ing-math/diss-joerg-kortemeyer/
[46] https://www.khdm.de/ag-bagym-math/entwicklung-des-mathematikinteresses-im-ersten-studienjahr/

nary and bridging courses for first-year engineering students and houses a research project on working and learning strategies in mathematics at university (for students with a major in mathematics *Arbeitsweisen und Lernstrategien im Mathematikstudium*) in the context of khdm[47].

- Universität Paderborn: Like Universität Hannover, it is part of khdm and therefore has a share in the projects WiGeMath, VEMINT, KoM@ING, and other research. Universität Paderborn has almost 20,000 students, of which 2,800 are enrolled in machine engineering courses alone[48], in addition to courses in electrical engineering and other STEM subjects. They also offer a learning centre[49] and bridging courses.
- Universität Ulm: Of Ulm's more than 11,000 students, 8,000 are enrolled in STEM subjects[50]. There is research on alternative evaluations and examination formats, e.g. in the bachelor course *Computational Science and Engineering*. The university offers bridging courses and a training camp for prospective students[51].
- Universität Würzburg: As one of the oldest universities in Germany (first founded 1402, closed 1413, re-founded 1582), Würzburg has about 28,000 students, with a minority of a few thousand studying STEM subjects. There is research on metacognition in the learning of mathematics (Mungenast, 2015).

Two partner universities, Carl von Ossietzky Universität Oldenburg and Philipps-Universität Marburg, are not included here because they do not offer engineering courses.

Lehre^n (http://www.lehrehochn.de/home/) is an alliance which selects a focus every year for a council (*Kolleg*). In 2013 the focus was on mathematics in engineering education. Six project teams from different universities were

[47] https://www.khdm.de/ag-bagym-math/diss-goeller/
[48] http://mb.uni-paderborn.de/presse/zahlen-daten-fakten/
[49] https://lama.uni-paderborn.de/en/lernzentrum.html
[50] http://www.uni-ulm.de/fileadmin/website_uni_ulm/studium/Studierendenstatistik/WS2015/Statistik6_WS15_16.pdf
[51] http://www.uni-ulm.de/misc/unitrain.html

chosen for the council[52] and are listed here; several more were invited to a poster presentation[53].

- University of Applied Sciences Aachen with the project *Fördern und Fordern – Installation eines semesterbegleitenden Anpassungskurses*, support and demand – installation of an accompanying adaptation course,
- Technical University Berlin with the project *Tumult*, a multimedia blended-learning project for first-year engineering students,
- Ruhr-Universität Bochum with the project MP^2-Mathe/Plus/Praxis,
- University of Applied Sciences Hamburg with the project *Themenwochen zur Verknüpfung der Mathematik, Elektrotechnik und Physik im ersten und zweiten Semester*, topic weeks for the conjunction of mathematics, electrical engineering, and physics in the first and second semester,
- Ostfalia University of Applied Sciences with the project *MF&FM – Mehr Feedback und formative Assessments in der Mathematik*, and
- Technical University Vienna with the project *AKMATH/GKMATH – Auffrischungs- und Grundkurs Mathematik an der Technischen Universität Wien*, refreshment and basic course in mathematics.

The aim of Lehre^n is to create a trustworthy surrounding for a community of practice to exchange ideas and concepts. Representatives of the participating projects met several times to enable contact and collaboration.

The dghd (*Deutsche Gesellschaft für Hochschuldidaktik*, German society for university education) unites a considerable number of societies for university education, for all kinds of subjects. In particular for mathematics in engineering courses, there is teachING-learnING.eu[54], with offices in Aachen, Bochum, and Dortmund. teachING-learnING.eu describes itself as competence and service centre for teaching and learning in engineering sciences (*Kompetenz- und Dienstleistungszentrum für das Lehren und Lernen in den Ingenieurwissenschaften*) and offers a broad variety of information, meetings,

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[52] http://www.lehrehochn.de/fileadmin/user_upload/mathing/LehreN_MathIngBroschüre2014.pdf
[53] www.hrk-nexus.de/uploads/media/Posterdokumentation.pdf
[54] http://www.teaching-learning.eu

conferences, workshops and working sessions for all groups involved in engineering, including not only teachers / lecturers, but also students, companies, and other experts[55].

There are other associations aiming at networking and exchanging of experience and expertise, like *Regionaltreffen Hilfsangebote Mathematik* (regional meeting for assistance in mathematics), whose member affiliations slightly differ from those listed above. They have either already been mentioned in this section, or they do not meet the filter criteria, i.e. they do not conduct research in mathematics education or do not aim at engineering students.

