

Chapter 2

Fuzzy Qualitative Trigonometry

2.1 Introduction

Trigonometry is a branch of mathematics that deals with the relationships between the sides and angles of triangles and with the properties and application of trigonometric functions of angles. It began as the computational component of geometry in the second century BC and plays a crucial role in domains such as mathematics and engineering. In order to bridge the gap between qualitative and quantitative descriptions of physical systems, we propose a fuzzy qualitative representation of trigonometry (FQT), which provides theoretical foundations for the representation of trigonometric properties. It is often desirable and sometimes necessary to reason about the behaviour of a system on the basis of incomplete or sparse information. The methods of model-based technology provide a means of doing this [1, 2]. The initial approaches to model-based reasoning were seminal but focused on symbolic qualitative reasoning (QR) only, providing a means whereby the global picture of how a system might behave could be generated using only the sign of the magnitude and direction of change of the system variables. This makes qualitative reasoning complementary to quantitative simulation. However, quantitative and qualitative simulation form the two ends of a spectrum; and semi-quantitative methods were developed to fill the gap. For the most part these were interval reasoners bolted on to existing qualitative reasoning systems (e.g. [3]), Blackwell did pioneering work on spatial reasoning on robots [4]; however, one exception to this was fuzzy qualitative reasoning which integrated the strengths of approximate reasoning with those of qualitative reasoning to form a more coherent semiquantitative approach than their predecessors [5, 6]. Model-based technology methods have been successfully applied to a number of tasks in the process domain. However, while some effort has been expended on developing qualitative kinematic models, the results have been limited [7–10]. The basic requirement for progressing in this domain is the development of a qualitative version of the trigonometric rules. Buckley and Eslami [11] proposed the definition of fuzzy trigonometry from the fuzzy perspective without consideration of the geometric meaning of trigonometry. Some progress has been made in this direction

by Liu [12], but as with other applications of qualitative reasoning, the flexibility gained in variable precision by integrating fuzzy and qualitative approaches is no less important in the kinematics domain. In this chapter an extension of the rules of trigonometry to the fuzzy qualitative case is presented, which will serve as the basis for fuzzy qualitative reasoning about the behaviour and possible diagnosis of kinematic robot devices.

Fuzzy qualitative reasoning combines the advantages of fuzzy reasoning and qualitative reasoning techniques. Research into the integration of fuzzy reasoning and qualitative reasoning has been carried out in both theory and application in the past two decades [5, 6, 13–18]. The use of fuzzy reasoning methods are becoming more and more popular in intelligent systems [19, 20], especially hybrid methods and their applications integrating with evolutionary computing [21–25], decision trees [26, 27], neural networks [28–32], data mining [13], and so on [33–37]. Qualitative reasoning is reviewed in [38–41]. The integration of fuzzy reasoning and qualitative reasoning (i.e., fuzzy qualitative reasoning) provides an opportunity to explore research (e.g., spatial reasoning) with both the advantages of fuzzy reasoning and qualitative reasoning. Some of fuzzy qualitative reasoning contributions can be found in [5, 6, 14–16]. Shen and Leitch [5] use a fuzzy quantity space (i.e., normalized fuzzy partition), which allows for a more detailed description of the values of the variables. Such an approach relies on the extension principle and approximation principle in order to express the results of calculations in terms of the fuzzy sets of the fuzzy quantity space.

Fuzzy reasoning has been significantly developed and has attracted much attention and exploitation from industry and research communities in the past four decades. Fuzzy reasoning is good at communicating with sensing and control level subsystems by means of fuzzification and defuzzification methods. It has powerful reasoning strategies utilising compiled knowledge through conditional statements so as to easily handle mathematical and engineering systems in model free manner. Fuzzy reasoning also provides a means of handling uncertainty in a natural way making it robust in significantly noisy environments. However, the fact that its knowledge is primarily shallow, and the questions over the computational overhead associated with handling grades of membership of discrete fuzzy sets must be taken into account if multi-step reasoning is to be carried out. On the other hand, Qualitative and Model-based Reasoning has been successfully deployed in many applications such as autonomous spacecraft support [42], Systems Biology [43] and qualitative systems identification [44]. It has the advantage of operating at the conceptual modelling level, reasoning symbolically with models which retain the mathematical structure of the problem rather than the input/output representation of rule bases (fuzzy or otherwise). These models are incomplete in the sense that, being symbolic, they do not contain, or require, exact parameter information in order to operate. Qualitative reasoning can make use of multiple ontologies, can explicitly represent causality, enable the construction of more sophisticated models from simpler constituents by means of compositional modelling, and infer the global behaviour of a system from a description of its structure [38, 41]. These features can, when combined with fuzzy values and operators, compensate for the lack of ability in fuzzy reasoning

