

## Chapter 2

# Literature Review

### 2.1 Geotechnical Site Characterization

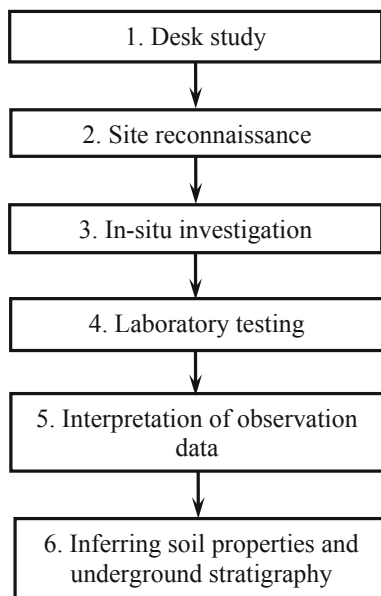
Geotechnical site characterization aims to delineate underground stratigraphy (including the number and thicknesses or boundaries of soil layers) and to estimate soil properties for geotechnical analysis and/or designs. As shown in Fig. 2.1, it is a multi-step process that can be divided into six stages: desk study, site reconnaissance, in situ investigation, laboratory testing, interpretation of observation data, and inferring soil properties and underground stratigraphy, as discussed in the following four subsections.

#### 2.1.1 Desk Study and Site Reconnaissance

Geotechnical site characterization often starts with desk study to collect the existing information about the specific site, including geological information, geotechnical problems and properties, site topography, groundwater conditions, meteorological conditions, existing construction and services, and previous land use (Clayton et al. 1995; Mayne et al. 2002). The existing information can be obtained from various useful sources, as shown in Table 2.1.

Geological information (e.g., geological history) is available from existing geological records (e.g., geological maps, reports, and publications), regional guides (e.g., Geotechnical Engineering Office 2000 for Hong Kong), air photographs, soil survey maps and records, textbooks, etc. The information of geotechnical problems and parameters (e.g., records of adverse ground conditions, soil classification and properties, and stratigraphy) can be collected from existing geotechnical reports (e.g., Kulhawy and Mayne 1990), peer-reviewed academic journals (e.g., geotechnical journals, engineering geology journals, and civil engineering journals), and previous ground investigation reports at or near the site.

**Fig. 2.1** Procedure of geotechnical site characterization (after Clayton et al. 1995; Mayne et al. 2002)



Information on site topography can be gathered from topographical maps and stereo air photographs. Well records, previous ground investigation reports, topographical maps, and air photographs provide information about groundwater conditions of the site (e.g., the groundwater level). Meteorological records provide information on meteorological conditions. Furthermore, information on existing construction and services and previous land use can be collected from topographical maps, air photographs, mining records, etc.

In addition to these sources of existing information, expertise of engineers (i.e., the domain knowledge of engineers, such as knowledge on geology and geotechnical engineering and local experience) also provides useful information for geotechnical site characterization. It is an integration of acquired information (including knowledge from education, information from professional training, and experience from deliberate practice) and engineers' comprehension (Vick 2002). Engineers' expertise is internally organized in knowledge patterns, each of which represents domain knowledge of a certain field or aspect (Vick 2002). The expertise patterns are internally formed when a person grows from an amateur (e.g., student) into a qualified engineer.

After desk study, site reconnaissance is carried out to confirm and supplement the information that is previously collected during desk study. Furthermore, the accesses and work conditions of the site are evaluated at the stage of site reconnaissance.

**Table 2.1** Summary of existing information (after Clayton et al. 1995)

Type	Source
Geology	Geological maps
	Geological reports
	Geological publications
	Regional guides
	Air photographs
	Soil survey maps and records
Geotechnical problems and properties	Geotechnical reports
	Academic journals (e.g., geotechnical journals, engineering geology journals, and civil engineering journals)
	Previous ground investigation reports
Site topography	Topographical maps
	Stereo air photographs
Groundwater conditions	Topographical maps
	Air photographs
	Well records
	Previous ground investigation reports
Meteorological conditions	Meteorological records
Existing construction and services	Topographical maps
	Plans held by utilities
	Mining records
	Construction press
Previous land use	Out-of-print topographical maps
	Out-of-print geological maps
	Air photographs
	Airborne remote sensing
	Archaeological society records
	Mining records

### 2.1.2 *In Situ Investigation*

In situ investigation work generally includes drilling and sampling, in situ testing, and groundwater investigation (Clayton et al. 1995; Mayne et al. 2002). Herein, four in situ tests are reviewed, including standard penetration test (SPT), cone penetration test (CPT), pressuremeter test (PMT), and vane shear test (VST).

SPT consists of driving a standard thick-walled sampler into ground at the bottom of a borehole through repeated blows of a standard hammer and measuring the number (i.e., SPT *N*-value) of blow counts to advance the sampler to a vertical distance of 300 mm after an initial seating drive of 150 mm (Clayton 1995; Mayne et al. 2002). SPT provides soil samples for soil classification and laboratory tests. It is relatively simple to perform and is suitable for many types of soils. The results

(SPT  $N$ -values) provided by SPT are, however, highly operator-dependent and are highly variable (e.g., Kulhawy and Trautmann 1996; Mayne et al. 2002).

CPT involves pushing a cylindrical steel probe into the ground at a constant rate and measuring the resistance to the penetration (Lunne 1997; Mayne et al. 2002). CPT generally provides three measurements: cone tip resistance, sleeve friction, and pore water pressure (e.g., piezocone test). During the past several decades, CPT has gained popularity around the world because it is fast and largely operator-independent and provides near-continuous measurements (e.g., Robertson and Campanella 1983a, b; Robertson 1990; Mayne et al. 2002; Phoon et al. 2003; Robertson 2009). However, CPT is not suitable for gravel and boulder deposits and does not allow retrieval of soil samples (e.g., Mayne et al. 2002).

PMT expands a long cylindrical probe radially into surrounding ground to measure the amount of volume of fluid and pressure used to inflate the probe (Mair and Wood 1987; Briaud 1992; Mayne et al. 2002). The measurements subsequently provide the relationship between the pressure and the deformation of soils. PMT has a strong theoretical background and provides a complete stress–strain curve (Mair and Wood 1987; Briaud 1992; Wang and O’Rourke 2007). The procedure of PMT is, however, relatively complicated, and it is quite time-consuming and expensive (Mayne et al. 2002).

VST consists of inserting a four-bladed vane into clay and rotating the device about the vertical axis by applying a torque (Mayne et al. 2002). The measured peak and residual values of the torque are used to calculate undrained shear strength and in situ sensitivity of the clay. The procedure of VST is relatively simple. VST provides a convenient way to evaluate the undrained shear strength for stability analysis of embankment, footing, and excavation in soft clay (Bjerrum 1973; Mesri 1989; Mayne et al. 2002). The undrained shear strength measured by VST, however, needs to be corrected by multiplying it with an empirical correction factor before it is used in calculation (Bjerrum 1973). In addition, application of VST is limited to soft or stiff clay, and the results from VST might be affected by sand lens and seams (Mayne et al. 2002).

### ***2.1.3 Laboratory Testing***

After in situ investigation work, soil samples are brought back to laboratory for further testing. The laboratory tests include measuring index properties (e.g., moisture content, specific gravity, unit weight, particle size distribution, Atterberg limits, and moisture–density relationship), strength and stiffness tests (e.g., unconfined compression test, direct shear test, and triaxial tests), permeability tests (e.g., constant head test and falling head test), and consolidation tests (e.g., oedometer test) (Clayton et al. 1995; Mayne et al. 2002). The laboratory tests provide measurements of index properties (e.g., water content, liquid limit, plastic limit, and shrinkage limit), strength and stiffness parameters (e.g., undrained shear strength, effective friction angle, Young’s modulus, and shear modulus), coefficient

of permeability, coefficient of compressibility, and preconsolidation stress. Laboratory tests generally result in more accurate measurements than those obtained from in situ tests, but they are usually more time-consuming and expensive than in situ tests (e.g., Mayne et al. 2002).

#### ***2.1.4 Interpretation of Observation Data and Inferring the Site Subsurface Conditions***

In situ and/or laboratory test measurements (i.e., measured property) might not be the soil properties that can be directly used in geotechnical analysis and/or designs (i.e., design property). The design property can be calculated from direct measurements by transformation models (including empirical correlations and/or theoretical relationships) between the measured property and the design property. Many transformation models between geotechnical properties are available in the geotechnical literature (e.g., Kulhawy and Mayne 1990; Phoon and Kulhawy 1999b). For example, Tables 2.2 and 2.3 summarize the availability of transformation models for clays and sands (Kulhawy and Mayne 1990; Phoon and Kulhawy 1999b), respectively.

For clays, both in situ (including SPT, CPT, PMT, and VST tests) and laboratory test results can be used to estimate soil properties on soil classification, consistency, in situ stress state (e.g., preconsolidation stress, overconsolidation ratio, and coefficient of horizontal soil stress), strength (e.g., undrained shear strength), and deformability (e.g., Young's modulus, constrained modulus, and coefficient of consolidation). In addition, laboratory tests of clays also provide measurements to evaluate unit weight, effective friction angle, Poisson's ratio, compression index, coefficient of secondary compression, and coefficient of permeability.

For sands, in situ (including SPT, CPT, and PMT tests) and/or laboratory test results can be used to estimate soil properties on soil classification, in situ stress state (e.g., coefficient of horizontal soil stress), strength (e.g., effective friction angle), deformability (e.g., Poisson's ratio, Young's modulus, compression index, constrained modulus, and subgrade modulus), permeability (e.g., coefficient of permeability), and liquefaction resistance (e.g., cyclic stress ratio). Several transformation models that will be used in this book are given in Appendix 2.1.

Based on the information available prior to the project (including existing information collected from various sources and the expertise of engineers), site observation data obtained from in situ and laboratory tests, and corresponding transformation models, geotechnical engineers estimate the site ground conditions (including soil properties and underground stratigraphy) for geotechnical analysis and/or designs. Finally, geotechnical engineers are responsible for producing a geotechnical site characterization report that records the information about the site and for providing some technical suggestions for geotechnical designs (Mayne et al. 2002).

**Table 2.2** Summary of availability of transformation models for clays provided by Kulhawy and Mayne (1990) (after Phoon and Kulhawy 1999b)

Property category	Soil property	Laboratory or theory correlation	In situ test correlation			
			SPT	CPT <sup>c</sup>	PMT	VST
Basic characterization	Classification	✓ <sup>a</sup>	× <sup>b</sup>	✓	×	×
	Unit weight	✓	×	×	×	×
	Consistency	×	✓	✓	×	×
In situ stress state	Preconsolidation stress	✓	✓	✓	✓	✓
	Overconsolidation ratio	✓	✓	✓	×	✓
	Coefficient of horizontal soil stress	✓	✓	✓	✓	×
Strength	Effective friction angle	✓	×	×	×	×
	Undrained shear strength	✓	✓	✓	✓	✓
Deformability	Poisson's ratio	✓	×	×	×	×
	Young's modulus	✓	×	×	✓	×
	Compression index	✓	×	×	×	×
	Constrained modulus	✓	✓	✓	×	×
	Coefficient of consolidation	✓	×	✓	×	×
	Coefficient of secondary compression	✓	×	×	×	×
Permeability	Coefficient of permeability	✓	×	×	×	×

<sup>a</sup>✓ = available<sup>b</sup>× = unavailable<sup>c</sup>Including piezocone test

### 2.1.5 Challenges in Geotechnical Site Characterization

Geotechnical site characterization relies on both the information available prior to the project (including the existing information collected from various sources and engineers' expertise) and site observation data (e.g., in situ and laboratory test results). It is a challenging task for geotechnical engineers to integrate systematically and rationally the information from the two sources (i.e., information available prior to the project and site observation data). This problem is further complicated by the fact that only a small portion of geotechnical materials is tested and the number of site observation data is usually limited during geotechnical site

**Table 2.3** Summary of availability of transformation models for sands provided by Kulhawy and Mayne (1990) (after Phoon and Kulhawy 1999b)

Property category	Soil property	Laboratory or theory correlation	In situ test correlation		
			SPT	CPT <sup>c</sup>	PMT
Basic characterization	Classification	✓ <sup>a</sup>	× <sup>b</sup>	✓	×
	Unit weight	✓	×	×	×
	Relative density	×	✓	✓	×
In situ stress state	Coefficient of horizontal soil stress	✓	×	✓	✓
Strength	Effective friction angle	✓	✓	✓	✓
Deformability	Poisson's ratio	✓	×	×	×
	Young's modulus	✓	✓	×	✓
	Compression index	✓	×	×	×
	Constrained modulus	✓	×	✓	×
	Subgrade modulus	✓	×	×	×
Permeability	Coefficient of permeability	✓	×	×	×
Liquefaction resistance	Cyclic stress ratio	×	✓	✓	×

<sup>a</sup>✓ = available<sup>b</sup>× = unavailable<sup>c</sup>Including piezocone test

characterization (Wang et al. 2010). Furthermore, the challenges in geotechnical site characterization become more profound because of the inherent spatial variability of soils and various uncertainties that arise during geotechnical site characterization, as discussed in the next section.

## 2.2 Uncertainties in Soil Properties

During geotechnical site characterization, various uncertainties are incorporated into the estimated soil properties (e.g., Christian et al. 1994; Kulhawy 1996; Phoon and Kulhawy 1999a; Baecher and Christian 2003), including inherent variability of soil properties, measurement errors, statistical uncertainty, and transformation uncertainty, as discussed in the following four subsections.

### 2.2.1 Inherent Variability

Geotechnical materials are natural materials, and their properties are affected by various factors during their formation process, such as properties of their parent

materials, weathering and erosion processes, transportation agents, and conditions of sedimentation (Vanmarcke 1977; Jaksa 1995; Phoon and Kulhawy 1999a; Baecher and Christian 2003; Mitchell and Soga 2005). Properties of geotechnical materials, therefore, vary inherently. Such inherent variability is independent of the state of knowledge about geotechnical properties and cannot be reduced as the knowledge improves. Therefore, it is categorized as “aleatory uncertainty” in nature (Baecher and Christian 2003).

