

Contents

1	Mechanics	1
1.1	Point Mechanics	1
1.1.1	Basic Concepts of Mechanics and Kinematics	1
1.1.2	Newton's Law of Motion	3
1.1.3	Simple Applications of Newton's Law	6
1.1.4	Harmonic Oscillator in One Dimension	13
1.2	Mechanics of Point Mass Systems	17
1.2.1	The Ten Laws of Conservation	17
1.2.2	The Two-Body Problem	19
1.2.3	Constraining Forces and d'Alembert's Principle	20
1.3	Analytical Mechanics	24
1.3.1	The Lagrange Function	24
1.3.2	The Hamilton Function	26
1.3.3	Harmonic Approximation for Small Oscillations	28
1.4	Mechanics of Rigid Bodies	33
1.4.1	Kinematics and Inertia Tensor	33
1.4.2	Equations of Motion	36
1.5	Continuum Mechanics	42
1.5.1	Basic Concepts	42
1.5.2	Stress, Strain and Hooke's Law	47
1.5.3	Waves in Isotropic Continua	49
1.5.4	Hydrodynamics	50
2	Electricity and Magnetism	61
2.1	Vacuum Electrodynamics	61
2.1.1	Steady Electric and Magnetic Fields	61
2.1.2	Maxwell's Equations and Vector Potential	66
2.1.3	Energy Density of the Field	68
2.1.4	Electromagnetic Waves	68
2.1.5	Fourier Transformation	69
2.1.6	Inhomogeneous Wave Equation	70

2.1.7	Applications	71
2.2	Electrodynamics in Matter	76
2.2.1	Maxwell's Equations in Matter	76
2.2.2	Properties of Matter	77
2.2.3	Wave Equation in Matter	79
2.2.4	Electrostatics at Surfaces.	80
2.3	Theory of Relativity.	83
2.3.1	Lorentz Transformation.	84
2.3.2	Relativistic Electrodynamics	87
2.3.3	Energy, Mass and Momentum	89
3	Quantum Mechanics.	93
3.1	Basic Concepts	93
3.1.1	Introduction	93
3.1.2	Mathematical Foundations	94
3.1.3	Basic Axioms of Quantum Theory	96
3.1.4	Operators	98
3.1.5	Heisenberg's Uncertainty Principle	100
3.2	Schrödinger's Equation	101
3.2.1	The Basic Equation.	101
3.2.2	Penetration	102
3.2.3	Tunnel Effect	104
3.2.4	Quasi-classical WKB Approximation	105
3.2.5	Free and Bound States in the Potential Well	106
3.2.6	Harmonic Oscillators	107
3.3	Angular Momentum and the Structure of the Atom.	110
3.3.1	Angular Momentum Operator.	110
3.3.2	Eigenfunctions of L^2 and L_z	111
3.3.3	Hydrogen Atom	112
3.3.4	Atomic Structure and the Periodic System	115
3.3.5	Indistinguishability	116
3.3.6	Exchange Reactions and Homopolar Binding.	118
3.4	Perturbation Theory and Scattering	120
3.4.1	Steady Perturbation Theory.	120
3.4.2	Unsteady Perturbation Theory.	122
3.4.3	Scattering and Born's First Approximation.	124
4	Statistical Physics	129
4.1	Probability and Entropy	129
4.1.1	Canonical Distribution	129
4.1.2	Entropy, Axioms and Free Energy	132
4.2	Thermodynamics of the Equilibrium	135
4.2.1	Energy and Other Thermodynamic Potentials.	135
4.2.2	Thermodynamic Relations	138

4.2.3	Alternatives to the Canonical Probability Distribution	140
4.2.4	Efficiency and the Carnot Cycle	141
4.2.5	Phase Equilibrium and the Clausius-Clapeyron Equation	143
4.2.6	Mass Action Law for Gases	146
4.2.7	The Laws of Henry, Raoult and van't Hoff	147
4.2.8	Joule-Thomson Effect	149
4.3	Statistical Mechanics of Ideal and Real Systems	150
4.3.1	Fermi and Bose Distributions	150
4.3.2	Classical Limiting Case $\beta\mu \rightarrow -\infty$	152
4.3.3	Classical Equidistribution Law	155
4.3.4	Ideal Fermi-Gas at Low Temperatures $\beta\mu \rightarrow +\infty$	156
4.3.5	Ideal Bose-Gas at Low Temperatures $\beta\mu \rightarrow 0$	157
4.3.6	Vibrations	160
4.3.7	Virial Expansion of Real Gases	161
4.3.8	Van der Waals' Equation	162
4.3.9	Magnetism of Localised Spins	164
4.3.10	Scaling Theory	169
5	Fractals in Theoretical Physics	189
5.1	Non-random Fractals	190
5.2	Random Fractals: The Unbiased Random Walk	192
5.3	'A Single Length'	194
5.3.1	The Concept of a Characteristic Length	194
5.3.2	Higher Dimensions	195
5.3.3	Additional Lengths that Scale with \sqrt{t}	195
5.4	Functional Equations and Scaling: One Variable	196
5.5	Fractal Dimension of the Unbiased Random Walk	197
5.6	Universality Classes and Active Parameters	197
5.6.1	Biased Random Walk	197
5.6.2	Scaling of the Characteristic Length	198
5.7	Functional Equations and Scaling: Two Variables	200
5.8	Fractals and the Critical Dimension	201
5.9	Fractal Aggregates	207
5.10	Fractals in Nature	210
6	Dynamical Systems and Chaos	215
6.1	Basic Notions and Framework	215
6.1.1	Phase Space	215
6.1.2	Continuous-Time Dynamical Systems	216
6.1.3	Flows and Phase Portraits	217
6.1.4	Insights from Dynamical Systems Theory	217
6.1.5	Some Examples	218

6.2	Fixed Points and Linear Stability Analysis	221
6.2.1	Fixed Points and Stability Matrix	221
6.2.2	Flow Linearization Theorem	222
6.2.3	The Different Types of Fixed Points	223
6.2.4	Constructing the Phase Portrait	225
6.2.5	Application: Anharmonic Oscillators	227
6.2.6	The Origin of Bifurcations	230
6.3	Attractors, Bifurcations and Normal Forms	231
6.3.1	Attractors	231
6.3.2	Conservative <i>Versus</i> Dissipative Systems	232
6.3.3	The Different Types of Bifurcations	233
6.3.4	Normal Forms and Structural Stability	235
6.4	Discrete-Time Dynamical Systems	235
6.4.1	Discrete-Time Evolution Equations	235
6.4.2	Linear Stability Analysis	236
6.4.3	Attractors and Bifurcations	237
6.4.4	Discretization by Poincaré Sections	237
6.5	Lyapunov Exponents and Deterministic Chaos	239
6.5.1	Lyapunov Exponents	239
6.5.2	Deterministic Chaos	240
6.5.3	Ergodic Theory	242
6.6	Routes to Chaos	243
6.6.1	Period Doubling and Subharmonic Cascade	243
6.6.2	Intermittency	245
6.6.3	Ruelle-Takens Scenario	246
6.6.4	Hamiltonian Systems and KAM Theorem	246
6.7	Conclusion	248
6.8	Problems	249
6.9	Further Reading	250
	Appendix A: Elementary Particles	253
	Appendix B: Answers to Questions	263
	Index	267

From Newton to Mandelbrot

A Primer in Theoretical Physics

Stauffer, D.; Stanley, H.E.; LESNE, A.

2017, XIV, 270 p. 84 illus., Hardcover

ISBN: 978-3-662-53683-4