# 2. Theoretical framework

In analysing how students[1] solve spatial geometrical tasks, I am interested in how language can play a role in the students' solving process. In a wider context, I intend to study the relationship between language and spatial ability for a deeper understanding of language and content learning in mathematics education. This section provides an overview of the theoretical frameworks in both domains of language and spatial ability, followed by theoretical considerations on the integration of both domains based on findings from previous literature.

In Section 2.1, the theoretical findings about spatial ability from psychology and mathematics education are presented. In order to better understand the concept of spatial ability and to account for the design of the main study later in this work, selected models of spatial ability and other important results from previous studies about spatial ability are discussed. The major findings about language and its use in mathematics learning and teaching from previous research, which includes the functions of language, the use of metaphors and different levels of language acquisition in mathematics classroom are presented in Section 2.2. In Section 2.3, the notion of spatial language as an interface between language and spatial ability is introduced by referring to important results from psycholinguistic studies. Theoretical approaches to the representation of mathematics knowledge, such as Bruner's stages of representation and Sfard's theoretical framework for investigating the role of algorithms in mathematical thinking, are presented in Section 2.4.

## 2.1 Spatial ability

Traditionally, spatial abilty has been extensively researched in the field of psychology (e.g., Thurstone 1950; Linn & Petersen, 1985). However, the past two decades have seen a renewed importance of spatial ability in mathematics education (e.g., Pinkernell, 2003; Casey et al., 2008; Büchter, 2011; Plath, 2014). The considerable amount of literature on spatial ability led to the establishment of a wide spectrum of definitions of spatial ability, some of which are introduced in the first part of this section. The next section, Section 2.1.2, addresses the cognitive processes playing an important role in solving spatial tasks. Based on the

[1] In the rest of this thesis, the terms 'students' is used to refer to students at primary or secondary level.



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different definitions of spatial ability, different models of spatial ability from psychology and mathematics education are introduced in Section 2.1.3. In Section 2.1.4, the role of spatial ability in mathematics classroom and its relationship to geometry learning and teaching based on theoretical discussions from previous literature and based on German mathematics curriculum are discussed. This is followed by the discussion of different strategies which, according to previous literature, students use during solving of spatial tasks in Section 2.1.5. The last part of this section, Section 2.1.6, is dedicated to the influence of sex on solving spatial tasks based on previous reported empirical results.

### *2.1.1 Definitions of spatial ability and approaches to its research*

One can find several definitions of the term *spatial ability*[2] in the literature, but spatial ability seems to be a collection of abilities for the manipulation of space, which are required to solve spatial tasks. Similar to the notion of problem in problem solving (cf. Pólya, 1949/1980; Kantowski, 1980; Carpenter, 1988), in this work, spatial tasks will be considered as non-routine tasks demanding spatial ability and other abilities and as tasks which require students to develop ways to overcome the obstacles in space in order to reach the goal of the spatial task. Due to the interchangeable use of the notions of *spatial ability*, *spatial knowledge*, *spatial intelligence*, *spatial thinking* and *spatial reasoning* in previous literature, a brief distinction between these notions will be undertaken for a better clarification of their use in this work. Whereas spatial ability encompasses different abilities required to navigate in space and solve spatial tasks (both in mental and real terms), spatial knowledge (perhaps also spatial intelligence often used in psychometric approach to spatial ability) seems to emphasise primarily the spatial information and positions in space stored and activated in the individual's cognitive system, whereby it can be stored using multiple representations (e.g., pictorial or symbolic). However, spatial knowledge can also encompass knowledge about strategies of dealing with spatial tasks and navigating in space from previous experience in solving spatial tasks. Whereas spatial reasoning emphasises the use of logic for solving spatial tasks, spatial thinking encompasses mental processes characterising the solving of a spatial task, in which students apply spatial reasoning and are required to develop strategies for reaching the goal addressed by the underlying spatial task.

[2] The plural of spatial ability, *spatial abilities*, emphasises the different components in spatial ability models.

Pinkernell (2003) defines spatial ability as the ability to mentally operate on spatial objects in space. Similarly, Thurstone (1938) views spatial ability as the ability to mentally operate on two and three-dimensional objects. In their definition of spatial ability, Linn and Petersen (1985) define spatial ability as referring to "skill[s] in representing, transforming, generating, and recalling symbolic, nonlinguistic information" (Linn & Petersen, 1985, p. 1482). Maier (1999) considers spatial ability not merely as the cognition and perception of visual information, but also mental processing of this information should belong to the domain of spatial ability. He defines spatial ability as the ability to view mental images in space and to think spatially (cf. Maier, 1999). The development of mental images, which do not necessarily require a reference object in real life, is an important spatial ability, however, a person with high spatial ability should also be capable to use these mental images and rearrange these mentally for developing new mental images from the ones available (cf. Maier, 1999).

From a mathematics education perspective, the notion of spatial ability should encompass not only the mental representation of space and its transformation in terms of Maier (1999) or Pinkernell (2003), but also in real actions. This includes spatial tasks which might also require motoric skills, such as navigating through a labyrinth or streets of a city, or moving or rotating a spatial object, or folding paper to solve particular spatial tasks. Hence in this present study, spatial ability shall be defined as a collection of abilities which change or support our perception of space and can be represented either mentally or in real terms using different representations of spatial knowledge.[3] The term *spatial abilities* will be used to emphasise the different abilities in spatial ability, especially concerning the assessment of spatial ability (see Section 2.1.3).

Similar to the different definitions of spatial ability, there are different approaches to and perspectives in research about spatial ability, depending on the underlying aims and goals of research. Linn and Petersen (1985) differentiate between four different research perspectives of spatial ability – strategic, cognitive, psychometric and the differential perspective. In the strategic research perspective, the test persons' strategies during the spatial task solving process are investigated qualitatively and identified, whereas the goal in the cognitive perspective is to identify and characterise cognitive processes during the spatial task solving pro-

[3] In contrast to Linn & Petersen's (1985) definition of spatial ability, in this thesis, spatial abilities can also consist of recalling linguistic or symbolic information.

cess (cf. Linn & Petersen, 1985). In the cognitive perspective, mental processes during the spatial task solving process are more important to characterise spatial abilities than the test results (cf. Linn & Petersen, 1985). In the third research perspective, the psychometric perspective, spatial ability is divided into subfactors or components by comparing correlations between different spatial tasks (cf. Linn & Petersen, 1985). Psychometric research was prominent among psychologists (e.g., Thurstone, 1938) who describe different factors of spatial ability in their models (see Section 2.1.3.3). In the differential perspective, research is characterised by a "comparison of spatial ability for different populations" (Linn & Petersen, 1985, p. 1480), whereby the performance in spatial ability tests are indicators for differences in spatial ability among different groups, for example, males versus females (cf. Linn & Petersen, 1985).

The foci in this present study include strategies and obstascles of students during the solving of spatial-verbal tasks which will be identified by analysing the use of language in order to understand how students solve spatial tasks. This means that the strategic and cognitive perspective of research on spatial ability are the most relevant for this present study.

### *2.1.2 Cognitive processes in solving spatial tasks*

There are several cognitive processes which are activated when students solve spatial tasks. In her study about how fourth-grade students solve spatial tasks, Plath (2014) states that perception of visual information, processing of information, generation of mental images and thinking are the most important cognitive processes taking place during the spatial task solving process. These cognitive processes should not be analysed as separate, but rather as mutually influential and as building up on each other, preferably in a cycle in the order they are introduced in the following sub-sections, to construct a complex system of cognitive processes (cf. Plath, 2014).

#### *2.1.2.1 Visual perception*

In general, the process of perception can be described as a process in which the person becomes aware of a particular knowledge in a particular environment. According to Lurija (1992), in modern psychology, perception is an active process consisting of several steps to be mastered:

> It [modern psychology] describes perception as an active process, in which information is searched, unique characteristics of an object are identified, these characteristics are compared to each other, adequate hypotheses are formulated and comparisons among these hypotheses and the initial data are made. (Lurija, 1992, p. 230)[4]

According to Frostig, Horne and Miller (1977) there are different types of visual perception. The most perceptive skills relevant for this present study are figure-ground perception, spatial relations perception and spatial position perception, which are explained below (the other ones can be found in Maier, 1999).

a) *Figure-ground perception*: This perceptive process deals with the ability to identify a figure or an object from an optical complex background or to identify parts of the object from a given object and isolate them (cf. Maier, 1999).

b) *Spatial relations perception*: The visual perception of spatial relations is the ability to visually identify and describe the relation between two or more spatial objects, for example, the ability to describe the spatial relation between two solids in geometry (cf. Maier, 1999).

c) *Spatial position perception*: In the perception of position in space, an individual shows his or her ability to identify and perceive the position of an object in space under consideration of his or her own body. Maier (1999) describes this perception as being close to the perception of spatial relations, the only difference being the importance of the body position for solving the spatial task successfully. The main difference between spatial position and spatial relations is that in spatial position one of the objects to which it is related to must be the observer's body.

Hence, visual perception in space is the perception of spatial information from the surrounding environment, which requires selection of information of specific spatial or geometrical characteristics of spatial objects, or spatial relations between two objects, or the spatial position of one's own body in space and relate it other spatial objects. The perception of such spatial information is needed for the creation of mental images in the cognitive system, which is discussed in the next section.

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[4] This quotation was translated from German to English and is reproduced without any further alterations from Lurija (1992, p. 230) by A.M.

*2.1.2.2 Mental images*

The notion of *mental images* and their formation is an important cognitive process when solving spatial tasks, and is the subject of different definitions in literature. Mental images are described as "the mental invention or recreation of an experience that in at least some respects resembles the experience of actually perceiving an object or an event, either in conjunction with, or in the absence of, direct sensory stimulation" (Finke, 1989, p. 2). Finke (1989) states that mental images are spatially arranged as objects are arranged in space, and that "imagined transformations and physical transformations exhibit corresponding dynamic characteristics" (Finke, 1989, p. 93). However, not every mental image, as an abstraction of a perceived external stimulus, does necessarily require a real object which corresponds one to one to it; as in the example of a mental image of a blue horse, which is formed by combining the images of 'blue' and 'horse' together to create a new mental image. Therefore, the process of mental image creation is an active process which enables the creation of new mental images from existing knowledge and enables new produced mental images to be used for the individuals' thinking and acting in the environment (cf. Plath, 2014).

*2.1.2.3 Information processing*

The processing of information during solving of spatial tasks is another important cognitive process following visual perception. There are two different approaches in psychology to explain how information is processed in the mind: propositional-code (cf. Pylyshyn, 1973) and dual-code theories (cf. Paivio, 1971). In the former theory, Pylyshyn (1973) proposes that information is coded as prepositions, which are defined as the smallest units of knowledge, and contain a relationship called the predicate and a number of arguments. Pylyshyn (1973) argues that propositions should not be equated with a string of words, and their representation in the mind cannot be solely described by pictures or words:

> A proposition is what a string of words may assert. A proposition is either true or false; the string of words is neither. A proposition may be asserted by a number of strings of words, in any language and in any modality. Furthermore, in the sense in which we use it when we speak of "propositional knowledge," it may involve no words of any kind. Thus when I look at the table in front of me, I see that there is a vase on it. I do not "see" patches of light or only an array or objects. My knowledge is en-

> riched by (among other things) the proposition asserted by a sentence such as "The vase is on the table", even though I did not utter (audibly or otherwise) this or any other sentence. (…) When we use the word "see," we refer to a bridge between a pattern of sensory stimulation and knowledge which is propositional. (Pylyshyn, 1971, p. 6)

Hence, Pylyshyn's (1971) model supports the idea that information is processed in a system of propositions, irrespective of whether the information is visual or verbal. In contrast, Paivio (1971) introduces a more widely acknowledged model of information processing which differentiates between the two kinds of information processing for verbal and imagery, the dual-code theory. The dual-code theory states that nonverbal visual information is processed in the image system and verbal information is processed in the verbal system (cf. Paivio, 1971). However, whilst both systems are not dependent on each other, they are interconnected (cf. Paivio, 1971). The following example should clarify the relationship between both systems in the dual-code theory. Consider an individual who is watching an elephant on television, this does not necessarily mean that he or she produces a mental image in the image system, but it could activate verbal information in the verbal system. Reciprocally, if an individual is exposed to the word 'elephant' it might lead to the activation of more verbal information and perhaps to the production of a corresponding mental image (cf. Plath, 2014). However, if the verbal information is very abstract, the individual is less likely to produce a mental image of the corresponding verbal information (cf. Paivio, 1971). According to Paivio (1971), information processes consist not only of the production of mental images or verbal information, but also of the connection between the image and verbal system. In terms of spatial information processing, Pavio's (1971) dual theory captures two important planes for representing and hence most likely also for processing spatial information: the visual level, which is essential for space perception and for storing mental images of spatial constellations or transformations which are not yet verbally processed, and the verbal level, which is vital for abstaction of spatial knowledge and its verbalisation.

