

Chapter 2

Principle and Mechanical Analysis on the Cold Rolling Precision Forming of Spline

Abstract Principle and process on the cold rolling precision forming of spline shafts have been introduced, mechanical analysis of the process has been introduced. The indexing and biting conditions, the rotating conditions, double-flank non-backlash meshing, the sliding of the contact point, and the slip-line field and the stress field are introduced and analyzed. Metal flow on the tooth surface of the workpiece and average pressure on the contact surface during the forming process have also been analyzed.

2.1 Principle and Process on the Cold Rolling Precision Forming of Spline Shafts

Figure 2.1 shows the cold rolling precision forming of spline shafts. The workpiece is restrained between the two rollers by the front and rear centers, and the two rollers rotate at the same direction synchronously and feed at the radial direction with uniform speed under the driven spindle. The two rollers are with the same parameters, and the workpiece rotates under the driven frictional moment produced by the roller when the feeding roller contacts with the workpiece. The tooth of the roller pressed into the surface of the workpiece results in the plastic deformation of the workpiece. After indentation of the tooth is formed, the roller and the formed spline tooth will mesh each other freely, and finally, the teeth of the spline shaft can be generated by the teeth of the rollers.

In the process of cold rolling precision forming, the main movements between the rollers and the workpiece include the following ones: rotation and radial feeding movement of the rollers, and the rotation movement of the workpiece, in which the movement of the rollers is the main movement and the movement of the workpiece is the passive movement. The teeth of the workpiece are formed through the free generation between rollers and the workpiece, and indexing equipment is not needed.

According to the movement characteristics of the spline shaft cold rolling precision forming, the forming process can be divided into four stages, as shown in Fig. 2.2. In the first stage, the rollers have not contacted with the static workpiece,

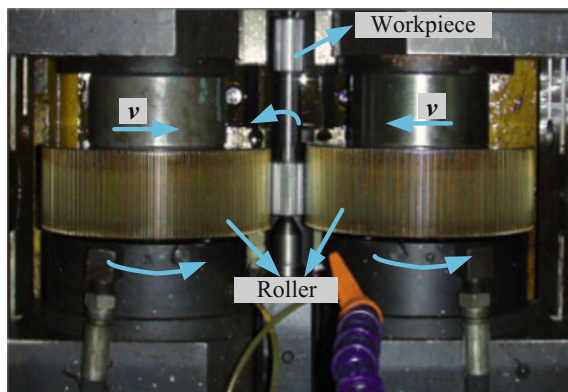


Fig. 2.1 Principle of the cold rolling precision forming process

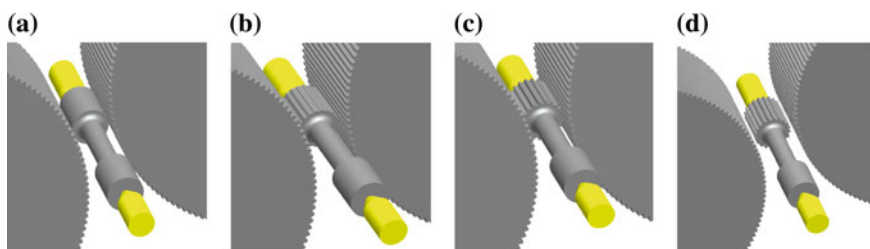


Fig. 2.2 Cold rolling precision forming process of spline shafts **a** stage before contacting; **b** indexing forming stage; **c** finishing stage; **d** exit stage

the two rollers feed rapidly to the workpiece synchronously, and it is called as a stage before contacting, as shown in Fig. 2.2a. In the second stage, the rollers press into and rotate together with the workpiece, and the tooth cogging was extruded in the workpiece by plastic deformation. The two rollers continuously feed at the radial direction to the position of the minimum center space. This stage can be called as the indexing forming period, as shown in Fig. 2.2b. This stage can be subdivided into two stages: The period from the contacting between the roller addendum and the workpiece to half a circle the workpiece rotates is called as the indexing stage. After indexing, the rollers will continuously feed to the minimum central distance, which is called as the pressing stage. Figure 2.2c is the finishing stage. In this period, the rollers will not feed anymore and just rolling freely together with the workpiece to finishing the tooth profile on the workpiece, but the central distance is kept constant. In the stage shown in Fig. 2.2d, the forming process finished, the rollers moved backward and disengaged with the workpiece.

The forming process of the tooth profile is completed in the stages of indexing, pressing, and finishing. In the two stages, the rollers contact with the workpiece, and local deformation occur continuously on the workpiece. In the period of

indexing, the circular billet contacts with the addendum circle of the rollers and rotates under the action of the frictional force, and the related movement between the roller and the workpiece is pure rolling of the roller addendum and the external circle of the billet. In the pressing period, the relationship between the billet and the roller is equivalent to the continuously modified gear transmission, and the moving process can be analyzed with the theory of modified gear transmission. In the final finishing stage, the moving relationship between the roller and the workpiece is similar to the double-side non-backlash meshing of a couple of gears. In different forming stages, the shape of the workpiece is different, and the deforming area and the moving characteristics are also different [1, 2].

2.2 Mechanical Analysis

According to the constant volume principle, the cross section of the spline shaft is equal before and after the cold rolling forming; that is, the cold rolling forming of spline can be considered as a plain strain state, there is no deformation along the axial direction during forming process, and the distance and strain along the axial direction are zero [3–5].

While the tooth number of the spline is an even number, the forming principle and the macroforce diagram are shown in Fig. 2.3. Four external forces are exerted on the workpiece by rollers, normal force P and frictional force F_f , which are applied on the workpiece symmetrically. The direction of normal force P will pass through the center of the roller, and the angle between the direction of P and the horizontal axis is defined as θ .