alone to deal with that kind of inference about complex systems. The computational cause-effect relations contained in qualitative models facilitates analysing and explaining the behaviour of a structural model. Based on a scenario generated from fuzzy reasoning's fuzzification process, fuzzy qualitative reasoning may be able to build a behavioural model automatically, and use this model to generate a behaviour description, acceptable by symbolic systems, either by abstraction and qualitative simulation or as a comprehensive representation of all possible behaviours utilising linguistic fuzzy values. Liu, Coghill and Brown had attempted two completely different approaches [45–47] based on fuzzy qualitative trigonometry [48]. Research reported in [45] proposed a normalised based qualitative representation from cognition perspective, it converts both numeric and sub-symbolic data into a normalisation reference where transfer of different types of data is carried out, the method was not implemented into spatial robots due to its costly computational cost and complex spatial relation though it was applied to planar robots. On the other hand, conventional robotics had been adapted with the fuzzy qualitative trigonometry, not only did it implement feasible for spatial robots but also it shows the promising potential for intelligent robotics [46]. A full account of fuzzy qualitative trigonometry is presented in this chapter with the goal of solving the intelligent connection problem (also known as symbol grounding problem) for physical systems, in particular robotic systems. This problem is one of the key issues in AI robotics [49] and relates to a wide range of research areas such as computer vision [50]. This chapter is organised as follows. Section 2.2 presents fuzzy qualitative (FQ) trigonometry. Section 2.3 derives FQ trigonometric functions. Section 2.4 proposes the derivative extension to FQ trigonometry. Section 2.5 addresses discussions and conclusions.

2.2 Fuzzy Qualitative Trigonometry

The membership distribution of a normal convex fuzzy number can be approximated by a 4-tuple fuzzy number representation. The representation of the 4-tuple fuzzy numbers is a better qualitative representation for trigonometry, because this representation has high resolution and good compositionality [5, 51]. The degree of resolution can be adjusted by the choice of fuzzy numbers. Such a representation provides the flexibility to carry out computation based on real numbers, intervals, triangular numbers and trapezoidal fuzzy intervals, which comprise nearly all of the computing elements in fuzzy qualitative reasoning. This representation has the ability to combine representations for different aspects of a phenomenon or system to create a representation of the phenomenon or systems as a whole. The computation of fuzzy numbers is based on its arithmetic operations. A four-tuple fuzzy number representation, namely $[a, b, \tau, \beta]$ with the condition $a \leq b$ and $a \times b \geq 0$, can be used to approximate the membership distribution of a normal convex fuzzy number. The four tuple set, $[a, b, \tau, \beta]$, can be defined by the following membership function, $\mu_a(x)$,

$$\mu_A(x) = \begin{cases} 0 & x < a - \tau \\ \tau^{-1}(x - a + \tau) & x \in [a - \tau, a] \\ 1 & x \in [a, b] \\ \beta^{-1}(b + \beta - x) & x \in [b, b + \beta] \\ 0 & x > b + \beta \end{cases} \quad (2.1)$$

Note that if $a = b$, Eq. 2.1 is a triangular fuzzy number with its membership as $\mu_A(x) = [\tau^{-1}(x - a + \tau), \beta^{-1}(a + \beta - x)]$, where $x \in [a - \tau, a + \beta]$. Besides, if $a = b$ and $\tau = \beta = 0$, it turns the 4-tuple fuzzy number into a real number. For instance $[1, 1, 0, 0]$ stands for the real number 1. In addition, the arithmetic operations on the 4-tuple parametric representation of fuzzy numbers are given in Appendix A, where $<_0$ is the partial order $<_\alpha$ when $\alpha = 0$, which was used in chapter [5]. The partial order has been given in a general form, i.e., $\{\odot_\alpha \mid (\odot \in \{>, <, =, \leq, \geq, \in, \subset, \supset, \subseteq, \supseteq\})\}$. For instance, partial order $<_\alpha$ is defined such that for m, n ($m \neq n$), we say m is α -less than n , $m <_\alpha n$, iff $a < b$, $a \in m_\alpha$, $b \in n_\alpha$ with m_α and n_α being the α -cuts of m and n , respectively. It is believed that fuzzy arithmetic is still a developing research topic, usage of the above-mentioned fuzzy operators is constrained by the fact that an assignment of α value is required in a calculation cycle.