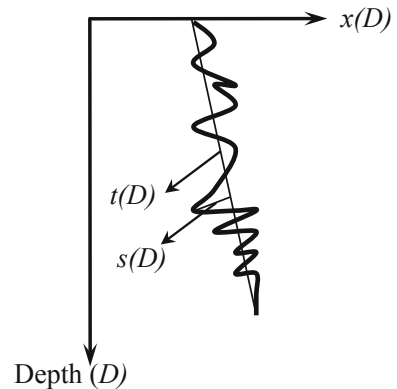
Soil properties inherently vary from one location to another location in both horizontal and vertical directions. The soil property at the same elevation is frequently simplified and represented by a single variable (i.e., fully correlated along horizontal direction). Such simplification is usually considered reasonable to some degree for at least two reasons: (1) The soils at the same elevation went through similar geological processes. Therefore, the values of a soil property at different locations, but with the same elevation, are somewhat close to each other, and the correlation of the soil property at different locations in horizontal direction is much stronger than that in vertical direction (e.g., Phoon and Kulhawy 1996, 1999a); (2) such simplification generally leads to conservative designs (e.g., Fenton and Griffith 2007; Klammler et al. 2010). The remaining part of this subsection, therefore, focuses only on spatial variation of soil properties in vertical direction.

Figure 2.2 shows spatial variation of a soil property  $x(D)$  along the depth  $D$  (i.e., in vertical direction), which can be decomposed into two components: the trend function  $t(D)$  of the soil property and the remaining fluctuating component  $s(D)$  (Lumb 1966; Vanmarcke 1977 and 1983; DeGroot and Baecher 1993; Phoon and Kulhawy 1999a; Phoon et al. 2003). The soil property can be written as

$$x(D) = t(D) + s(D) \quad (2.1)$$

in which  $t(D)$  represents the mean value of  $x(D)$  at a given depth;  $s(D)$  represents the variation of  $x(D)$  at the given depth and has a mean of zero and a standard deviation of  $\sigma_s(D)$ . Probability theory and statistics have been applied to model the inherent spatial variability of soil properties since 1970s (Lumb 1966 and 1974; Vanmarcke

**Fig. 2.2** Spatial variability of soil properties along the depth (after Phoon and Kulhawy 1999a)





1977 and 1983; DeGroot and Baecher 1993; Jaksa 1995; Fenton 1999a, b; Wang et al. 2010). Detailed modeling of Eq. (2.1) is further discussed in the following two subsections.

### 2.2.1.1 Lumb's Formulation

Lumb (1966) proposed three different forms of spatial variation of soil properties along the depth: (1) Both  $t(D)$  and standard deviation  $\sigma_s(D)$  of  $s(D)$  are spatially constant; (2)  $t(D)$  varies linearly along the depth, but  $\sigma_s(D)$  is spatially constant; and (3)  $t(D)$  varies linearly along the depth and  $\sigma_s(D)$  increases with the depth. Lumb (1966) examined the spatial variation of soil properties of four soils (i.e., silty sand, clayey silt, sandy clay, and marine clay) and showed that (1) the tangent of effective friction angles of silty sand and clayey silt has spatially constant mean and standard deviation along the depth; (2) the mean of the compression index of sandy clay increases linearly with the depth, while its standard deviation remains constant along the depth; (3) the mean and standard deviation of the undrained shear strength of marine clay increase as the depth increases; and (4) Gaussian distributions are in good agreement with distributions of soil properties of the four soils except for compression index of sandy clay, the distribution of which is close to a lognormal distribution. The mean and standard deviation of soil properties were examined by Lumb (1966), but the correlation of the variation of a soil property at different locations was not considered. By this simplification, the soil property that is normally distributed has the following representation (e.g., Lumb 1966)

$$x(D) = t(D) + \sigma_s(D)z \quad (2.2)$$

in which  $z$  is a standard Gaussian random variable. If the soil property is lognormally distributed, it can be written as (e.g., Ang and Tang 2007; Au et al. 2010)

$$x(D) = \exp(t(D)_N + \sigma(D)_N z) \quad (2.3)$$

in which  $t(D)_N$  and  $\sigma_s(D)_N$  are the mean and standard deviation of the logarithm  $\ln(x(D))$  of  $x(D)$ .

### 2.2.1.2 Random Field Theory

Vanmarcke (1977 and 1983) developed the random field theory to characterize the spatial variability of geotechnical materials, by which the correlation of a soil property at different locations (i.e., autocorrelation) is taken into account rationally. In the context of random field theory, a soil property within a statistically homogenous soil layer is described by a series of random variables with the same mean and standard deviation, and the autocorrelation among these random variables depends on the correlation length (also sometimes known as “scale of fluctuation”)

(Vanmarcke 1977 and 1983; Fenton and Griffiths 2008). The correlation length is a separation distance, within which the soil property shows a relatively strong correlation from point to point. In the space domain, a soil property that is normally distributed can be written as (Ang and Tang 1984; Fenton and Griffiths 2008; Wang et al. 2010)

$$\underline{x}(\underline{D}) = \mu \underline{1} + \sigma \underline{L}^T \underline{Z} \quad (2.4)$$

in which  $\underline{x}(\underline{D})$  is a vector of the soil property of interest at different  $n_D$  depths;  $\underline{D}$  is a depth vector;  $\mu$  and  $\sigma$  are, respectively, the mean and standard deviation of the soil property, which are spatially constant;  $\underline{1}$  is a vector with  $n_D$  components that are all equal to one;  $\underline{Z}$  is a standard Gaussian vector with  $n_D$  components; and  $\underline{L}$  is a  $n_D$  by  $n_D$  upper triangular matrix obtained by Cholesky decomposition of the correlation matrix  $\underline{R}$  satisfying

$$\underline{R} = \underline{L}^T \underline{L} \quad (2.5)$$

in which the  $(i, j)$ th entry is the correlation coefficient (i.e.,  $\rho_{ij}$ ) of the soil property at the  $i$ th and  $j$ th depths and is calculated from a correlation function  $f_\rho$ , which is a function of the correlation length and describes the correlation of the variation of a soil property at different locations. For a given correlation structure (or correlation function), a random field is uniquely determined by the mean, standard deviation, and correlation length. Based on a set of measurements (e.g., cone tip resistance measured by a CPT test), the mean, standard deviation, and correlation length of the measured properties can be estimated (e.g., Vanmarcke 1977; Jaksa 1995; Fenton 1999b). Note that applying random field theory to describe (or model) the inherent spatial variability of soil properties involves two important issues: the statistical homogeneity (or stationarity) and the correlation function, which are discussed in the following two subsections, respectively.

### 2.2.1.3 Statistical Homogeneity and Data Transformation

Generally speaking, statistical homogeneity (or stationarity) means that the mean and standard deviation of the soil property of interest are spatially constant, and the autocorrelation only depends on the separate distance between two locations rather than the absolute positions (e.g., Vanmarcke 1983; Fenton 1999a, b). Statistical homogeneity is a significant prerequisite for conventional statistical analysis (e.g., calculating mean and standard deviation) on a set of observation data. If the observation data are not stationary, data transformation techniques can be used to transform the observed data into stationary data, such as detrending techniques for non-constant trend component and variance transformation techniques (e.g., logarithmic transformation and the Box–Cox transformation) for non-constant standard deviation (e.g., Jaksa 1995).

Detrending techniques, such as regression analysis and normalization method (e.g., Jaksa 1995; Phoon et al. 2003), are widely used to remove the obvious trend component. Regression analysis gives the best fit of a predefined trend function based on the observation data. To some degree, the choice of the trend function is a decision on how much of the spatial variability in observation data is considered as a deterministic function, and correspondingly, the residual component in the measurements is considered statistically and is modeled as a random process (i.e., a random field in space domain) (Baecher 1987). The trend function can be linear or polynomial, but, as pointed out by Lumb (1974), there are rarely sufficient data to estimate any form more complicated than a linear trend in routine site characterization. The normalization method can also be used to remove the trend in direct measurements for many geotechnical properties. For example, normalization by the effective overburden stress accounts for the effect of confinement that generally increases with the depth (Wroth 1984; Houlsby 1988; Robertson 1990, 2009; Phoon et al. 2003). Note that normalization by the effective overburden stress requires information on soil unit weights and groundwater conditions, which might not be available.

The adequacy of detrending is of great significance for the assumption of stationarity. This can be examined by many methods, such as visual inspection of the autocovariance function (e.g., Box and Jenkins 1970; Jaksa 1995), the run test (e.g., Alonso and Krizek 1975; Campanella et al. 1987), Kendall's  $\tau$  tests (e.g., Ravi 1992), and Bartlett's test (e.g., Phoon et al. 2003). It is also worthwhile to point out that the detrending process leads to a decrease in the estimated correlation length (e.g., Phoon et al. 2003), because the detrending process removes the large-scale fluctuation in nature (Fenton 1999a).

#### 2.2.1.4 Correlation Function and Correlation Length

The correlation between the variations of a soil property at different locations can be characterized by a correlation function. In general, there are two types of correlation functions: the finite-scale model, in which the correlation dies out rapidly as the separate distance is greater than the correlation length, and the fractal model, in which the correlation remains significant over a very large distance (Fenton 1999a). Although Fenton (1999b) examined 143 sets of CPT data and concluded that the correlation of soil properties follows the fractal model in nature, the finite-scale model is commonly used in the analysis of geotechnical data (e.g., Lumb 1974; Vanmarcke 1977; DeGroot and Baecher 1993; Jaksa 1995; Phoon et al. 2003).

The major advantage of the finite-scale model is that usually only one model parameter, i.e., correlation length, needs to be determined. In addition, Fenton (1999a) also noted that for a given site, there may be little difference between a properly selected finite-scale model and a real fractal model over a finite domain. Equation (2.6) gives four commonly used finite-scale models for the analysis of geotechnical data (Vanmarcke 1977)

$$f_\rho = \begin{cases} \exp\left(-\frac{2|\Delta D|}{\lambda}\right) & \text{(a)} \\ \exp\left[-\pi\left(\frac{\Delta D}{\lambda}\right)^2\right] & \text{(b)} \\ \left(1 + \frac{4|\Delta D|}{\lambda}\right) \exp\left(-\frac{4|\Delta D|}{\lambda}\right) & \text{(c)} \\ \exp\left(-\frac{|\Delta D|}{\lambda}\right) \cos\left(\frac{\Delta D}{\lambda}\right) & \text{(d)} \end{cases} \quad (2.6)$$

in which  $\Delta D$  is the separate distance between two depths;  $\lambda$  is the correlation length. The correlation length can be estimated by choosing a theoretical correlation model given by Eq. (2.6) to fit the empirical autocorrelation function estimated from observation data (e.g., Jaksa 1995) or by maximum-likelihood methods (e.g., DeGroot and Baecher 1993; Fenton 1999a, b). Because the available observation data for a specific site are usually limited, choosing an appropriate correlation function and determining the correlation length are challenging tasks for a specific site. Fenton (1999a) argued that the correlation model developed for one site is applicable for another site that has similar geological conditions. Among the four finite-scale models in Eq. (2.6), the single exponential correlation function given by Eq. (2.6a) is most widely used in analysis of soil data (e.g., Lumb 1974; DeGroot and Baecher 1993; Lacasse and Nadim 1996; Phoon et al. 2003; Wang et al. 2010). Phoon et al. (2003) observed that, among the four models in Eq. (2.6), the single exponential correlation function (i.e., Eq. (2.6a)) leads to the most stringent criteria for the identification of stationarity using Bartlett statistics.

The statistically homogenous soil layers and correlation functions of soil properties are usually determined using in situ and laboratory test data (i.e., measured soil properties) (e.g., Fenton 1999b; Phoon et al. 2003). Phoon et al. (1995) and Phoon and Kulhawy (1999a) summarized the inherent spatial variability of many soil properties (including both measured and design soil properties), some of which are shown in Table 2.4. Research is, however, relatively limited that addresses directly inherent spatial variability of design soil properties (e.g., effective friction angle) using random field theory.

### 2.2.2 Measurement Errors

Measurement errors are unavoidable during in situ and laboratory tests (Christian et al. 1994; Kulhawy 1996). As pointed out by Kulhawy (1996) and Kulhawy and Trautmann (1996), measurement errors arise from three sources: equipment errors (e.g., inaccuracies of the measuring devices and variations in equipment geometries and systems used for routine testing), procedural–operator errors (e.g., limitations in existing test standards and how these standards are followed by operators), and random testing error (i.e., the remaining scatter in measurements that is not assignable to specific testing parameters and is not caused by inherent variability of soils). The total measurement error can be estimated from the square root of the sum

**Table 2.4** Summary of inherent spatial variability of soil properties (After Phoon and Kulhawy 1999a)

Test type	Soil type	Soil property	Mean		Coefficient of variation (%)		Correlation length in vertical direction (m)	
			Range	Average	Range	Average	Range	Average
Strength test	Clay	Undrained shear strength (UC) <sup>a</sup>	6–412 kPa	100 kPa	6–56	33	0.8–6.1	2.5
		Undrained shear strength (UU) <sup>b</sup>	15–363 kPa	276	11–49	22		
		Undrained shear strength (CIUC) <sup>c</sup>	130–713 kPa	405	18–42	32		
	Sand	Effective friction angle	35°–41°	37.6°	5–11	9	N/A	N/A
CPT	Clay	Tangent of effective friction angle	0.65–0.92	0.744	5–14	9	N/A	N/A
		Cone tip resistance	0.5–2.1 MPa	1.6 MPa	5–40	27	0.1–2.2	0.9
		Corrected cone tip resistance	0.4–2.6 MPa	1.3 MPa	2–17	8	0.2–0.5	0.3
	Sand	Cone tip resistance	0.4–29.2 MPa	4.1 MPa	10–81	38	0.1–2.2	0.9
SPT	Clay	SPT <i>N</i> -value	7–63	32	37–57	44	N/A	2.4
	Sand	SPT <i>N</i> -value	7–74	35	19–62	54	N/A	N/A
PMT	Clay	Limit pressure	0.4–2.8 MPa	10.1	10–32	15	N/A	N/A
	Sand	Limit pressure	1.6–3.6 MPa	2.3 MPa	23–50	40	N/A	N/A
VST	Clay	Young's modulus from pressuremeter tests	5.2–15.6 MPa	9.0 MPa	28–68	42	N/A	N/A
		Undrained shear strength (VST) <sup>d</sup>	6–375 kPa	105 kPa	4–44	24	2.0–6.2	3.8

<sup>a</sup>Undrained compression test  
<sup>b</sup>Unconsolidated undrained triaxial test  
<sup>c</sup>Consolidated isotropic undrained triaxial compression test  
<sup>d</sup>Vane shear test

of the equipment, procedural–operator, and random testing errors (Orchant et al. 1988; Kulhawy and Trautmann 1996). Phoon et al. (1995) summarized the measurement errors of several laboratory tests, including strength tests (e.g., triaxial test, laboratory vane shear test, direct shear test, and Atterberg limit tests). In addition, Kulhawy and Trautmann (1996) summarized the measurement errors of several in situ tests (e.g., SPT, CPT, PMT, and VST) and estimated the total measurement errors of these in situ tests. The total measurement errors of SPT, CPT, PMT, and VST range from 15 % to 45 %, from 5 % to 25 %, from 10 % to 25 %, and from 10 % to 20 %, respectively. Note that these values can be used only as approximate guidelines for estimating measurement errors because they are evaluated based on limited available data and the evaluation relies on subjective judgments (Kulhawy and Trautmann 1996).