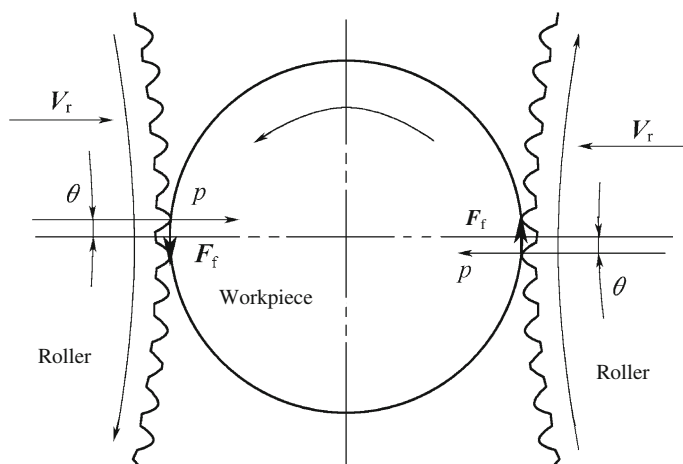


Fig. 2.3 Force diagram of the spline cold rolling forming process

2.3 Analysis on the Cold Rolling Forming Process of the Spline Tooth

2.3.1 Indexing and Biting Conditions [6]

In the cold rolling precision forming of spline shafts, the indexing operation is realized by the relative free rolling of the rollers and the workpiece. After contacting with the roller addendum, the circular billet will be driven to rotate along with the rollers, and prescriptive number of teeth will be extruded on the external circle of the billet. The moving parameters should match certain geometrical conditions in the process. In the first stage of the spline cold rolling forming, biting rotating and precision indexing should be guaranteed. The success of the cold rolling precision forming depends on the rotating conditions and precision of the index.

2.3.2 Rotating Conditions

Figure 2.3 shows that at the initial stage of contacting indexing, the external circle of the billet contacts with the addendum of the rollers. The movement can be considered as a frictional wheel transmission between the addendum cylinder of the rollers and the external cylinder of the billet, and the contact area can be simplified as a microplane.

According to the principle of friction, the relationship between the macronormal force P and the macrofrictional force F_f is as follows:

$$\mu = F_f/P \quad (2.1)$$

where μ is the sliding friction coefficient between the roller addendum and the workpiece surface.

The two rollers are mounted symmetrically relative to the workpiece. The forces applied to the workpiece have composed two moments rotating the workpiece. The moment formed by the friction force on the tooth flanks is M_f , and the moment formed by the bilateral normal force is M_p . While the friction moment of the contact area M_f is larger than the normal force moment M_p , the workpiece will rotate, and thus, the rotating condition is as follows:

$$M_f \geq M_p \quad (2.2)$$

That is

$$F_f d_z \cos \theta > P d_z \sin \theta \quad (2.3)$$

where d_z is the diameter of the billet, θ is the angle between the normal force and the connecting line of the two roller centers, $\theta < \pi/Z_2$, and Z_2 is the tooth number of the roller.

From Eqs. (2.1) and (2.3), in order to guarantee the rotation of the workpiece in the preliminary stage of the cold rolling precision forming, the friction coefficient must satisfy the following condition:

$$\mu > \tan \theta \quad (2.4)$$

For the biting rotation of cross rolling, based on the geometrical parameters of the workpiece and the roller, through the control of the relative reduction, the value of θ can meet the biting condition in an appropriate scope.

Define the ratio of the workpiece reduction and the workpiece diameter after the compression $\Delta s'/d$ as the limit relative reduction, and then, the rotation biting condition is as follows [7]:

$$\frac{\Delta s'}{d} \leq \frac{1}{4} \cdot \frac{1 + d/d_{a2}}{\frac{(1 + d/d_{a2})^2}{\mu^2} + (d/d_{a2})^2} \quad (2.5)$$

where d_{a2} is the diameter of the roller addendum.

Taking the limit relative reduction ratio $\Delta s'/d$ and the ratio between the workpiece diameter after rotating for half a circle and the diameter of the roller addendum d/d_{a2} as the parametric variable, the curve obtained from the relation of Eq. (2.5) under different frictional coefficient μ is shown in Fig. 2.4.

The influence of coefficient μ on the rotating condition is more significant than d/d_{a2} . d/d_{a2} decreases with the increase in $\Delta s'/d$; that is, the rotating condition is better with a large ratio of the roller and the workpiece diameter, the relative reduction is larger, and the selection range of the feed rate is more extensive. While d/d_{a2} increased from different values (1, 0.5, 0.4, 0.2, and 0.1) to 0.05, the increased percentage of $\Delta s'/d$ is shown in Table 2.1.

Fig. 2.4 Relationship between the limit reduction ratio $\Delta s'/d$, friction coefficient μ and d/d_{a2}

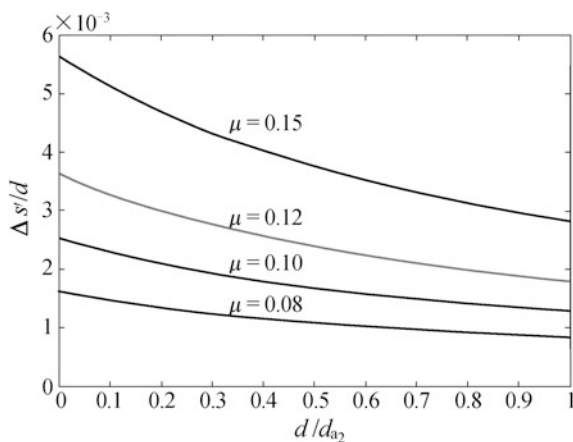


Table 2.1 Variable percentage of the limit relative reduction

μ	d/d_{a2}				
	0.1 (%)	0.2 (%)	0.4 (%)	0.5 (%)	1.0 (%)
0.12	4.71	14.33	34.49	43.09	91.16
0.3	4.84	14.57	34.31	44.28	94.76
0.5	4.97	15.07	36.04	46.81	102.37

The table shows that the increased percentage of the limit reduction decreased from the lower right to the top left corner of the table, under conditions $d/d_{a2} < 2$, and the influence becomes weakened, but is still in the controllable scopes. If the diameter of the roller is selected from the perspective of the rotating condition, the diameter of the roller should be more than 5 times of the workpiece diameter.

From Eq. (2.5), the rotating biting condition can be realized by the control of the reduction value from the perspective of process parameter selection. From the aspect of die design, there exist the other two improved rotating biting conditions.

From the geometrical relationship, it is known that the value of θ is not larger than half of the angle between the two teeth of the roller. That is, $\theta < \pi/z_2$.