*2.1.2.4 Thinking process*

After the perception of spatial objects on a visual level, processed information arising from mental images requires thinking as a cluster of mental actions for real actions (cf. Aebli, 1980). In her neuropsychological approach to the notion of thinking, Lurija (1992) describes thinking as a set of processes which can be

categorised in three phases. According to Lurija (1992), thinking processes are triggered by the subjects' need to consider a task or problem and find a solution for it urgently, and when the subject is confronted with a situation, which does not have any ready solution (innate or habitual). In the second phase of thinking processes, subjects analyse the task requirements and components, identify the most important characteristics and compare these with each other (cf. Lurija, 1992). The third phase, also known as strategic thinking, consists of the subject's choice of a solution among different possibilities, and the development of a plan for finding the solution of the problem (cf. Lurija, 1992). Hence, the phases of thinking processes in Lurija (1992) coincide with the notion of problem solving (i.e. development of strategies) and metacognition in mathematics education.

Based on Freud's (1982) definition of thinking as inner experimental test action, Spering and Schmidt (2009) point out that thinking, which is defined as mental monitoring of reality, does not necessarily induce real actions, since an action developed from thinking processes can be performed mentally. The objects of thinking processes or thoughts are dependent on the individual and his or her knowledge, and on the context or task in which the subject needs to make up a plan, decide, and reason to solve a task (cf. Spering & Schmidt, 2009). However, objects of thoughts can also be thoughts themselves, whereby the subject is able to analyse his or her way of thinking and analyse thinking processes to process more compex reasoning[5] or search for errors in reasoning (cf. Spering & Schmidt, 2009).

In this section, four cognitive processes which are essential for solving spatial tasks have been introduced. However, this should not exclude that other processes can play an important role in solving spatial tasks. For instance, in developmental psychology, language is regarded to be a major factor influencing thinking processes. Vygotsky (1993) considers language as an inner speech which is regarded as a medium for supporting thinking processes. A detailed account of the relationship between thinking processes and language is discussed in Section 2.2.2. Vygostky's (1993) approach to thinking process supports the assumption that thinking processes should not be analysed separately from the other cognitive processes. However, in a spatial context, thinking processes should be an essential aspect for processing visual or verbal spatial information and manipulat-

[5] In this thesis, thinking denotes all thinking process which are produced either conscious or unconscious, whereas reasoning should refer to conscious thinking processes which involve the use of logic.

ing it, whether the leaner uses mental or real actions to reach the goal stipulated by the underlying spatial task. Another factor that influences spatial thinking is the subject's spatial knowledge or several spatial abilties, which are discussed in the upcoming section.

### *2.1.3 Selected spatial ability models*

As mentioned in the first section of this chapter, spatial ability has been an object of research among both psychologists and mathematics educators. Due to the earlier extensive research in psychology (mainly starting during the 1930s), some models of spatial ability in mathematics education are strongly influenced by the psychological approach. For a better understanding of the diversity in spatial abilities, four models of spatial ability, two from the psychological perspective (cf. Thurstone, 1950; Linn & Petersen, 1985) and two from the perspective of mathematics educators (cf. Maier, 1999; Pinkernell, 2003), chosen due to their clarification and adequacy of the notion of spatial abilities in previous literature (e.g., Büchter, 2011), are introduced in the upcoming subsections. As already mentioned in Section 2.2.1, spatial ability is considered as a component of intelligence in psychology, which is dominated by the need to describe and classify its inner structure and its complexity into sub-factors or spatial abilities (e.g., Thurstone, 1950). In contrast, recent spatial ability models (e.g., Pinkernell, 2003) in mathematics education tend to focus more on the role of spatial ability in learning and teaching mathematics, especially in geometry classes. This comparison shows the wide spectrum of definitions and competencies associated with the notion of spatial ability in psychology and mathematics education.

#### *2.1.3.1 Thurstone's spatial ability model*

Thurstone's (1938) work on intelligence theory, in which he formulated the different primary components of intelligence in *principal landmarks in mental ability* (cf. Thurstone, 1938) was an important initial step to emphasise and classify the psychological core of spatial ability. In his factor model, space or spatial abilities are considered as one of the primary mental abilities, together with other factors such as verbal, word fluency, number, perception, memory and reasoning (cf. Thurstone, 1938).

In his work about investigating test persons in terms of abilities in visual thinking, Thurstone (1950) was also one of the psychologists to offer a model of spatial ability based on the differentiation between three factors of spatial abilities:

*visualization*, *spatial relations* and *spatial orientation*. *Visualization* is defined as the "ability to imagine the movement or internal displacement among the parts of a configuration that one is thinking about" (Thurstone, 1950, p. 518). The factor *spatial relations* is "the ability to recognize the identity of an object when it is seen from different angles" (Thurstone, 1950, p. 518). Thurstone (1950) states that wheras the objects or configurations are rigid in the factor of spatial relations, the factor of visualization implies movement of object or configuration. Finally, Thurstone (1950) defines *spatial orientation* as the ability to "think about those spatial relations in which the body orientation of the observer is an essential part of the problem" (Thurstone, 1950, p. 519). Thus, in contrast to spatial relations, in spatial orientation the position of the person solving the spatial task is important for the successful accomplishment of the underlying task.

Thurstone's (1950) spatial ability model was one among the first to describe the different factors of spatial ability and integrate them in a model. It served as a basis for extensive discussion about the nature of spatial abilities and influenced other spatial ability models, not only in psychology, but also in mathematics education (e.g., Maier's model of spatial ability in Section 2.1.3.3).

*2.1.3.2 Linn and Petersen's spatial ability model*

Linn & Petersen (1985) offer another approach to the structure of spatial abilities by focusing on the different strategies students employ during the solving of spatial tasks. In their meta analysis consisting of 172 studies, they presented three components of spatial abilities and reference tests, which are used to measure the corresponding spatial ability (see Table 1). As Table 1 shows, this model is based on the three components – *spatial perception*, *mental rotation* and *spatial visualization* – which denote orientation by using the gravitational vertical or horizontal, mental rotation and multistep analytic procedure respectively.

One of the advantages of Linn & Petersen's (1985) model are the straightforward definitions of each component and the measurement of each component via the corresponding cognitive tests (see Table 1). Büchter (2011) describes Linn and Petersen's (1985) model as cognitive-psychologically and psychometrically plausible and uses this model together with the corresponding reference tests to measure students' spatial abilities to investigate the relationship between spatial ability and mathematics performance. This model is used not only to describe the concept of spatial ability, but also to provide assessment of student spatial abilities through reference tests.

| Component | Definition | Reference test |
|---|---|---|
| Spatial Perception | Spatial perception is defined as the spatial ability "to determine spatial relationships with respect to the orientation of their own bodies, in spite of distracting information." (Linn & Petersen, 1985, p. 1482) | Water Level Task (WLT) (cf. Piaget & Inhelder, 1958); Rod and Frame Test (RFT) (cf. Witkin, Dyk, Faterson, Goodenough & Karp, 1962) |
| Mental Rotation | Mental rotation is the spatial ability to "rotate a two or three dimensional figure rapidly and accurately" (Linn & Petersen, 1985, p. 1483) | Mental Rotation Test (MRT) (cf. Linn & Petersen, 1985); Flags and Cards (c.f. French, Elstrom & Price, 1963) |
| Spatial Visualization | Spatial visualization is the component "associated with those spatial ability tasks that involve complicated, multistep manipulations of spatially presented information" (Linn & Petersen, 1985, p. 1484). | Different Aptitude Set – Spatial Relations Subtest (DAT:SR) (cf. Bennett, Seashore & Wesman, 1973); Paper Folding (cf. Linn & Petersen, 1985) |

**Table 1 Components of Linn and Petersen's (1985) spatial ability model**

### *2.1.3.3 Maier's spatial ability model*

Psychological spatial ability models have also served as a basis for mathematics educators researching the domain of spatial ability. Maier's (1999) spatial abilty model integrates spatial-visual components of different psychological models (e.g., Thurstone, 1950; Linn & Petersen, 1985). Maier's (1999) model consists of six spatial-visual components: *spatial perception* and *mental rotation* according to Linn and Petersen (1985) (see Section 2.1.3.2) and *spatial visualization*, *spatial relations*, und *spatial orientation* according to Thurstone (1950) (see Section 2.1.3.1) and *factor K*. The last component, *factor K*, is based on the factor kinesthetic imagery in Michael, Guilford, Fruckter and Zimmermann (1957)'s three factor model of spatial ability. They define it as representing "merely a left-right discrimination with respect to the location of the human body" (Michael et al. 1957, p. 191 as citied in Maier, 1999, p. 45).

Regarding the importance of spatial abilities in mathematics classroom, Maier (1999) emphasised that the three components, *visualization*, *spatial relations* and to greater extent *spatial orientation* are the most relevant in mathematics classroom (as well as in everyday life and work). The importance of these three factors is also visible in his model, in which the areas representing these factors are enlarged (see Figure 1). The smaller areas in the model representing the other factors of spatial abilities (*mental rotation*, *spatial perception* and *factor K*) in the model have been described as rather special and specific (cf. Maier, 1999).

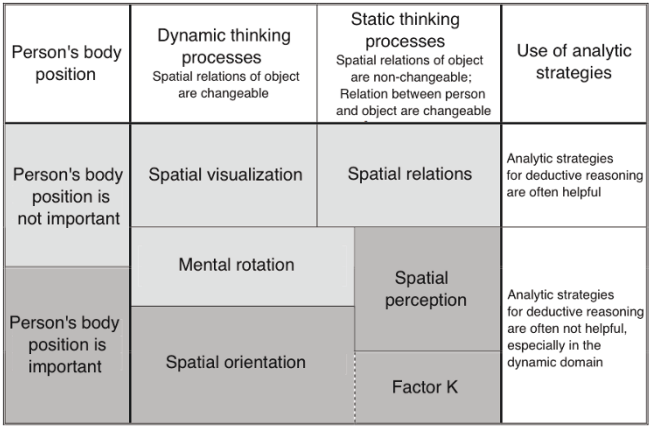



**Figure 1 A visualisation of the spatial ability model called "the (spatial) map" according to Maier (1999, p. 71)[6]**

Maier's (1999) model shows the acknowledgment of psychological spatial ability models in mathematics education, especially how psychological approaches to spatial abilities can be integrated in mathematics education. In particular, Maier (1999) shows the importance of Linn and Petersen's (1985) model in mathematics education and considers Thurstone's (1950) notion of spatial orientation, which have been neglected or not explicity mentioned in other spatial ability models in mathematics education. Based on this model, Maier (1999) also shows how spatial abilities are important in geometry by describing which spatial abilities are required for which geometrical task (see Section 2.1.4). However, due to the lack of consideration of geometrical knowledge explicitly in his model, another spatial ability model in mathematics education is introduced in the next section.

### *2.1.3.4 Pinkernell's spatial ability model*

Pinkernell (2003) developed an integrative model for spatial abilities after concluding that the psychological models are not enough to deliver appropriate results in the field of mathematics education. Pinkernell's model describes various spatial abilities which are allocated in three categories, *spatial-visual operations* (*Räumlich-visuelles Operieren), geometrical thinking* (*Geometrisches Denken)*

[6] The diagram in Figure 1 was translated from German to English and is reproduced without any further alterations from Maier (1999, p. 71) by A.M.

and *visual abilities (Visualisierungskompetenz)*. *Spatial-visual operations* consist of mental actions on spatial objects and real actions in space. Abilities belonging to spatial-visual operations include the ability to mentally construct or reproduce objects, to transform their spatial-visual characteristics, for example, their position or form, to model, to draw and to describe real space object and its transformation (cf. Pinkernell, 2003). Pinkernell's (2003) second category of spatial abilities, *geometrical thinking*, includes abilities such as recognising and understanding spatial objects, describing them and their transformation under consideration of their geometrical characteristics. Abilities within this category include the reduction of solids to their geometrical form, pattern or structure to facilitate problem-solving processes. The last category, *visual abilities*, is the interpretation and construction of different representation forms of spatial-visual objects, such as models, graphs and the description of configurations using the correct terms for the underlying concepts (cf. Pinkernell, 2003).