It is also worthwhile to note that measurement errors arise from a lack of knowledge about test equipments and procedures, and they are, therefore, categorized as “epistemic uncertainty” in nature (Baecher and Christian 2003). As the knowledge on test equipments and procedures improves, the measurement errors can be reduced. In addition, both measurement errors and inherent variability of soils contribute to the scatter of observation data. It is, however, difficult to discern the fluctuation resulted from inherent variability and that arising from measurement errors.

### 2.2.3 *Statistical Uncertainty*

As mentioned in Sect. 2.1.5 “Challenges in geotechnical site characterization,” the number of project-specific test results is usually limited during geotechnical site characterization (Nawari and Liang 2000; Baecher and Christian 2003; Wang et al. 2010). Different sets of test results with a relatively small sample size might result in significantly different statistics of soil properties. In other words, statistics of soil properties estimated from limited test results are uncertain. Such uncertainty is known as “statistical uncertainty” (Christian et al. 1994; Kulhawy 1996; Baecher and Christian 2003). Statistical uncertainty arises from insufficient observation data and is commonly considered to be included in measurement errors (Kulhawy 1996). It decreases as the observation data increases and is categorized as “epistemic uncertainty” in nature (Baecher and Christian 2003).

### 2.2.4 *Transformation Uncertainties*

Design soil property that is directly used in design can be estimated from in situ and laboratory test results (i.e., measured property) by transformation models, such as

**Table 2.5** Four transformation models and their uncertainties (After Phoon and Kulhawy 1999b)

Soil type	Design property	Measured property	Relationships <sup>b, c, d</sup>	Uncertainty $\sigma_\varepsilon$
Clay	Undrained shear strength (UU) <sup>a</sup> , $S_{u,UU}$	SPT $N$ -value	$\log\left(\frac{S_{u,UU}}{p_a}\right) = \log(0.29) + 0.72 \log(N_{SPT}) + \varepsilon$	0.15
	Undrained shear strength (VST), $S_{u,VST}$	Plasticity index ( $PI$ )	$\frac{S_{u,VST}}{\sigma'_p} = (0.11 + 0.0037PI)(1 + \varepsilon)$	0.25
	Young's modulus measured by pressuremeter test $E_{PMT}$	SPT $N$ -value	$\log\left(\frac{E_{PMT}}{p_a}\right) = \log(19.3) + 0.63 \log(N_{SPT}) + \varepsilon$	0.37
Sand	Effective friction angle, $\phi'$ ( $^\circ$ )	Cone tip resistance $q_c$	$\phi' = 17.6 + 11.0 \log\left(\frac{q_c/p_a}{\sqrt{\sigma'_{v0}/p_a}}\right) + \varepsilon$	2.8 $^\circ$

<sup>a</sup>Unconsolidated undrained triaxial test<sup>b</sup> $p_a$  is the atmospheric pressure<sup>c</sup> $\sigma'_p$  is preconsolidation stress<sup>d</sup> $\sigma'_{v0}$  is vertical effective stress

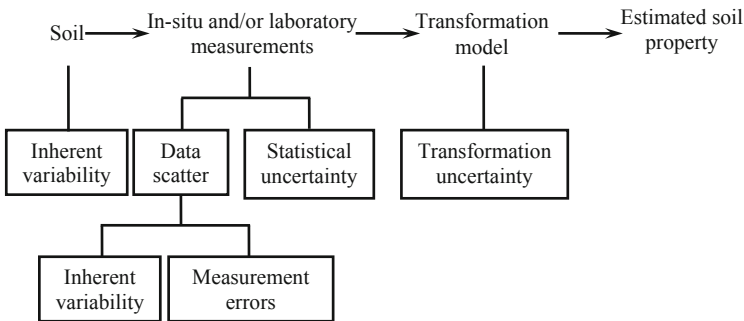
empirical and theoretical correlations between the measured property and design property (e.g., Kulhawy and Mayne 1990; Phoon and Kulhawy 1999b). Empirical correlations are often obtained from empirical data fitting, and hence, they are associated with some uncertainties because of data scatter and inaccuracy of the best fit (e.g., Kulhawy and Mayne 1990; Phoon and Kulhawy 1999b). Theoretical correlations are also associated with some uncertainties due to idealizations and simplifications in the theory (Phoon and Kulhawy 1999b). The uncertainties associated with the transformation model are collectively referred to as “transformation uncertainty”  $\varepsilon$ . Transformation uncertainty can be modeled as a random variable with a mean of zero and standard deviation of  $\sigma_\varepsilon$  that indicates the magnitude of the transformation uncertainty and reflects the degrees-of-belief on the corresponding transformation model (Phoon and Kulhawy 1999b). Phoon et al. (1995) and Phoon and Kulhawy (1999b) summarized the transformation uncertainty of several commonly used transformation models, four of which are shown in Table 2.5.

Transformation uncertainty arises from a lack of knowledge about the relationship between the measured property and the design property, and it is therefore categorized as “epistemic uncertainty” and can be reduced as the knowledge about the relationship improves (e.g., Baecher and Christian 2003; Zhang et al. 2004).

### 2.2.5 Uncertainty Propagation

The inherent variability and various epistemic uncertainties (including measurement errors, statistical uncertainty, and transformation uncertainty) that arise during site characterization are incorporated into estimated design properties, as shown in Fig. 2.3. The total variability (including inherent variability, measurement errors, statistical uncertainty, and transformation uncertainty) of estimated design properties can be estimated by a second-moment approach developed by Phoon et al. (1995) and Phoon and Kulhawy (1999a, b). When the epistemic uncertainties (i.e., measurement errors, statistical uncertainty, and transformation uncertainties) are large, the total variability of estimated design soil properties is large. This generally leads to conservative designs by reliability-based design (RBD) approaches (e.g., Phoon et al. 1995). As the knowledge improves, the epistemic uncertainties are reduced and the total variability of estimated design soil properties is reduced. The conservatism in designs resulted from RBD approaches then is reduced. This is reasonable in the sense that the design should be less conservative as the knowledge on soil properties accumulates.

It is, however, worthwhile to point out that the epistemic uncertainties that arise from insufficient knowledge do not contribute to the actual response of geotechnical structures. In contrast, the inherent variability of soils affects significantly the actual response of geotechnical structures (e.g., Phoon et al. 1995; Fenton and Griffiths 2002 and 2003; Fenton et al. 2005; Hicks 2005; Klammler et al. 2010). To understand probabilistically the actual response of geotechnical structures, it is meaningful to characterize explicitly the inherent variability of design soil properties for RBD. Research is, however, rare that addresses directly and explicitly the inherent variability of design soil properties using probability theory.



**Fig. 2.3** Uncertainties in estimated soil properties (after Phoon and Kulhawy 1999a)



## 2.3 Bayesian Approach

Bayesian approach provides a formal framework to integrate systematically information from different sources, such as observation data and knowledge available prior to collecting the observation data (i.e., prior knowledge) (e.g., Ang and Tang 2007; Sivia and Skilling 2006). It can be used to estimate probability distributions of model parameters of a system based on prior knowledge and observation data and to update the probability of an event using new information obtained (Baecher and Christian 2003).

### 2.3.1 Bayesian Mathematical Framework

Let  $\Theta = [\theta_1, \theta_2, \dots, \theta_{n_m}]$  denote a set of random variables that represent the  $n_m$  uncertain model parameters of a model  $M(\Theta)$  of the system concerned. Under a Bayesian framework, the updated distribution  $P(\Theta|Data)$  of model parameters  $\Theta$  is written as (e.g., Ang and Tang 2007; Wang et al. 2010)

$$P(\Theta|Data) = KP(Data|\Theta)P(\Theta) \quad (2.7)$$

in which  $Data$  is the observation data;  $K = 1/P(Data)$  is a normalizing constant that is independent of model parameters;  $P(\Theta)$  is the prior distribution of model parameters that reflects prior knowledge on model parameters in the absence of observation data; and  $P(Data|\Theta)$  is the likelihood function that reflects the model fit with observation data.

### 2.3.2 Prior Distribution

In general, there are two types of prior distribution: uninformative prior distribution and informative prior distribution (Siu and Kelly 1998; Baecher and Christian 2003), as discussed in the following two subsections.

#### 2.3.2.1 Uninformative Prior Distribution

Uninformative prior distributions suggest that there is no prevailing prior knowledge on the possible values of model parameters and all possible values of model parameters are considered equally likely (Jeffreys 1983; Baecher and Christian 2003). For mutually independent model parameters, the prior distribution  $P(\Theta)$  is further written as

$$P(\Theta) = \prod_{i=1}^{n_m} P(\theta_i) \quad (2.8)$$

in which  $P(\theta_i)$ ,  $i = 1, 2, \dots, n_m$  is the prior distribution of the model parameter  $\theta_i$ . Equation (2.9) gives three commonly used uninformative prior distributions of  $\theta_i$ .

$$P(\theta_i) = \begin{cases} 1/(\theta_{i,\max} - \theta_{i,\min}) & \text{for } \theta_i \in [\theta_{i,\min}, \theta_{i,\max}] & \text{(a)} \\ \text{constant} & \text{for } \theta_i \in [-\infty, +\infty] & \text{(b)} \\ 1/\theta_i & \text{for } \theta_i \in [0, +\infty) & \text{(c)} \end{cases} \quad (2.9)$$

Equation (2.9a) gives an uninformative prior distribution of  $\theta_i$  with a range from the minimum  $\theta_{i,\min}$  to the maximum  $\theta_{i,\max}$ . This prior distribution is particularly useful when a reasonable range of  $\theta_i$  is available. Equation (2.9b) gives an uninformative prior distribution of  $\theta_i$  that ranges from negative infinity to positive infinity. Both Eq. (2.9a) and (2.9b) have been criticized because they are not invariant to transformation of variables (Jeffreys 1983; Siu and Kelly 1998; Baecher and Christian 2003). It is also noted that the integral on Eq. (2.9b) is infinite, which is improper since the integral on a probability density function (PDF) should be unity (Baecher and Christian 2003). In addition, Eq. (2.9b) has two singularities at both endpoints.

Equation (2.9c) gives an uninformative prior distribution of  $\theta_i$  which ranges from zero to positive infinity and is uniform in a log scale (i.e.,  $\log(\theta_i)$  is uniformly distributed). Equation (2.9c) is also known as “Jeffreys prior,” and it is invariant to power and scale transformation of variables (Jeffreys 1983; Sivia and Skilling 2006). Note that Eq. (2.9c) also has a singularity at  $\theta_i = 0$ .

Uninformative prior distributions are widely used for the cases with a large number of observation data (e.g., Beck and Yuen 2004; Zhang 2011), in which the observation data tend to dominate the results of Bayesian approach so that the effect of prior knowledge is not so important. Research is, however, rare that explores systematically the effect of uninformative prior knowledge (e.g., Eq. (2.9a)) when the observation data are relatively limited, e.g., geotechnical site characterization.

### 2.3.2.2 Informative Prior Distribution

Informative prior distribution can be estimated from prior knowledge about model parameters, and it reflects the degrees-of-belief (or confidence level) of prior knowledge on model parameters. When limited prior knowledge on  $\theta_i$  (e.g., the mean and standard deviation of the system parameter  $\theta_i$  of interest) is available and it is testable (i.e., it can be determined whether or not there is a probability distribution consistent with the prior knowledge), the maximum entropy method can be used to obtain the prior distribution from the prior knowledge (e.g., Siu and Kelly 1998; Sivia and Skilling 2006). Within the maximum entropy method, the

information entropy  $H_I$  is used as a measure of uncertainty of the prior PDF  $P(\theta_i)$  of  $\theta_i$ , and it is defined as (Shannon and Weaver 1949)

$$H_I = - \int P(\theta_i) \ln[P(\theta_i)] d\theta_i, \quad i = 1, 2, \dots, n_m, \quad (2.10)$$

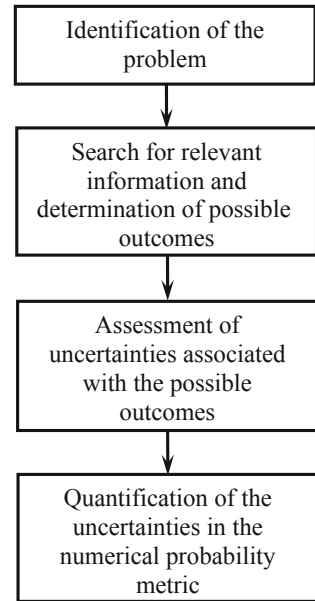
Then, the prior PDF  $P(\theta_i)$  of  $\theta_i$  is obtained by maximizing the information entropy  $H_I$  subject to constraints of the testable information provided by prior knowledge and normalization (e.g., Sivia and Skilling 2006). The principle of maximum entropy provides a reasonable approach for generating prior distribution based on limited prior knowledge that specifies some probability constraints.