Therefore, Eq. (2.4) can be modified as follows:

$$\mu > \tan \frac{\pi}{z_2} \quad (2.6)$$

In the rotating frictional condition given by Eq. (2.6), the friction coefficient and the tooth number of the roller can both be guaranteed by the design of the rollers. Two design schemes of the roller are proposed to ensure reliable rotation.

(1) The method to limit the minimum tooth number of the rollers.

The increase in the friction coefficient between the roller and the billet is beneficial to the rotation of the billet. But in order to obtain the required processing precision, the external cylinder surface of the billet is always machined by finishing turning, and the tooth of the rollers should be treated by grinding. Process lubricant should be used in the cold rolling precision forming to prolong the lifetime of the roller, and the value of the friction coefficient μ is about 0.1–0.2 [8]. Therefore, in order to realize the rotation of the workpiece, the minimum tooth number should be restricted, from Eq. (2.5):

$$Z_2 > \frac{\pi}{\arctan \mu} \quad (2.7)$$

In the design of rollers, the minimum tooth number should be verified according to Eq. (2.6). The tooth number of the roller will also influence the accumulated error of the formed spline tooth pitch.

(2) The method of increasing the friction coefficient.

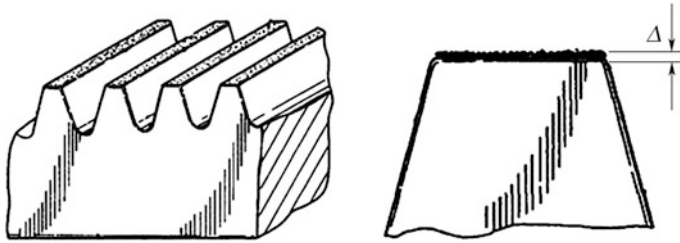


Fig. 2.5 Coating on the addendum of the roller

At the initial indexing stage of the cold rolling precision forming of spline shafts, the external circle contacts with the addendum of the roller and rotates under the driven friction force. Biting rotation can be realized by increasing the friction coefficient between the workpiece and roller addendum. Coating technology has already been widely used long time ago in the field of cutting tool manufacturing. According to an American patent [9, 10], in the cold rolling of spline with rack dies, the plane rollers were coated with WC or TiC for a thickness of $\Delta = 0.035\text{--}0.050\text{ mm}$ to improve the lifetime of the dies, as shown in Fig. 2.5. The biting rotation condition can be improved by increasing the friction coefficient between the roller and the billet. Because the coating layer has a higher hardness, the wear resistance of the roller can be increased and thus increases the lifetime of the roller.

The forming rotating biting condition should all be considered on the period of cold rolling machine selection, die design, and process parameter planning.

The two schemes mentioned above can be used for the selection of the roller parameters and as the rotating conditions in the manufacturing of the rollers. Equation (2.5) can be used as the rotating conditions realized through the control of the process reduction in the determination of cold rolling machine and dies.

2.3.3 Geometrical Indexing Conditions

At the preliminary stage of the spline shaft cold rolling forming process, the addendums of the rollers contacts with the external circle of the workpiece billet firstly, and the rollers rotate along with the workpiece. To indexing accurately on the workpiece, sliding should not happen on the contact area of the roller addendum and the external circle of the workpiece, and the diameter of the billet should be in a special dimensional relationship with the diameter of the roller addendum. Therefore, the geometrical indexing condition can be deduced.

According to the meshing principle, while the roller rotates for one tooth, the workpiece will rotate for one tooth correspondingly; therefore, the chord length $\overline{AB}$ between the two tooth addendums should be equal to the chord length $\overline{AB}$ on the

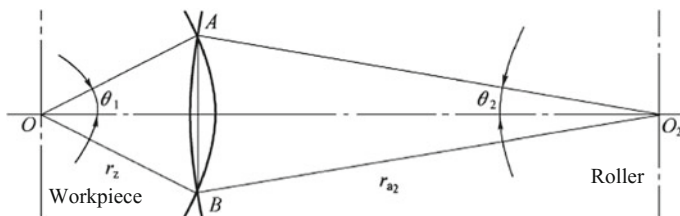


Fig. 2.6 Indexing of the roller on the workpiece billet

external circle of the billet corresponding to the adjacent tooth cogging center of the workpiece billet, and the geometrical relationship is shown in Fig. 2.6.

Thus

$$r_z \sin \theta_1 = r_{a2} \sin \theta_2 \quad (2.8)$$

where r_z is the radius of the billet before rolling forming. θ_1 is half of the angle between the two adjacent teeth on the workpiece, $\theta_1 = \pi/Z$. θ_2 is half of the angle between the adjacent teeth on the roller, $\theta_2 = \pi/Z_2$. And r_{a2} is the radius of the addendum circle of the roller.

In Eq. (2.8), the relation between the billet radius before rolling and the addendum circle radius of the roller is involved. In practical application, it can usually be expressed with the diameters as follows:

$$d_z \sin \theta_1 = d_{a2} \sin \theta_2 \quad (2.9)$$

While determining the cold rolling precision forming parameters, the diameter of the spline shaft billet d_z and the addendum circle diameter of the roller d_{a2} should meet the geometrical indexing condition.

Because cold rolling precision forming is a kind of chipless production method, once the formed spline parameters have been determined, the theoretical diameter d_z of the billet should be calculated according to the volume constant principle before and after cold rolling forming. In the design of rollers, the required indexing condition of Eq. (2.9) can be satisfied through addendum modification to obtain suitable diameter of the addendum circle.

The diameter of the billet d_z is a very important parameter in the cold rolling forming of spline shaft, which has a great influence on the precision of the forming process.

Because the shape of the spline is complex, there are many parameters describing the shape of the spline, such as modulus, number of the tooth, pressure angle, diameter of the addendum circle, and diameter of the root circle. According to the given parameters of the drawing, calculation of the spline shaft cross-sectional area is rather complex based on the constant volume principle. Presently, empirical formula or trial-and-error method is often used in production to determine the diameter of the billet [11, 12]. Zhang Dawei and Li Yongtang et al.

provided an exact solving method of the billet diameter with numerical simulation calculation and also deduced the calculation formula of the billet diameter of cold rolling precision forming process [13, 14]:

$$d_z = \sqrt{d_f^2 + \frac{4}{\pi} A_{th} Z} = \sqrt{d_f^2 + \frac{Z}{\pi} \left[\frac{d_b^2}{3} (\tan^3 \alpha_a - \tan^3 \alpha_f) + d_a s_a - d_f s_f \right]} \quad (2.10)$$

where

d_f is the root circle diameter of the spline.