FQ trigonometry actually is the FQ description of conventional trigonometry. The unit circle of conventional trigonometry has been modified by the introduction of quantity spaces for orientation and translation. Conventional trigonometric functions (e.g. a sine function) have been abstracted to obtain FQ versions of these operations. For a good survey on FQ reasoning please see [48]. The quantity space for every variable in the system is a finite and convex discretisation of the real number line. FQ quantity space Q consists of an orientation component Q^a and a translation component Q^d . It can be described as,

$$\begin{aligned} Q^a &= [QS_a(\theta_1), \dots, QS_a(\theta_i), \dots, QS_a(\theta_m)] \\ Q^d &= [QS_d(l_1), \dots, QS_d(l_j), \dots, QS_d(l_n)] \end{aligned} \quad (2.2)$$

where $QS_a(\theta_i)$ denotes the state of an angle θ_i , $QS_d(l_j)$ denotes the state of a distance l_j , m and n are the number of the elements of the two components. The position measurement of $P(QS_a(\theta_i), QS_d(l_j))$ determined by both the characteristics of the fuzzy membership functions of $QS_a(\theta_i)$ and $QS_d(l_j)$. The geometric meaning of FQ trigonometry is demonstrated in a proposed FQ unit circle, in which the motion is described by an orientation component and a translation component. A FQ unit circle is nothing but a conventional trigonometric circle whose axes are replaced by unit quantity spaces. The position state of a FQ point is defined by the projections of the point into FQ axes. For instance, let the X axis projection of a FQ point P be X_p , and the Y axis projection as Y_p , its FQ position can be described as $P = (X_p, Y_p)$. It means that a point position in FQ coordinates is described by fuzzy sets, hence, fuzzy arithmetic and its characteristics can be applied to calculation of FQ states in FQ coordinates. In accordance with Cartesian coordinates convention, the position, $P_o(X_o, Y_o) = ([0, 0, 0, 0], [0, 0, 0, 0])$, is the FQ origin, where the axes intersect. Counterclockwise is the positive orientation, the number of qualitative orientation

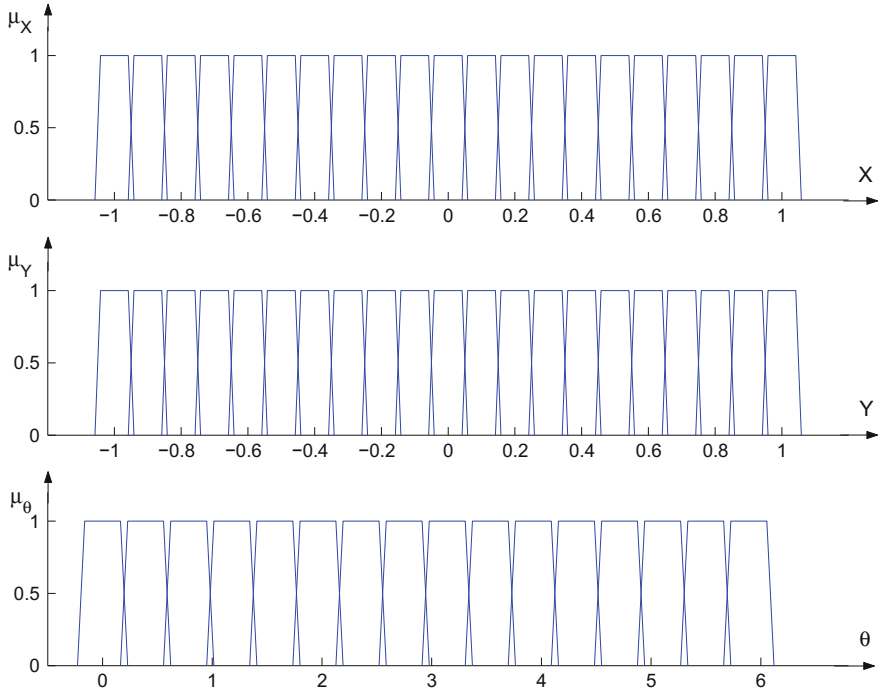


Fig. 2.1 Quantity spaces described by fuzzy membership functions ($p = 17$, $q_x = 23$ and $q_y = 23$)

states in a full circle starting from the X axis is denoted by p , the number of qualitative translation states is denoted by q_{axis} , (i.e. q_x , q_y). Though different functions can be employed to generate fuzzy numbers of a quantity space, two conditions must hold. First, fuzzy numbers in each component must be origin symmetric. Second, special real numbers (e.g., 0 , $\pi/2$, π , $3\pi/2$ in an orientation, and -1 , 0 , -1 in a translation) are the centres of some certain fuzzy numbers in their corresponding quantity space. For instance, a right angle in conventional trigonometry is corresponded to fuzzy number $QS_a(p/4 + 1)$, whose centre is equal to $\pi/2$. Please note that, for a 4-tuple fuzzy number (e.g., $[a, b, \tau, \beta]$), its centre is defined by $((a + b)/2) \times 2\pi$. Note that the fuzzy numbers in the orientation quantity space need to be multiplied by 2π before they are applied to the arithmetic in Appendix A.