When previous data are available, statistical analysis can be used to estimate prior distribution of model parameters, such as calculating the mean and standard deviation and maximum-likelihood function and fitting previous data with specified distributions by regression analysis (Berger 1985; Maritz and Lwin 1989; Carlin and Louis 1996; Siu and Kelly 1998). Such statistical analysis is a pragmatic approach for constructing prior distributions using previous data. It generally takes the previous data from various sources as the evidence with equal weight. This is, however, not well justified because the quality (or credibility) of data from different sources may be significantly different. In addition, it is difficult to incorporate systematically subjective judgments in the statistical analysis, which is often an essential component of prior knowledge, e.g., engineering judgments in geotechnical engineering.

Subjective probability provides an operational tool to measure the degrees-of-belief of engineers' opinions and to express quantitatively their engineering judgments (e.g., Baecher 1983; Vick 2002; Garthwaite et al. 2005; Silva et al. 2008). Subjective probability assessment methods are, therefore, another useful approach to construct informative prior distributions (Clemen 1996; Siu and Kelly 1998). They are usually developed based on cognitive models, in which a series of cognitive activities of the cognitive process are divided into several stages (e.g., Beach et al. 1986; Koriatic et al. 1980; Smith et al. 1991; Vick 2002). For example, Fig. 2.4 shows a stage cognitive model that consists of four stages (Vick 2002): identification of the problem, search for relevant information and determination of possible outcomes, assessment of uncertainties associated with the possible outcomes, and quantification of uncertainties in the numerical probability metric. During subjective probability assessment, various cognitive heuristics are involved, such as availability heuristic, anchoring and adjustment heuristic, and representativeness heuristic (Tversky and Kahneman 1974; Vick 2002). These heuristics are generally useful tools that help the assessor make decisions in a probabilistic manner, but they might, simultaneously, lead to some cognitive biases, as discussed in what follows.

According to the availability heuristic, the likelihood of an event is estimated by the ease with which similar events are recalled (Garthwaite et al. 2005). The event with relatively high probability is easy to be recalled. Conversely, an event with small probability is often difficult to be retrieved from memory. The availability

**Fig. 2.4** A stage cognitive model (after Vick 2002)



heuristic is often a useful indicator for assessing the probability of an event, but it can also lead to availability bias because of missing some information that is relatively difficult to be acquired (Clemen 1996; Garthwaite et al. 2005). In addition, the availability heuristic possibly results in hindsight bias because of the inclination to exaggerate the likelihood of occurrence of an event after it occurs and confirmation bias due to the inclination to ignore disconfirming evidence during searching for information in memory (Vick 2002).

The assessor sometimes estimates the probability of an event by choosing an initial probability value roughly, which is known as “anchor,” and adjusts the probability value according to knowledge about the event of interest to obtain the final probability value. This is referred to as “anchoring and adjustment heuristic.” This heuristic helps the assessor overcome cognitive limitation of human information processing capacity (Vick 2002; Garthwaite et al. 2005). Insufficient adjustment, however, might occur, so that the probability estimated from the heuristic might be biased toward the anchor. This subsequently leads to overconfidence bias (Vick 2002).

The representativeness heuristic evaluates the probability of an event by the extent to which the event is similar to some other events belonging to a particular category (Clemen 1996; Vick 2002). The probability of the event concerned belonging to the category increases as the similarity between the event of interest and other related events of the category increases. In general, the representativeness heuristic is a useful tool to estimate probability based on prior knowledge. Several cognitive biases, however, sometimes arise from it because the base rate of the

event concerned is ignored and there is the insensitivity to sample size of the category (Garthwaite et al. 2005).

In addition to cognitive biases, the outcomes (e.g., prior distribution herein) obtained from subjective probability assessment methods are also influenced by cognitive limitations, such as limited capacity of processing information that might lead to ignorance of useful information, limited good calibration range of subjective probability (e.g., from 0.2 to 0.8 by Fischhoff et al. 1977), and limited cognitive discrimination range of subjective probability (e.g., from 0.01 to 0.99 by Hogarth 1975; Fischhoff et al. 1977; Vick 1997 and 2002). Because of the cognitive biases and limitations, outcomes of subjective probability assessment might deviate from the actual beliefs of the assessor and be inconsistent with basic probability axioms (Vick 2002). To enable meaningful and coherent outcomes, subjective probability assessment often requires formally organized external procedures (Vick 2002; Baecher and Christian 2003). It is sometimes difficult to satisfy such a requirement due to the limitation of resources in geotechnical site characterization, particularly for projects with medium or relatively small sizes.

### 2.3.2.3 Confidence Level of Prior Knowledge

The standard deviation or coefficient of variation (COV) of the prior distribution indicates the degrees-of-belief (or confidence level) of the prior knowledge about model parameters. When the COV is relatively small, the confidence level of the prior knowledge on model parameters is relatively high, and the prior knowledge is relatively informative. As the COV increases, the confidence level of prior knowledge on model parameters decreases, and the prior knowledge becomes more and more uninformative. Siu and Kelly (1998) also noted that informative prior knowledge is likely to have significant impact on the results of Bayesian approach when observation data are relatively limited.

### 2.3.3 Likelihood Function

Constructing a likelihood function requires a model  $M(\Theta)$  that represents the system concerned and describes how the observation data are generated from the system. Within the model  $M(\Theta)$ , model parameters  $\Theta$  can be generally divided into two types:  $\Theta_S$  representing the unknown input parameter of the model used to describe behaviors of the system and  $\Theta_U$  representing modeling errors associated with the model. The uncertainties in  $\Theta_S$  and the modeling errors  $\Theta_U$  result in uncertainties in predications of the model. The goodness of fit of model predications with the observation data for a given set of model parameters is defined by the likelihood function. Based on the applications of the Bayesian approach, the likelihood function can be developed in three ways:

- (1) Consider the uncertainties in input parameters  $\Theta_S$  only when developing the likelihood function. For example, Jung et al. (2008) and Cetin and Ozan (2009) developed the likelihood functions considering uncertainties in input parameters of empirical models of soil classification. Yan et al. (2009) constructed the likelihood function for selecting the most appropriate regression model between the compression index and various other soil properties (e.g., liquid limit, plastic index) with the consideration of uncertainties of regression models.
- (2) Consider modeling errors  $\Theta_U$  only when developing the likelihood function. For example, Zhang et al. (2004) derived the likelihood function for reducing modeling errors associated with the empirical transformation model between the SPT  $N$ -values and undrained shear strength. Zhang et al. (2009) developed the likelihood function for characterization of modeling errors associated with deterministic models of slope stability analysis. Zhang et al. (2010a, b) constructed the likelihood function for probabilistic back analysis of slope stability with the consideration of modeling errors associated with deterministic models of slope stability analysis. In addition, Ching et al. (2010) derived the likelihood function for reducing uncertainties in estimated soil properties with the consideration of modeling errors associated with empirical transformation models.
- (3) Consider both uncertainties in input parameters  $\Theta_S$  and modeling errors  $\Theta_U$  when developing the likelihood function. Wang et al. (2010) and Cao et al. (2011) developed the likelihood function for probabilistic characterization of soil properties based on random field theory, in which both uncertainties in model parameters of the random field and modeling errors associated with the transformation model were considered. The Bayesian approach developed by Wang et al. (2010) and Cao et al. (2011) provides proper probabilistic characterization of soil properties using both prior knowledge and limited site observation data (i.e., CPT data). Their approach, however, only applies to a single and predefined statistically homogenous soil layer and provides no information on the number or thicknesses/boundaries of statistically homogenous soil layers.

### 2.3.4 Posterior Distribution

The posterior distribution in Eq. (2.7) is a joint distribution of model parameters. To obtain the posterior marginal PDF of one model parameter, integration on the posterior distribution over the space of the other model parameters is required. Since the posterior distribution might be very complicated, analytical integration is often infeasible. Numerical integration may be performed using a multidimensional grid over the space of other model parameters, but it must be performed repeatedly for a number of values of the model parameters concerned so as to yield

information about the whole marginal distribution of the model parameter. The computational complexity has been recognized as one key limitation of the Bayesian approach (e.g., Zhang et al. 2009; Wang et al. 2010). Traditionally, conjugated prior distributions (e.g., Siu and Kelly 1998; Baecher and Christian 2003) have been used to bypass the computational problems, but this nevertheless introduces artificial limitations to the choice of prior distribution.

Laplace asymptotic approach (Bleistein and Handelsman 1986) has been widely used in Bayesian system identification to bypass the computational complexity when a relatively large number of observation data are available (e.g., Papadimitriou et al. 1997; Beck and Katafygiotis 1998; Katafygiotis and Beck 1998; Katafygiotis et al. 1998; Wang et al. 2010). By Laplace asymptotic approach, the posterior distribution is approximate to a joint Gaussian PDF, in which the mean vector is equal to the most probable values (MPV) of model parameters. Under this approximation, the determination of posterior mean values of model parameters is reduced to finding the MPV by numerically minimizing the objective function  $f_{\text{obj}} = -\ln[P(\Theta|Data)]$ . The covariance matrix of the posterior distribution is provided by the inverse of the Hessian matrix of  $f_{\text{obj}} = -\ln[P(\Theta|Data)]$ . Note that Laplace asymptotic approach generally requires a relatively large number of observation data so that the posterior distribution peaks sharply and the MPV of model parameters are identified with relative ease by the optimization procedure (i.e., minimizing the objective function  $f_{\text{obj}} = -\ln[P(\Theta|Data)]$ ). Wang et al. (2010) have successfully applied the Laplace asymptotic approach to obtain the posterior distribution of random field model parameters using CPT data and showed that the results obtained from the Laplace asymptotic approach are in good agreement with those obtained from numerical integration. This indicates that CPT tests provide sufficient data for Laplace asymptotic approach to identify the MPV of random field model parameters.

### 2.3.5 Updating the Probability of an Event

Bayesian approach can also be used to update the probability of an event  $A$  using the new evidence  $B$  on  $A$ . The updated probability  $P(A|B)$  of  $A$  given the new evidence  $B$  is written as (e.g., Siu and Kelly 1998; Ang and Tang 2007)

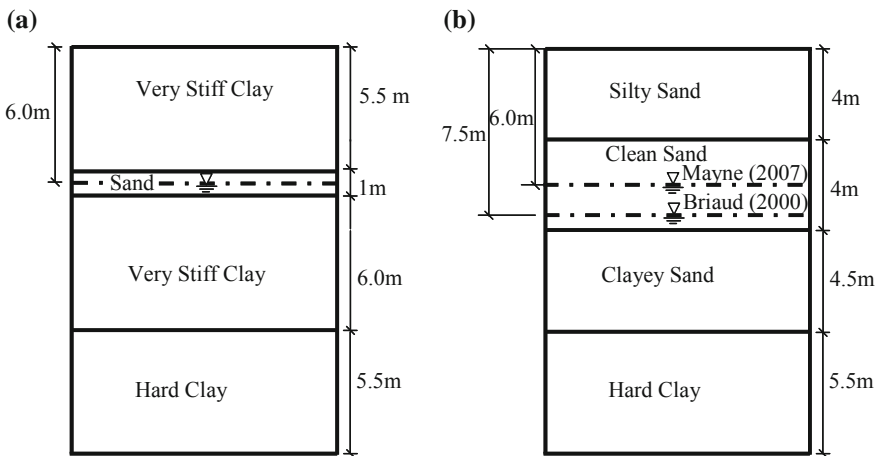
$$P(A|B) = P(B|A)P(A)/P(B) \quad (2.11)$$

in which  $P(A)$  is the probability of  $A$  prior to the collection of the new evidence  $B$ ;  $P(B)$  is the probability of the new evidence  $B$ , which is independent of  $A$ ; and  $P(B|A)$  reflects the information on  $A$  provided by the new evidence  $B$ . Equation (2.11) has been applied to calibration of estimated failure probability in liquefaction evaluation (e.g., Juang et al. 1999) or slope reliability assessment (e.g., Cheung and Tang 2005), probabilistic assessment of soil–structure interaction (e.g., Schuster et al. 2008), and model class selection (e.g., Beck and Yuen 2004; Yan

et al. 2009; Yuen 2010). For example, in model class selection,  $A$  represents a model class and  $B$  represents the new evidence (e.g., observation data). Using Eq. (2.11), the plausibility (or occurrence probability) of the model class  $A$  given the new evidence  $B$  is determined.

## 2.4 National Geotechnical Experimentation Site (NGES) at Texas A&M University

To provide well-characterized reference sites for the development and evaluation of geotechnical design and construction methods and in situ testing methods, five US National Geotechnical Experimentation Sites (NGES) were established by US Federal Highway Administration (FHWA) (Dimillio and Prince 1993; Benoit 2000). Extensive in situ and laboratory test results obtained from NGES are available in the geotechnical literature (e.g., Briaud 2000; Benoit 2000). The test results obtained from the NGES at Texas A&M University (TAMU) are used to illustrate the probabilistic approaches for geotechnical site characterization developed in this book. The NGES at TAMU is therefore introduced briefly below. The NGES at TAMU is underlain by Pleistocene fluvial and overbank deposits and Eocene-aged Spiller Member of the Crockett Formation and includes a clay site and a sand site (Briaud 2000). Figure 2.5a, b shows the stratigraphy of the clay site and sand site, respectively. As shown in Fig. 2.5a, the clay site is comprised of a top stiff clay layer extending from the ground surface to 5.5 m deep, a thin sand layer from the depth of 5.5 m to the depth of 6.5 m, another stiff clay layer down to 12.5 m deep, and a hard clay layer thereafter. The groundwater level at the clay site



**Fig. 2.5** Soil profiles of the clay site and sand site of NGES at Texas A&M University (after Briaud 2000). **a** Clay site. **b** Sand site



is about 6.0 m deep (Briaud 2000). As shown in Fig. 2.5b, the sand site is underlain by a silty sand layer extending from the ground surface to 4.0 m deep, a clean sand layer from the depth of 4.0 m to the depth of 8.0 m, a clayey sand layer from 8.0 m deep to 12.5 m deep, and a hard clay layer thereafter. The groundwater level at the sand site is around 7.5 m deep by Briaud (2000) (or about 6.0 m deep by Mayne 2007). Briaud (2000) summarized results of in situ tests (including SPT, CPT, and PMT) and laboratory tests performed in the clay site and sand site, and Mayne (2007) reported an additional set of CPT test results obtained from the sand site.