A_{th} is the section area of one of the teeth on the spline.

Z is the tooth number of the spline.

d_b is the base circle diameter of the spline, $d_b = m z \cos \alpha$.

d_a is the addendum circle diameter of the spline.

α_f is the pressure angle of the root circle on the spline, $\alpha_f = \arccos\left(\frac{d_b}{d_f}\right)$.

α_a is the pressure angle of the addendum circle on the spline, $\alpha_a = \arccos\left(\frac{d_b}{d_a}\right)$.

S_f is the tooth thickness of the root circle on the spline, $s_f = d_f \left(\frac{s}{d} + \text{inv} \alpha - \text{inv} \alpha_f \right)$.

S_a is the tooth thickness of the addendum circle on the spline, $s_a = d_a \left(\frac{s}{d} + \text{inv} \alpha - \text{inv} \alpha_a \right)$.

Because the formula is relatively complex and is not convenient for manual calculation, it is programmed in the study, and the billet diameter before cold rolling can be easily obtained after inputting corresponding tooth parameters.

2.4 Double-Flank Non-backlash Meshing

At the initial stage of indexing, after the tooth cogging is pressed on the billet, the cold rolling process can be considered and analyzed as a double-flank non-backlash meshing movement [15]. The state of double-flank non-backlash meshing is different from the single flank meshing of common gear transmission, and researches on this field are very limited. During the cold rolling forming process, the contact area changes instantly, the contacting state of the roller and the workpiece is very complex, and the forces applied on every tooth vary significantly, which has a great influence on the forming quality of the tooth on the workpiece.

2.4.1 Contact Area in the Forming Process

If the movements of the roller and workpiece are considered as a meshing movement in the cold rolling precision forming process, in the axial section, the contact

of every instant can be considered as point contact, and during the meshing process the roller rotating for a tooth, the number of the contacting points on both sides of the tooth flank is different. The number of the contact points has a close relation with the overlap coefficient, which increases with the increasing tooth number, and is always not an integer number; therefore, the number of the contact points during the meshing process is changing. For example, when the overlap coefficient is in the range of 1–2, single-tooth and double-teeth contact area are all exist on the left and right meshing line. In the cold rolling forming of spline shafts, the contact conditions at the meshing instant of the roller and the workpiece are shown in Fig. 2.7.

In the contact process of the roller and the workpiece shown in Fig. 2.7, a tooth of the workpiece must experience the following period shown in Fig. 2.7a–f from entering into the contact area to disengage from it. In Fig. 2.7a, the tooth ② has just entered into the contact area, the former tooth ① is still not disengaged from the contact, the number of the contact points is 4, and there are two couples of teeth in contact on the left and right meshing lines. At the position of (b), tooth ① on the left meshing line disengaged from contact, there are still two couples of teeth in contact, and the total number of the contact points is 3. After rotating to the position of (c), tooth ① has completely disengaged from contact, only tooth ② are in the contact state on the left and right meshing lines, and the number of the contact points is 2. At the position of (d), the rear tooth ③ enters into the contact area and

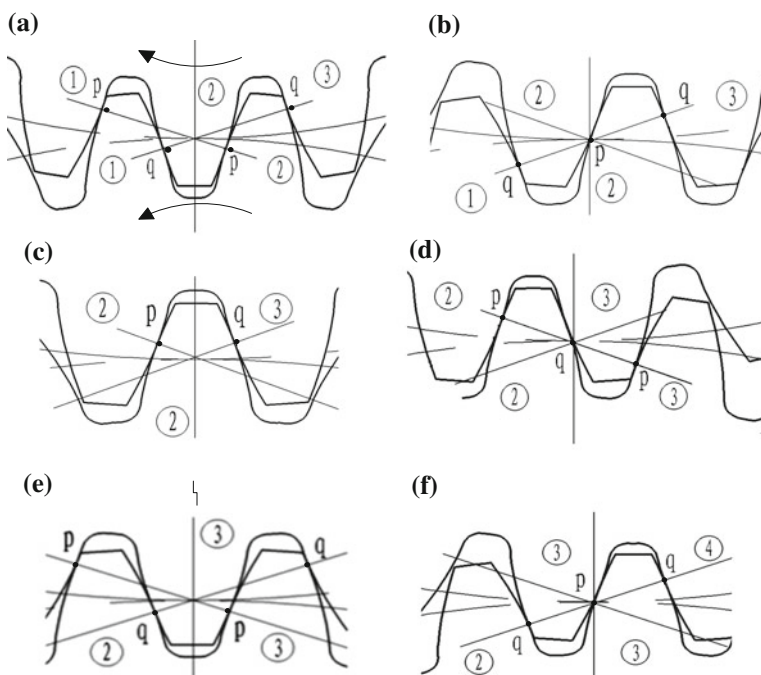


Fig. 2.7 Instant variation of the contact points on the tooth surface

tooth ② and tooth ③ on the left meshing line are in contact, while only tooth ② is in the contact state on the right meshing line, which is just opposite to the contact condition of Fig. 2.7b, and the number of the contact number is 3. At the position of (e), there are both two couples of teeth in contact, which is the same condition as (a) and the contact number is 4, but the contacting teeth are changed from ① and ② to ② and ③. Continuously rotating with this sequence, and at the position of (f), tooth ② will disengage from the contact area and the contact condition is the same as tooth ① which will disengage from the contact area in Fig. 2.7b.

The number of the contact points and the variation in the position is shown in Fig. 2.7, when the workpiece or the roller rotates for one tooth (such as from tooth ② to tooth ③). Therefore, the following conclusions can be drawn:

- (1) The number of the contact points and positions is changed periodically, and the period of rotating for one tooth is called as a cycle. The total number of the contact points on the left and right meshing lines is equal to the sum of the contacting teeth on the two lines, and each couple of meshing teeth is corresponded to one contact point.
- (2) The contact points p and q on both sides of the tooth flank are moving points at opposite moving directions, and while one of the points moving toward the dedendum of the tooth, the other one will inevitably moving toward the addendum of the tooth, and vice versa. No matter how the roller rotates, the two contact points are always moving on the meshing line with the continuous rotating of the roller and the workpiece.