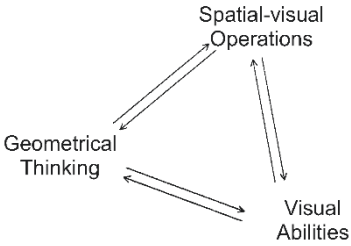



**Figure 2 A visualisation of Pinkernell's (2003, p. 57) spatial ability model**[7]

Therefore, Pinkernell's (2003) model provides a collection of abilities which can be directly associated with in the solving of spatial task in mathematics education. The first category in this model addresses both internal and external representations of spatial objects; the latter having been ignored in psychological models of spatial ability. Additionally, the second category deals with geometrical thinking, which plays a major role for the application and fostering of spatial abilities in mathematics classrooms. The different representation forms and communication, which are essential for teaching and learning mathematics, are also addressed in the last category of Pinkernell's (2003) model. This model rep-

[7] The diagram in Figure 2 was translated from German to English and is reproduced without any further alterations from Pinkernell (2003, p. 57)

resents the notion of spatial abilities in this thesis due to Pinkernell's (2003) success in integrating both fundamental elements from psychological and mathematics education models about spatial abilities. It is suitable from the perspective of mathematics education since it allows an analysis of student behavior in solving of spatial tasks and highlights the importance of spatial abilities in the interplay among the three categories (see Figure 2).

### *2.1.4 Spatial abilities in German mathematics curriculum and classroom*

The learning and training of spatial abilities is an important part of geometry lessons in mathematics. However, their development and training in mathematics classroom is not directly addressed in mathematics curriculum, but rather represented in conjunction to knowledge, competencies, or tasks in spatial geometry. Examples of such competencies formulated in the German mathematics curriculum include students' ability to produce nets of solids, to mentally operate with solids, and to classify geometrical objects (cf. Federal German Ministry of Education, 2004). The reluctance to address spatial abilities explicitly in mathematics curriculum leads to the assumption that spatial abilities seem to be more of a requirement for geometry learning in mathematics lessons, rather than a topic to learn by itself.

Maier (1999) provides an extensive summary of geometrical content which shows the development of spatial abilities in spatial and plane geometry in mathematics classrooms according to his model (see Section 2.1.3.3). Spatial-geometrical tasks which belong to the factor of spatial perception include discovering of symmetry of solids or figures and identifying solids' or figures' height (cf. Maier, 1999). Spatial abilities within the component of visualization and spatial relations can be developed by using building cubes to build solids, describing everyday-life objects and recognising basic shapes, drawing or folding geometrical figures, solids and eventually concepts (e.g., parallel lines), estimating and comparing length, areas and volume etc. (cf. Maier, 1999). Adequate geometry topics for Maier's (1999) spatial componenent of rotations are dealt with at secondary level, for instance, when introducing the concepts of rotation, inversion in a point and solids of revolution. The fourth spatial component of spatial orientation can be developed in geometry by expressing spatial relations between locations of objects and persons (e.g., to the right or left of…) at primary level, and orientation in a grid coordinate or in trigonometry at secondary level (cf. Maier, 1999).

The above-mentioned tasks in geometry lessons show how spatial abilities and their development are integrated within geometrical learning in mathematics classes in primary and secondary schools. The integration of spatial abilities in geometry classroom has been an issue in mathematics education for a long time. For instance, Ilgner (1974) argued that the development of spatial abilities in mathematics classrooms is not only the focus in particular geometry topics or tasks, but it should be rather considered as a methodological principle for geometry lessons in general to support both geometrical and spatial thinking in mathematics. The consideration of geometrical thinking as part of spatial abilities opens new perspectives to the approach to spatial abilities, which have been neglected in spatial ability models adopted from psychology (e.g., Linn & Petersen, 1985). As Pinkernell's (2003) model shows, abilities such as constructing models and describing properties or symmetries of spatial objects are also important for the research, development and assessment of student's spatial abilities in mathematics education. As a matter of fact, Pinkernell's (2003) model will be considered as an important theoretical framework for the notion of spatial abilities in this present research.

### *2.1.5 Strategies for solving spatial tasks*

When solving spatial tasks, students need to develop or choose among strategies to solve the tasks, which demand particular spatial abilities.[8] The existence of different strategies for solving spatial tasks has also been recorded in previous studies about strategy choice in solving spatial tasks: "the preceding brief review of the processing strategies used by individuals on standard spatial tests makes it clear that different individuals attempting the same spatial task do, in fact, use a wide range of strategies" (Clements, 1983, p. 15).

After a brief introduction of the notion of strategy in the first part of this section, different strategies for solving spatial tasks from previous research is presented. The selected strategies for solving spatial tasks include Barrat's (1953) analytic and holistic dichotomy and Gittler's (1984) three-dimensional and two-dimensional thinking.

[8] The fact that solving of spatial tasks requires strategies rather than just activation of knowledge indicates the similarity between the notion of spatial task in the present study to the notion of (spatial) problem in the domain of problem-solving.

*2.1.5.1 Definition of strategies*

The term *strategy* is often used in the domain of problem solving in mathematics education, but few mathematics educators give an extensive definition of this concept. Schoenfeld (1983) describes two types of decision moments that can take place during the process of problem solving, which can be considered as building the core of his notion of strategies. These moments are induced by two main questions: *what shall be done to solve the problem?* and *how can this be achieved to solve the problem?* (cf. Schoenfeld, 1983 as citied in Fülöp, 2015). In the former ones, which are called strategic decisions, aims and goals are formulated and decisions are taken via actions. In the latter types of decisions, the focus is on the implementation of the decisions which have been taken in strategic decisions (cf. Fülöp, 2015). Based on Schoenfeld (1983)'s previous work, Fülöp (2015) provides a more explicit definition of strategies, which she defines as "an approach that is not domain specific and is of general character, which focuses on the goal and the task as a whole, but which is flexible enough to allow for several ways to proceed when solving a problem" (Fülöp, 2015, p. 49). Fülöp (2015) adds that strategies can be considered as abstractions and as inventions, since they are subjective and imaginary.

In their work about how children use strategies, Siegler and Jenkins (1989) give a similar definition of strategy as "any procedure that is nonobligatory and goal directed" (Siegler & Jenkins, 1989, p. 11) and "as relatively grand entities that encompass a variety of means toward an end" (Siegler & Jenkins, 1999, p. 11). According to Siegler and Jenkins (1989), strategies consist of both conscious or unconscious activities, whereas in comparison, a consciously adopted strategy is called *plan*, which is a "behavior that is voluntary, self-conscious, and intended" (Scholnick & Friedman, 1987, p. 5 as citied in Siegler & Jenkins, 1989, p. 12). Regarding the activation of strategies, different strategies can be competing against each other for activation, if the goal has not been reached yet (cf. Siegler & Jenkins, 1989).

Based on these theoretical assumptions, the term *strategy* is used in this present study to denote an action which is needed by a subject to reach the goal in a particular task, and therefore the development and choice of strategies is influenced by the nature of the task (cf. Plath, 2014) and by the subject's knowledge and experience (cf. Rott, 2011). Based on Siegler and Jenkins (1989)'s assumption,

an action should then be denoted as strategy, when there is the possibility of choice, otherwise it should be regarded as a demand of the task. In the next section, different types of strategies used in solving spatial tasks from previous litertaure are introduced and discussed.

*2.1.5.2 Holistic and analytic strategies*

A well-known differentiation in strategies of solving spatial tasks is Barrat's (1953) difference between whole (holistic strategies) and part approach (analytic strategies) in mental rotation tasks. Holistic strategies or *spatial manipulation* (cf. Burin, Delgado & Prieto, 2000), are strategies in which learners mentally transform or manipulate the whole object or its parts or move around the object in order to solve the spatial task (cf. Barrat, 1953). In analytic strategies, also known as *feature comparison* (cf. Burin et al., 2000), learners compare details of the orginal object and then decide whether the original and the comparing object are identical (cf. Barratt, 1953). Analytic strategies tend to require the learner to focus on specific characteristics of the object, describe these verbally analytic or use logical reasoning to solve the underlying spatial task (cf. Barratt, 1953).

The following phrases are examples of students' verbal thinking in solving a spatial task using holistic or analytic strategy respectively:

1) "I'd try to put figures on top of one another and twist them around" (Barratt, 1953, p. 22).
2) "What I did was orient myself by some identifying mark as the curved line at the bottom; then I noticed another marking symbol as either going to the left or right when I had it in this position" (Barratt, 1953, p. 22).

In the first phrase, the student describes the movement of the figure(s), which indicates the use of a holistic strategy. In the second one, the student identifies symbols or marks, and compares the details when the figure is in different spatial positions. Moreover, the use of holistic strategies (as in phrase (1)) shows that a verbalisation of thinking is still possible but to a limited extent, since the student uses spatial imagery mapping during the movement to solve the spatial task, rather than particular characteristics which could be verbalised more easily.

Some researchers (e.g., Maier, 1999; Plath, 2014) seem to agree that the nature of the spatial task influences the strategy choice among analytic and holistic. For

instance, as Maier (1999) points out in his spatial ability model (see Section 2.1.3.3), analytic strategies are expected to be used more in spatial tasks requiring the spatial components *visualization* or *spatial relations*. Spatial tasks assigned to *mental rotation* or *spatial orientation* should activate holistic strategies for their solving (cf. Maier, 1999) (see Figure 1). Other researchers (e.g., Grüßing, 2002) argument that the strategy choice between analytic and holistic is influenced by the students' mathematics performance. In her study about students solving spatial tasks, Grüßing (2002) concludes that there is a tendency that high-performance students use analytic strategies more efficiently than low-performance students (cf. Grüßing, 2002). In other studies about strategic thinking in general, researchers (e.g., Schwank, 2003) state that the choice of strategies is highly dependent on the subject's preferences, rather than solely on the nature of task.

Another differentiation between two types of strategies in solving spatial tasks, *three-dimensional* and *two-dimensional*, are introduced in the subsequent subsection.

#### *2.1.5.3 Three-dimensional and two-dimensional strategies*

*Three-dimensional* and *two-dimensional* strategies are two other strategies which were identified during the solving of written spatial tasks in Gittler's (1984) study about how learners solve three-dimensional cube tests. In *three-dimensional* strategies, learners solve the underlying spatial task by developing three-dimensional mental models, which are used, transformed, moved etc., for solving the spatial task (cf. Gittler, 1984). In contrast, learners using *two-dimensional* strategies produce two dimensional images of the three-dimensional spatial object to facilitate the solving of the spatial task (cf. Gittler, 1984), hence neglecting the third dimension in space. For an example of three-dimensional and two-dimensional strategies, consider an item from the *3D Cube Test* from Gittler (1984), a test for measuring spatial abilities whereby students must control whether one of the given six cubes corresponds to the original cube (see Figure 3).

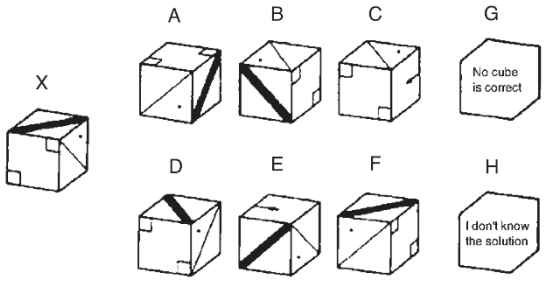



**Figure 3 3D Cube test item from Gittler's spatial ability assessment test (cf. Gittler, 1984, p.144)**[9]

In the test item in Figure 3, the students are required to find out which cubes A to C correspond to the left most cube. A student who applies three-dimensional strategies is expected to develop a three-dimesional mental image of the cube and mentally rotate it and compare the spatial relations between the different patterns on the surfaces. In contrast, students who use two-dimensional strategies would rather perceive the cube as two-dimensional, as depicted in the test item on the paper (see Figure 3), and neglect the existence of the three not directly visible cube surfaces in the background. Therefore, in this case students would rotate the cube as a figure constructed from the three visible parts (which can be perceived as three adjacent parallelograms in Figure 3), and propose Cube A in Figure 3 as a solution. In his study, Gittler (1984) concludes that the majority of the test persons do not create three dimensional images of the three-dimensional objects and manipulate these, but instead they tend to use two-dimensional thinking to solve the spatial task.

With regard to the previous differentiation between analytic and holistic strategies in Section 2.1.5.2, three-dimensional and two-dimensional strategies can be described using holistic and analytic strategy groups respectively. In three-dimensional thinking, students develop a mental image of the underlying spatial object wholly, which coincides with the holistic approach to solving spatial tasks. In comparison, students applying analytic strategies focus on details of the object, in the case of two-dimensional strategies, on part of the three-dimensional object. For instance, students applying holistic strategies in the task

[9] The original name of the test in German is *Dreidimensionaler Würfeltest* and it was translated by A. M.

visualised in Figure 3 manipulate a three-dimensional model of the corresponding cube, whilst students using an analytic approach develop a two-dimensional model and rely more on the properties, such as the spatial relations between the three-surfaces and the patterns on them.