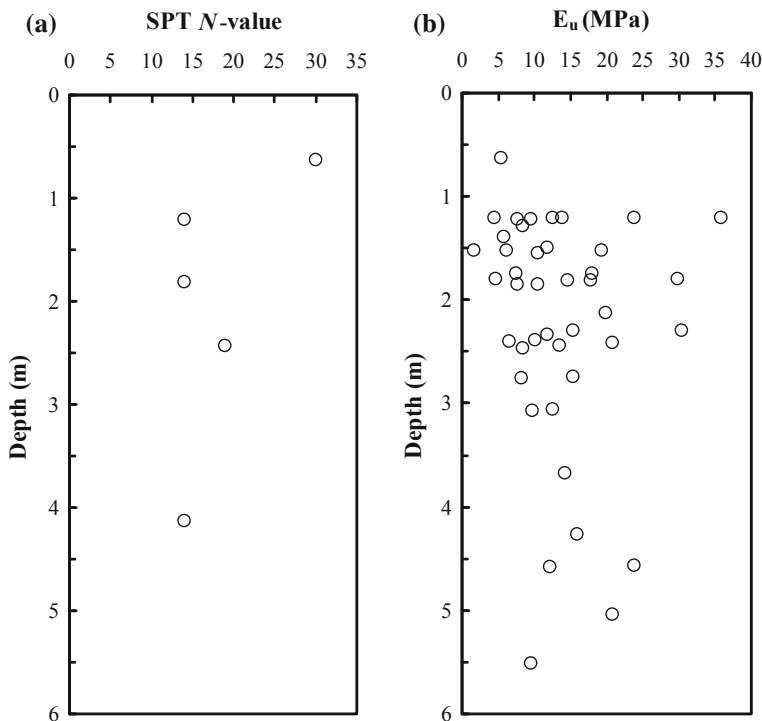
Table 2.6 provides the average soil properties of different soil layers of the clay site and sand site reported by Briaud (2000), including water content, particle size distribution, natural unit weight, Atterberg limits, strength (i.e., undrained shear strength, effective stress cohesion, and effective friction angle), coefficient of

**Table 2.6** Summary of soil properties of NGES at Texas A&M University (After Briaud 2000)

Soil property	Clay site				Sand site			
Soil type	Very stiff clay <sup>a</sup>	Sand	Very stiff clay <sup>b</sup>	Hard clay	Silty sand	Clean sand	Clayey sand	Hard clay
Water content (%)	24.4	N/A	24.5	26.4	12	17	25	26
Percent of grains (by weight) smaller than 0.075 mm (%)	N/A	N/A	N/A	N/A	17.4	4	30	N/A
Mean grain size (mm)	N/A	N/A	N/A	N/A	0.2	0.25	0.075	N/A
Natural unit weight (kN/m <sup>3</sup> )	19.6	N/A	19.5	18.9	20	17	18.3	20
Plastic limit (%)	20.9	N/A	22	22.7	N/A	N/A	22	20
Liquid limit (%)	53.7	N/A	60	56	N/A	N/A	41	61
Undrained shear strength (kPa)	110	N/A	140	160	N/A	N/A	N/A	235
Effective stress cohesion (kPa)	13	N/A	57	N/A	N/A	N/A	N/A	N/A
Effective friction angle (°)	20	N/A	26.5	N/A	34	31	N/A	N/A
Coefficient of permeability (m/year)	0.0005	N/A	0.00007	N/A	0.05	N/A	N/A	N/A
SPT <i>N</i> -value (bl/0.3 m)	12	N/A	32	N/A	15	19	22	55
Cone tip resistance (MPa)	2.0	N/A	6.0	N/A	7.0	9.0	10.0	N/A
Young's modulus measured by pressuremeter test (MPa)	15.0	N/A	35.0	230.0	9.0	9.0	33.0	N/A
Limit pressure (MPa)	0.75	N/A	2.2	6.5	0.8	1.0	1.9	N/A

<sup>a</sup>The top soil layer at the clay site

<sup>b</sup>The third soil layer at the clay site



**Fig. 2.6** Standard penetration test (SPT)  $N$ -value and undrained Young's modulus measured by pressuremeter tests at the clay site of the NGES at Texas A&M University. **a** SPT  $N$ -values. **b** Undrained Young's modulus. (after Briaud 2000)

permeability, SPT  $N$ -values, cone tip resistance, and Young's modulus and limit pressure measured by pressuremeter tests. Figure 2.6a shows 5 SPT  $N$ -values versus depth obtained from the SPT tests performed within the top stiff clay layer (Briaud 2000). Pressuremeter tests were also carried out in the top clay layer (Briaud 2000). Figure 2.6b shows in total 42 measurements of the undrained Young's modulus  $E_u$  from pressuremeter tests performed at different depths in the top clay layer (Briaud 2000). The SPT  $N$ -values and undrained Young's modulus  $E_u$  obtained from the clay site are used to illustrate and validate the probabilistic approach developed in Chap. 5 of this book.

In addition, Akkaya and Vanmarcke (2003) summarized inherent spatial variability of soil properties at the NGES, and they concluded that (1) for the clay site (see Fig. 2.5a), the mean value, coefficient of variation, and correlation length of cone tip resistance range from 0.14 MPa to 0.27 MPa, from 50 % to 82 %, and from 3.3 m to 9.0 m, respectively, and their respective mean values are 0.2 MPa, 65 %, and 5.4 m; (2) for the sand site (see Fig. 2.5b), the mean value, coefficient of variation, and correlation length of cone tip resistance range from 7.0 MPa to 9.0 MPa, from 30 % to 76 %, and from 1.6 m to 6.4 m, respectively, and their

respective mean values are 8.4 MPa, 50 %, and 3.3 m. Note that the average cone tip resistance (i.e., from 7.0 to 9.0 MPa) of the sand site compares favorably with that reported by Briaud (2000) (see Table 2.6), while the average cone tip resistance (e.g., from 0.14 to 0.27 MPa) of the clay site is much less than that reported by Briaud (2000) (see Table 2.6).

## 2.5 Probabilistic Slope Stability Analysis

Various uncertainties exist in slope engineering, such as inherent spatial variability of soil properties, uncertainties in subsurface stratigraphy, and modeling errors associated with geotechnical models. Probability theory and statistics provide a rational vehicle to account for these uncertainties in slope stability analysis. In the context of probability theory and statistics, reliability of slope stability is frequently measured by a “reliability index” ( $\beta$ ) or slope failure probability ( $P_f$ ), which is defined as the probability that the minimum factor of safety ( $FS$ ) is less than unity (i.e.,  $P_f = P(FS < 1)$ ). For reference, Table 2.7 lists  $\beta$  and  $P_f$  for representative geotechnical components and systems and their expected performance levels. The value of  $\beta$  commonly ranges from 1 to 5, corresponding to  $P_f$  varying from about 0.16 to  $3 \times 10^{-7}$ . Geotechnical designs require a  $\beta$ -value of at least 2 (i.e.,  $P_f < 0.023$ ) for an expected performance level better than “poor.” Relatively small  $P_f$  (information on the tail of probability distribution) is of great interest to geotechnical practitioners. The reliability index  $\beta$  and slope failure probability  $P_f$  can be estimated by various solution methods, such as the first-order second-moment method (FOSM) (e.g., Tang et al. 1976; Christian et al. 1994; Hassan and Wolff 1999), first-order reliability method (FORM, also referred to as Hasofer–Lind method) (e.g., Low and Tang 1997; Low et al. 1998; Low 2003), direct Monte Carlo simulation (MCS) method (e.g., El-Ramly et al. 2002; Griffiths and Fenton 2004; El-Ramly et al. 2005), and subset simulation (e.g., Au 2001; Au and Beck 2001 and 2003; Au et al. 2009, 2010; Wang et al. 2009). These methods are briefly reviewed in the following subsections.

**Table 2.7** Relationship between reliability index ( $\beta$ ) and probability of failure ( $P_f$ ) (after U.S. Army Corps of Engineers 1997)

Reliability index $\beta$	Probability of failure $P_f = \Phi(-\beta)^a$	Expected performance level
1.0	0.16	Hazardous
1.5	0.07	Unsatisfactory
2.0	0.023	Poor
2.5	0.006	Below average
3.0	0.001	Above average
4.0	0.00003	Good
5.0	0.0000003	High

<sup>a</sup> $\Phi(\odot)$  = Standard normal cumulative distribution function

### 2.5.1 First-Order Second-Moment Method (FOSM)

Let  $g(\underline{x})$  denote the performance function (or deterministic model) of slope stability used to calculate  $FS$  of slope stability, in which  $\underline{x} = [x_1, x_2, \dots, x_k]$  is a set of random variables representing uncertain model parameters in the performance function. Consider, for example, that  $FS$  is normally distributed. By FOSM, the reliability index  $\beta$  is calculated as (e.g., Cornell 1969; Ang and Tang 1984; Baecher and Christian 2003)

$$\beta = \frac{\mu_{FS} - 1}{\sigma_{FS}} \quad (2.12)$$

in which  $\mu_{FS}$  and  $\sigma_{FS}$  are the mean and standard deviation of  $FS$ , respectively.  $\mu_{FS}$  is the value of  $g(\underline{x})$  calculated at mean values  $\mu_i, i = 1, 2, \dots, k$  of random variables  $x_i, i = 1, 2, \dots, k$ , i.e.,  $\mu_{FS} = g(\mu_1, \mu_2, \dots, \mu_k)$ , and  $\sigma_{FS}$  is calculated as

$$\sigma_{FS} = \sqrt{\sum_{i=1}^k \sigma_i^2 \left( \frac{\partial g}{\partial x_i} \right)^2 + \sum_{i=1}^k \sum_{j \neq i}^k \rho_{ij} \sigma_i \sigma_j \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j}} \quad (2.13)$$

in which  $\sigma_i, i = 1, 2, \dots, k$  are standard deviations of random variables  $x_i, i = 1, 2, \dots, k$ , respectively;  $\frac{\partial g}{\partial x_i}, i = 1, 2, \dots, k$  are partial derivatives of the performance function with respect to  $x_i, i = 1, 2, \dots, k$ , respectively;  $\rho_{ij}$  is the correlation coefficient between two different uncertain parameters  $x_i$  and  $x_j$ , where  $i \neq j$ . Then, the slope failure probability  $P_f$  is calculated as  $1 - \Phi(\beta)$ , in which  $\Phi$  is the cumulative distribution function of a standard Gaussian random variable (e.g., Baecher and Christian 2003).

FOSM is a relatively simple approach for performing probabilistic slope stability analysis. It, however, requires an analytical model of slope stability analysis, which is differentiable for all uncertain variables (i.e.,  $x_i, i = 1, 2, \dots, k$ ) involved in the model. In addition, FOSM usually uses a predefined critical slip surface of slope failure and does not account for uncertainties of the critical slip surface.

### 2.5.2 First-Order Reliability Method (FORM)

By FORM, the reliability index  $\beta$  is interpreted as a measure of the distance between the peak of the multivariate distribution of the uncertain variables (e.g., the joint distribution of  $\underline{x} = [x_1, x_2, \dots, x_k]$ ) and the critical point on the failure boundary in a dimensionless space. It is calculated as (e.g., Hasofer and Lind 1974; Ang and Tang 1984; Low and Tang 1997; Baecher and Christian 2003)

$$\beta = \min_{x_i \in \Omega_F} \sqrt{\left[ \frac{x - \underline{\mu}}{\underline{\sigma}} \right]^T \underline{\rho}^{-1} \left[ \frac{x - \underline{\mu}}{\underline{\sigma}} \right]} \quad (2.14)$$

in which  $\Omega_F$  represents the failure domain;  $\underline{\mu}$  is a mean vector of uncertain variables, i.e.,  $\underline{\mu} = [\mu_1, \mu_2, \dots, \mu_k]$ ;  $\underline{\sigma}$  is a standard deviation vector of uncertain variables, i.e.,  $\underline{\sigma} = [\sigma_1, \sigma_2, \dots, \sigma_k]$ ; and  $\underline{\rho}$  is the correlation matrix of uncertain variables. Note that the equivalent mean values and standard deviations of random variables should be used in Eq. (2.14) when uncertain variables are not normally distributed (Ang and Tang 1984).

Equation (2.14) has been implemented in a spreadsheet environment with the aid of the built-in optimization tool “Solver” for probabilistic slope stability analysis by Low and Tang (1997), Low et al. (1998), and Low (2003). Center coordinates and radius of circular slip surfaces were also considered as additional optimization variables by Low (2003) so that variation of potential critical slip surfaces was implicitly factored in the analysis. However, because of the limitation of the optimization tool used, FORM tends to overestimate  $\beta$  and underestimate  $P_f$  (Wang et al. 2009).

### 2.5.3 Direct Monte Carlo Simulation (Direct MCS)

MCS is a numerical process of repeatedly calculating a mathematical or empirical operator, in which the variables within the operator are random or contain uncertainties with prescribed probability distributions (e.g., Ang and Tang 2007; Wang 2011; Wang et al. 2011). The numerical result from each repetition of the numerical process is considered as a sample of the true solution of the operator, analogous to an observed sample from a physical experiment. Consider, for example, probabilistic slope stability analysis in which the mathematical operator (i.e., the performance function  $g(\underline{x})$ ) involves calculation of the factor of safety,  $FS$ , and judgment of occurrence of failure (i.e.,  $FS < 1$ ). The direct MCS starts with the characterization of probability distributions of uncertainties concerned, as well as the slope geometry and other necessary information, followed by the generation of  $n_{MC}$  sets of random samples according to the prescribed probability distributions. Using the deterministic model  $g(\underline{x})$  and the  $n_{MC}$  sets of random samples,  $n_{MC}$  possible values of  $FS$  are obtained. Then, statistical analysis is performed to estimate  $P_f$  or  $\beta$ , with the slope failure defined as  $FS < 1$  (e.g., Baecher and Christian 2003).