The above analysis is suitable for the finishing stage of the cold rolling precision forming, the teeth of the workpiece have been formed, and the central distance between the roller and the workpiece will not change any more. During the relative rolling process of the roller and the workpiece, the overlap coefficient is changing, and the number of the contact points is varied periodically. It can be seen from the moving of the contact point on both flanks that the force exerted on the instant contact point is different, and the plastic deformation on both sides of the tooth is also different. With a big modulus of the cold rolling spline, tooth error will be induced in conditions of a large tooth height.

In the process of the radial feeding of the roller, the tooth profile of the workpiece is not perfect. If the overlap coefficient is less than 1, disengagement between the roller and the workpiece will occur, this kind of contact condition is more complex, and this period can be considered as a transitional process, which has not great influence on the final precision of the formed components. In the cold rolling forming process of spline shafts, because of the variation in the number of contact points and the instant meshing and disengagement, the spindle of the machine will be impacted; vibration of the machine, unqualified accuracy of the components, and even failure of the forming process will also be caused. Therefore, damping vibration attenuation structure must be designed in the cold rolling forming machine to reduce the impact applied by the forming force on the spindle transmission system.

2.4.2 Position of the Contact Points

In the spline shaft cold rolling forming process, non-backlash meshing between the rollers and the workpiece exists on double flanks; the position of the contact points on the spline tooth flank can be determined on the basis of gear meshing theory, and the relation between the position parameters can be obtained [16]. The rollers are rotating at a constant angular velocity, the position at each moment is fixed, and variation in the contact point position on the tooth surface with time can be expressed by the rotating angle of the roller or the workpiece.

From the above analysis, there are always contact points existing on the lateral surface of the tooth, and complete analytical model of the contact point position can be obtained by expressing the position of contact point on the meshing rollers with the rotating angle of the rollers or workpiece.

2.4.2.1 Position of the Contact Point on the Bilateral Tooth Surfaces of the Spline

The position the symmetrical line of the spline tooth coinciding with the connecting line of the two roller centers can be taken as the reference. With the rotation of the workpiece, the positions of the contact points p and q at both sides of the tooth flanks can be expressed by four different forms as shown in Fig. 2.8. That is, the contact point is on the right meshing line and the left side of the reference position, as shown in Fig. 2.8a; the contact point is on the right meshing line and the right side of the reference position, as shown in Fig. 2.8b; the contact point is on the left meshing line and the left side of the reference position, as shown in Fig. 2.8c; and the contact point is on the left meshing line and the right side of the reference position, as shown in Fig. 2.8d.

While the workpiece is rotating counterclockwise around its center O_1 , the deviation angle φ_{qh} with the central reference position is positive; while the workpiece is rotating clockwise around the center, the deviation angle φ_{qh} is negative and $\varphi_{qh} = 0$ at the reference position. The contact condition shown in Fig. 2.8a is taken as an example for the analysis of the relationship between different geometrical parameters. The right side of the formed spline intersects with the right meshing line at the left of the reference position, and the intersection point is q . According to the meshing principle, the contact point of the spline and roller moves along the meshing line; at the reference position, the intersection point q of the right side of the spline tooth and the meshing line is at position 1, and the intersection point will move to position 2 when the spline rotates for a left deviation angle φ_{qh} . That is, the contact point q will move from position 1 to position 2 after the spline rotates for a left deviation angle φ_{qh} .

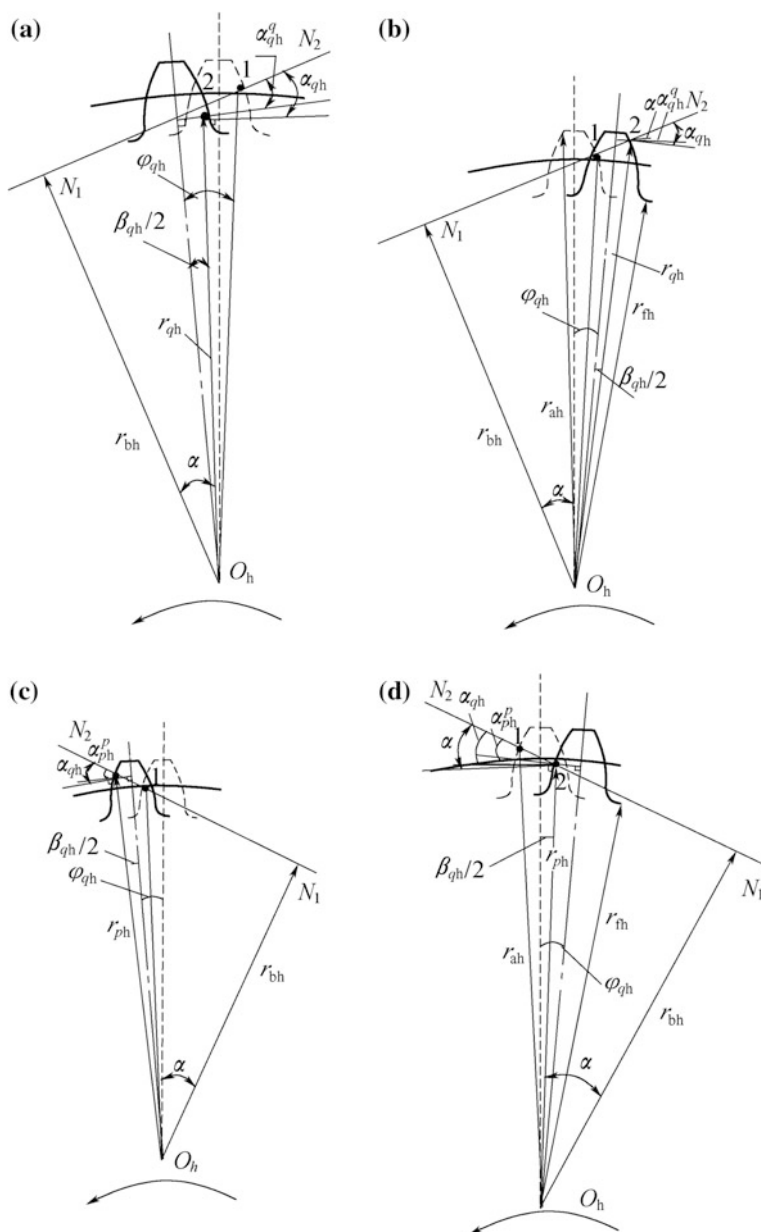


Fig. 2.8 Position of the contact points on the spline tooth **a** *right meshing line* and *left* of the reference; **b** *right meshing line* and *right* of the reference; **c** *left meshing line* and *left* of the reference; **d** *left meshing line* and *right* of the reference

The spline pressure angle of the contact point q on position 2 is as follows:

$$\alpha_{qh} = \arccos(r_{bh}/r_{qh}) \quad (2.11)$$

where

r_{bh} is the radius of the spline base circle and

r_{qh} is the radius of the circle on the spline at the contact point q .