### *2.1.6 Sex differences in spatial ability performance*

There is a considerable amount of literature on students' spatial abilities which shows evidence that there are differences between male and female participants when solving spatial tasks (e.g., Rost, 1977; Linn & Petersen, 1985; Geiser, Lehmann and Quaiser-Pohl, 2006; Büchter, 2011). In their meta analysis study about test persons solving different spatial tasks (different tasks for different spatial components according to their spatial ability model, see Section 2.1.3.2), Linn & Petersen (1985) show that there are sex differences when it comes to performance in tasks requiring spatial abilities. Following an analysis of the achieved results under consideration of sex, it was concluded that male test persons' performance was significantly better in tests of spatial perception and mental rotation than that of the female test persons (cf. Linn & Petersen, 1985). No sex differences were found in the comparison of males' and females' results of the spatial visualization tests.

In his research about sex differences in mathematics performance and spatial abilities, Büchter (2011) shows that the sex differences in mathematics performance can be statistically completely accounted for by sex differences in spatial abilities. In particular, Büchter (2011) states that the component of mental rotation in spatial abilities plays a major role in highlighting the sex differences. Similarly, other studies, like that of Masters and Sanders (1993), confirm the sex differences in spatial ability performance when learners solve tasks regardings the spatial component of mental rotation.

Other researchers (e.g., Rost, 1977) investigated students' spatial abilities and verbal competencies in their quantitive studies. In his research about sex differences in solving spatial tasks, Rost (1977) concludes that whilst female participants show better performance in verbalisations, the male counterparts achieve better results in tasks requiring more logical reasoning and spatial abilities including spatial orientation. Burton, Henninger and Hafetz (2005) conclude similar results in their quantitative research about sex differences and evaluation of the relations between spatial abilities, verbal fluency and finger-length ratios.

Whereas the male participants in their study performed better in tasks demanding mental rotation, female participants performed better in verbal fluency (cf. Burton et al., 2005). Similarly, Geiser, Lehmann and Quaiser-Pohl (2006) point out that there are sex differences concerning verbal and spatial abilities: "whereas females outperform males on measures of verbal fluency, males outperform females on certain tests of spatial ability" (Geiser et al., 2006, p. 683).

In their study about the relationship between verbal and spatial processes, Lanca and Kirby (1995) investigated students' memory and learning of contour maps in four different groups: a verbal learning group, a spatial learning group, a combined verbal and spatial group and a study-only control group. The results of their study indicate better male performance in learning spatial instructions and in three-dimensional map information. Female students in non-spatial task groups tended to memorise two dimensional maps better than in spatial task groups, however, no effect of spatial abilities could be found in the case of map memory (c.f. Lanca & Kirby, 1995). Sex differences in spatial tasks were also identified in the study of Coluccia, Iosue and Bradimonte (2007) about the relationship between map drawing skills and spatial orientation abilities. Coluccia et al. (2007) suggest that males use other strategies for learning maps than females. In particular, males tended to make more use of a holistic approach, whereas females tend to focus more on local features, thus preferring analytic approaches (cf. Coluccia et al., 2007).

However, there are studies, such as the one from Caldera et al. (1999), which do not show any significant differences between male and female participants when solving spatial tasks. In their study about strategies in spatial visualization tasks, Burin, Delgado and Pieto (2000) also conclude that sex does not play a significant role when students use strategies to solve spatial tasks.

The fact that a considerable high number of the studies show that there are sex differences in performance of solving spatial tasks, leads to the questioning about an adequate explanation for this phenomenon. Bergvall, Sorby and Worthen (1994) explain that genetical and environmental factors are responsible for the sex differences in spatial abilities. Whilst Linn and Petersen (1985) reject the genetic reason for such sex differences, the assumption that environmental factors are responsible for the sex differences is emphasised in some empirical studies about sex differences in spatial abilities (e.g., Vasta, Knott & Gaze, 1996). Vasta, Knott and Gaze (1996) state that the fact that spatial abilities can

be trained by suitable tasks and experiences indicate that the subject's environment could play a more important role than sex (cf. Vasta, Knott & Gaze, 1996). However, the deviation of two strands in research of spatial ability, the strand which argues that sex does play an important role in solving spatial tasks, and the other strand which denies this claim, show that disagreement and contradications in this area of research. However, since a substantial amount of literature indicates possible sex differences in solving of spatial tasks, which mostly concerns the test items of mental rotation or spatial orientation (e.g., Linn & Petersen, 1985; Coluccia et al., 2007, Büchter, 2011), it is assumed that sex may play an important role in solving spatial tasks. Some studies (e.g., Geiser et al., 2006; Burton et al., 2010) have researched both spatial and verbal competenices of studies in solving spatial tasks quantitatively. Most of these studies show the tendency that the female participants show better verbal abilities, whereas the male counterparts have performed better in spatial tasks, especially in tasks requiring mental rotation or spatial orientation. Such findings show that students' sex as an impacting factor is worth further investigation when analysing how students solve spatial tasks and their use of language in the task solving process.

### *2.1.7 Summary: Spatial ability*

Spatial ability entails different abilities depending on the approach, whether psychological or mathematics educational, and on the intentions, such as analysis of internal processes or learning and task solving processes. However, researchers (e.g., Maier 1999; Plath, 2014) seem to agree that visual perception, construction of mental images, processing information, and thinking are cognitive processes which are characteristic for solving spatial ability tasks.

From a mathematics educational perspective, I believe that both mathematics educational and psychological models of spatial abilities are important for investigating the notion of spatial abilities. Whereas the models in psychology offer a way to assess students' spatial ability performance and structure the important components which constitute spatial ability, models in mathematics education offer perspectives to integrate spatial abilities in learning and teaching of mathematics, such as the verbalisation of properties and construction of solids. The consideration of Linn and Petersen's (1985) model for assessment of students' spatial ability performance and Pinkernell's (2003)'s model for spatial task design plays an important role in the theoretical framework about spatial abilities in this present study. In this study, the notion of spatial abilities is considered as a

collection of abilities, which encompasses not only mental abilities (i.e. spatial thinking), such as mental manipulation of spatial objects, but also spatial-motoric skills, such as performing real actions on building cubes, and spatial-linguisic skills, such as describing a spatial object.

Following the definition of the notion of strategy as a goal-oriented procedure and flexible regarding its use in solving tasks, an overview of the most common strategies for solving spatial tasks from previous literature have been discussed in Section 2.1.5. It was agreed upon by researchers that the use of strategies is highly dependent on the subject's thinking and experiences, rather than only on the nature of the spatial task. However, the subject's sex could also play a role in solving spatial tasks, as numerous previous studies about sex differences in spatial abilities have shown (see Section 2.1.6). Findings from previous research on spatial abilities (e.g., Geiser et al., 2006; Burton et al., 2010) show that males outperform females when it comes to solving spatial tasks linked to mental rotation or spatial orientation, whereas females are better at verbal competencies in general. The latter competencies are addressed in general in the next section about the use of language in learning and teaching mathematics.

## 2.2 Language in mathematics classroom

The central role of language in learning and teaching mathematics has been increasingly recognised in the field of mathematics education. Research about language in mathematics education has pointed out the complex relationship between language and mathematics education, which has been summarised by Durkin (1991) in the following phrases:

> Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language. Such observations could be made of most school curricula, but the interweaving of mathematics and language is particularly intricate and intriguing. (Durkin, 1991, p. 3).

This section provides an overview of the interweaving of mathematics and language based on results from previous literature. After giving a definition of language from several disciplines, the relationship between language and thinking based on Vygotski's (1934/1993) theoretical framework are discussed. This is followed by the description of two important functions of language in mathematics learning and teaching.

Regarding the use of language in mathematics classroom, there are different varieties of language spoken by students and teachers in mathematics, which is described in Section 2.2.4. Section 2.2.5 focuses on the use of metaphors for learning and teaching mathematics. This is followed by a discussion of the importance of language in German mathematics curricula in the last section of this sub-chapter.

### *2.2.1 Definition of the notion of language*

Language is one of the most important aspects in learning and teaching of knowledge and is a phenomenon which accompanies us throughout everyday life to communicate and understand each other and to represent objects and situations in the world. The linguist Ferdinand de Saussure (1916) differentiates between three aspects of language: *langage,* which refers to the abstract concept of human language as a biological propert*; langue*, which denotes a specific language system and their set of rules for generating *langage*, for example, the German or English language; and *parole,* which denotes the concrete use of speech in a particular language system (cf. de Saussure, 1916)[10]. However, due to the global use of language in different disciplines there are various definitions of and approaches to the notion of language. In structuralism and formal theories of grammar, language can be considered as a system of signs determined by syntactical rules to generate words and sentences for communication, whereby these syntactical rules are a finite set and language specific (cf. Chomsky, 1995). From a cognitive and neuropsychologist perspective, language compromises the mental processes in the human brain significant for language acquisition and development and understanding of language (cf. Lesser, 1989). One of the basic assumptions of this view of language is the innateness of language, which means that language is produced in the brain rather than learnt through experience (cf. Lesser, 1989).

The communicative aspect of language plays an important role in sociolinguistics and pragmatics. In this domain, language is defined as a communication system consisting of signs and meanings used by human beings to express themselves and to communicate with other human beings (cf. Lyons, 1981). In con-

[10] In this thesis, the words *parole* and *spoken language* are used interchangeably. The words *langue* and *language* should retain the same meaning as in de Saussure's (1916) theoretical framework.

trast to formal theories of grammar language, in sociolinguistics, the rules of language are considered to be the result of the process of communication among its speakers, thus communication is a major catalyst for the development of syntactic rules of language (cf. van Valin, 2001). The importance of communication in social context is embedded in the notion of discourse, which Bruner (1966/1971) describes as:

> Discourse consists, in its essentials, of an *addresser*, and an *addressee*, a contact that joins them, a message passing between them, a context in which the message refers and a linguistic code that governs the way in which messages are put together and things referred to. (Bruner, 1971, p. 106)

In this present study, the sociolinguistic view of the concept of language is the most adequate, because language is vital for communication and as developing in a social environment through experience and interaction with other individuals. Although syntactical structures of language are important for developing appropriate communication, these should not be regarded as the only important units to construct language, because one should not neglect acceptability, which is a feature often observed in speech and which does not hinder communication. For instance, consider the phrase *a giraffe is a type of plant,* which is grammatically correct, but not considered as appropriate in speech due to its underlying meaning, hence not acceptable. Therefore, the aspect of acceptability shows how social interaction influences the use and meaning of language, which justify Evans and Levinson's (2009) claim that many aspects of language are shaped by communication in a social context.

Other concepts of language which are important for the present study are language proficiency and language acquisition. Language proficiency denotes a speaker's language skills or competencies, which are the ability to speak, read, write, listen and understand a particular language, which can be the first, second or a foreign language. Moreover, language acquisition describes the learning process of understanding a language and of communicating in the same language by generating words and sentences. This process requires learners to develop phonetic, morphological, syntactical and semantical features of a language. Therefore, language proficiency can be regarded as an indicator for a students' language acquisition. In the context of language use in education, language and

mathematics education researchers differentiate between everyday language and academic language in language acquisition in classrooms. A more detailed description of these levels of acquisition in mathematics education is discussed in Section 2.2.4.

### *2.2.2 Language and thinking*

Language plays an important role in the development of thinking, since language is a medium for generalising and abstracting concepts, which are used to organise experiences, and act as mediator between the subject and the object of thought (cf. Vygotsky, 1993). Lew Vygotsky was one of the first psychologists who investigated the relationship between language and thinking extensively and systematically using a psychological experimental approach. Among his works, *Thought and Language* provides an extensive description of the inner connection between speech and cognitive concepts. Vygotsky (1934/1993) describes the relationship between thought and language as a complex dynamic process characterised by movement from thought to word and vice versa. However, in this process, thought is not just expressed via language, but emerges through the use of words (cf. Vygostky, 1993). The connection between thought and language leads to the analysis of meanings of words, which is described as the union of word and thought:

> Word meaning is an elementary 'cell' that cannot be further analyzed and represents the most elementary form of the unity between thought and word. The meaning of a word represents such a close amalgram of thought and language that it is hard to tell whether it is a phenomenon of speech or a phenomenon of thought (…). It is a phenomenon of verbal thought, or meaningful speech – a union of word and thought (Vygotsky, 1993, p. 212)

Hence, the discussion of the relationship between language and thinking demands a deeper investigation of word meanings. Vygostky (1934/1993) points out the need to differentiate between two types of speech, internal and external speech, for a deeper understanding of word meaning. [11] The first type of speech

[11] The notion of language in Vygostky's (1934) work seems to concide with the notion of *langage* in de Saussure's (1916) work. However, in contrast to de Saussure's (1916) notion of *parole*, which denotes the external speech in concrete terms, Vygostky includes the mental verbal representations in the cognitive system in his notion of speech.