A MATLAB toolbox named XTRIG has been developed by [48] to implement the proposed FQ trigonometry. XTRIG has a function for the generation of 4-tuple fuzzy numbers, say, a fuzzy number, $[a, b, \tau, \beta]$, the function is $b - a = \kappa_0 \tau$ with the condition $\tau = \beta$. κ_0 is a threshold parameter to define the shape of fuzzy numbers. For instance, let the number of the quantity space of an orientation p be 17 for a FQ trigonometric circle, those of a translation, q_x and q_y , be 23 each, and set κ_0 as 5, XTRIG generates the corresponding quantity spaces of the orientation Q^a and translation Q^d , which are shown in Fig. 2.1. It shows that the fuzzy-number generation

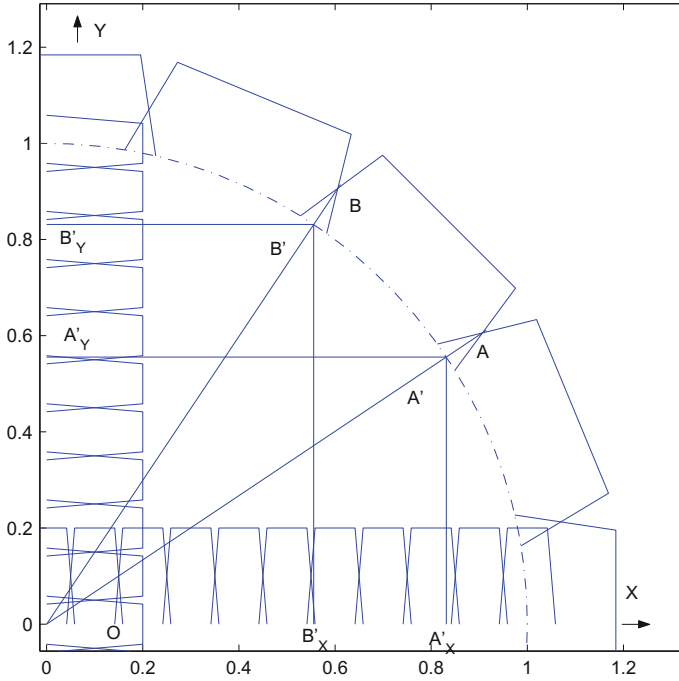


Fig. 2.2 The relation between qualitative translation and orientation

function distributes the 16 fuzzy numbers in the translation component in the range $[0 \ 1)$, and two sets of the 21 fuzzy numbers in the range $[-1 \ 1]$. Figure 2.2 shows the conversion from a fuzzy qualitative angle to its FQ position, where the qualitative angle is the third orientation state in the orientation quantity space Q^a (i.e., $[0.099 \ 0.151 \ 0.0104 \ 0.0104]$). The conversion is carried out as follows. Firstly, we calculate the positions of crossing points A , B between the fuzzy number (i.e., the third orientation angle) and its adjacent fuzzy numbers; Secondly, we calculate crossing point positions A' , B' between the unit circle curve and the two line segments OA and OB ; Thirdly, we obtain two line segments $A'_x B'_x$ and $A'_y B'_y$ by projecting points A' , B' into X and Y coordinates, respectively. Finally we generate two sets of fuzzy numbers which totally or partially are in the range $A'_x B'_x$ and $A'_y B'_y$. The set of fuzzy numbers are within $A'_x B'_x$ is the FQ position on X axis of the orientation angle (i.e., $QS_a(3)_x$); the other set within $A'_y B'_y$ is for Y axis projection (i.e., $QS_a(3)_y$). The FQ position is,

$$P_{QS_a(3)} = (P_{QS_a(3)_x}, P_{QS_a(3)_y}) \quad (2.3a)$$

where

$$P_{QS_a(3)_X} =_0 \begin{bmatrix} 0.5583 & 0.6417 & 0.0167 & 0.0167 \\ 0.6583 & 0.7417 & 0.0167 & 0.0167 \\ 0.7583 & 0.8417 & 0.0167 & 0.0167 \end{bmatrix} \quad (2.3b)$$

$$P_{QS_a(3)_Y} =_0 \begin{bmatrix} 0.5583 & 0.6417 & 0.0167 & 0.0167 \\ 0.6583 & 0.7417 & 0.0167 & 0.0167 \\ 0.7583 & 0.8417 & 0.0167 & 0.0167 \end{bmatrix} \quad (2.3c)$$

The FQ values of the third orientation angle in the X and Y axes are denoted by a set of three 4-tuple fuzzy numbers and a set of five fuzzy numbers generated from its translation quantity space when its α cut is defaulted as zero. The conversion methodology can also be applied to the conversion from fuzzy numbers in its translation quantity space to those in its orientation quantity space.