The tooth width angle on the circle of the contact point is as follows:

$$\beta_{qh} = \frac{s_h}{r_h} - 2(inv\alpha_{qh} - inv\alpha) \quad (2.12)$$

where

r_h is the radius of the spline pitch circle, $r_h = mz_h/2$ and

s_h is the tooth width of the spline on the pitch circle, $s_h = m_h/2$.

From the geometrical relation shown in Fig. 2.8a, the deviation angle φ_{qh} is as follows:

$$\varphi_{qh} = \beta_{qh}/2 + \alpha - \alpha_{qh} \quad (2.13)$$

where α is the meshing angle of the roller and the spline ($^\circ$), which can be replaced approximately by the pressure of the pitch circle.

Equations (2.10), (2.11), and (2.12) can be simplified as follows:

$$\varphi_{qh} = \frac{s_h}{2r_h} - \tan\left(\arccos\frac{r_{bh}}{r_{qh}}\right) + \tan\alpha \quad (2.14)$$

Here, the radius of the meshing point is expressed as follows:

$$r_{qh} = \frac{r_{bh}}{\cos\left[\arctan\left(\varphi_{qh} - \frac{s_h}{2r_h} - \tan\alpha\right)\right]} \quad (2.15)$$

N_1N_2 is the right meshing line of the roller and the workpiece, and further deduced according to Fig. 2.8a, the angle between the meshing line and the normal line of the spline tooth center line on the contact point of the spline positive tooth flank can be expressed as follows:

$$\alpha_{qh}^q = \alpha_{qh} - \beta_{qh}/2 = \alpha_{qh} - \frac{1}{2}\left[\frac{s_h}{r_h} - 2(inv\alpha_{qh} - inv\alpha)\right] \quad (2.16)$$

If the final meshing point on the tooth dedendum is F, the radius of the circle at F is r_F , the corresponding chord length is t_F , and then, the tooth height at the contact point is as follows:

$$x_{qh} = r_{qh} \cos(\beta_{qh}/2) - \sqrt{r_{Fh}^2 - (t_{fh}/2)^2} \quad (2.17)$$

It can be seen from the equations listed above, α , r_h , and s_h are the parameters of the formed spline, and the position of the contact point can be described by the parameters such as the radius r_{qh} of the meshing circle at the contact point, the angle α_{qh}^q of the meshing line and the normal line of the spline tooth central line at the contact point, and the tooth height x_{qh} . All of these parameters have a unique determined relation with the deviation angle φ_{qh} between the workpiece and the reference position.

At the contact state shown in Fig. 2.8b, φ_{qh} is negative. When contacting at the left tooth flank, φ_{ph} , r_{ph} , x_{ph} , and α_{ph}^p have the same relation as shown in Eqs. (2.10)–(2.16). Also, because $\varphi_{ph} = \varphi_{qh}$, if φ_{qh} is determined, among the scope of the rotation angle of the spline, the position of the contact points p and q on the left and right tooth flanks can be determined.

2.4.2.2 The Position Parameters of the Contact Point on the Rollers

The position parameters of the corresponding contact points on the roller teeth are shown in Fig. 2.9.

The value of r_{qh} on the contact point of the spline tooth can be decided by φ_{qh} , and the geometrical relation is as follows:

$$\rho_{qh} = \sqrt{r_{qh}^2 - r_{bh}^2} \quad (2.18)$$

where ρ_{qh} is corresponded to N_1Q in Fig. 2.9, which is the curvature radius of the contact point on the spline, while QN_2 is the curvature radius at the contact point of roller, and

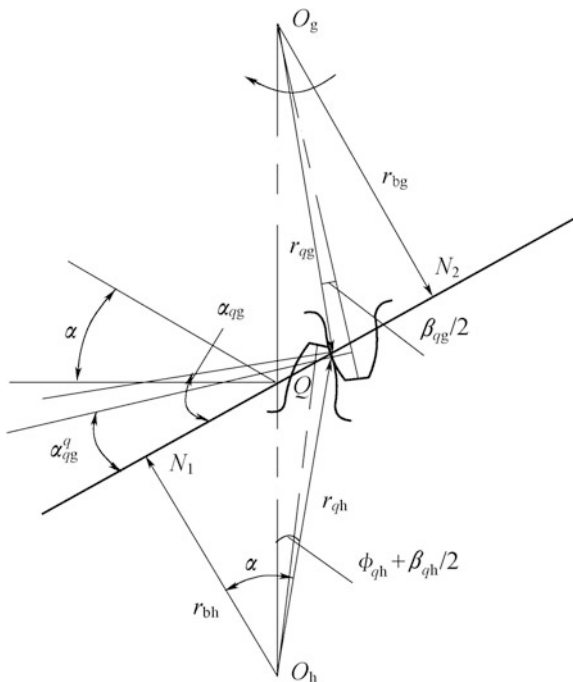
$$\rho_{qg} = \overline{N_1N_2} - \rho_{qh} \quad (2.19)$$

Calculated with the standard center distance without modification:

$$\rho_{qg} = \left(\frac{r_h + r_g}{2} \right) \sin \alpha - \rho_{qh} \quad (2.20)$$

From $r_{qg} = \sqrt{\rho_{qg}^2 + r_{bg}^2}$, the circle radius r_{qg} at the contact point of the roller tooth can be solved. Similar to the derivation of the contact position on the spline flanks, the position parameters (α_{qg} , β_{qg} , φ_{qg} , α_{qg}^q , and x_{qg}) of the contact point on the left and right tooth flanks of the roller can be obtained. After the mathematical models of the contact point, position parameters have been built, and the boundary conditions can be conveniently determined in the theoretical calculation and numerical simulation process.