*inner speech* is required by the use of language and which serves as an instrument of thinking. *Inner speech* denotes the semantics of speech and can be considered as a truncated form of the external speech in which the addressee is oneself. One of the characteristics of inner speech observed in the Vygotsky's (1934/1993) experiments is its syntactical fragmentation when compared to the external speech. In particular, during the observation of students' inner speech, which was carried out by an analysis of students' speech addressed to themselves (i.e. egocentric speech according to Piaget (1951)), a tendency of subject and object omission, whereas students reduce an expression to its predicate, could be noticed (cf. Vygostky, 1993). To investigate the area of inner speech more deeply, Vygotsky (1934/1993) states that inner speech develops from egocentric speech, a concept developed from Piaget (1951) and which Vygotsky (1993) considers as a transitional process from a child's social collective activity to a more invidualised one. Egocentric speech is based on the concept of a child's egocentrism, in which children between the ages of three and five focus on themselves and think that other human beings have their same thoughts, feelings and experiences (cf. Vygotksy, 1993). Vygotsky (1993) considers egocentric speech as an essential part of the social learning process and considers it as means to examine childrens' inner speech by using experimentation and observation to externalise inner speech. This process of externalisation of inner speech can be facilitated by the design and implementation of an outer activity in which the child is encouraged to engage in social speech (i.e. speech addressed to others rather than oneself) which would enable the objective functional analysis of inner speech (cf. Vygotsky, 1993). An important criterion for the existence of social speech is the feeling of being understood, which is only present if a social situation can be created during the experiment (cf. Vygotsky, 1993).

The second type of speech, *external speech*, is defined as speech generated for other addressees rather than oneself and therefore focuses more on the phonetic aspect of speech (cf. Vygostky, 1993). Vygostky (1993) argues that it is wrong to consider the inner speech as being existent before the external speech or as being the reproduction of external speech in memory. However, he suggests that external and inner speech can be considered as opposites having entirely separate speech functions (cf. Vygotsky, 1993)). Although abbreviation can also be observed in external language, Vygostky (1934/1993) describes this phenomenon as rule for internal speech and as an exception for external speech. Another difference between *inner* and *external speech* is the agglutination of words in the

former speech to combine them, which is also a typical characteristic of a child' egocentric speech (cf. Vygotsky, 1993).

Vygotsky (1934/1993) emphasises that inner speech is a speech which should not be considered merely as an interior aspect of external speech. He summarises the external speech as the verbalisation of thoughts into words and thus materialising them, whilst internal speech transforms speech into thoughts (cf. Vygotsky, 1993). However, one should not consider the existence of the corresponding units in both speech and thoughts and there are no separate units in thinking as in speech: "The two processes are not identical, and there is no rigid correspondence between the units of thought and speech" (Vygotsky, 1993, p. 249). Consider, for instance, a situation in which a child is playing football – this corresponds to one thought, but several units of speech, for example, *child*, *play* and *football*.

A possible way to describe the process of verbal thought or verbalised thinking is as a development of thought which is triggered by a motive, shaped in the inner speech and in word meanings and ends in words or external speech (cf. Vygostky, 1993). Nevertheless, the relationship between language and thinking remains complex, since the process of verbal thought might involve movements back and forth along the different phases (cf. Vygotsky, 1993). However, a simplification of the verbal thought sheds light on the complex relationship between language and thinking. The investigation of this binary interrelationship shows that the understanding of language requires the understanding of thoughts. Vice versa, the understanding of language is not sufficient without deciphering and understanding of the underlying thoughts, because most often there is always an intention for the use of language. This leads us to the two functions of language in the mathematics educational context, which are described in the next section.

### *2.2.3 The functions of language*

Language plays an important role in mathematics learning, just as in every learning process. In previous research about language use in mathematics classroom (e.g., Maier & Schweiger, 1999; Wessel, 2015) two different functions of language – the communicative function and the cognitive function of language – were established. The former denotes the use of language for understanding each other, by focusing on communicating one's thoughts, views and considerations to other persons (cf. Maier & Schweiger, 1999).

Language in its cognitive function is used to acquire new knowledge through the mediation of information with the help of conceptual representations (cf. Maier & Schweiger, 1999). Elements of language and conceptual learning induce the construction of new knowledge and enable or facilitate its activation in the cognitive system (cf. Maier & Schweiger, 1999). Wessel (2015) points out that the background of the cognitive function of language is based on the assumption that language is a cognitive tool which facilitates thinking and understanding processes. Consider the following task for a deeper understanding of the communicative and the cognitive functions of language: *"draw a square, a rectangle, and a quadrilateral on a piece of paper and write down the characteristics of each figure".* On a communicative level, the function of this task is to communicate that the student has to solve this particular task which can be conveyed by spoken or written form of language. On a cognitive level, the students learn to understand the different notions *square, rectangle* and *quadrilateral* and differentiate between them in order to derive their different characteristics by using language and activating and manipulating verbal information in their cognitive system.

Both functions of language do overlap and should not be considered as separate entities (cf. Wessel, 2015). Maier and Schweiger (1999) describe the communicative role of language as being an amplifying factor on the cognitive function, because the aim of the former is to support the goal of the cognitive function. This supports the claim that communication is vital for the construction of cognitive processes, which can be seen as a joint approach from the sociolinguistic and cognitive perspective of the concept of language.

### *2.2.4 Levels of language acquisition*

As already mentioned in Section 2.2.1, language acquisition is the process of acquiring the different aspects of a particular language. The process of language acquisition does not only refer to the acquisition of a new foreign language, or of the mother tongue during childhood, but it also includes the learning of new registers within the language which an individual can speak fluently. In sociolinguistics, a register can be defined as a variety of language[12] which is used in a

[12] In this thesis, the term *variety of language* denotes a language with different characteristics (mostly on a semantic or syntactical level) within *langue* (cf. de Saussure, 1916) deriving from its use in different contexts, in this case, in mathematics teaching and

particular environment for a particular purpose. An example of register is legal English or legal German, which does not only include specific vocabulary, but also complex sentence structure and other linguistic aspects typical in such registers.

Different registers or varieties of language can also be observed in the different languages used for learning and teaching in classrooms:

> The talk of teachers and students draws together – or bridges – the 'everyday' language of students learning through English as a second language, and the language associated with the academic registers of school which they must learn to control. (Gibbons, 2006, p. 1)

A popular theoretical framework about the use of language for different purposes in educational context is Cummins' (1979) differentiation between two levels of language acquisition: *Basic Interpersonal Communication Skills* (BICS) and *Cognitive Academic Language Proficiency* (CALP). BICS denotes the abilities to communicate in a language in everyday-life situations with other speakers of the same language. This type of language is characterised by the use of non-verbal language (such as gestures) to facilitate communication and understanding (cf. Cummins, 1979). Moreover, social commucation in BICS tends to be context-embedded, which means that a common context is required for the speakers to understand each other. Such a language variety tends to be used to communicate with friends and relatives and does not require complex syntactical structures or specific vocabulary which is not used in everyday life situations (cf. Cummins, 1979).

In contrast, CALP can be understood as an extension of BICS, because it denotes the ability to use oral and written language to additionally understand academic concepts and ideas. This level of language acquisition is characterised by the extensive use of complex syntactical structures and academic vocabulary (cf. Cummins, 1979). The vocabulary used is influenced by the topic or subject of the conversation or situation, for example, mathematics, physics, biology, chemistry, etc. This supports the idea that the level of proficiency in CALP has a ma-

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learning. This differs from the traditional association of the term varieties of language in linguistics, which tends to include dialects, sociolects and other varieties spoken by cultural groups.

jor impact on students' performance in school. In contrast to BICS, a context is not necessary in CALP and non-verbal language is not as important as in BICS:

> CALP is said to occur in 'context-reduced' academic situations. Where high order thinking skills (e.g., analysis, synthesis, evaluation) are required in Curriculum, language is 'disembedded' from a meaningful, supportive context (Baker, 2006, p. 174).

In mathematics educational context, CALP can be translated to mathematics language which is the language commonly used in mathematics textbooks, in teachers' or mathematics education researchers' language or in academic texts by mathematics educators.[13] Halliday (1978) describes mathematics language as "the meaning that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes" (Halliday, 1978, p. 195). However, due to the different areas of mathematics mathematics language should be considered as a generalised term to denote the collection of different sub-registers used in mathematics classroom, for example, spatial sub-register in geometry classes, functional sub-register in calculus, etc.

Cummin's (1979) distinction between BICS or everyday language and CALP or mathematics language has served as a basis for the investigation of different varieties of language in learning and teaching context in mathematics classroom. However, both varieties of language should not be considered as entirely separate entities, but rather as a continuum (cf. Gibbons, 2006; Clarkson, 2009; Prediger & Wessel, 2011), in which the degree of abstraction increases (from everyday to mathematics language) or decreases (from mathematics to everyday language). An assumption which supports the representation of everyday and mathematics language as a continuum is the fact that a solid knowledge of everyday language serves as fundamental basis for the acquisition of academic language, in particular mathematics language. However, as Leisen (2011) points out, there are other varieties of language, such as instruction language, i.e. the language

[13] In models about levels of language acquisition in mathematics education, e.g. from Prediger and Wessel (2011), researchers differ between three levels of language acquisition: everyday language (*Alltagssprache*), academic language (*Bildungssprache*) and technical language (e.g., mathematics language) (*Fachsprache*), the last two being more closely related in terms of their properties. In this work, CALP in mathematics education shall refer to the language used for expressing mathematical ideas in mathematics classroom, hence, it will be referred to as mathematics language.

used in learning and teaching in educational context, regardless of the subject. Other language varieties include visual language, which serves for visualising and explaining knowledge and procedures using graphs, pictures, diagrams, etc., and symbolic or formal language, which is used to represent knowledge and procedures using symbols, formulas, etc. (cf. Leisen, 2011). The latter varieties of language show how different representations of mathematical knowledge (see Section 2.4.1) influence the kind of language used in mathematics classroom.

### *2.2.5 Metaphors in mathematics classroom*

The use of metaphors in learning mathematics was pointed out by different studies about language in mathematics classroom (e.g., Pimm, 1981; Sfard, 1998; Lakoff & Núñez, 2000; Malle, 2009; Font, Godino, Planas & Acevedo, 2010). According to Lakoff and Johnson (1980), metaphors are devices used for exploration and understanding of a particular idea through another one. A similar description of metaphors can be found in linguistics, whereby it is defined as a rhetorical device that transfers a meaning from one word to another. However, metaphors should not only be considered just as figures of speech or an embellishment in a discourse, but rather as medium to enable and support thinking about abstract ideas (cf. Lakoff & Núñez, 2000). Sfard (1998) states that metaphors are important objects of analysis in learning mathematics due to following reasons:

> Their [metaphors] special power stems from the fact that they often cross the borders between the spontaneous and the scientific, between the intuitive and the formal. Conveyed through language from one domain to another, they enable conceptual osmosis between everyday and scientific discourses, letting our primary intuitition shape scientific ideas and the formal conceptions feed back into the intuition. (Sfard, 1998, p. 4)

Mathematics and mathematical discourse is based on the use of metaphors, which Malle (2009) describes as words or phrases with a 'transferred' meaning, i.e. the meaning originates from another non-mathematical field (cf. Malle, 2009). An example of a metaphor in mathematics education is the phrase "an isosceles triangle is like a human standing on two feet, therefore both sides are equal" (Malle, 2009, p. 10). Malle (2009) claims that most of the mathematical terms originate from everyday language (for example, *tree*, *root*, *area*, *divide*, *break-down*, *real*, *similar*, *proportion,* etc.) and other ones, like *function*, *integral*, *differentiate*, *prime*, *trigonometry*, *diagonal,* etc. also originate from everyday language of either Latin or Greek. However, sometimes the meaning has on-

ly been partially transferred from everyday to mathematics language, as in the case of the meaning of the mathematical term *root* from the Latin word *radix,* which means base, foundation or origin (cf. Malle, 2009).