The three biggest German organisations involved in mathematics education, DMV (Deutsche Mathematiker Vereinigung[56]), GDM (Gesellschaft für Didaktik der Mathematik[57]), and MNU (Mathematisch-Naturwissenschaftlicher Unterricht[58]) founded a commission for the transition from school to university, *Mathematik-Kommission Übergang Schule-Hochschule*. They are involved in the communication with the Ministries of Education in all of the 16 German *Länder* and compose statements concerning the standards of education for mathematics and their implications for schools, universities and the future of mathematics education in Germany. The three societies participating in the commission are themselves involved in assembling and discussing of curricula and educational guidelines. The members of this commission are active in their societies and at their universities and work on various projects in these institutions. The *Kommission Übergang Schule-Hochschule* itself, however, does not conduct any projects of their own.

2.1.6 Engineering courses at RUB

This section covers the specific course conditions at Ruhr-Universität Bochum (RUB). It gives an overview and details on course modules, examination regulations, and the work expected from the students. As such, it provides background to the project undertaking at hand, which must consider local actualities. Whenever these are relevant for the project design, it will be indicated in Chapter 4.

[55] http://www.teaching-learning.eu/ueber_uns/ueber_uns.html
[56] dmv.mathematik.de
[57] www.didaktik-der-mathematik.de
[58] www.mnu.de

Courses and course modules

Ruhr-Universität Bochum (RUB)[59] offers different courses in engineering. There is Mechanical Engineering (*Maschinenbau*, MB), Civil Engineering (*Bauingenieurwesen*, BI), Environmental Engineering and Resource Management (*Umwelttechnik und Ressourcenmanagement*, UTRM), Electrical Engineering and Information Technology (*Elektro- und Informationstechnik*, ET/IT), all of which are based on a three-year *Bachelor of Science* course and continue in *Master of Science* courses. There are various foci such as mechanics, energy and process technology, engineering informatics, construction technology and automation engineering, automobile engineering, micro engineering or material engineering, to name but those for the *Master of Science* course in mechanical engineering[60].

Admittance to engineering courses at Ruhr-Universität Bochum has varied in recent years. Due to local changes in school leaving examinations[61], all engineering courses now have locally valid *numeri clausi*. Depending on the average mark in the school leaving exam *(Abitur)*, aspiring students either get a place immediately or have to wait up to three semesters. The requirements for getting a place without waiting are moderate, though, as the average mark[62] necessary ranks from 2.9 (MB) to 3.1 (BI and UTRM) or may even be invalidated as in ET/IT where all applicants were accepted (cf. http://www.ruhr-uni-bochum.de/zsb/nc_werte.ws12.htm, retrieved 08/12/2013). These conditions are easily met by the majority of school leavers. In the years 2010-2012, universities were allowed to dispense with 60% of their places according to their own selection procedure (*Hochschuleigenes Auswahlver-*

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[59] Ruhr-Universität Bochum was founded in 1965 and is one of Germany's ten largest universities with more than 42,000 students from 130 countries, of which one third study Natural Sciences or Engineering, cf. www.rub.de.

[60] For civil engineering, students can choose between structural engineering, computational engineering, geo technology and tunnel construction, water and environmental management, and traffic engineering. Students aspiring to a degree in UTRM are offered management of processes and products, of energy, of infrastructure and traffic, or of water and soil.

[61] In 2005 North Rhine-Westphalia introduced 12 instead of 13 years of school, meaning increased numbers of school leavers in 2013.

[62] The best possible mark is 0.7, the theoretically possible worst pass mark is 4.0. According to the Ministry for School and Education in North Rhine-Westphalia, average Abitur pass marks deliver an average of 2.50 (in 2012), 2.52 (in 2011) and 2.56 (in 2010) with statistic deviations of 0.67, 0.66 and 0.66 respectively (cf. http://www.standardsicherung.schulministerium.nrw.de/abitur/, retrieved 08/12/2013).

fahren), as long as they gave away 20% of their places to the best students of each year, and 20% to those who had waited long enough. For their 60%, Ruhr-Universität Bochum decided for a mixture of average *Abitur* mark, waiting time, and lottery. In combination with a promotion policy to fill places otherwise left vacant, all this amounted to a rather heterogeneous level of qualifications for students accepted into engineering courses. One vital downside has to be mentioned, though: The complexity of these university entrance procedures not uncommonly resulted in delayed starts, as some students got their admission several weeks after lectures had started. Particularly in the natural sciences and technical subjects, where content builds on content, late starters are severely disadvantaged.