Monte Carlo simulation (MCS) is gaining popularity in the reliability analysis of slope stability due to its robustness and conceptual simplicity. It can be performed together with various types of deterministic models, such as limit equilibrium

methods (e.g., El-Ramly et al. 2002, 2005, and 2006), finite element method (e.g., Griffiths and Fenton 2004; Xu and Low 2006; Griffiths et al. 2009; Hicks and Spencer 2010), and response surface method (e.g., Xu and Low 2006). It, however, suffers from a lack of efficiency and resolution at small probability levels and does not offer insights into the relative contributions of uncertainties to the failure probability (Baecher and Christian 2003).

### 2.5.4 Subset Simulation

Recently, an advanced Monte Carlo simulation called “subset simulation” has been applied to probabilistic slope stability analysis to improve the efficiency and resolution of the MCS (Au et al. 2009; Wang et al. 2009; Au et al. 2010). Subset simulation makes use of conditional probability and Markov Chain Monte Carlo Simulation (MCMCS) method to efficiently compute small tail probability (Au 2001; Au and Beck 2001 and 2003). It expresses a small probability event as a sequence of intermediate events  $\{F_1, F_2, \dots, F_m\}$  with larger conditional probability and employs specially designed Markov Chains to generate conditional samples of these intermediate events until the final target failure region is achieved. For the slope stability problem, let  $Y = 1/FS$  be the critical response (Au et al. 2009; Wang et al. 2009; Au et al. 2010). The probability of  $Y = 1/FS$  being larger than a given value  $y$  (i.e.,  $P(Y = 1/FS > y)$ ) is of interest, and let  $0 < y_1 < y_2 < \dots < y_{m-1} < y_m = y$  be an increasing sequence of intermediate threshold values. The sequence of intermediate events  $\{F_1, F_2, \dots, F_m\}$  are chosen as  $F_i = \{Y > y_i, i = 1, 2, \dots, m\}$  for these intermediate threshold values. By sequentially conditioning on the event  $\{F_i, i = 1, 2, \dots, m\}$ , the failure probability can be written as

$$P_f = P(F_m) = P(F_1) \prod_{i=2}^m P(F_i|F_{i-1}) \quad (2.15)$$

where  $P(F_1)$  is equal to  $P(Y > y_1)$  and  $P(F_i|F_{i-1})$  is equal to  $\{P(Y > y_i|Y > y_{i-1}): i = 2, \dots, m\}$ . In implementations,  $y_1, y_2, \dots, y_m$  are generated adaptively using information from simulated samples so that the sample estimate of  $P(F_1)$  and  $\{P(F_i|F_{i-1}): i = 2, \dots, m\}$  always corresponds to a common specified value of conditional probability  $p_0$  ( $p_0 = 0.1$  is found to be a good choice) (Au 2001; Au and Beck 2001 and 2003; Au et al. 2010). The efficient generation of conditional samples is pivotal in the success of subset simulation, and it is made possible through the machinery of MCMCS (e.g., Beck and Au 2002; Robert and Casella 2004). The procedures of subset simulation and MCMCS are described in the following two subsections, respectively.

### 2.5.4.1 Subset Simulation Procedure

Subset simulation starts with direct MCS, in which  $N$  MCS samples are generated. The  $Y$  values of the  $N$  samples are calculated and ranked in an ascending order. The  $(1 - p_0)N$ th value in the ascending list of  $Y$  values is chosen as  $y_1$ , and hence, the sample estimate for  $P(F_1) = P(Y > y_1)$  is always  $p_0$ . In other words, there are  $p_0N$  samples with  $F_1 = Y > y_1$  among the samples generated by direct MCS. Then, starting from the  $p_0N$  samples with  $F_1 = Y > y_1$ , MCMCS is used to simulate  $(1 - p_0)N$  additional conditional samples given  $F_1 = Y > y_1$ , so that there are a total of  $N$  samples with  $F_1 = Y > y_1$ . The  $Y$  values of the  $N$  samples with  $F_1 = Y > y_1$  are ranked again in an ascending order, and the  $(1 - p_0)N$ th value in the ascending list of  $Y$  values is chosen as  $y_2$ , which defines the  $F_2 = Y > y_2$ . Note that the sample estimate for  $P(F_2|F_1) = P(Y > y_2|Y > y_1)$  is also equal to  $p_0$ . Similarly, there are  $p_0N$  samples with  $F_2 = Y > y_2$  and these samples provide “seeds” for the application of MCMCS to simulate additional  $(1 - p_0)N$  conditional samples with  $F_2 = Y > y_2$  so that there are  $N$  conditional samples with  $F_2 = Y > y_2$ . The procedure is repeated  $m$  times until the probability space of interest (i.e., the sample domain with  $Y > y_m$ ) is achieved. Note that the subset simulation procedures contain  $m + 1$  steps, including one direct MCS to generate unconditional samples and  $m$  steps of MCMCS to simulate conditional samples. The  $m + 1$  steps of simulations are referred to as “ $m + 1$  levels” in subset simulation (Au and Beck 2001 and 2003; Au et al. 2010), and in total, these  $m + 1$  levels of simulations generate  $N + m(1 - p_0)N$  samples.

### 2.5.4.2 Markov Chain Monte Carlo Simulation (MCMCS)

MCMCS method is a numerical process that simulates a sequence of samples of random variables  $\underline{x}$  as a Markov Chain with the joint PDF of random variables as the Markov Chain’s limiting stationary distribution (e.g., Beck and Au 2002; Robert and Casella 2004). It provides a feasible way to generate  $n_{\text{MCMC}}$  samples from an arbitrary PDF, particularly when the PDF is complicated and is difficult to express analytically and explicitly. There are several MCMCS methods, such as Metropolis algorithm (Metropolis et al. 1953), original Metropolis–Hastings (MH) algorithm (Hastings 1970), and modified Metropolis–Hastings (MMH) algorithm (Au 2001; Au and Beck 2001 and 2003). For example, the original MH algorithm is described below.

Consider, for example, random variables  $\underline{x} = [x_1, x_2, \dots, x_k]$  with a prescribed joint PDF  $P(\underline{x})$ . In MH algorithm, the Markov Chain starts with an initial state  $\underline{x}_1$  that is predefined by the user (e.g., conditional failure samples generated in the previous subset simulation level). Then, the  $i$ th state of the  $\underline{x}$  Markov Chain,  $\underline{x}_i$ ,  $i = 2, 3, \dots, n_{\text{MCMC}}$ , (e.g.,  $\underline{x}_2$  for the second state) is generated from its previous state  $\underline{x}_{i-1}$  (e.g., the initial state  $\underline{x}_1$  for  $\underline{x}_2$ ). A candidate sample  $\underline{x}_i^*$ ,  $i = 2, 3, \dots, n_{\text{MCMC}}$ , for the  $i$ th state is generated from the proposal PDF  $f(\underline{x}_i^*|\underline{x}_{i-1})$  (e.g., a normal distribution or uniform distribution centered at the previous state  $\underline{x}_{i-1}$ ). The candidate sample  $\underline{x}_i^*$

is, however, not necessarily to be accepted as the  $i$ th state of the  $\underline{x}$  Markov Chain (i.e.,  $\underline{x}_i$ ). The chance to accept the candidate sample  $\underline{x}_i^*$  as the  $\underline{x}_i$  depends on the “acceptance ratio,”  $r_a$ , which is calculated as

$$r_a = \frac{P(\underline{x}_i^*)}{P(\underline{x}_{i-1})} \times \frac{f(\underline{x}_{i-1}|\underline{x}_i^*)}{f(\underline{x}_i^*|\underline{x}_{i-1})} \quad \text{for } i = 2, 3, \dots, n_{\text{MCMC}} \quad (2.16)$$

in which  $P(\underline{x}_i^*)$  and  $P(\underline{x}_{i-1})$  are PDF values of  $\underline{x}_i^*$  and  $\underline{x}_{i-1}$ , respectively;  $f(\underline{x}_i^*|\underline{x}_{i-1})$  is the conditional PDF value of  $\underline{x}_i^*$  given  $\underline{x}_{i-1}$ ;  $f(\underline{x}_{i-1}|\underline{x}_i^*)$  is the conditional PDF value of  $\underline{x}_{i-1}$  given  $\underline{x}_i^*$ . Using PDF of  $\underline{x}$  and the predefined proposal PDF, the acceptance ratio of  $\underline{x}_i^*$  is obtained from Eq. (2.16). When  $r_a$  is greater than unity, the candidate sample  $\underline{x}_i^*$  is accepted as the  $i$ th state of the  $\underline{x}$  Markov Chain (i.e.,  $\underline{x}_i$ ). When  $r_a$  falls within  $[0, 1]$ , the probability to accept  $\underline{x}_i^*$  as  $\underline{x}_i$  is  $r_a$ .

In implementation, a random number  $u$  is generated from a uniform distribution with a range from zero to one. If  $u$  is less than  $r_a$ ,  $\underline{x}_i^*$  is accepted as  $\underline{x}_i$ , i.e.,  $\underline{x}_i = \underline{x}_i^*$ . Otherwise,  $\underline{x}_i^*$  is rejected, and  $\underline{x}_i$  is taken as the previous state  $\underline{x}_{i-1}$ , i.e.,  $\underline{x}_i = \underline{x}_{i-1}$ . For example, the  $r_a$  value for the candidate sample  $\underline{x}_2^*$  is calculated from Eq. (2.16) using  $\underline{x}_1$  and  $\underline{x}_2^*$ , and the second state  $\underline{x}_2$  is then determined accordingly by comparing the values of  $r_a$  and  $u$ . Starting from the initial state  $\underline{x}_1$ , the procedure described above is repeated  $n_{\text{MCMC}} - 1$  times to generate  $n_{\text{MCMC}} - 1$  samples of  $\underline{x}$ , i.e.,  $\underline{x}_i, i = 2, 3, \dots, n_{\text{MCMC}}$ . This leads to a Markov Chain that is comprised of  $n_{\text{MCMC}}$   $\underline{x}$  samples (including the initial sample).

The original MH algorithm reduces to the Metropolis algorithm when the proposal PDF is symmetric, i.e.,  $f(\underline{x}_i^*|\underline{x}_{i-1}) = f(\underline{x}_{i-1}|\underline{x}_i^*)$ . Both the Metropolis algorithm and the original MH algorithm are not applicable for high-dimensional problems because the acceptance ratio becomes exponentially small as the dimension  $k$  increases and most of candidate samples are rejected (Au 2001; Au and Beck 2001). The MMH algorithm was proposed for high-dimensional problems by Au (2001) and Au and Beck (2003). In MMH algorithm, the random vector  $\underline{x}$  with  $k$  independent components is simulated by generating the candidate sample of  $\underline{x}$  component by component, so that the acceptance ratio of individual component of the candidate sample of  $\underline{x}$  does not decrease to zero when the dimension increases (Au 2001). In this book, MCMCS method is applied in both subset simulation (see Chaps. 7 and 8) and the probabilistic approach developed for site characterization (see Chap. 5).

## Appendix 2.1: Several Empirical Correlations Reported by Kulhawy and Mayne (1990)

This appendix summarizes several empirical correlations reported by Kulhawy and Mayne (1990). Table 2.8 shows empirical correlations between standard penetration test (SPT)  $N$ -value and effective friction angle of sands, the relative density of



**Table 2.8** Relationship between SPT  $N$ -value and effective friction angle of sand (after Kulhawy and Mayne 1990)

SPT $N$ -value	Relative density	Approximate effective friction angle $\phi'$ (°)	
		a	b
0–4	Very loose	<28	<30
4–10	Loose	28–30	30–35
10–30	Medium	30–36	35–40
30–50	Dense	36–40	40–45
>50	Very dense	>41	>45

<sup>a</sup>After Peck et al. (1974)<sup>b</sup>After Meyerhof (1956)

which ranges from very loose to very dense. When the SPT  $N$ -value is less than 4, the effective friction angle is less than  $28^\circ$  by Peck et al. (1974) or less than  $30^\circ$  by Meyerhof (1956). The effective friction angle increases as the SPT  $N$ -value increases. When the SPT  $N$ -value is greater than 50, the effective friction angle is greater than  $41^\circ$  by Peck et al. (1974) or greater than  $45^\circ$  by Meyerhof (1956).

Table 2.9 gives an empirical correlation between cone tip resistance measured by cone penetration test (CPT) and effective friction angle of sands, the relative density of which ranges from very loose to very dense. As the cone tip resistance increases from less than 2.0 MPa to larger than 20 MPa, the effective friction angle increases from less than  $30^\circ$  to larger than  $45^\circ$ .

Table 2.10 summarizes the minimum and maximum of the typical dry unit weight of sands, including silty sand, clean sand, micaceous sand, and silty sand and gravel. The minimum and maximum of the dry unit weight range from  $13.6 \text{ kN/m}^3$  to  $14.0 \text{ kN/m}^3$  and from  $20.0 \text{ kN/m}^3$  to  $22.9 \text{ kN/m}^3$ , respectively.