Based on Lakoff & Johnson's (1980) previous work, Lakoff & Núñez (2000) define conceptual metaphors as mechanisms, which support the understanding of abstract ideas through concerete terms. Metaphors are perceived to be projections from source domains to target domains, in which the sources' properties and characteristics are assigned to the target (cf. Lakoff & Núñez, 2000; Font et al., 2010). There are two types of conceptual metaphors used in mathematics: *grounding* and *linking* metaphors (cf. Lakoff & Núñez, 2000). Whilst grounding metaphors are metaphors which project a source outside the field of mathematics to a target within mathematics, in linking metaphors both source and target orginate within mathematics, but tend to be anchored in different fields of mathematics (cf. Font et al., 2010). An example of a grounding metaphor is the phrase, "classes are containers" (Núñez, 2000, p. 13). In this example, properties of the source *container* are projected to the notion of class (see Table 2).

| Source domain | Target domain |
|---|---|
| Container schemas | Classes |
| Interiors of container schema | Classes |
| Objects in an interior | Class members |
| Being an object in an interior | The membership relation |
| An interior of one container schema | A subclass in a larger class |

**Table 2 The grounding metaphor *classes are containers* (Núñez, 2000, p. 13)**

The linking between contexts and situations in everyday life and abstract ideas is a fundamental principle for learning of mathematics and constructing abstract mathematical knowledge in the classroom. Therefore, metaphors are often used to construct and reconstruct mathematical knowledge, especially when considering that "(…) a larger number of the most basic, as well as the most sophisticated, mathematical ideas are metaphorical in nature" (Lakoff & Núñez, 2000, p. 364). Metaphors are important tools to facilitate mathematics learning, since the desirable meaning of mathematical terms in mathematics language can be transferred from everyday language (cf. Malle, 2009). However, a heavy reliance on such a meaning transfer to learn mathematics can also lead to barriers in teaching mathematics (cf. Malle, 2009). For instance, most students (especially at the beginning of secondary level) tend to understand the mathematical concept of *diagonal* as *oblique* or *slanting*, as in the case of use of the word *diagonal* in eve-

ryday language, sometimes also in geometry classes (for example, in typical examples of rectangles and parallelograms) (cf. Malle, 2009). Whereas the use of such a metaphor is helpful, it might also be hindering, since not every slanting line is a diagonal (cf. Malle, 2009). Several mathematics education researchers (e.g., Sfard, 1998; Malle, 2009) have pointed out how metaphors can act as obstacles in mathematics learning and teaching. However, most researchers in mathematics education (e.g., Pimm, 1981; Lakoff & Núñez, 2000; Malle 2009) seem to agree that metaphors are important elements in mathematical discourse to learn and teach mathematical knowledge, and for supporting the understanding of abstract mathematical ideas. This present study shows the importance of metaphors in mathematical discourse and illustrates the limitations of metaphors when communicating and constructing mathematical-spatial knowledge.

### 2.2.6 Roles of language in German mathematics curriculum and classroom

The three main roles of language in mathematics education upon which researchers (e.g., Prediger, 2013; Wessel, 2015; Schütte, 2009; Knapp, 2006) seem to agree are: language as a learning medium, language as a learning goal or language as a barrier in learning and teaching of mathematics (cf. Prediger, 2013).

Language as a learning medium means that language is used to represent and communicate content knowledge via linguistic elements (cf. Knapp, 2006). Wessel (2015) states that the student language acts as a learning medium in the process of learning content. She emphasises the didactical principles of Wagenschein (1989) and Winter (1996), who argue that students should be competent in their mother tongue and then in the respective academic language and that knowledge should be first represented in everyday language and then in the academic language respectively (cf. Wessel, 2015). The emphasis on language as a learning medium is also documented in German mathematics curriculum as a goal of mathematics classes:

> Cognitive processes related to content knowledge, concept formation, and the assessment and evaluation of mathematical issues and problems are mediated via language, as well as the presentation of learning results and its discussion in communication. (North-Rhine Westphalia Ministry of Education, 2011, p. 10)[14]

[14] This section was translated by A. M. and is reproduced without any further alterations from the curriculum documents of North-Rhine Westphalia Ministry of Education (2011).

However, the teacher's mathematical language or language used in textbooks can also act as a learning medium. For instance, tasks in current mathematics textbooks do not only provide calculations, but also provide or demand descriptions or explanations. To solve or understand tasks with descriptions or explanations learners are not only required to understand the mathematical language but also to produce language.

Language as a learning goal is based on the belief that the aim of mathematics lessons should not only be the learning of content knowledge, but also of linguistic knowledge associated to language used in mathematics classrooms (cf. Prediger, 2013). Language learning has been gaining importance in mathematics lessons, as the following excerpt from the German mathematics curriculum shows:

> The acquisition of mathematics content learning is linked intensively to the development of language skills. (…). Such language skills [e.g., presentation of learning results, communication etc.] do not develop just from the everyday language competencies, but they should be initiated and further developed in language-sensitive mathematics lessons. (North-Rhine Westphalia Ministry of Education, 2011, p.10)[15]

There are several specific goals which teachers need to consider in order to develop a language-sensitive mathematics lesson. For instance, teachers should ensure that secondary students describe phenomena by using mathematics language, that they use the right tenses when describing results, processes or algorithms and that they use adequate and complex sentence structure in order to represent relations (cf. North-Rhine Westphalia Ministry of Education, 2011). Another focus on learning of language skills in German mathematics curriculum can be observed in the high importance of argumentation and communication for mathematics learning (cf. Federal German Ministry of Education, 2004). In particular, students at the end of lower secondary level are required to communicate mathematical facts comprehensible, and to be able to justify them (cf. North-Rhine Westphalia Ministry of Education, 2014). Further students' abilities at this level, formulated as reasoning and communication skills in mathematics curriculums in North-Rhine Westphalia, include reading skills, such as the ability to extract, structure and evaluate mathematical information from texts, pictures and

[15] This section was translated by A. M. and is reproduced without any further alterations from the curriculum documents of North-Rhine Westphalia Ministry of Education (2011).

tables, communication skills, such as the ability to communicate and present mathematical problem solving processes in one's own words and to use mathematical terms, and other cognitive-linguistic skills, such as the ability to construct concept networks and relation networking (cf. North-Rhine Westphalia Ministry of Education, 2014). Therefore, students are not only expected to learn new vocabulary in mathematics, but also to learn how to communicate, argument and reason, which are also important aspects of language learning.

However, language can also act as a barrier in learning processes in mathematics learning and teaching (cf. Radford & Barwell, 2016). Clarkson (1992) states that language is a barrier in mathematics learning for multilingual students with low language proficiency in their first language (L1) or second language (L2), who often obtain lower achievements than students with high language proficiency. Therefore, mathematics lessons should also provide language support along with content learning for reducing obstacles in mathematics learning which can arise from linguistic issues.

The above mentioned three roles of language in mathematics learning and teaching show the high influence of language in mathematics lessons, and show the need to integrate language learning and content learning in mathematics education. Whereas the curriculum addresses the need for a higher language awareness and the need for integrative language and content learning in mathematics, the implementation of language and content learning in mathematics lessons is not only influenced by the mathematical topic (e.g., calculus, geometry, probability, etc.), but also by the diverse needs of students with different linguistic backgrounds. Therefore, in mathematics lessons with the focus of language and content learning, teachers need to mediate between the development of different students' language proficiency and the particular mathematical topic.

### *2.2.7 Summary: Language in mathematics classroom*

In Section 2.2, an overview of the theoretical framework about language in mathematics classroom was provided. From a sociolinguistic perspective of language, language is developed and constructed by communication in mathematics lessons between teachers and students. Along with the communicative function of language, language and its cognitive processes is a means for learning and understanding mathematics knowledge (see Section 2.2.3). As it can be observed from Vygotsky's (1934/1993) work, the relationship between thinking and lan-

guage is rather complicated to describe, but the meaning of language in communication helps to understand better how language and thinking processes are interrelated. Regarding the use of language in mathematics classrooms, there are mainly two different varieties of language or levels of acquisition in mathematics lessons: everyday language and mathematics language from the students' and teachers' perspective respectively (see Section 2.2.4). Both of these levels of language acquisitions make use of metaphors in order to describe abstract mathematical ideas. Based on the theoretical framework of Lakoff and Núñez (2000), metaphors can be considered as projections from a source to a target in mathematics. Depending on whether the metaphor's source orginates outside mathematics or inside mathematics, Lakoff and Núñez (2000) differentiate between grounding and linking metaphors (see Section 2.2.5). Due to the high resemblance of students' language to everyday language, in comparison to teachers' language, which is more a mathematical language, one can hypothesise that students are more likely to use grounding metaphors. The difference between students' and teachers' language can be observed in the need to develop language and content integrated learning in mathematics classroom in particular to support low language proficiency students in their content learning (see Section 2.2.6). However, language and content integrated learning in mathematics classroom is also required for the further development of language skills (regardless of the students' language proficiency) and to raise language-awareness among students.

## 2.3 Interplay of spatial ability and language

After outlining the current research on spatial ability and on language separately, the relationship between both, which has not been focused on enough in recent mathematics education research, is discussed from a mathematics education perspective. The reason for the negligence of this relationship in research might be the standpoint in traditional psychology in which cognitive processes are likely to be considered as separate entities:

> Interfunctional relations in general have not as yet received the attention they merit. The atomistic functional modes of analysis prevalent during the past decade treated psychic processes in isolation. Methods of research were developed and perfected with a view to studying separate functions, while their interdependence and their organization in the structure of consciousness as a whole remained outside the field of investigation. (Vygotsky, 1934/1993, p. 1)

However, the previous theoretical background manifests both spatial ability and language in its cognitive function as cognitive processes in mathematics education. Therefore, this section presents detailed results from previous research about possible intersections between the domains of language and spatial ability. The first section, Section 2.3.1, provides theory on the phenomenon of spatial language from psychology and linguistics research, which is the strongest argument for considering the link between both domains. The next section, Section 2.3.2, presents an attempt to support the connection between both domains by focusing on the role of language in solving spatial tasks.

### *2.3.1 Spatial language*

The phenomenon of spatial language seems to provide a promising direction for research on the relationship between language and spatial abilities. Being more intensively researched in the area of psycholinguistics, some of the most common definitions of spatial language are as *language of objects and places* (cf. Landau & Jackendoff, 1993), *language of space* or *spatial description* (cf. Levinson, 1996; Levinson, 2003; Taylor & Tversky, 1996). The commonality in these definitions is that spatial language is the language used when describing spatial phenomena, whether objects or places. From a mathematics education perspective, spatial language can be defined as the register used in spatial geometry.

In his approach to language in space, Lehmann (2013) uses the notion of *spatial construction* which he defines as the linguistic coding of spatial orientation in a language system. Lehmann's (2013) notion of spatial construction or spatial language emphasises the use of language in order to orientate oneself in space and enriches space by the communicative and cognitive functions of language. However, according to Lehmann (2013), the origin of spatial construction is non-linguistic, because the non-linguistic substrate serves as a universal basis for space construction in different languages worldwide. Moreover, the process of space construction takes place differently depending on the language used (cf. Lehmann, 2013). Similarly, Levinson (1996) points out the importance of language for our development of spatial conceptions by referring to the fact that "many languages do not use the planes through the body to derive spatial coordinates, i.e. they have no left/right/front/back spatial terms" (Levinson, 1996, p. 356). For instance, Australian Aboriginal languages, such as Guugu Yimithirr and Thaayorre, do not have any words for relative directions, such as right or

left, but instead use cardinal directions for spatial orientation (cf. Levinson, 1994). Hence, spatial conceptions and spatial language can be regarded as influencing each other to a certain extent.

Cognitive linguists, such as Coventry, Tenbrink and Bateman (2009), indicate the vital importance of spatial language which they describe as enabling us "to find objects and places outside the immediate stimulus reach of speakers, making use of a wealth of spatial relations, spatial perspectives, and spatial objects" (Coventry et al., 2009, p. 1). Moreover, they highlight the importance of spatial language to occur in a dialogue rather than in a monologue, since the interaction in a dialogue gives students the opportunity to participate more actively and creates a less artificial and less restricted setting (cf. Coventry et al., 2009). This kind of setting enables the participants to find common reference frames, identify the required spatial properties and objects and to give feedback to each other to develop and enhance spatial language (cf. Coventry et al., 2009).

Several researchers (e.g., Coventry et al., 2009; Levinson, 1996; Landau & Jackendoff, 1993) emphasise the importance of spatial language to understand the underlying spatial concepts. Levinson (1996) states that an analysis of spatial language is useful to examine the speakers' understanding of spatial concepts (cf. Levinson, 1996). Similarly, Landau and Jackendoff (1993) point out that an appropriate analysis of spatial language enables new insights and knowledge about spatial thinking and the understanding of the corresponding spatial representations. Their concept of spatial representations is defined as "a level of mental representation devoted to encoding the geometric properties of objects in the world and the spatial relationships among them" (Landau & Jackendoff, 1993, p. 217). This definition of spatial representations can be considered as comprising several characteristics of the components of spatial ability in Pinkernell's (2003) model (mental and real spatial visual operations and geometrical thinking) (see Section 2.1.3.4), and of cognitive process in spatial tasks (such as spatial relations perception and mental images) (see Section 2.1.2). Furthermore, Landau & Jackendoff (1993) analyse the relationship between spatial representations and other information and representations, which is visualised in Figure 4. They state that spatial representations, which is used synonymously with spatial information or knowledge, are formed from visual, auditory or haptic information and act as an input for motor information and even for language or linguistic representation. The double arrow between language and spatial representations in Figure 4

shows the reciprocal effect of these two representations, which must exist to account for the phenomenon of spatial language. In order to gain spatial information from language, Landau & Jackendoff (1993) presuppose that "if linguistic judgements can be based on spatial properties of objects then the information involved in the linguistic judgement must be able to pass through the interface between spatial representation and language" (Landau & Jackendoff, 1993, p. 256).

