

Contents

1	Introduction	1
2	Auxiliary Material and Notation	3
3	The Continuous Setting	9
3.1	Hardy spaces	9
3.1.1	Boundary values of holomorphic functions	9
3.1.2	Integral formulas	13
3.1.3	Fourier series of the boundary functions	14
3.2	The Hilbert transform	17
3.3	Riemann-Hilbert problems	23
3.3.1	Linear Riemann-Hilbert problems	24
3.3.2	Nonlinear Riemann-Hilbert problems	25
3.4	Obtaining the Hilbert transform from a Riemann-Hilbert problem	26
3.4.1	Choice of the problem	26
3.4.2	Solutions of the problem	27
3.4.3	Counterexamples for $u \notin \mathcal{C}^{1+\alpha}$	31
4	Circle Packings	35
4.1	First examples and ideas	35
4.2	Basic definitions	39
4.2.1	Complex	39
4.2.2	Circle packing	42
4.3	Manifold structure	44
4.3.1	Contact function	45
4.3.2	Angle sums and branch structures	46
4.3.3	Parametrization of \mathcal{D}_b	49
4.3.4	Normalization	50
4.4	Discrete harmonic functions on circle packings	52
4.5	Discrete analytic functions	55
4.6	Maximal packings	56

4.7	Some results on discrete analytic functions	57
4.7.1	Discrete maximum principles	58
4.7.2	Approximation of the Riemann Mapping	58
5	Discrete Hilbert Transform	61
5.1	Discrete boundary value problems	62
5.1.1	Definition and examples	62
5.1.2	Linearization of boundary value problems	67
5.2	Proof of the maximal packing conjecture	68
5.2.1	The transformed packing	70
5.2.2	Differential of $\tilde{\omega}$ at the transformed packing	71
5.2.3	Basis for the kernel of $\tilde{\mathcal{J}}$	76
5.3	Discrete Hilbert transform	79
5.3.1	Difficulties of the Schwarz problem	80
5.3.2	Discretization of the nonlinear problem	82
5.3.3	Linearization of the discrete operator	86
6	Numerical Results and Future Work	89
6.1	Test computations	89
6.2	Eigenvalues of the discrete transform	91
6.3	Elimination of constants	94
6.4	Curvature of the Circle Packing manifold	96
6.5	Local frames	97
	Bibliography	101

A Discrete Hilbert Transform with Circle Packings

Volland, D.

2017, XI, 102 p. 27 illus., 10 illus. in color., Softcover

ISBN: 978-3-658-20456-3