Table 2.11 summarizes typical effective friction angles of sands. Typical effective friction angles of loose and dense uniform sands with round grains are  $27.5^\circ$  and  $34.0^\circ$ , respectively. Typical effective friction angles of loose and dense well-graded sands with angular grains are  $33.0^\circ$  and  $45.0^\circ$ , respectively. Typical effective friction angles of loose and dense sandy gravels are  $35.0^\circ$  and  $50.0^\circ$ , respectively. In addition, typical effective friction angles of loose and dense silty

**Table 2.9** Relationship between cone tip resistance  $q_c$  and effective friction angle of sand (after Kulhawy and Mayne 1990)

Normalized cone tip resistance $q_c/p_a^b$	Relative density	Approximate effective friction angle $\phi'$ (°) <sup>a</sup>
<20	Very loose	<30
20–40	Loose	30–35
40–120	Medium	35–40
120–200	Dense	40–45
>200	Very dense	>45

<sup>a</sup>After Meyerhof (1956)<sup>b</sup> $p_a$  is the standard atmospheric pressure

**Table 2.10** Typical soil unit weight of sand (after Kulhawy and Mayne 1990)

Soil type	Dry unit weight (kN/m <sup>3</sup> ) <sup>a</sup>	
	Minimum	Maximum
Silty sand	13.6	20.0
Clean, fine to coarse sand	13.3	21.7
Micaceous sand	12.0	18.8
Silty sand and gravel	14.0	22.9

<sup>a</sup>After Hough (1969)

**Table 2.11** Typical values of effective friction angle of sand (after Kulhawy and Mayne 1990)

Soil type	Effective friction angle $\phi'$ (°) <sup>a</sup>	
	Loose	Dense
Sand, round grains, uniform	27.5	34.0
Sand, angular grains, well graded	33.0	45.0
Sandy gravels	35.0	50.0
Silty sand	27.0–33.0	30.0–34.0
Inorganic silt	27.0–30.0	30.0–34.0

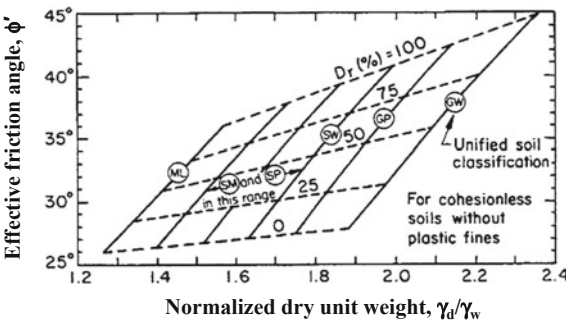
<sup>a</sup>After Terzaghi and Peck (1967)

sands range from 27.0° to 33.0° and from 30.0° to 34.0°, respectively. For loose and dense inorganic silts, their typical effective friction angles range from 27.0° to 30.0° and from 30.0° to 34.0°, respectively.

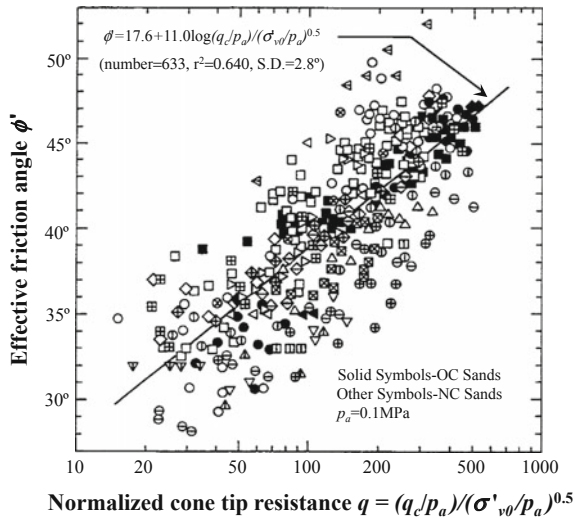
Figure 2.7 shows variation of effective friction angle of soils as a function of dry unit weight, relative density, and soil type. The effective friction angle increases as the dry unit weight and relative density increase. Figure 2.8 shows an empirical correlation between cone tip resistance  $q_c$  measured by CPT and the effective friction angle  $\phi'$  of sands, which is a semilog regression equation

$$\phi' = 17.6 + 11.0 \log \left( \frac{q_c/p_a}{\sqrt{\sigma'_{v0}/p_a}} \right) \tag{2.17}$$

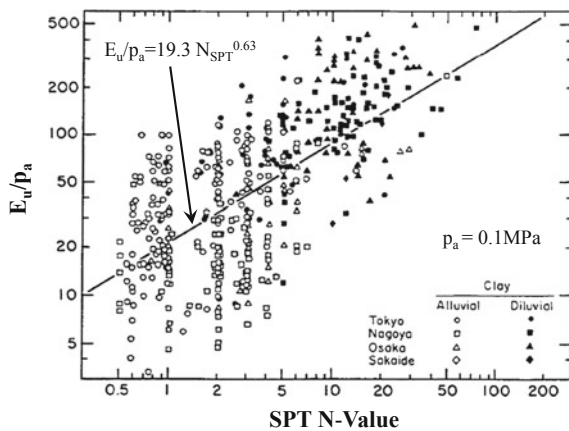
**Fig. 2.7** Relationship between the normalized dry unit weight and effective friction angle (after Kulhawy and Mayne 1990)



**Fig. 2.8** Regression between normalized cone tip resistance and effective friction angle (after Kulhawy and Mayne 1990 and Wang et al. 2010)



**Fig. 2.9** Regression between SPT  $N$ -value and undrained Young's modulus of clay (after Ohya et al. 1982; Kulhawy and Mayne 1990; Phoon and Kulhawy 1999b and Wang and Cao 2013)



in which  $\sigma'_{v0}$  and  $p_a$  are vertical effective stress and standard atmospheric pressure (i.e., 0.1 MPa), respectively. Figure 2.9 shows an empirical correlation between SPT  $N$ -value (i.e.,  $N_{SPT}$ ) and undrained Young's modulus  $E_u$  of clay, and it is written as

$$E_u/p_a = 19.3 N_{SPT}^{0.63} \quad (2.18)$$

In Eq. (2.18), the undrained Young's modulus  $E_u$  of clay is measured by pressuremeter tests.

## References

- Akkaya, A.D., and E.H. Vanmarcke. 2003. Estimation of spatial correlation of soil parameters based on data from the Texas A&M University NGES. In *Probabilistic site characterization at the national geotechnical experimentation sites*, Geotechnical Special Publication No. 121, 29–40.
- Alonso, E.E., and R.J. Krizek. 1975. Stochastic formulation of soil properties. In *Proceedings of 2nd international conference on applications of statistics and probability in soil and structure engineering*, 9–32. Aachen, Germany.
- Ang, A.H.S., and W.H. Tang. 2007. *Probability concepts in engineering: Emphasis on applications to civil and environmental engineering*. New York: Wiley.
- Ang, A.H.S., and W.H. Tang. 1984. *Probability concepts in engineering planning and design*, vol. II. New York: Wiley.
- Au, S.K. 2001. *On the solution of first excursion probability by simulation with applications to probabilistic seismic performance assessment*. Ph.D. thesis. California, USA: California Institute of Technology.
- Au, S.K., and J.L. Beck. 2001. Estimation of small failure probabilities in high dimensions by subset simulation. *Probabilistic Engineering Mechanics* 16(4): 263–277.
- Au, S.K., and J.L. Beck. 2003. Subset simulation and its applications to seismic risk based on dynamic analysis. *Journal of Engineering Mechanics* 129(8): 1–17.
- Au, S.K., Z. Cao, and Y. Wang. 2010. Implementing advanced Monte Carlo simulation under spreadsheet environment. *Structural Safety* 32(5): 281–292.
- Au, S.K., Y. Wang, and Z. Cao. 2009. Reliability analysis of slope stability by advanced simulation with spreadsheet. In *Proceedings of the second international symposium on geotechnical safety and risk*, 275–280. Gifu, Japan, June 2009.
- Baecher, G.B. 1983. Professional judgment and prior probabilities in engineering risk assessment. In *Proceedings of fourth international conference on application of statistics and probability in soil and structural engineering*, 635–650. Universita di Firenze (Italy), Pitagora Editrice.
- Baecher, G.B. 1987. *Statistical analysis of geotechnical data*. Report No. GL-87-1. Vicksburg, VA: U.S. Army Engineer Waterways Experiment Station.
- Baecher, G.B., and J.T. Christian. 2003. *Reliability and statistics in geotechnical engineering*, 605 pp. Hoboken, New Jersey: Wiley.
- Beach, L., V. Barnes, and J. Christensen-Szalanski. 1986. Beyond heuristics and biases: A contingency model of judgmental forecasting. *Journal of Forecasting* 5(3): 143–157.
- Beck, J.L., and S.K. Au. 2002. Bayesian updating of structural models and reliability using Markov chain Monte Carlo simulation. *Journal of Engineering Mechanics* 128(4): 380–391.
- Beck, J.L., and L.S. Katafygiotis. 1998. Updating models and their uncertainties. I: Bayesian statistical framework. *Journal of Engineering Mechanics* 124(4): 455–461.
- Beck, J.L., and K.V. Yuen. 2004. Model selection using response measurements: Bayesian probabilistic approach. *Journal of Engineering Mechanics* 130(2): 192–203.
- Benoit, J. 2000. The United States National Geotechnical Experimentation Sites program: The first decades. In *National geotechnical experimentation sites*, Geotechnical Special Publication No. 93, 1–25.
- Berger, J.O. 1985. *Statistical decision theory and Bayesian analysis*. Springer.
- Bjerrum, L. 1973. Problems of soil mechanics and constructions on soft clays. In *Proceedings of 8th international conference on soil mechanics and foundation engineering*, Moscow, Vol. 3, 111–159.
- Bleistein, N., and R. Handelsman. 1986. *Asymptotic expansions of integrals*. New York: Dover.
- Box, G.E.P., and G.M. Jenkins. 1970. *Time series analysis: Forecasting and control*, 553 pp. San Francisco: Holden-Day.
- Briaud, J.L. 2000. The National Geotechnical Experimentation Sites at Texas A&M University: Clay and sand. A summary. In *National geotechnical experimentation sites*, Geotechnical Special Publication No. 93: 26–51.

- Briaud, J.L. 1992. *The pressuremeter*. London, UK: Taylor & Francis.
- Campanella, R.G., D.S. Wickremesinghe, and P.K. Robertson. 1987. Statistical treatment of cone penetrometer test data. In *Proceedings of 5th international conference on applications of statistics and probability in soil and structure engineering*, Vancouver, 1011–1019.
- Cao, Z., Y. Wang, and S.K. Au. 2011. CPT-Based probabilistic characterization of effective friction angle of sand. In *Geotechnical risk assessment and management*, Geotechnical Special Publication No. 224, 403–410.
- Carlin, B.P., and T.A. Louis. 1996. *Bayes and Empirical Bayes methods for data analysis*. London: Chapman and Hall.
- Cetin, K.O., and C. Ozan. 2009. CPT-Based probabilistic soil characterization and classification. *Journal of Geotechnical and Geoenvironmental Engineering* 135(1): 84–107.
- Cheung, R.W.M., and W.H. Tang. 2005. Realistic assessment of slope reliability for effective landslide hazard management. *Geotechnique* 55(1): 85–94.
- Ching, J., K.K. Phoon, and Y.C. Chen. 2010. Reducing shear strength uncertainties in clays by multivariate correlations. *Canadian Geotechnical Journal* 47(1): 16–33.
- Christian, J.T., C.C. Ladd, and G.B. Baecher. 1994. Reliability applied to slope stability analysis. *Journal of Geotechnical Engineering* 120(12): 2180–2207.
- Clayton, C.R.I. 1995. *The Standard Penetration Test (SPT): Methods and use*. Report 143. UK: Construction Industry Research and Information Association.
- Clayton, C.R.I., M.C. Matthews, and N.E. Simons. 1995. *Site investigation*. Cambridge, Mass., USA: Blackwell Science.
- Clemen, R.T. 1996. *Making hard decisions: An introduction to decision analysis*. Pacific Grove: Duxbury Press.
- Cornell, C.A. 1969. A probability-based structural code. *Journal of the American Concrete Institute* 66(12): 974–985.
- DeGroot, D.J., and G.B. Baecher. 1993. Estimating autocovariance of in situ soil properties. *Journal of the Geotechnical Engineering Division* 119(GT1): 147–166.
- DiMillio, A.F., and G. Prince. 1993. *National Geotechnical Experimentation Sites*.
- El-Ramly, H., N.R. Morgenstern, and D.M. Cruden. 2002. Probabilistic slope stability analysis for practice. *Canadian Geotechnical Journal* 39: 665–683.
- El-Ramly, H., N.R. Morgenstern, and D.M. Cruden. 2005. Probabilistic assessment of stability of a cut slope in residual soil. *Geotechnique* 55(1): 77–84.
- El-Ramly, H., N.R. Morgenstern, and D.M. Cruden. 2006. Lodalen slide: A probabilistic assessment. *Canadian Geotechnical Journal* 43(9): 956–968.
- Fenton, G. 1999a. Estimation for stochastic soil models. *Journal of Geotechnical and Geoenvironmental Engineering* 125(6): 470–485.
- Fenton, G. 1999b. Random field modeling of CPT data. *Journal of Geotechnical and Geoenvironmental Engineering* 125(6): 486–498.
- Fenton, G., and D.V. Griffiths. 2008. *Risk assessment in geotechnical engineering*. Hoboken, New Jersey: Wiley.
- Fenton, G.A., and D.V. Griffiths. 2007. Reliability-based deep foundation design. In *Probabilistic applications in geotechnical engineering*, Geotechnical Special Publication No. 170, 1–12.
- Fenton, G.A., and D.V. Griffiths. 2003. Bearing capacity prediction of spatially random  $c$ - $\phi$  soils. *Canadian Geotechnical Journal* 40(1): 54–65.
- Fenton, G.A., and D.V. Griffiths. 2002. Probabilistic foundation settlement on spatially random soil. *Journal of Geotechnical and Geoenvironmental Engineering* 128(5): 381–390.
- Fenton, G.A., D.V. Griffiths, and M.B. Williams. 2005. Reliability of traditional retaining wall design. *Geotechnique* 55(1): 55–62.
- Fischhoff, B., P. Slovic, and S. Lichtenstein. 1977. Knowing with certainty: The appropriateness of extreme confidence. *Journal of Experimental Psychology: Human Perception and Performance* 3(4): 552–564.
- Garthwaite, P.H., J.B. Kadane, and A.O. Hagan. 2005. Statistical methods for eliciting probability distributions. *Journal of the American Statistical Association* 100(470): 680–700.
- Geotechnical Engineering Office. 2000. *Geoguide 2: Guide to site investigation*. Hong Kong.