Fig. 2.9 Position of the contact point on the roller



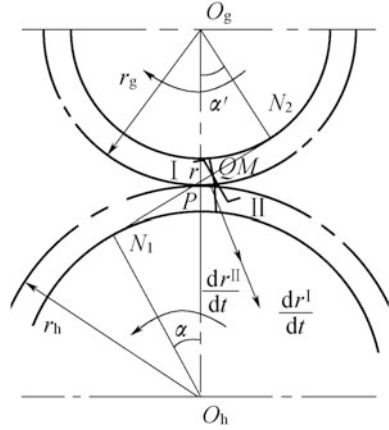
2.5 Sliding of the Contact Point

In the cold rolling precision forming of spline shafts, the main motion of the dies (rollers) is the rotation and radial feeding of the roller. The workpiece is non-backlash meshed with the roller on double flanks. The tooth flank bearing the extrusion force and driving the workpiece to rotate is called as the active flank. Restrained by the teeth of the roller, the other tooth flank contacted by the flowing metal is called as the driven flank. Based on the engagement of the roller and the workpiece, the relative sliding of the contact point is analyzed, and the sliding motion mathematical models of the contact point on the active tooth flank and the driven flank of the workpiece have been set up.

2.5.1 Sliding Motion of the Contact Point on the Active Tooth Flank

The meshing and sliding between the workpiece and the roller on the active tooth flank are shown in Fig. 2.10. In the figure, surface I is the active flank of the forming spline, and II is the tooth flank contacting with it. On the right meshing line, the contact point is M , and the pitch point is P . Let $r = \overline{PM}$; when point M is

Fig. 2.10 Sliding of the contact point on the active tooth flank



located on the right side of point P , r is positive; when the point M locates on the left side of P , then the value of r is negative.

According to the meshing theory, for the tooth profile of involute spline, the curvature radii at the contact point (meshing point) of flank I and II are as follows:

$$\rho_h = \overline{N_1 P} + r = r_h \times \sin \alpha + r \quad (2.21)$$

$$\rho_g = \overline{N_2 P} - r = r_g \times \sin \alpha - r \quad (2.22)$$

The sliding coefficient of the contact point on spline tooth I is as follows:

$$\chi_1 = \frac{\frac{d_1 r^I}{dt} - \frac{d_2 r^{II}}{dt}}{\frac{d_1 r^I}{dt}} = 1 - i_{21} \times \frac{\rho_h}{\rho_g} \quad (2.23)$$

where $\frac{d_1 r^I}{dt}$ is the moving speed of contact point M on the spline plane relative to tooth flank I (m/s) and

$\frac{d_1 r^{II}}{dt}$ is the moving speed of contact point M on the spline plane relative to tooth flank II (m/s). Substituting Eqs. (2.21) and (2.22) into (2.23), obtain:

$$\chi_1 = 1 + i_{21} - \left(\frac{r_g \times \sin \alpha + r_h \times \sin \alpha}{r + r_h \times \sin \alpha} \right) \quad (2.24)$$

In the meshing process of the workpiece and the roller, the active tooth flank located on the right meshing line always meshes from the tooth addendum and then enters the tooth dedendum along the contact line. Therefore, the direction of the moving speed of contact point M relative to tooth flank I is always along the tangent line of the workpiece tooth flank and pointed to the tooth dedendum. From the positive and negative of the sliding coefficient, the relationship between the sliding

direction and the speed direction can be obtained. While contact point M is on the right of the pitch point P , $r > 0$, $\chi_1 > 0$; while contact point M is on the left of the pitch point P , $r < 0$, $\chi_1 < 0$; while the contact point M is overlap with the pitch point P , then $r = 0$, $\chi_1 = 0$.

2.5.2 Sliding Motion of the Contact Point on the Driven Tooth Flank

Similar to the contacting on the active tooth flank, the contact sliding between the driven tooth flank of the workpiece and the roller is shown in Fig. 2.11. In the figure, surface III is the driven tooth flank corresponding to the active flank I on the workpiece and surface IV is the tooth flank of the roller contacted with the driven tooth flank III. On the left meshing line, the contact point is M' , and the pitch point is P' . Let $r' = \overline{P'M'}$; while contact point M' is on the right side of the pitch point P' , the value of r' is positive; while contact point M' is on the left side of the pitch point P' , the value of r' is negative.

The curvature radius of the contact point (meshing point) of flank III and flank IV is as follows:

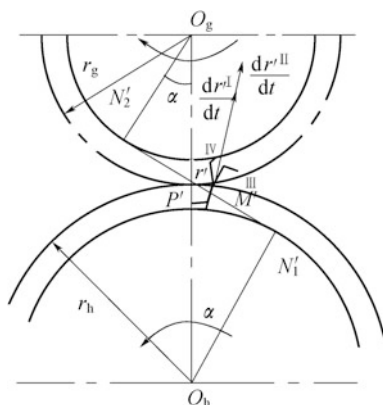
$$\rho'_h = \overline{N_1 P} - r' = r_h \times \sin \alpha - r' \quad (2.25)$$

$$\rho'_g = \overline{N_2 P} + r' = r_g \times \sin \alpha + r' \quad (2.26)$$

Accordingly, the sliding coefficient of the contact point on the spline tooth flank is as follows:

$$\chi'_1 = \frac{\frac{d_1 r'^I}{dt} - \frac{d_2 r'^{II}}{dt}}{\frac{d_1 r'^I}{dt}} = 1 - i'_{21} \times \frac{\rho'_g}{\rho'_h} \quad (2.27)$$

Fig. 2.11 Sliding of the contact point on the driven tooth flank



From Eqs. (2.25), (2.26), and (2.27), obtain:

$$\chi'_1 = 1 + i_{21} + \left(\frac{r_g \times \sin \alpha + r_h \times \sin \alpha}{r' - r_h \times \sin \alpha} \right) \quad (2.28)$$

Contrary to the active tooth flank, the driven tooth surface III on the left meshing line is always meshing from the tooth dedendum and then transition to the addendum along the left meshing line; that is, the direction of the relative moving speed $\frac{d_1 r'^1}{dt}$ of the contact point M' to tooth flank III is always along the tangent of the tooth surface and pointed to the tooth addendum.