2.3 Fuzzy Qualitative Trigonometric Functions

FQ trigonometric functions are derived from conventional trigonometric functions based on the extension principle, which allows the extension of classical mathematical operators to the fuzzy domain. It means that an arithmetic operation performed between n fuzzy sets will yield a fuzzy set of the same form. Each trigonometric function is derived and illustrated using FQ trigonometry and the quantity spaces of the FQ coordinates. For simplicity, the symbols of quantitative trigonometric functions are used to describe their counterparts in FQ trigonometry but with qualitative variables instead (e.g., $QS_i(j)$, where $i \in \{a, d\}$ and $j \in \{p, q\}$). In order to give a clear explanation of FQ trigonometric functions, let us consider the example for the third FQ angle in Fig. 2.2 as a general case of the i th orientation angle (i.e., $QS_a(i)$). $P_{QS_a(i)}$ stands for the FQ position of the intersection of $QS_a(i)$ to a unit circle curve, and its projected FQ positions on the axes of X and Y are $QS_d(|A'_x B'_x|)$ and $QS_d(|A'_y B'_y|)$. For now we are ready to define FQ trigonometric functions,

$$\sin(QS_a(i)) =_\alpha \frac{QS_d(|A'_y B'_y|)}{[1 \ 1 \ 0 \ 0]} =_\alpha QS_d(|A'_y B'_y|) \quad (2.4a)$$

$$\cos(QS_a(i)) =_\alpha \frac{QS_d(|A'_x B'_x|)}{[1 \ 1 \ 0 \ 0]} =_\alpha QS_d(|A'_x B'_x|) \quad (2.4b)$$

$$\sec(QS_a(i)) =_\alpha \frac{[1 \ 1 \ 0 \ 0]}{QS_d(|A'_x B'_x|)} =_\alpha \frac{[1 \ 1 \ 0 \ 0]}{\cos(QS_a(i))} \quad (2.4c)$$

$$\csc(QS_a(i)) =_{\alpha} \frac{[1 \ 1 \ 0 \ 0]}{QS_d(|A'_y B'_y|)} =_{\alpha} \frac{[1 \ 1 \ 0 \ 0]}{\sin(QS_a(i))} \quad (2.4d)$$

$$\arcsin(QS_d(|A'_y B'_y|)) =_{\alpha} QS_a(i) \quad (2.4e)$$

$$\arccos(QS_d(|A'_x B'_x|)) =_{\alpha} QS_a(i) \quad (2.4f)$$

where $\sin(QS_a(i))$, $\cos(QS_a(i))$, $\sec(QS_a(i))$ and $\csc(QS_a(i))$ FQ functions are straightforward as shown in Fig. 2.2. The i th fuzzy qualitative position's arcsine and arccosine functions shown in Eqs. 2.4e and 2.4f, i.e., $\arcsin(QS_d(i))$ and $\arccos(QS_d(i))$, are the inverse of its sine and cosine functions given in Eqs. 2.4a and 2.4b. Refer to Fig. 2.2, if points B'_y and A'_y are the crossing points of the i th fuzzy number and its adjacent fuzzy numbers in Y coordinate, the fuzzy numbers in the orientation quantity space within arc $\widehat{A'B'}$ are its corresponding qualitative angle. FQ arcsine and arccosine functions, as their quantitative counterparts have, are constrained that $\arcsin(QS_d(i)) \subseteq_{\alpha} [-\frac{p}{4} - 1, \frac{p}{4} + 1]$, and $\arccos(QS_d(i)) \subseteq_{\alpha} [1, \frac{p}{2} + 2]$. Further, the tangent of $QS_a(i)$, written as $\tan(QS_a(i))$, is defined as the ratio of the opposite to the adjacent sides, $QS_d(|A'_y B'_y|) / QS_d(|A'_x B'_x|)$. It can clearly see that the tangent of $QS_a(i)$ is equal to the sine of $QS_a(i)$ divided by the cosine of $QS_a(i)$. The same can apply to cotangent function, then we obtain

$$\tan(QS_a(i)) =_{\alpha} \frac{QS_d(|A'_y B'_y|)}{QS_d(|A'_x B'_x|)} =_{\alpha} \frac{\sin(QS_a(i))}{\cos(QS_a(i))} \quad (2.5a)$$

$$\cot(QS_a(i)) =_{\alpha} \frac{QS_d(|A'_x B'_x|)}{QS_d(|A'_y B'_y|)} =_{\alpha} \frac{\cos(QS_a(i))}{\sin(QS_a(i))} \quad (2.5b)$$