Table 2.9: Overview of Compulsory Courses for the First Semester in MB

Day	Scheduled Lectures	Times
Monday	lecture in Mechanics A, tutorial in Mathematics A, lecture in Mathematics A	10.00 – 18.00
Tuesday	tutorial in Mechanics A, lecture in Basics of Construction Technology, lecture in Materials I	08.00 – 16.00
Wednesday	lecture in Mathematics A, lecture in Physics A	14.00 – 18.00
Thursday	lecture in Mechanics A, lecture in Basic Chemistry, tutorial in Physics	10.00 – 16.00
Friday	practical course in Materials, lecture in Basic Chemistry	10.00 – 18.00

Note. Sessions last 90 – 120 minutes. Times given represent attendance time on campus, free sessions included.

An overview over the compulsory lectures, tutorials and practical courses is given in Table 2.9 (slight variations are possible from one year to another). It amounts to almost 30 hours of lectures and tutorials every week. The lecture and tutorial in mathematics represent a workload of 9 credit points. According to the European Credit Transfer System (ECTS), this translates into a workload of 270 hours per semester (Europäische Kommission, 2009). The lecture and tutorial add up to less than 100 hours, therefore students are expected to do two thirds of their work in mathematics outside of regular university sessions. This is obviously much more than they are used to doing from school. And if this rule were applied to all subjects (not only mathematics), it would mean an immense workload, presumably more than one would expect when starting a university course. In comparison, the OECD average was 1749 hours per year in 2010, and Germans worked 1419 hours per year[63], which translates to 35 to 45 hours per week, depending on the number of working weeks in a year.

All this results in the fact that a considerable number of average or below average students are confronted with demands that not only weigh heavy on their intellectual skills but which also demand the vast majority of their waking hours. It is not surprising that the problems occurring are difficult to overcome.

Examination regulations in mathematics

As mentioned above, there are three different engineering courses whose participants all attend the mathematics lectures MP^2-Math/Plus/Practice focuses on: Mechanical Engineering (*Maschinenbau*, MB), Civil Engineering (*Bauingenieurwesen*, BI) and Environmental Engineering and Resource Management (*Umwelttechnik und Ressourcenmanagement*, UTRM). For all three, the lecture *Mathematics 1* (*Higher Mathematics A* for BI and UTRM) is a required course in the first semester and has to be followed up by *Mathematics 2* and *Mathematics 3 and Numerical Mathematics* (*Higher Mathematics B* and *Higher Mathematics C and Basics in Numerical Mathematics*, respectively) in the subsequent semesters.

All students are registered automatically for the written exams in mathematics (Ruhr-Universität Bochum, 2009a, 2009b, 2009c, §5 (14), §5 (11), §5 (11) respectively) and cannot unregister (§5 (18), §5, (17), §5 (17)). If they fail an attempt, students are automatically registered for the next possible regular examination date (§5 (17), §5, (15), §5 (15)). What is more, all students only have the limited number of four attempts in order to pass the written exam (§5 (20), §12 (1), §12 (1)), plus an oral examination which means the restriction of a mere pass grade. Each failed attempt counts, non-attendance counts as failure (§5 (22), §5, (16), §5 (16)). Students are automatically registered for the mathematics lectures of the second and third semester even if they have not passed the first semester yet. Similar regulations apply for other obligatory first

[63] http://dx.doi.org/10.1787/888932505564

year courses, such as Physics, Mechanics and Chemistry (depending on the exact engineering course attended) which themselves are regarded as serious obstacles.

The written examination in *Mathematics 1* or *Mathematics A* respectively itself consists of 10 to 12 exercises, graded in a points system. It is possible to include multiple choice exercises, too, so some lecturers do so as it relieves the strain of grading several hundred papers. In order to avoid passing by coincidence, in some years minus points for wrong answers in the multiple choice questions were introduced. To get a pass grade, students have to accumulate a minimum of 50% of the points possible at the written exam. The bonus points acquired before with the help of homework or mini exams count for a little extra. Nevertheless, the pass rates have a longstanding tradition of around 50%[64]. The lecturers in mathematics and the department of engineering have decided that no technical tools whatsoever are allowed, meaning that in spite of working with graphing calculators at school, students have to compute everything by hand at university.

All these regulations mean that students must come to grips with their workload in their first year at university, otherwise they face the danger of having to cope with twice as much in their second year. As the official workload for a mathematics course is 270 hours per semester, of which two thirds are to be spent in private study (see section 2.1.6), this borders on the impossible.

