Preface

This book was originally published in Japanese by Saiensu-sha, Tokyo, Japan in May 2003. Then, the first English edition was published by Springer in 2006 with some revision from Japanese version. It has been 10 years since the publication of the first English edition. During this decade, so many remarkable progresses have been made in the area of quantum information theory. So, I decided to publish the second English edition with considerable revision to include these latest progresses.

I believe that the most important progress among this decade is the resolution of the additivity problem. This problem was proposed as a problem equivalent to various kinds of additivity problems in entanglement theory and channel capacity. To include this progress, I have added Sect. 8.13: Violation of superadditivity of entanglement formation. Since this topic needs special knowledge for classical information, I have also added Sect. 2.6: Large Deviation on Sphere. Although Sect. 2.6 is important for the understanding of quantum information, it seems that its content is partially misunderstood among quantum information researchers. So, this section is also helpful for understanding quantum information. Further, since this topic affects the channel capacity, I rewrote Sect. 9.2: C-Q Channel Coding with Entangled Inputs.

The second most important progress is considerable progress on quantum hypothesis testing. This progress has been made by so many authors. Although Chernoff bound, Hoeffding bound, and Han–Kobayashi bound in this topic had not been exactly solved at that time, these exact forms have been completely solved during this decade. To reflect this progress, I completely rewrote Chap. 3. Since the quantum Han–Kobayashi bound is closely related to the new type of quantum Rényi relative entropy, the new Chap. 3 also discusses it. Further, to make Chap. 3 more self-contained, I have moved Section: Information Quantities in Quantum Systems to Chap. 3. The content of this chapter is employed in Chap. 4 because the hypothesis testing is closely related to channel coding. Hence, I also rewrote Chap. 4 partially. Recently, this relation has been of interest for many researchers. In this revision, I emphasize this relation more strongly while this relation was discussed in the first edition. I also summarize its history in Sect. 4.10.

The third most important progress is considerable progress in conditional Rényi entropy. To discuss this issue, I newly added Sect 2.1.5: Conditional Rényi entropy and Sect 5.6: Conditional Rényi Entropy and Duality. This progress has made notable influence on entropic uncertainty relation, secure random number generation, entanglement measure, and the duality relation between coherence and information leakage. Since these four areas also have greatly progressed during this decade, I summarize them in the following new sections: Sect. 7.3: Entropic Uncertainty Relation, Sect. 8.14: Secure Random Number Generation, Sect. 8.8: Maximally Correlated State, and Sect. 8.15: Duality Between Two Conditional Entropies. Further, using the contents of Sect. 8.15, I simplified the proof in Sect. 9.6: Channel Capacity for Quantum-State Transmission. Also, based on the previously gained knowledge, I newly added Subsect. 9.6.3: Decoder with assistance by local operations.

Other topics have advanced recently, and we can list discord, Bregman divergence, and matrix convex function, among them. The first edition discussed discord, however, its treatment is not perfect. So, in the second edition, I have completed it in new Sect. 8.10: Discord. To deal with recent progress of Bregman divergence, I added Sect. 2.2.2: Bregman divergence. Recently, extremal point decomposition of matrix convex functions was completed. This decomposition brings us a more detailed analysis of quantum f-relative entropy. So, to include the decomposition, I rewrote Appendix A.4: Convex Functions and Matrix Convex Functions. Then, I have newly added Sect. 6.7: Relative Modular Operator and Quantum f-Relative Entropy.

As one of the features of this book, I have discussed the axiomatic approach, while the first edition emphasizes mainly in entanglement measure. However, this approach is also important in the entropy theory. To clarify this relation, I have newly added Sect. 2.5: Continuity and Axiomatic Approach. Also, I have added several descriptions related to this approach.

In this edition, I additionally have included around 120 new exercises, so that this edition totally has 450 exercises. I also have completed solutions for all exercises for readers' convenience. Since each chapter can be understood separately, I have organized the second edition so that each chapter contains solutions for exercises and proofs of theorems in that chapter. In particular, since Chap. 2 is composed of knowledge from classical information and has a distinguished description from existing textbooks, this chapter might be useful for readers interested only in classical information. Recently, I have published another book "Introduction to Quantum Information Science" with S. Ishizaka, A. Kawachi, G. Kimura, and T. Ogawa, which is more introductory. Since this book is more mathematically oriented, I changed the title to "Quantum Information Theory: A Mathematical Foundation."

I am grateful to Prof. Fumio Hiai, Prof. Francesco Buscemi, Dr. Motohisa Fukuda, Mr. Kengo Takeuchi, and Mr. Kosuke Ito for their comments. I would like to express my appreciation for their cooperation. I would also like to thank Dr. Claus E. Ascheron of Springer Science+Business Media for his encouragement and patience during the preparation of the manuscript.