where

$$\cos^2(QS_a(i)) + \sin^2(QS_a(i)) =_{\alpha} QS_d(p) \quad (2.5c)$$

With the setting of the example in Fig. 2.2, we obtain the following results,

$$\begin{aligned} \cos(QS_a(114)) &=_0 \begin{bmatrix} 0.7583 & 0.8417 & 0.0167 & 0.0167 \\ 0.8583 & 0.9417 & 0.0167 & 0.0167 \\ 0.9583 & 1.0417 & 0.0167 & 0.0167 \end{bmatrix} \\ \sec(QS_a(14)) &=_0 \begin{bmatrix} 4.1379 & 6.3158 & 0.2670 & 0.7430 \\ 2.9268 & 3.8710 & 0.1361 & 0.2670 \\ 2.2642 & 2.7907 & 0.0823 & 0.1361 \\ 1.8462 & 2.1818 & 0.0551 & 0.0823 \\ 1.5584 & 1.7910 & 0.0395 & 0.0551 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \tan(QS_a(3)) &= {}_0 \begin{bmatrix} 0.6634 & 0.8462 & 0.0323 & 0.0415 \\ 0.8876 & 1.1266 & 0.0415 & 0.0552 \\ 1.1818 & 1.5075 & 0.0552 & 0.0772 \end{bmatrix} \\ \cot(QS_a(3)) &= {}_0 \begin{bmatrix} 1.1818 & 1.5075 & 0.0552 & 0.0783 \\ 0.8876 & 1.1266 & 0.0415 & 0.0561 \\ 0.6634 & 0.8462 & 0.0323 & 0.0423 \end{bmatrix} \\ \arcsin(QS_d(19)) &= {}_0 \begin{bmatrix} 0.0990 & 0.1510 & 0.0104 & 0.0104 \\ 0.1615 & 0.2135 & 0.0104 & 0.0104 \end{bmatrix} \\ \arccos(QS_d(20)) &= {}_0 \begin{bmatrix} 0.0365 & 0.0885 & 0.0104 & 0.0104 \end{bmatrix} \end{aligned}$$

The example above indicates two characteristics of FQ trigonometry. First, FQ trigonometric functions have period characteristic, e.g., $\cos(QS_a(114)) = \cos(QS_a(2 + 16 \times 7))$. Secondly, FQ tangent and cotangent functions are calculated based on FQ sine, cosine functions and the fundamental trigonometric identity. The use of the latter ensures the elimination of those fuzzy numbers that are the results generated by FQ sine and cosine functions but they lose their geometric meaning.

2.4 Derivatives of Fuzzy Qualitative Trigonometry

Combining FQ trigonometry with the extension principle [52] and the principle of convex and normal fuzzy sets' mapping [53], we have the lemmas below for the derivatives of FQ trigonometric functions.

Lemma I *If $y = d^n(x)$ is a n order derivative function, then with A being a fuzzy set in quantity space X ($x \in \bigcup FQT_\alpha(Q_X)$), $d^n()$ maps from A to a fuzzy set B in quantity space Y ($y \in \bigcup FQT_\alpha(Q_Y)$) such that*

$$\mu_B(y) = \sup_{x \in d^{-n}(y)} \mu_A(x) \quad (2.6)$$

where $FQT_\alpha()$ is one of FQT functions, α stands for α -cut and $Q_x, Q_y \subset Q$.

Fuzzy numbers herein are fuzzy sets of the real line $\mathbb{R}$ with a normal, fuzzy convex and continuous membership function of bounded support. In order to demonstrate the proposed extension, we have implemented it into XTRIG MATLAB toolbox. Recall the example used in Fig. 2.1. For first-order derivative, Lemma I can be rewritten as,

$$\mu_B(y) = \sup_{x \in d^{-1}(y)} \mu_A(x) \quad (2.7)$$

For instance, first order derivative of $\cos(QS_a(3))$ can be calculated as,

$$\begin{aligned} d^1(\cos(QS_a(3))) &=_{\alpha} -\sin(QS_a(3)) \\ &=_{\alpha} \begin{bmatrix} -0.7119 & -0.7966 & 0.0169 & 0.0169 \\ -0.8136 & -0.8983 & 0.0169 & 0.0169 \\ -0.9153 & -1.000 & 0.0169 & 0 \end{bmatrix} \end{aligned} \quad (2.8)$$

With the definition of the support of a fuzzy set, $\text{supp}(A)$, Lemma I can be further developed as a lemma below for the mapping of a fuzzy set's support.