The aim of these regulations is, naturally, to put pressure on students to apply themselves, to work hard, and to not waste time. The rules also discourage pro forma students who only enrol but do not attend courses, as they will quickly collect failed attempts and therefore must leave the course.

Lectures, homework, and tutorials

As depicted in Table 2.9, in *Mathematics 1* respectively *Mathematics A* the students are expected to attend four hours of lectures every week and two hours of tutorials. The lectures are held in lecture halls which seat more than

[64] According to Heublein, Schmelzer, and Sommer (2008); Heublein et al. (2012); Heublein, Richter, Schmelzer, and Sommer (2014), the dropout rate in engineering courses varies between 30% and 50% (meaning graduation rates between 50% and 70%), depending on the type of university, with a decreasing tendency. It is therefore legitimate to estimate the pass rates of single examinations lower.

Table 2.10: Contents of Mathematics Lecture in the First Semester

1 Sets, Numbers and Functions [✓]	9 Linear Algebra
2 Vectors ✓	10 Limits [✓]
3 Straight Lines and Planes ✓	11 Elementary Functions ✓
4 Systems of Linear Equations ✓	12 Differentiation ✓
5 Matrices [✓]	13 Applications of Differentiation ✓
6 Determinants	14 Integration ✓
7 Eigenvalues and Eigenvectors	15 Applications of Integration [✓]
8 Quadrices	16 Differential Equations

Note. ✓: usually covered in depth at school, [✓]: covered superficially

800 students with the help of microphones and screen presentations (individual lecturers may prefer chalk and blackboard). They follow the traditions of mathematics lectures in so far as the lecturers usually present the material from an axiomatic point of view (see section 2.1.2). In mathematics lectures for engineering students, however, proofs are often omitted and instead the stress is put on examples and applications relevant for engineering. The lectures are complemented by a printed (or printable) script and subsequent uploads of the transcriptions from the presentations during the lectures. And of course there are several books around which especially cater for engineering students and their demands on mathematics, e.g. Papula (2011), Papula (2010) or Meyberg and Vachenauer (2003).

The depth of the mathematics taught to engineering students in their first year, however, is reasonable. Several branches are also taught at school and students should, theoretically, be familiar with the concept of derivation and integration. The themes are shown in detail in Table 2.10[65], together with indications if they are commonly dealt with at school. In an ideal world, a considerable part of the content of the first semester would not really be new for the students. In reality, however, the school curriculum allows variations, different foci and omissions, owing to the homogeneity accepted in modern classrooms.

There is homework, usually four exercises per week which can be handed in in order to be graded which in turn can lead to a few bonus points counting

[65] http://www.ruhr-uni-bochum.de/imperia/md/content/mathematik/service-zentrum/skripte/mbbi1_2012.pdf, pp. 2-3

for the final written examination. Additional analogous exercises are discussed in the tutorials, in comparably small groups of no more than 40 students, in compliance with constructivist learning theories, see section 2.1.2. The tutorials are held by teaching assistants, research assistants or senior students. The steps in the solutions are mostly explained in detail and more slowly in the tutorials than would be possible during the lecture. The majority of students prefer to work on their mathematics homework in groups, at least at some stage during the solution process. This is encouraged by the fact that it is traditionally allowed to hand in homework in groups of up to three students. Although this may also foster mindless copying of exercises, it also opens the perspective for fruitful cooperative learning. Both the engineering and the mathematics department have rooms available in their libraries and cafeterias where students can sit together and discuss their tasks and homework. Hence many of them spend a good part of their time on campus, even outside lecture times.

It has become a custom, too, to have mini exams two or three times during the semester. These usually last 15 to 20 minutes and are held at the beginning of a lecture. They consist of four short exercises, sometimes multiple choice, and cover the lecture contents of three to four weeks. Students are granted bonus points that count for the final examination. In order to prevent cheating in the crowded lecture halls, there are up to three different sets of tasks. The mini exams are graded by the teaching assistants and discussed in subsequent session of the tutorials. The mini exams are regarded as serious tests as they are written under similar conditions (independent performance, no calculators or other technical tools) and are therefore the starting point of MP^2-Math/Plus, see chapter 4.3.1.

Some lecturers additionally offer collective tutorials where solutions for the weekly homework tasks are presented. This may differ from one year to the other according to attendance and personal resources. The number of uploads in the accompanying e-learning course also varies substantially, depending on the preference of the lecturers and the time on their hands. A perfectly-elaborated example is the year 2012/2013 where every single lecture was accompanied by a pre-learning video of five to seven minutes which introduced the topic of the oncoming lecture and ended with a multiple choice question which again awarded some bonus points for the final examination.