Nagoya, Japan Masahito Hayashi

Preface to the First English Edition

This book is the revised and English edition of the Japanese book *Introduction to Quantum Information Theory*, which systematically describes quantum information theory and was originally published by Saiensu-sha, Tokyo, Japan in May 2003. The study of information processing based on the physical principles of quantum mechanics began in the 1960s. Recently, some protocols of quantum information processing have been demonstrated experimentally, and their theoretical aspects have been examined more deeply and mathematically. In particular, the field that is concerned with their theoretical bounds is called quantum information theory and has been studied by many researchers from diverse viewpoints.

However, only Holevo's book *Probabilistic and Statistical Aspects of Quantum Theory*, which was published back in 1980 (English version in 1982), places a heavy emphasis on the mathematical foundation of quantum information theory. Several books concerning quantum information science have been published since the late 1990s. However, they treat quantum computation, the physical aspects of quantum information, or the whole of quantum information science and are not mainly concerned with quantum information theory. Therefore, it seemed to me that many researchers would benefit from an English book on quantum information theory, and so I decided to publish the English version of my book. I hope that it will make a contribution to the field of quantum information theory.

This book was written as follows. First, the author translated the original Japanese version in cooperation with Dr. Tim Barnes. Next, the book was revised through the addition of many new results to Chaps. 8–10 and a historical note to every chapter. Several exercises were also added, so that the English version has more than 330 exercises. Hence, I take full responsibility for the content of this English version. In this version, theorems and lemmas are displayed along with the names of the researchers who contributed them. However, when the history of the theorems and lemmas is not so simple, they are displayed without the contributing researchers' names and their histories are explained in a historical note at the end of the given chapter.

I am indebeted to Prof. Masanao Ozawa and Dr. Tohya Hiroshima for their feedback on the Japanese version, which been incorporated into the English version. I am also grateful to (in alphabetical order) Dr. Giulio Chiribella, Mr. Motohisa Fukuda, Prof. Richard Gill, Dr. Michael Horodecki, Dr. Satoshi Ishizaka, Dr. Paolo Perinotti, Dr. Toshiyuki Shimono, and Dr. Andreas Winter, for reviewing the technical aspects of the English version. Further, Dr. Tomohisa Hayakawa, Mr. Daichi Isami, Mr. Takashi Okajima, Mr. Tomotake Sasaki, Mr. Taiji Suzuki, Mr. Fuyuhiko Tanaka, and Mr. Ken'ichiro Tanaka used the draft of the English version in their seminar and verified its contents. Miss Rika Abe commented on the nontechnical parts of the book. Further, Mr. Motohisa Fukuda helped me in compiling the references. I would like to express my appreciation for their cooperation.

I also would like to thank Prof. Hiroshi Imai of the University of Tokyo and the people associated with the ERATO Quantum Computation and Information Project for providing the research environments for this English version. I would like to express my gratitude to Dr. Glenn Corey and editorial staffs of Springer for good excellent editing process. I would also like to thank Dr. Claus E. Ascheron of Springer Science+Business Media for his encouragement and patience during the preparation of the manuscript.

Hongo, Tokyo, Japan Masahito Hayashi
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Preface to the Japanese Edition

This textbook attempts to describe quantum information theory, which is a presently evolving field. It is organized so that the reader can understand its contents with very elementary prior knowledge. This research field has been developed by many researchers from various backgrounds and has matured rapidly in the last 5 years.

Recently, many people have considered that more interdisciplinary activities are needed in the academic world. Hence, education and research must be performed and evaluated on a wide scope. However, since the extreme segmentation of each research area has increased the difficulty of interdisciplinary activities. On the other hand, quantum information theory can in some sense form a bridge between several fields because it deals with topics in a variety of disciplines including physics and information science. Hence, it can be expected to contribute in some way to removing the segmentation of its parent fields. In fact, information science consists of subfields such as computer science, mathematical statistics, and Shannon's information theory. These subfields are studied in separate contexts.

However, in quantum information theory, we must return to the fundamentals of the topic, and there are fewer boundaries among the different fields. Therefore, many researchers now transcend these boundaries.

Given such a starting point, the book was written to enable the reader to efficiently attain the interdisciplinary knowledge necessary for understanding quantum information theory. This book assumes only that the reader has knowledge of linear algebra, differential and integral calculus, and probability/statistics at the undergraduate level. No knowledge of quantum mechanics is assumed.

Some of the exercises given in the text are rather difficult. It is recommended that they be solved in order to acquire the skills necessary for tackling research problems. Parts of the text contain original material that does not appear elsewhere. Comments will be given for such parts.

The author would like to thank Prof. Hiroshi Imai of the University of Tokyo, Prof. Shun-ichi Amari of the Brain Science Institute at RIKEN, Prof. Kenji Ueno of Kyoto University, and the people associated with the ERATO Quantum Computation and Information Project, the Brain Science Institute at RIKEN, and the Department of Mathematics at Kyoto University for providing me with the means to continue my research. The author also wishes to thank Prof. Hiroshi Nagaoka of the University of Electro-Communications, Prof. Akio Fujiwara of Osaka University, Prof. Keiji Matsumoto of the National Institute of Informatics, and Dr. Tomohiro Ogawa of the University of Tokyo for helpful discussions and advice. This text would not have been possible without their enlightening discussions.

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