Lemma II *With the convention in Lemma I, the following is obtained,*

$$d^n(\text{supp}(A)) = \text{supp}(B). \quad (2.9)$$

Furthermore, applying Lemma II to the example, we have the fuzzy sets' support version of FQ trigonometric first-order derivative as below,

$$\begin{aligned} d^1(\text{sup}(\cos(QS_a(3)))) &=_{\alpha} \text{sup}(d^1(\cos(QS_a(3)))) =_{\alpha} \text{sup}(\sin(QS_a(3))) \\ &=_{\alpha} \text{sup}\left(\begin{bmatrix} -0.7288 & -0.8135 \\ -0.8305 & -0.9152 \\ -0.9322 & -1.000 \end{bmatrix}\right) \\ &=_{\alpha} [-0.7288 \ -1.000] \end{aligned} \quad (2.10)$$

It shows that the support of the derivative output has two representations: local support and global support. The local support consists of the support values of the fuzzy numbers of the output; the global support is the support of a qualitative state. In the XTRIG toolbox, the column dimension is used to tell the representation of a fuzzy number from that of its support.

Lemma III *Quantity spaces in a unit circle must be x-axis or y-axis symmetric, or origin symmetric.*

Lemma above is the condition that Lemmas I and II hold; actually it is also the condition for FQ trigonometry. The characteristic of x-axis or y-axis symmetry is required for a translation quantity space, and that of origin symmetry is for an orientation quantity space.

Say A is a fuzzy set, $[A]^{\alpha}$ is a compact subset of $\mathbb{R}$ and $M(A)$ is a closed interval bounded by the lower and upper possibilistic mean values $[M_*(A), M^*(A)]$ of A [54], then it leads to the following definition,

Definition I α -centre of a fuzzy set is its possibilistic mean value.

$$C(A_\alpha) = \frac{M^*(A_\alpha) + M_*(A_\alpha)}{2} \quad (2.11)$$

where

$$M^*(A_\alpha) = \frac{\int_0^1 Pos[A \geq b(\alpha)] \times \max[A]^\alpha d\alpha}{\int_0^1 Pos[A \leq b(\alpha)] d\alpha}$$

$$M_*(A_\alpha) = \frac{\int_0^1 Pos[A \leq a(\alpha)] \times \min[A]^\alpha d\alpha}{\int_0^1 Pos[A \leq a(\alpha)] d\alpha}$$

and Pos denotes possibility.

Definition II α power support of a fuzzy set is the proportion of its α -power to its α -height.

$$PS(A_\alpha) = \begin{cases} \int_R \mu_A^\alpha(x) dx & \alpha = 1 \\ \frac{\int_R \mu_A^\alpha(x) dx}{1-\alpha} & \alpha \neq 1 \end{cases} \quad (2.12)$$

Definition III A fuzzy qualitative state $QS(A)$ in a α -level of a fuzzy set $\mu_A(X)$ can be described by its α -centre and scaled α power support.

$$QS(A) = {}_\alpha [C(A), \gamma PS(A)] \quad (2.13)$$

where $\gamma \in [0, 1]$ is a scaling factor. Hence, we have two representations for a qualitative state, four-tuple fuzzy numbers and centre-power-support fuzzy intervals. For instance, given a four tuple fuzzy number $A = [a \ b \ \tau \ \beta]$, its centre-power-support representation can be obtained below with default value $\alpha = 0$,

$$QS(A) = {}_\alpha [C(A), \gamma PS(A)]$$

$$= {}_\alpha \left[\frac{a+b}{2}, b - a + \frac{\tau+\beta}{2} \right]$$

Let us consider $QS_d(3) = [0.2034 \ 0.2881 \ 0.01069 \ 0.01069]$, its fuzzy qualitative state at α_0 level is $[0.2458, \ 0.1016\gamma]$. That is, the state is a 1D interval range with 0.2458 as its centre, 0.1016γ as its scalable radius. A fuzzy qualitative state's linguistic meaning or precision can be controlled by the combination of the adjustment of its scaling factor and fuzzy quantity spaces. Further, arithmetic for FQ states at a α level can be defined in terms of midpoint-radius (MR) representation and Definition III, for literature on interval MR representation, please see [55, 56].

Definition IV For $QS(A) =_{\alpha} [C(A) \gamma_A PS(A)]$ ($QS(A) \in Q$), $QS(B) =_{\alpha} [C(B) \gamma_B PS(B)]$, ($QS(B) \in Q$), γ_A and γ_B are scaling factors, respectively, we define

$$QS(A) + QS(B) =_{\alpha} [C(A) + C(B), \gamma_A PS(A) + \gamma_B PS(B)] \quad (2.14a)$$

$$QS(A) - QS(B) =_{\alpha} [C(A) - C(B), \gamma_A PS(A) + \gamma_B PS(B)] \quad (2.14b)$$

$$QS(A) \times QS(B) =_{\alpha} [C(A) \times C(B), |\rho(C(A))| \times \gamma_B PS(B) + |C(B)| \gamma_A PS(A) + \gamma_A \gamma_B PS(A) PS(B)] \quad (2.14c)$$

where

$$\begin{cases} \rho = 1 & \text{strong MR multiplication} \\ \rho = \frac{2}{3} & \text{weak MR multiplication} \end{cases}$$

and

$$\frac{I}{QS(A)} =_{\alpha} \left[\frac{C(B)}{D}, \frac{\gamma_B PS(B)}{D} \right] \quad (2.14d)$$

where

$$D =_{\alpha} C^2(B) - \gamma_B^2 PS^2(B), \text{ and } QS(B) \neq \mathbf{0}$$

and

$$\frac{QS(A)}{QS(B)} =_{\alpha} QS(A) \times \frac{I}{QS(B)} \text{ for } QS(B) \neq \mathbf{0} \quad (2.14e)$$