The university's SZMA helpdesk provides extra support for students attending a course in service mathematics seeking specific support: The department of mathematics at RUB has been aware of the problems students from other departments face when they attend lecture in mathematics, and therefore founded a *Service Centre for Mathematics and Applications Servicezentrum für Mathematik und Anwendungen*, SZMA in 2007[66], which offers help and support. Among other things, the SZMA offers training courses for tutors and a helpdesk. This helpdesk is open three hours every weekday afternoon[67] and staffed by teaching assistants, tutors and senior students employed for marking homework. Students can visit the helpdesk without an appointment (preferably when someone associated with their mathematics lecture is on duty) and get their questions answered.

The conditions described boil down to the fact that the demands in mathematics on engineering students are considerable – particularly when it comes to choosing the modalities which work for them and to studying independently.

2.2 Learning Strategies

This section is part of the broad Chapter 2 which contains the theoretical background relevant for our study. It presents the information on learning strategies, as the last step before the research approach and objectives are addressed in Chapter 3.

When aiming at an improved learning output, it is imperative to look closely at the processes of learning, i.e. of building competence. In agreement with e.g. Wild (2005) and Rach and Heinze (2011), a promising perspective is recognised in exploring general and meta-level skills in terms of learning strategies, whose investigation allows for revealing both the cognitive dispositions as well as affective barriers and pathways – and finally the interrelations between them. As shown in section 2.1.5, numerous research and instruction projects that attend to these issues can be found in Germany alone (for an overview of international projects, see Dunn et al., 2012). The causality between adequate learning strategies and successful learning seems well established (for example cf. Erdem Keklik & Keklik, 2013), although the perspective of how to sustainably

[66] http://www.ruhr-uni-bochum.de/ffm/szma/
[67] The opening times were expanded to four hours every weekday in 2013.

encourage learning strategies, to identify justified combinations of interventions, and to understand the influence of motivation needs to be further explored.

Apart from a talent for abstract thought and formalities, mastering a university course containing mathematics needs a combination of broader capacities, like general skills and attitudes such as self-organisation, perseverance and frustration tolerance, as well as subject-specific abilities (cf. Pintrich, Smith, Garcia, & McKeachie, 1993; Weinstein & Palmer, 2002; Wild, 1994). It is students' meta-level learning behaviour[68] that is crucial, taking account of the words of de Guzmán et al. (1998), who state:

> Students' success is linked to a great extent to their capacity of developing "meta-level" skills allowing them, for instance, to self-diagnose their difficulties and to overcome them, to ask proper questions to their tutors, to optimise their personal resources, to organise their knowledge, to learn to use it in a better way in various modes and not only at a technical level (p. 760).

Thus, when looking for a suitable catalogue of learning strategies, it is essential to search for a comprehensive inventory that includes these skills. Accordingly, learning strategies are understood as all kinds of planned and conscious learning behaviour and the attitudes behind it, involving observable actions (e.g. solving tasks, asking questions, taking notes) as well as thought processes (e.g. planning, reflecting) on the basis of both cognitive and affective-motivational dispositions[69]. This perspective is supported by Blömeke, Gustafsson, and Shavelson (2015) who contribute to modelling competence "as a process, a continuum with many steps in between" (p. 7). In particular they emphasise the following approach:

> Thus, we suggest that *trait* approaches recognize the necessity to measure behaviorally, and that *behavioral* approaches to competence recognize the role of cognitive, affective and conative resources. At this time, we encourage research on competence in higher education emanating from either perspective and paying attention particularly to the steps in between (p. 7).

[68] However, de Guzmán et al. (1998) do not mean metacognition in the sense described in section 2.1.2 when characterising meta-level learning behaviour, but rather learning strategies in terms of Wild (2005), see Figure 2.9.

[69] This extends to a lack of planning and conscious actions as well, as it also presents a characteristic of an individual's learning strategy.