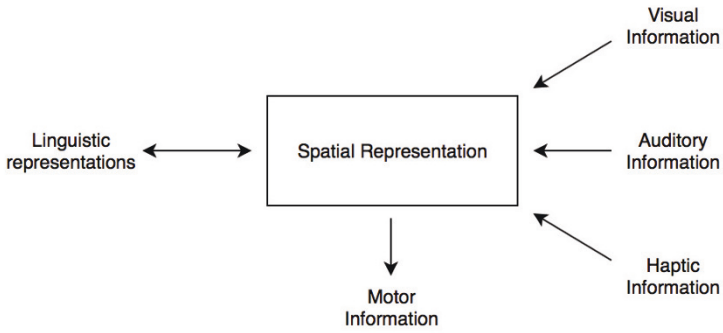



**Figure 4 The relationship between spatial representation and language according to Landau and Jackendoff (1993, p. 256).**

Therefore, for a better understanding of the relationship between language and spatial ability, one has to analyse deeper the phenomenon of spatial language, which can be simplified as the language used to talk about objects and about places (cf. Landau & Jackendoff, 1993).

*2.3.1.1 Functions of spatial language*

Based on the theoretical framework of Levinson (2009) and Landau and Jackendoff (1993), there are two main functions of spatial language in discourse: *object description* and *object localization*. The former function of spatial language is to describe objects by referring to their underlying spatial characteristics and representations required for their perception in space. Landau & Jackendoff (1993) state that shape is the most important factor for identifying objects, especially when it comes to choosing names for a particular object, which has not been assigned a common name yet. For the identification of objects based on shape, the spatial representations must provide enough information about the shape so that an individual is able to differentiate between the objects which be-

long to a particular category based on the shape (cf. Landau & Jackendoff, 1993). There are several objects which might be described by the same shape, for example, a football and a basketball, however, a more detailed shape description is needed to identify one ball from another. Another useful way to describe objects might be by focussing on the distinct parts of that objects and naming them separately by focussing on their shape and then to agree on a suitable name for the whole object depending on the names of parts (cf. Landau & Jackendoff, 1993). A corresponding example is shown in Figure 5(a), where a particular object is broken down in four parts (represented by the different shadings), each of which can be assigned a particular name, such as *arms*, *head*, etc… and a possible name for the whole object could be a *cube with a head and two arms*.

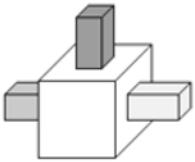


**Figure 5(a) Breakdown for describing a spatial object**

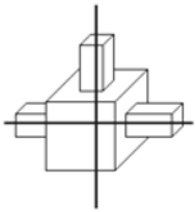


**Figure 5(b) Oriented axes for describing a spatial object**

A further way to describe spatial objects is to use oriented axes to describe its spatial parts. The oriented axes denote the regions of an object based on spatial orientation, such as the top or bottom of an object, etc. (cf. Landau & Jackendoff, 1993). Figure 5(b) shows an example of how oriented axes can be used to differentiate between the different spatial parts of a particular object. The horizontal axis in Figure 5(b) distinguishes between the top and bottom of the spatial object, whereas the vertical axis serves as left and right spatial orientation. The use of such oriented axes to describe parts of objects can be regarded as spatial orientation within the object, which is described more in the next sub-section, Section 2.3.1.2.

The second function of spatial language is to localise an object in space. Spatial language which is used for object localization is characterised by several linguistic phenomena which represent the cognitive process in spatial tasks such as figure-ground, spatial relation and spatial position perception. Figure-ground per-

ception has been described as the ability to identify a figure or an object from an optical complex background or to identify parts of the object from a given object and isolate them (see Section 2.1.2.1). When describing the location of spatial objects or the spatial relation between two spatial objects, the notions of *figure* and *ground*, are important concepts in order to distinguish between what is localised with regards to what (cf. Talmy, 1983). *Figure* denotes the object which is to be located and is synonym with theme (cf. Carroll, 1988) and reference object or trajectory (cf. Langacker, 1987). Talmy (1983) describes the figure as "a moving or conceptually moveable object whose site, path, or orientation is conceived as variable the particular value of which is the salient issue" (Talmy, 1983, p. 232). In order to locate a figure in space, the locating entity *ground*, also known as *relatum* in Carroll's (1988) work, has to be introduced (cf. Jordens & Lalleman, 1988). In comparison to the figure, the ground is the reference object and is considered to be stationary in the frame of reference. Moreover, the ground's properties should be known in order to characterise the figure's spatial properties (cf. Talmy, 1983). For instance, in the phrase "the book is on the shelf", the book is the figure and the shelf is the ground, whereby the shelf is used to identify the location of the book in space.

The localisation of an object in space can be dependent on the position of the describing person in space, who acts as the centre in spatial discourse (cf. Levinson, 2003, 2006). On a linguistic level, elements in speech which can indicate that the person is the figure or the ground in space are referred to as spatial deixis. Levinson (2006) defines spatial deixis as a linguistic operation in which an individual uses his or her own body or speech to act as a reference point to identify the figure. Spatial deixis are phrases or words which require contextual information bound to a particular spatial situation in order to understand their spatial meaning (cf. Levinson, 2006). According to Levinson (2006), there are three different types of spatial deixis which might be useful when localising an object in space: *central*, *compositional* deixis and *optional origo*. *Central* deixis are demonstrative pronouns, for example, *this*, or adverbs, for example, *here*, in which the reference object is dependent from the contextual frame (cf. Levinson, 2006). *Compositional* deixis are deixis, which are added in a spatial description, which is already defined and given (cf. Levinson, 2006). For instance, if one compares the utterances, *the nail at the edge of the table*, and, *the key is at this edge of the table*, the latter includes a more specific description of the spatial position of the nail in relation to the table and in the second utterance it is clear

which edge of the table is meant. *Optional Origo* are deixis used in descriptions, in which the describing person acts as a centre of the coordinate system in space (cf. Levinson, 2006). An example of use of optional origo is the utterance, *the dog is at the left side of the box*, which is only true from the perspective of the describing person, which acts as the centre of the spatial situation (cf. Levinson, 2006). The first type of dexis, *central,* do not necessarily require a verbally predefined spatial relation, but it requires an additional dimension, mostly a visual configuration which can be supported by gestures. In contrast, *compositional* and *optional origo* can be based on a wholly verbal definition of the spatial relation, which can be understood without a visualisation of the space in which the objects described can be found. In the case of the use of deixis in different registers of language, central deixis seem to be more common in everyday language due to their requirement of a context. At first sight, both compositional and optional origo seem to exhibit characteristics of mathematics language i.e. requiring specific vocabulary (e.g. spatial prepositions), however, they are still context-oriented, but to a lesser extent than central deixis due to their higher degree of verbalisation. Therefore, spatial deixis can be regarded as elements which strenghten the aspect of how language and spatial-visual content are deeply interrelated and show the necessity of language to orientate oneself in space in a social environment. The issue of how spatial orientation can be represented in language is discussed in the upcoming section.

*2.3.1.2 Spatial orientation and language*

Although, at first sight, spatial orientation seems to be a wholly cognitive process, its concepts and operations can be given a social dimension by lexical and grammatical structures of languages (cf. Lehmann, 2013). When describing the localisation of the reference object, Levinson (1996) differentiates between three different frames of reference, which are also referred to as strategies for spatial orientation: *absolute*, *relative*, and *intrinsic* frames of reference.

In the absolute frame of reference, the individual chooses an invariable, context-free reference point, for instance, the four compass directions together with the directions “up” (sky) and “down” (ground) (cf. Levinson, 1996). An example of absolute frame of reference can be found in the phrase *the city hall is east of the church.* In this phrase, *east* is absolute and does not allow any interferences of the person’s body position in the description of the spatial relation.

In the second frame of reference, *intrinsic*, an individual uses the structure of another object as a ground for spatial orientation: "in the intrinsic frame of reference the figure object is located with respect to what are often called *intrinsic* or *inherent* features of the ground object" (Levinson, 1996, p. 366). Consider the phrase *the central station is to the left of the cathedral*, were the structure of the ground *cathedral* is used to localise the position of the central station. If the ground does not have any structure at all, then this frame of reference cannot be applied in context. However, in this case the cathedral has a front, most probably where the main entrance door is located, and this serves for specifying the position of the central station. Whereas the intrinsic frame of reference describes the relation between two spatial objects, the next frame of reference can describe ternary relations.

In the *relative* frame of reference, individuals orientate themselves in space by choosing their body as the ground for orientation. In this case, the individual considers the structure of his body for orientation, such as the front and back side, the upper and lower side, and the right and left side. For example, consider the situation where a player is in front of a basketball stand and the ball is next to the stand. Let us assume that at first a player sees the ball to the left of a basketball stand. If the player turns 90 degrees anticlockwise around the stand from his original position, the ball would be behind the stand, thus the person is describing the spatial relation under consideration of their body position. As a matter of fact, this frame of reference can be considered as more flexible because the reference point can change its position or orientation in space.

Levinson (1996) states that whereas reference frames provide an adequate way to describe spatial orientation, they do not hinder ambiguity provided by the pragmatics of language. Consider the use of *behind* in the following phrases, *the car is behind the bus,* in which *behind* is semantically nonspecific concerning the reference frames. It could either mean that view of the car is obstructed by the bus (which could be equivalent to *the car is (driving) in front of the bus on the same lane and I am (driving) behind the bus on the same lane*) or the car is on the right or left-hand side of the rear of the bus from a passer-by's point of view. Therefore, when analysing spatial language, one should not only consider the conventional meaning of the elements carrying spatial meaning, but also the context in which these elements are used.

After a description of spatial language from a more linguistic point of view, the next section addresses the use of spatial language in mathematics learning and teaching. This is mainly done by focussing on current German mathematics curriculum specifications, since spatial language in German mathematics educational research has not been extensively researched.

#### *2.3.1.3 Spatial language in German mathematics curriculum*

Talking about space should be emphasised and supported in mathematics classroom, especially in geometry lessons, where students are required to describe objects and their location. In terms of curricular demands, the German mathematics curriculum provides just a few content-based specifications on space and figures with the emphasis on developing and learning spatial language in mathematics classroom. Such specifications in curricula address the need for students to be able to describe geometrical structures in the environment and to describe and reflect on characteristics and spatial relations between geometrical objects (e.g., position-relation; cf. Federal German Ministry of Education, 2004). Therefore, in general, the acquisition of spatial language is underestimated in German mathematics curriculum specifications.

Few German and international researchers (e.g., Wollring, 2012) have attempted to address the importance of spatial language in mathematics lessons. Wollring (2012) points out that vocabulary in the German spatial language is mostly borrowed and it has already got another meaning in everyday language. The phenomenon of borrowing in mathematics language from other varieties of language can also be interepreted as use of metaphors in the context of mathematics education (see Section 2.2.5). Consider, for instance, the term *Ecke* in the German language: *Ecke* in everyday language means corner, whereas in German spatial language, *Ecke* refers to a vertex of a solid in spatial geometry. A similar phenomenon can be observed in the English spatial language, where *edge* in everyday language can mean the border, e.g., *a path runs along the edge of the field*. This requires students to reconstruct and negotiate new meanings of known words in the new spatial context to learn spatial language successfully. Wollring (2012) states that merely the meaning of linguistic elements for expressing spatial orientation requires little, if any, negotiation in the students' spatial concept formation, for example, the distinction between left and right. The above facts show the need to raise awareness about the need for developing spatial language

in mathematics classroom. The fact that many notions in spatial language are borrowed from everyday language increases the ambiguity and hence difficulties among students' understanding, especially if such new meaning are not explicitly negotiated in mathematics classrooms.

Overall, spatial language seems to play an important role in mathematics learning, especially in geometry lessons, in which students are required to describe objects and their locations, which are the functions of spatial language, described in Section 2.3.1.1. The amalgation of spatial knowledge and linguistic elements in spatial language provides an appropriate approach for investigating the relationship between content and language learning in the present study. In this present work, spatial language denotes a variety of language which is not only limited to describing objects, but it should also include the language used to solve spatial tasks, to verbalise spatial thinking or strategies, and to orientate oneself in space in a social context.

### *2.3.2 Spatial task strategies and language*

Following an overview of spatial language, a further attempt to provide another approach to the relationship between language and spatial abilities by focussing on language and the strategies for solving spatial tasks is provided in this section. As already described in Section 2.1.5, one of the most common differentiations in strategies for solving spatial tasks is *analytic* vs. *holistic* (cf. Barrat, 1953). In analytic strategies, learners focus more on specific characteristics of the object and use logical reasoning to solve the spatial task. In contrast to mental transformation and manipulation of objects in holistic strategies, spatial characteristics of objects and the underlying logical reasoning can be verbalised more easily (see Section 2.1.5.2). The stronger connection of analytic strategies to verbalisation and holistic strategies to visualisation can be observed in Schultz's (1991) categorisation of strategies for solving spatial tasks in *visual-holistic* and *verbal-analytic.* Hence, spatial language, whether in inner, external[16] or even in written form, seems to play an important role in analytic strategies of solving spatial tasks.