- Griffiths, D.V., and G.A. Fenton. 2004. Probabilistic slope stability analysis by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering* 130(5): 507–518.
- Griffiths, D.V., J. Huang, and G.A. Fenton. 2009. Influence of spatial variability on slope reliability using 2-d random fields. *Journal of Geotechnical and Geoenvironmental Engineering* 135(10): 1367–1378.
- Hassan, A.M., and T.F. Wolff. 1999. Search algorithm for minimum reliability index of earth slopes. *Journal of Geotechnical and Geoenvironmental Engineering* 125(4): 301–308.
- Hasofer, A.M., and N.C. Lind. 1974. Exact and invariant second-moment code format. *Journal of the Engineering Mechanical Division* 100(EM1): 111–121.
- Hastings, W.K. 1970. Monte Carlo sampling methods using Markov chains and their applications. *Biometrika* 57: 97–109.
- Hicks, M.A. 2005. Risk and variability in geotechnical engineering. *Geotechnique* 55(1): 1–2.
- Hicks, M.A., and W.A. Spencer. 2010. Influence of heterogeneity on the reliability and failure of a long 3D slope. *Computers and Geotechnics* 37: 948–955.
- Hogarth, R.M. 1975. Cognitive processes and the assessment of subjective probability distributions. *Journal of the American Statistical Association* 70(350): 271–289.
- Hough, B.K. 1969. *Basic soils engineering*, 634 pp. New York: Ronald Press.
- Houlsby, G. 1988. *Discussion session contribution*. Birmingham: Penetration Testing in the U.K.
- Jaksa, M.B. 1995. *The influence of spatial variability on the geotechnical design properties of a stiff, overconsolidated clay*. Ph.D. thesis. University of Adelaide.
- Jeffreys, H. 1983. *Theory of probability*. Oxford, New York: Oxford University Press.
- Juang, C.H., D.V. Rosowsky, and W.H. Tang. 1999. Reliability-based method for assessing liquefaction potential of soils. *Journal of Geotechnical and Geoenvironmental Engineering* 125(8): 684–689.
- Jung, B.C., P. Gardoni, and G. Biscontin. 2008. Probabilistic soil identification based on cone penetration tests. *Geotechnique* 58(7): 591–603.
- Katafygiotis, L.S., C. Papadimitriou, and H.F. Lam. 1998. A probabilistic approach to structural model updating. *Soil Dynamics Earthquake Engineering* 17(7–8): 495–507.
- Katafygiotis, L.S., and J.L. Beck. 1998. Updating models and their uncertainties: Model identifiability. *Journal of Engineering Mechanics* 124(4): 463–467.
- Klammler, H., M. McVay, D. Horhota, and P. Lai. 2010. Influence of spatially variable side friction on single drilled shaft resistance and LRFD resistance factors. *Journal of Geotechnical and Geoenvironmental Engineering* 138(8): 1114–1123.
- Koriat, A., S. Lichtenstein, and B. Fischhoff. 1980. Reasons for confidence. *Journal of Experimental Psychology: Human Learning and Memory* 6(2): 107–118.
- Kulhawy, F.H., and P.W. Mayne. 1990. *Manual on estimating soil properties for foundation design*, 306 pp. Report EL 6800. Palo Alto: Electric Power Research Institute.
- Kulhawy, F.H. 1996. From Casagrande's 'Calculated Risk' to reliability-based design in foundation engineering. *Civil Engineering Practice* 11(2): 43–56.
- Kulhawy, F.H., and C.H. Trautmann. 1996. Estimation of in-situ test uncertainty. In *Uncertainty in the geologic environment: From theory to practice*, Geotechnical Special Publication, 58(I): 269–286.
- Lacasse, S., and F. Nadim. 1996. Uncertainties in characterizing soil properties. In *Uncertainty in the geologic environment: From theory to practice*, Geotechnical Special Publication, 58(I): 49–75.
- Low, B.K. 2003. Practical probabilistic slope stability analysis. In *Proceeding of 12th Panamerican conference on soil mechanics and geotechnical engineering and 39th U.S. rock mechanics symposium*, 2777–2784. Cambridge, Massachusetts: M.I.T. Verlag Gluckauf GmbH Essen.
- Low, B.K., R.B. Gilbert, and S.G. Wright. 1998. Slope reliability analysis using generalized method of slices. *Journal of Geotechnical and Geoenvironmental Engineering* 124(4): 350–362.
- Low, B.K., and W.H. Tang. 1997. Reliability analysis of reinforced embankments on soft ground. *Canadian Geotechnical Journal* 34(5): 672–685.

- Lumb, P. 1974. Application of statistics in soil mechanics. In *Soil Mechanics—New Horizons*, Chapter 3, ed. Lee, I.K., 44–111. New York.
- Lumb, P. 1966. The variability of natural soils. *Canadian Geotechnical Journal* 3(2): 74–97.
- Lunne, T., P.K. Robertson, and J.J.M. Powell. 1997. *Cone penetration testing in geotechnical practice*, 317 pp. London, U.K.: Blackie-Academic Publishing.
- Mair, R.J., and D.M. Wood. 1987. *Pressuremeter testing: Methods and interpretation*. London: Ciria.
- Maritz, J.S., and T. Lwin. 1989. *Empirical Bayes methods*. London: Chapman and Hall.
- Mayne, P.W. 2007. *Cone penetration testing: A synthesis of highway practice*, National Cooperative Highway Research Program (NCHRP) Synthesis 368. Washington, D.C.: Transportation Research Board.
- Mayne, P.W., B. Christopher, and J. DeJong. 2002. *Subsurface investigations—geotechnical site characterization*, No. FHWA NHI-01-031. Washington, D.C.: Federal Highway Administration, U.S. Department of Transportation.
- Meyerhof, G.G. 1956. Penetration tests and bearing capacity of cohesionless soils. *Journal of the Soil Mechanics and Foundations Division* 82(SM1): 1–19.
- Mesri, G. 1989. Stability analysis with the simple and advanced  $\phi = 0$  method for a failed dike. *Discussion Soils and Foundations* 23(4): 133–137.
- Metropolis, N., A. Rosenbluth, M. Rosenbluth, and A. Teller. 1953. Equations of state calculations by fast computing machines. *Journal of Chemical Physics* 21(6): 1087–1092.
- Mitchell, J.K., and K. Soga. 2005. *Fundamentals of soil behavior*. Hoboken, New Jersey: Wiley.
- Nawari, N.O., and R. Liang. 2000. Reliability of measured geotechnical properties. In *Performance confirmation of constructed geotechnical facilities*, Geotechnical Special Publication No. 94, 322–335.
- Ohya, S., T. Imai, and M. Matsubara. 1982. Relationships between N value by SPT and LLT pressuremeter results. In *Proceedings of 2nd European symposium on penetration testing*, Vol. 1, 125–130. Amsterdam.
- Orchant, C.J., F.H. Kulhawy, and C.H. Trautmann. 1988. *Reliability-based design of foundations for transmission line structures: Critical evaluation of in-situ test methods*. Report TR-5507(2). Palo Alto: Electric Power Research Institute.
- Papadimitriou, C., J.L. Beck, and L.S. Katafygiotis. 1997. Asymptotic expansions for reliability and moments of uncertain systems. *Journal of Engineering Mechanics* 123(12): 1219–1229.
- Peck, R.B., W.E. Hansen, and T.H. Thornburn. 1974. *Foundation engineering*, 514 pp. New York: Wiley.
- Phoon, K.K., and F.H. Kulhawy. 1999a. Characterization of geotechnical variability. *Canadian Geotechnical Journal* 36(4): 612–624.
- Phoon, K.K., and F.H. Kulhawy. 1999b. Evaluation of geotechnical property variability. *Canadian Geotechnical Journal* 36(4): 625–639.
- Phoon, K.K., and F.H. Kulhawy. 1996. On quantifying inherent soil variability. In *Uncertainty in the geologic environment: From theory to practice*, Geotechnical Special Publication No. 58 (I): 326–340.
- Phoon, K.K., F.H. Kulhawy, and M.D. Grigoriu. 1995. *Reliability-based design of foundations for transmission line structures*. Report TR-105000. Palo Alto: Electric Power Research Institute.
- Phoon, K.K., S.T. Quek, and P. An. 2003. Identification of statistically homogeneous soil layers using modified Bartlett statistics. *Journal of Geotechnical and Geoenvironmental Engineering* 129(7): 649–659.
- Ravi, V. 1992. Statistical modeling of spatial variability of undrained strength. *Canadian Geotechnical Journal* 29: 721–729.
- Robert, C., and G. Casella. 2004. *Monte Carlo statistical methods*. Springer.
- Robertson, P.K. 2009. Interpretation of cone penetration tests—a unified approach. *Canadian Geotechnical Journal* 46: 1337–1355.
- Robertson, P.K. 1990. Soil classification using the cone penetration test. *Canadian Geotechnical Journal* 27(1): 151–158.

- Robertson, P.K., and R.G. Campanella. 1983a. Interpretation of cone penetration tests. Part I: Sand. *Canadian Geotechnical Journal* 20(4): 718–733.
- Robertson, P.K., and R.G. Campanella. 1983b. Interpretation of cone penetration tests. Part II: Clay. *Canadian Geotechnical Journal* 20(4): 734–745.
- Schuster, M.J., C.H. Juang, M.J.S. Roth, and D.V. Rosowsky. 2008. Reliability analysis of building serviceability problems caused by excavation. *Geotechnique* 58(9): 743–749.
- Shannon, C.E., and W. Weaver. 1949. *The mathematical theory of communication*. Urbana, IL: University of Illinois Press.
- Silva, F., T.W. Lambe, and W.A. Marr. 2008. Probability and risk of slope failure. *Journal of Geotechnical and Geoenvironmental Engineering* 134(12): 1691–1699.
- Siu, N.O., and D.L. Kelly. 1998. Bayesian parameter estimation in probabilistic risk assessment. *Reliability Engineering and System Safety* 62(1–2): 89–116.
- Sivia, D.S., and J. Skilling. 2006. *Data analysis: A Bayesian tutorial*. New York: Oxford University Press.
- Smith, G.F., P.G. Benson, and S.P. Curley. 1991. Belief, knowledge, and uncertainty: A cognitive perspective on subjective probability. *Organizational Behavior and Human Decision Processes* 48(2): 291–321.
- Tang, W.H., M.S. Yucemen, and A.H.S. Ang. 1976. Probability based short-term design of slope. *Canadian Geotechnical Journal* 13: 201–215.
- Terzaghi, K., and R.B. Peck. 1967. *Soil mechanics in engineering practice*, 729 pp. New York: Wiley.
- Tversky, A., and D. Kahneman. 1974. Judgment under uncertainty: Heuristics and biases. *Science* 185: 1124–1131.
- U.S. Army Corps of Engineers. 1997. Engineering and design: introduction to probability and reliability methods for use in geotechnical engineering. *Engineer Technical Letter 1110-2-547*. Washington, D.C.: Department of the Army.
- Vanmarcke, E.H. 1977. Probabilistic modeling of soil profiles. *Journal of Geotechnical Engineering* 103(11): 1127–1246.
- Vanmarcke, E.H. 1983. *Random fields: Analysis and synthesis*. Cambridge: MIT Press.
- Vick, S.G. 2002. *Degrees of belief: Subjective probability and engineering judgment*. Reston, Virginia: ASCE Press.
- Vick, S.G. 1997. Dam safety risk assessment: New directions. *Water Power and Dam Construction* 49(6).
- Wang, Y. 2011. Reliability-based design of spread foundations by Monte Carlo simulations. *Geotechnique* 61(8): 677–685.
- Wang, Y., S.K. Au, and F.H. Kulhawy. 2011. Expanded reliability-based design approach for drilled shafts. *Journal of Geotechnical and Geoenvironmental Engineering* 137(2): 140–149.
- Wang, Y., S.K. Au, and Z. Cao. 2010. Bayesian approach for probabilistic characterization of sand friction angles. *Engineering Geology* 114(3–4): 354–363.
- Wang, Y., and Z. Cao. 2013. Probabilistic characterization of Young's modulus of soil using equivalent samples. *Engineering Geology* 159(12): 106–118.
- Wang, Y., Z. Cao, S.K. Au, and Q. Wang. 2009. Reliability analysis of a benchmark problem for slope stability. In *Proceedings of the second international symposium on geotechnical safety and risk*, 89–93. Gifu Japan, June 2009.
- Wang, Y., and T.D. O'Rourke. 2007. Interpretation of secant shear modulus degradation characteristics from pressuremeter tests. *Journal of Geotechnical and Geoenvironmental Engineering* 133(12): 1556–1566.
- Wroth, C.P. 1984. The interpretation of in-situ soil tests. *Geotechnique* 34(4): 449–489.
- Xu, B., and B.K. Low. 2006. Probabilistic stability analyses of embankments based on Finite-Element Method. *Journal of Geotechnical and Geoenvironmental Engineering* 132(11): 1444–1454.
- Yan, W.M., K.V. Yuen, and G.L. Yoon. 2009. Bayesian probabilistic approach for the correlations of compression index for marine clays. *Journal of Geotechnical and Geoenvironmental Engineering* 135(12): 1932–1940.



- Yuen, K.V. 2010. Recent development of Bayesian model class selection and applications in civil engineering. *Structural Safety* 32(5): 338–346.
- Zhang, F. 2011. *Bayesian ambient modal identification incorporating multiple setups*. Ph.D. thesis. Hong Kong: City University of Hong Kong.
- Zhang, J., W.H. Tang, and L.M. Zhang. 2010a. Efficient probabilistic back-analysis of slope stability model parameters. *Journal of Geotechnical and Geoenvironmental Engineering* 136 (1): 99–109.
- Zhang, J., L.M. Zhang, and W.H. Tang. 2009. Bayesian framework for characterizing geotechnical model uncertainty. *Journal of Geotechnical and Geoenvironmental Engineering* 135(7): 932–940.
- Zhang, L.L., J. Zhang, L.M. Zhang, and W.H. Tang. 2010b. Back analysis of slope failure with Markov chain Monte Carlo simulation. *Computers and Geotechnics* 37(7–8): 905–912.
- Zhang, L.M., W.H. Tang, L.L. Zhang, and J.G. Zheng. 2004. Reducing uncertainty of prediction from empirical correlations. *Journal of Geotechnical and Geoenvironmental Engineering* 130 (5): 526–534.

Probabilistic Approaches for Geotechnical Site  
Characterization and Slope Stability Analysis

Cao, Z.; Wang, Y.; Li, D.

2017, XVI, 190 p. 55 illus., 7 illus. in color., Hardcover

ISBN: 978-3-662-52912-6