As depicted in section 2.1.2, research on the significance of learning strategies in mathematics education has its roots in contributions highlighting the role of affect, motivation and beliefs (McLeod, 1992; Hannula, Evans, et al., 2004; Leder et al., 2002; Leder & Grootenboer, 2005; Roesken & Casper, 2011; Zan et al., 2006), as all cognitive processes involve affective stances that moderate the tension between modes of intuitive and analytical thinking (e.g. Fischbein, 1987; Stavy & Tirosh, 2000). It can be said, therefore, that this has led to a fortified interest in certain kinds of learning strategies. In the context of mathematics, overcoming motivational and affective barriers with the help of meta-skills, e.g. self-regulation, has become an important issue. What is more, mathematics demands the use of effective planning as well as organised and consistent work (cf. Rach & Heinze, 2011). More than many other subjects, mathematics is cognitively challenging and needs motivational perseverance, thus representing an ideal research area for the influence of interventions addressing learning strategies, both on a general and a meta-level. The goals when assessing students' learning behaviour are various, however: taking an inventory, describing the development, comparing or improving learning behaviour (cf. Lovelace & Brickman, 2013), and so are the research interests, in mathematics popularly performance prediction or the identification of at-risk students. Apart from more time-consuming methods (like observing groups of students or individual students when learning mathematics, using thinking-aloud and videotaping, or keeping track of the learning process by convincing learners to write detailed learning logs), this has led to a great variety of questionnaires in many languages.

2.2.1 Questionnaires for assessing learning strategies

Questionnaires with different focal points (according to the background of the authors and their research interests) originate from this variety (cf. Pintrich et al., 1993; Weinstein & Palmer, 2002). Schellings (2011) gives a comprehensive overview from an international and a Dutch perspective. Though her work is based on the text-heavy learning of history, the general categories of learning behaviour can be applied to other subjects as well. Differentiating between motivational and cognitive aspects when dealing with learning strategies is a widely accepted concept (cf. Nenniger, 1999) and is in keeping with the understanding of affective aspects as a key issue.

In the following, approaches to capture learning strategies which have influenced subsequent research fundamentally will be outlined. The selection includes only those which reflect the importance of affective and motivational issues. Pintrich et al. (1993) developed a questionnaire “to measure college undergraduates’ motivation and self-regulated learning” (Artino, 2005, p. 3), the Motivated Strategies for Learning Questionnaire (MSLQ). The MSLQ measures motivation and self-regulated learning in general and for a particular course by means of six motivation and nine learning strategies subscales. Initially, Pintrich and de Groot (1990) started by postulating a five latent factor structure comprising expectancy, value, affect, learning strategies, and self-regulation. The items that were developed for operationalising these constructs later formed the basis for the 15 subscales (six for motivation, nine for learning strategies) mentioned above. MSLQ has been applied in many research studies (Duncan & McKeachie, 2005), partly aiming at developing a new conceptualisation with respect to the significance of the single sub-scales (Dunn et al., 2012; Hilpert, Stempien, van der Hoeven Kraft, & Husman, 2013). MSLQ’s reliability has turned out to be “robust” and its predictive validity to actual course performance is considered “reasonable” (Pintrich et al., 1993, p. 801).

The Approaches to Studying Inventory (ASI) by N. Entwistle and Ramsden (1983) and its refinements (ASSIST by Tait, Entwistle, & McCune, 1998, ALSI by N. Entwistle & McCune, 2004) feature the main distinction of categorising learning behaviour as being of either strategic (deep) or of apathetic (surface) approach. The dichotomy forms the inventory’s two main factors which in turn contain up to 16 subscales, depending on the version of the questionnaire. Although the authors do not group their items into motivational and (meta-)cognitive scales, the object of research is nearly identical to that of MSLQ users. A specific feature of ASI and its variations is the idea to measure not only the desired learning behaviour (strategic approach), but also what is hypothesised as less success-oriented (apathetic approach). This produces a multifarious picture of learning behaviour.

Another well-known instrument to capture students learning strategies is the Learning and Study Strategies Inventory (LASSI) by Weinstein and Palmer (2002). LASSI covers thoughts, behaviour, attitudes and beliefs in relation to successful learning that can also be fostered by interventions. Its ten scales are classed into affective strategies, goal strategies, and comprehension monitoring strategies, thus covering cognitive, metacognitive (particularly self-regulative), affective and motivational aspects. LASSI is not only used for research purposes, but is also recommended to students to use for themselves in order to get feedback on their strengths and weaknesses. LASSI's reliability coefficients (Cronbach's α) for its different scales are reported to score between .86 and .68, the lowest often being considered insufficient (Weinstein, Schulte, & Palmer, 1987). Its validity to academic performance depends on the specific scale, e.g. Cano (2006) found, using multiple regression, that two scales (namely *Affective Strategies* and *Goal Strategies*) contributed to academic performance, whereas one (*Comprehension Monitoring Strategies*) did not.