A similar argument is proposed by Schwank (2003) in her theory about thinking styles and the role of language. Schwank (2003) differentiates between two types

[16] The terms *internal* and *external* are used synonymously with internal and external speech according to Vygostky (1993) (see Section 2.2.2).

of thinking styles: predicative and functional. The predicative thinking is more of a static modelling where the focus is on determining same or similar characteristics (for example, color or form) in invariant and structural connections. In contrast, functional thinking emphasises the transformation within thinking processes, thus focussing on the dynamic aspect of thinking. With regard to the role language in thinking processes, Schwank (2003) adds that language is important for solving tasks requiring predicative thinking processes. The recognition of characteristics and structural connections in predicative thinking processes in mathematics tend to be expressed using concepts which are easier to verbalise if learners can activate the corresponding knowledge adequately (cf. Schwank, 2003). In contrast to predicative thinking, learners solving a task using functional thinking processes are often unable to verbalise their procedural approach to the task adequately, not only due to the increasing cognitive demands of verbalisations, but also because language does not seem to be the main cognitive tool in accomplishing the respective task (cf. Schwank, 2003).

Schwank's (2003) approach to the two thinking styles is analogous to Barrat's (1953) distinction between analytic and holistic strategies for solving spatial tasks. The focus of predicative thinking on identifying features and structural relationships is typical for analytic strategies of spatial tasks. Similarly, holistic strategies are characterised by the dynamic process of transformation, which complies with the definition of functional thinking according to Schwank (2003). The emphasis on importance of language in both predicative thinking and analytic strategies makes the high similarity between Schwank's (2003) thinking styles and Barrat's (1953) spatial strategies plausible. However, Schwank's (2003) theoretical framework is more general in scope, since the objects of thought are not explicitly defined, as in the case of analytic and holistic strategies, in which the object of thought are spatial objects or their mental representation.

Language seems to play a more important role in certain strategies of solving spatial tasks over others. In particular, language used in analytic strategies to solve spatial tasks should exhibit certain characteristics, such as geometrical-spatial characteristics of spatial objects. Therefore, in addition to the overview of spatial language given in Section 2.3.1, the notion of spatial language should be extended by the language which is used for solving spatial tasks, negotiating spatial meaning in discourse and verbalising spatial thinking, whether it is of predicative/analytic or functional/holistic nature. Based on assumptions from previ-

ous literature (e.g., Schwank, 2003; Plath, 2014), students should find it easier to verbalise spatial thinking of an analytic nature, thus the verbalisation of holistic spatial thinking in spatial tasks is expected to be more challenging. Such facts strengthen the second approach to the relationship between language and spatial ability in this thesis, i.e. language is useful for solving spatial ability tasks in certain circumstances, based on the individual's thinking approach, whether analytic or holistic.

### *2.3.3 Summary: Interplay of language and spatial ability*

In this section, I described the interplay of language and spatial ability based on theoretical findings from previous literature. Two approaches of linking language with spatial abilities were discussed. The first approach emphasised the use of spatial language in order to verbalise spatial thinking for understanding the underlying spatial conceptions. This focussed on the structure of spatial language and its underlying functions in spatial discourse: to describe spatial objects, their location in space or their location with regard to other spatial objects in space and to orientate oneself in space. The second approach was based on the potential use of language in developing strategies to solve spatial tasks. Previous literature shows that language could be influential in the development of analytic strategies for solving spatial tasks (cf. Section 2.3.2). It was agreed that the notion of spatial language should involve language used to solve spatial tasks, which can be realised either in oral or written form. Moreover, spatial language encompasses the aspect of verbalising spatial thinking or strategies, thus an analysis of spatial language provides a promising approach to investigate how students solve spatial tasks and their spatial knowledge. The different ways of how mathematical (also spatial) knowledge can be represented in mathematics education is discussed in the next section.

## 2.4 Representations of mathematical knowledge

In this section, different ways of representing spatial knowledge are discussed. In the first theoretical framework, the internal representations of mathematical conceptions, i.e. the nature of mathematical knowledge according to Sfard (1991), are introduced. This is followed by the second theoretical framework developed by Bruner (1966/1971), which focuses on the external modes of representations of mathematical knowledge.

### *2.4.1 Sfard's dual nature of mathematical conceptions*

In her work about concept development in mathematics education, Sfard (1991) investigates the nature of mathematics conceptions, which are defined as "the whole cluster of internal representations and associations evoked by the [mathematical] concept – the concept's counterpart in the internal, subjective 'universe of human knowing'" (Sfard, 1991, p. 3). Sfard (1991) emphasises the need to focus on the cognitive processes from which mathematical concepts emerge and proposes two conceptions of mathematical concepts: *structural* and *operational* conceptions. Most mathematical concepts are known for their high degree of abstraction, whose mathematical definition, existence and properties are predefined by mathematicians and do not allow room for adjustments (cf. Sfard, 1991). The reference to the static nature of conceptions regarding mathematical notions is referred to as *structural*:

> Seeing a mathematical entity as an object means being capable of referring to it as if it was a real thing – a static structure, existing somewhere in space and time. It also means being able to recognize the idea 'at a glance' and to manipulate it as a whole, without going into details. (Sfard, 1991, p. 4).

In contrast, the other way of internalising mathematical concepts is by perceiving the mathematical concept as an array of actions in which the concept emerges. This means that instead of a static entity, an *operational* conception of a mathematical concept is of a dynamic nature and encompasses the operations and processes in developing the underlying concept:

> In contrast, interpreting a notion as a process implies regarding it as a potential rather than actual entity, which comes into existence upon request in a sequence of actions. Thus, (...) the operational [conception] is dynamic, sequential, and detailed. (Sfard, 1991, p. 4)

For a deeper understanding of the two conceptions of mathematical concepts, consider the concept of symmetry in geometry. Symmetry can be regarded as a geometrical characteristic of a form or object as in *a square is symmetric*. However, it can also be conceived as a process or set of actions as in the phrase *a shape is symmetric if it can be divided into two or more identical pieces.* In the former phrase of structural nature, the focus is on a static property of being symmetric or not. In the latter phrase, the focus is on the operational conception

of symmetry, i.e. the process of dividing an object into identical parts, hence on the actions which should be performed by learners to develop or learn the concept of symmetry in an operational approach.

Sfards' (1991) theory evinces similarities with Schwank's (2003) thinking styles, which have been described in Section 2.3.2. The predicative and functional thinking style seems to conform with the structural and operational conceptions of mathematical concepts in Sfard's (1991) framework. However, whereas the focus in Schwank's (2003) theory is on thinking processes, Sfard (1991) elaborates more on the underlying mathematical concepts as results of the predicative and functional thinking style. Therefore, Sfard's (1991) theoretical work about the nature of mathematical concepts provides a suitable framework for this present study, in which students' spatial conceptions (as conceptions of spatial objects) in solving spatial tasks are investigated. In Section 2.3.2, I have indicated the plausible connection between the thinking styles from Schwank (2003) and Barrat's (1953) distinction between analytic and holistic strategies. Sfard's (1991) structural and operational conceptions can also be associated with analytic and holistic strategies respectively, whereby analytic and holistic strategies can be regarded as (mental) actions performed in spatial tasks which are of structural or static and operational or dynamic nature respectively. In the subsequent section, three representations regarding the external representation of mathematical concepts in mathematics education are discussed.

### *2.4.2 Bruner's modes of representation*

In his research in developmental psychology, Bruner (1966/1971) proposed that mathematical knowledge can be represented in three modes of representation: the enactive, iconic, and symbolic. In the enactive mode, knowledge is represented by a set of actions which are required to achieve the underlying goal. The iconic mode is characterised by the focus on the pictures and graphics to represent knowledge. The symbolic representation is characterised by "a set of symbolic or logical propositions drawn from a symbolic system that is governed by rules or laws for forming and transforming prepositions" (Bruner, 1971, p. 45) and in contrast to other modes, it enables the representation of knowledge with a high level of abstraction. Regarding the use of the different representations, Bruner (1971) argues that there is no optimal sequence of the modes of representations during learning processes:

> If it is true that the usual course of intellectual development moves from enactive through iconic to symbolic representation of the world, it is likely that an optimum sequence will progress in the same direction. Obviously, this is a conservative doctrine. For when the learner has a well-developed symbolic system, it may possible to by-pass the first two stages [the enactive and iconic]. (Bruner, 1971, p. 49)

Hence, rather than a sequence, Bruner's (1966/1971) modes of representation should be represented as interdependent entities, whereby knowledge can be represented differently in each mode and transferred from one to another. Bruner's (1966/1971) theoretical framework can be applied to describe the different representations of spatial knowledge. In the enactive representation, a set of real actions to represent spatial knowledge in solving spatial tasks can be the rotation of cubes or other spatial objects to visualise a particular rotation or movement. Students can also use a picture of the spatial object (e.g., supported by arrows) or a sequence of pictures (e.g., the spatial object in different positions) in order to visualise the rotation or movement of the spatial object in the iconic mode of representation. In the symbolic representation, spatial knowledge can be expressed in terms of mathematical symbols, words and phrases, such as representing the notion of rotation by using the degree(s) notation, by explicitly describing the involved actions of movement or by using rotation matrices. Regarding the representation based on the nature of spatial tasks, spatial tasks of a symbolic representation can include describing spatial objects and spatial relations and understanding their underlying spatial meaning based on words and phrases.

Most researchers (e.g., Bruner, 1971; Bruner et al., 1988; Plath 2014) agree that all three representations are important for development and learning of mathematical concepts. Figure 6 visualises a possible model for the representation modes in spatial tasks based on Bruner's (1966/1971) theoretical framework.

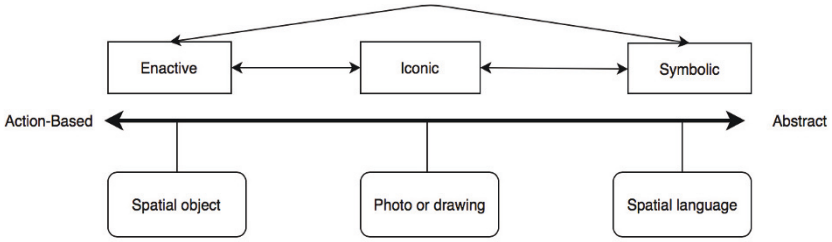



**Figure 6 A visualisation of Bruner's (1966/1971) stages of representation under consideration of spatial knowledge**

The double arrows between the three representations of knowledge, *symbolic*, *iconic* and *enactive* (cf. Bruner, 1971), in Figure 6 indicate the possibility of change in representation of knowledge. A symbolic representation of spatial knowledge can be represented by the highly abstract phenomenon of spatial language, whereas photos or drawings (for example, of spatial objects) can be used to represent spatial knowledge in iconic and enactive mode on a less abstract level respectively (see Figure 6).

*2.4.3 Summary: Representations of mathematical knowledge*

Two models about representing spatial knowledge were introduced in Section 2.4.1 and Section 2.4.2. First, Sfard's (1991) model about the internal representations in mathematics knowledge was introduced, wherein her theory was applied to the mental processes which can affect how spatial knowledge is represented in spatial tasks or spatial solving processes. However, the internal representation in Sfard's (1991) model is not enough to represent the whole complex phenomenon of spatial knowledge, since it does not reveal much about how and in which way spatial knowledge can be externalised. Therefore, Bruner's (1971) work about the external representations of knowledge – symbolic, iconic and enactive modes of representations – was considered as crucial for illustrating the different ways of externalising spatial knowledge. Hence, both models can be used for representing spatial knowledge both from an internal cognitive perspective and from an external social perspective, as it is illustrated in Figure 7.

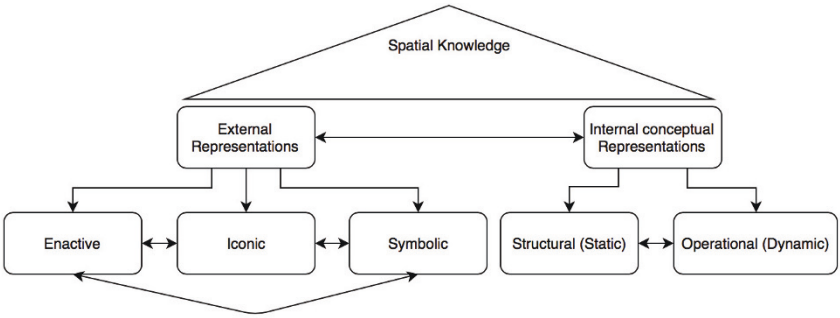



**Figure 7 Overview of the internal and external representations of spatial knowledge**

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