Due to Rump's overestimation theory in [57] that the overestimation of MR interval arithmetic compared to power set operations is uniformly bounded by a factor 1.5 in radius, we introduce an index to distinguish strong and weak multiplication in terms of interval MR representation, see Eq. 2.14c. Strong multiplication with the index as 1 includes the overestimation caused by interval MR multiplication, on the other hand, weak multiplication with the index as 2/3 does not. Generally speaking, interval computation in MR representation is faster, especially matrix operations, than infimum-supremum representation. This overestimation characteristic provides a method of controlling tradeoffs for the selection of interval representations for interval-related applications, e.g., real-time humanoid systems. The algorithms in MR representation and its conversion on four-tuple-fuzzy-number-based computation have been implemented into the XTRIG toolbox. We use MR-based computation in the rest of this chapter, not only because its faster computation nature is suitable

for robotics application, but also because MR-based representation is more flexible to control precision in data conversion among numerical, symbolic and hybrid data, thanks to the fact that fuzzy quantity space with domain background allows convert parameters among different types of data.

2.5 Conclusion

A fuzzy qualitative description of conventional trigonometry has been presented in this chapter. The unit circle of conventional trigonometry has been modified by the introduction of fuzzy partitions for orientation and translation. Conventional trigonometric functions (e.g., a sine function) and rules (e.g., the sine rule) have been abstracted to give fuzzy qualitative versions of these operations. In addition, fuzzy qualitative versions of the conventional triangle theorems are also provided in FQT. Examples have been given throughout the chapter to demonstrate FQT's ability. One of these focuses on robot kinematics and explains how contributions could be made by a new type of FQT-based kinematics to achieve the intelligent connection of low-level sensing and control tasks to high-level symbolic tasks.

The proposed fuzzy qualitative variables become linguistic variables or symbolic variables when they are modified with descriptive symbols [58–60]. Besides, methods such as aggregation operators [61] can be used to select specific symbols from a set of fuzzy qualitative states. Hence, fuzzy qualitative states in FQT can be denoted by symbolic terms which provide a promising base for behavioural description in robotics [62]. For example, Jenkins and Mataric [63] have provided a skill-level interface named behaviour vocabulary for a humanoid robot. However, the connection between the behaviour vocabulary and low-level sensing and control tasks is still uncertain. FQT is an abstraction of conventional trigonometry into the domain of fuzzy logic and qualitative reasoning. This version of trigonometry can replace the role that conventional trigonometry plays in robot kinematics; so a general robot kinematics can be derived based on FQT. The new type of kinematics handles fuzzy qualitative states that allow access to both numerical data and symbolic data. Further, a FQT-based robot kinematics could provide a transit layer for communicating variables, even variable-based cognitive functions of knowledge-level tasks and numerical sensing and motion control [49, 50]. From an implementation perspective, a robotic system with this type of kinematics can be easily fitted into a conventional fuzzy model which consists of a fuzzification unit, knowledge base, inference engine and defuzzification unit. The FQT-based reasoning could play a crucial role in an inference engine whose variables are supported by a knowledge base. The output of an inference engine is able to access knowledge-based systems, e.g., symbolic planning subsystems. The fuzzification and defuzzification units are able to provide low-level sensing and control tasks, it is obvious this role can be easily replaced by other techniques, e.g., fuzzy clustering.

Fuzzy qualitative reasoning seeks to harness the strengths of fuzzy reasoning and qualitative reasoning. It is in a position to play a crucial role in closing the gap between

low-level sensing and control tasks and high-level symbolic description. Our future work will focus on the development of an intelligent architecture using the FQT in the domain of AI robotics. The architecture is composed of a low-level handler, knowledge-based handler, and inference handler. The low-level handler provides an interface to numerical data; it is composed of a fuzzification unit and defuzzification unit, in each unit different techniques can be applied such as fuzzy clustering. A knowledge-based handler not only provides a knowledge base to support the system inferences, but also facilitates human supervision (e.g., cognitive inputs). FQT is the core of the inference handler which should be able to, scalable and in parallel interface both low-level handler and knowledge-based handler.

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