All questionnaires described so far resort to self-assessment of student behaviour. It must be conceded that this entails the weakness that "the learner's perceptions of his or her strategies are measured" (Schellings, 2011, p. 94), which need not coincide precisely with the strategies themselves. In this context it is interesting to compare self-reported learning behaviour (especially concerning metacognition) to the results gained with other methods, e.g. *Thinking Aloud* (David, 2013). Considering affective aspects, however, the learner's subjective perspective is what counts. Other problems, like assessing the sufficiency or efficiency of study time or effort, might be harder to overcome.

2.2.2 Origin and structure of the LIST questionnaire

For our research at Ruhr-Universität Bochum in Germany, the decision fell for the German LIST questionnaire (Learning Strategies at University, Wild and Schiefele, (1994)), which is based on the same classification as MSLQ and takes up aspects from LASSI as well. LIST was invented for measuring learning strategies of medium generality, between learning styles and learning tactics (Wild, 2000). The instrument distinguishes between cognitive, metacognitive and resource-related learning strategies and comprises dimensions of learning strategies grouped accordingly. This mirrors the acceptance of this taxonomy in the German-speaking community (Wild, 2000). Just like its English predecessors, this approach originates from educational research and thus is not subject-specific. However, in university mathematics education the instruments are frequently used to assess students' learning behaviour on a general level while combining the results with subject-related measures (Lovelace & Brickman, 2013).

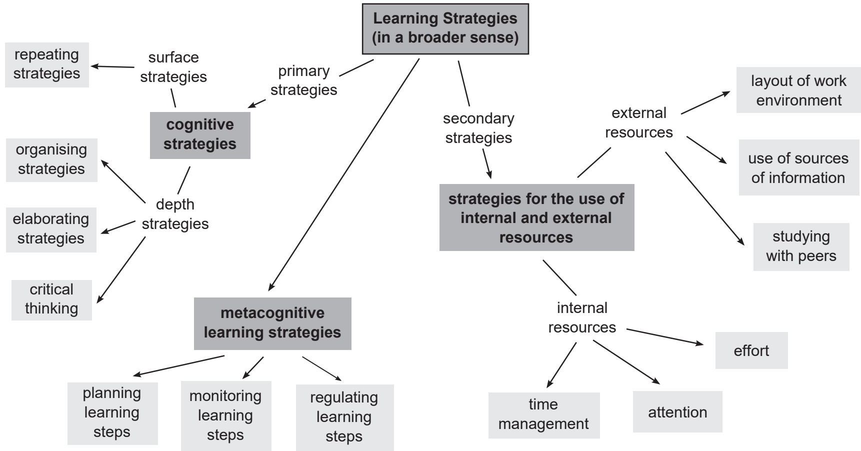

Figure 2.9: Overview of Learning Strategies (Wild, 2005, p. 194), translation by author

LIST covers various learning strategies, which are divided into cognitive, metacognitive, and resource-oriented scales, see Figure 2.9. According to Wild (2005), cognitive (primary) learning strategies encompass surface learning (repeating, rote-learning) as well as deep learning strategies (e.g. re-organising of and elaborating on subject matter[70]). Metacognitive learning strategies cover planning, monitoring, and regulating the next steps in the learning process. Resource-oriented learning strategies are subdivided into those using external resources (e.g. sources of information, working with peers) and those employing internal resources (effort, attention, and time management). The version of LIST employed in our research can be found in Appendix A, page 239, along with an English translation in Appendix B, page 254, see also Table I.1 on page 280.

The LIST questionnaire for measuring learning strategies in academic studies was first compiled in the 1990s (Wild & Schiefele, 1994) and has since been modified and tested several times. LIST has been applied in the context of many subjects, mathematics among them (cf. Liebendörfer et al., 2014, for an overview), with overall satisfying results with regard to reliability (Wild and Schiefele (1994) found Cronbach's α between .64 for *Metacognition* and .90

[70] Particularly elaborating strategies are regarded as central for the learning of mathematics, see Göller et al. (2013).

for *Attention*) and validity (Wild, 2000, 2005). In the following, how LIST takes up scales and items from MSLQ and LASSI is explored in detail, in order to understand its origins and to illustrate its structure.

Apart from *Motivation*, the scales from LIST are derived directly from MSLQ, although the number of items varies. Some items in LIST are translations of MSLQ items. In comparison to LIST, MSLQ seems very differentiated in terms of motivation, it incorporates six *Motivation* scales (*Intrinsic Goal Orientation*, *Extrinsic Goal Orientation*, *Task Value*, *Control of Learning Beliefs*, *Self-Efficacy for Learning and Performance*, *Test Anxiety*) comprising 31 items. LIST does not have items with the label *Motivation* as such, but LIST's six items on *Attention* (which are all reverse coded) and eight items on *Effort* more or less cover this aspect, for example "I work late at night or at the weekends if necessary". And other LIST scales, in particular the resource-oriented ones, are meant to measure the degree of motivation a student possesses when preparing for an important exam, with items like "I fix the hours I spend daily on learning in a schedule". The main difference between the two questionnaires is that MSLQ puts more emphasis on including different aspects of motivation as *Goal Orientation*, or *Control of Learning Beliefs*. For LIST, on the other hand, the aim was to clearly keep apart cognitive and motivational aspects.

LASSI (Weinstein & Palmer, 2002) also separates cognitive aspects, but has much less communalities with LIST. LASSI scales partly cover the same contents though holding different names, e.g. *Concentration* and *Attitude* (LASSI) compared to *Attention* (LIST). The numbers of items in a scale are different, too: There are 3 to 8 in LIST (if *Metacognitive Strategies* are divided into three scales, 4 to 8 if not), 3 to 12 (3 to 8) in MSLQ, and a constant 8 items in all LASSI scales. This results in considerable differences in analogous scales between LIST and MSLQ: LIST has 31 in *Cognitive Strategies* whereas MSLQ has 19 in the respective scales. According to the inventors of LIST, scales were expanded in order to reach better reliability (Wild, 2000). All three questionnaires use Likert scales, ranging from 5 points (LIST) over 6 (LASSI) to 7 (MSLQ). An overview on how LIST is based on MSLQ and LASSI is provided in Table 2.11.

Table 2.11: Synoptic Table for LIST, MSLQ, and LASSI

LIST		MSLQ		LASSI	
Cognitive Strategies	Critical Checks (8 items)	Learning Strategies	Critical Thinking (5 items)	Affective^{A} Strategies, Goal^{G} Strategies, Comprehension^{C} Monitoring Strategies, see also below	
	Elaborating (8 items)		Elaboration (6 items)		Information Processing^{C} (8 items)
	Organizing (8 items)		Organization (4 items)		Selecting Main Ideas^{G} (8 items)
	Repeating (7 items)		Rehearsal (4 items)		
Resource-related Strategies	Attention (6 items)		Motivation$*$ (see below)		Attitude^{A} (8 items)
	Effort (8 items)		Effort Regulation (4 items)		Concentration^{A} (8 items)
	Learning Environment (6 items)		Time / Study Environmental Management (8 items)		
	Time Management (4 items)				Time Management^{A} (8 items)
	Peer Learning (7 items)		Peer Learning (3 items)		
	Using Works of Reference (4 items)		Help Seeking (4 items)		Study Aids^{C} (8 items)
Metacogn. Strat.	Planning (4 items)		Metacognitive Strategies (12 items)		Self-Testing^{C} (8 items)
	Monitoring (4 items)				
	Regulating (3 items)				

A = affective strategies, G = goal strategies,
C = comprehension monitoring strategies

Synoptic Table for LIST, MSLQ, and LASSI, continued.

<table>
<tr><th colspan="2">LIST</th><th colspan="2">MSLQ</th><th colspan="2">LASSI</th></tr>
<tr><td rowspan="2">Resource-rel. Strat.</td><td rowspan="2">Attention
(see above)</td><td rowspan="6">Motivation</td><td>Control of Beliefs*
(4 items)</td><td rowspan="7">Affective Strategies[A], Goal Strategies[G]</td><td rowspan="2"></td></tr>
<tr><td>Self-Efficacy for Learning and Performance*
(8 items)</td></tr>
<tr><td rowspan="5"></td><td rowspan="5"></td><td>Intrinsic Goal Orientation
(4 items)</td><td rowspan="2">Motivation[A]
(8 items)</td></tr>
<tr><td>Extrinsic Goal Orientation
(4 items)</td></tr>
<tr><td>Task Value
(6 items)</td><td></td></tr>
<tr><td>Test Anxiety
(5 items)</td><td>Anxiety[G]
(8 items)</td></tr>
<tr><td></td><td></td><td>Test Strategies[G]
(8 items)</td></tr>
<tr><td colspan="2">77 items</td><td colspan="2">81 items</td><td colspan="2">80 items</td></tr>
</table>

A = affective strategies, G = goal strategies

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