The relationship between the sliding direction and the speed direction can be obtained by analyzing the positive and negative of the sliding coefficient on the contact point χ'_1 . While the contact point M' is overlapping with the pitch point P' , $r' = 0$, $\chi'_1 = 0$; while M' is within the pitch circle, $r' > 0$, $\chi'_1 < 0$, the sliding direction of tooth surface III relative to tooth surface IV is the same with $\frac{d_1 r'^1}{dt}$; while contact point M' is outside of the pitch circle, $r' < 0$, $\chi'_1 > 0$, the sliding direction of tooth surface III relative to tooth surface IV is opposite to $\frac{d_1 r'^1}{dt}$.

2.5.3 Metal Flow on the Tooth Surface of the Workpiece

Under the action of the rolling force, metals on the tooth surface of the workpiece reach the plastic yield conditions and macroslip of the metals occurs on the surface of the workpiece. Because of the restriction of the roller tooth surface, metals can only flow along the tooth flank of the roller. On the surface of the deforming area, the metal flow direction is consistent with the relative sliding of the contact point. The macro characteristics of the metal flow on the tooth surface can be obtained through the analysis of the sliding conditions of the contact points on the active and driven tooth flank of the workpiece.

According to the above analysis, for the active tooth flank of the workpiece, while $\chi_1 > 0$, that is, when $\frac{d_1 r^I}{dt} > \frac{d_2 r^{II}}{dt}$, the sliding direction of surface I relative to surface II is opposite to $\frac{d_1 r^I}{dt}$; the direction of the friction shearing force on tooth surface I is the same with $\frac{d_1 r^I}{dt}$. Because the metal flow direction coincides with the direction of the shearing force, the direction of $\frac{d_1 r^I}{dt}$ is just the metal flow direction on tooth surface I under this situation. While $\chi_1 < 0$, that is, $\frac{d_1 r^I}{dt} < \frac{d_2 r^{II}}{dt}$, the relative sliding direction between flank I and II is the same as the direction of $\frac{d_1 r^I}{dt}$. Because the friction shear on the tooth surface I of the workpiece is contrary to the direction of $\frac{d_1 r^I}{dt}$, the metal flow direction on the tooth surface of the workpiece is opposite to the direction of $\frac{d_1 r^I}{dt}$.

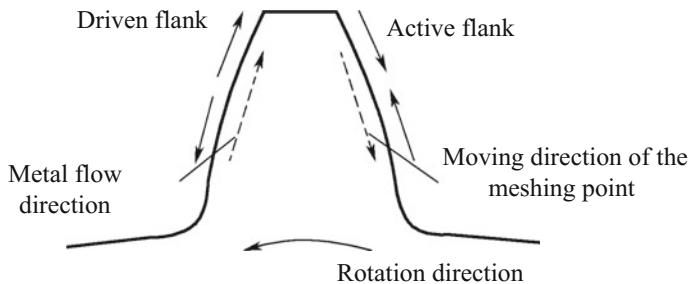


Fig. 2.12 Metal flow conditions on the surface of the workpiece

Accordingly, the analysis method of the metal flow direction on the driven tooth surface is similar to the method of the active surface. While $\chi'_1 > 0$, the metal flow direction on surface III is the same as $\frac{d_1 r'^1}{dt}$; while $\chi'_1 < 0$, the metal flow direction on surface III is contrary to $\frac{d_1 r'^1}{dt}$.

During the double-flank non-backlash meshing process, the sliding direction of the contact points on the surface of the workpiece is different from that of the roller contacting surface, which results in the difference of the metal flow direction on the tooth flank of the workpiece, and the metal flow direction is shown in Fig. 2.12.

On the active tooth flank, metals will flow from the addendum and dedendum of the tooth to the surrounding of the pitch point. While on the driven tooth flank, metals will flow from the pitch point to the addendum and dedendum. The metal flow rule will result in the inclination of the tooth profile on the formed cold rolling forming components to a certain side. Under conditions, the modulus of the formed spline is bigger, because the height of the tooth is larger; the forming force is correspondingly very large. The inclination of the tooth will be more serious, which will result in the disqualification of the components due to the large error of the tooth profile. Therefore, cold rolling precision forming process is suitable for the forming of spline shafts with a modulus less than 2.

2.6 Mechanical Analysis on the Cold Rolling Precision Forming of Spline

2.6.1 Basic Assumption of the Stress Analysis

The plastic forming zone of cold rolling precision forming spline varies instantly with the rotation of the workpiece and the roller. In order to analyze the stress of the contacting zone, discrete modeling analysis method according to the contact position is proposed. Based on the built mathematical model of the contact zone, the contacting zone is dispersed according to the angle of the workpiece tooth

central line relative to the reference position. At every position, mechanical analysis is carried out using computer-aided calculation method according to the slip-line field theory of plane strain.

On the study of the stress state in the plastic deformation of a point and the solving of the stress components, the following assumptions are proposed:

- (1) There is no deformation in the axial direction;
- (2) The volume is invariable in the plastic deformation;
- (3) The material of the workpiece is ideal rigid-plastic material; and
- (4) The normal force and the friction force are evenly distributed in the contact area between addendum of the roller and the workpiece, as well as between the flank of the roller and the workpiece.

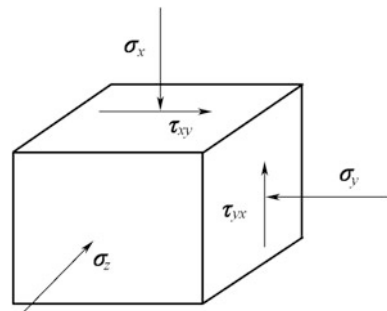
The coordinate system of the stress analysis is chosen as follows: taking the center of roller as the zero point; the line connecting the center of roller and the tiptop of roller addendum as y-axis; and the axial direction of the workpiece or the roller as z-axis. xOy plane is the main stress plane, and stress along z-direction is the medium main stress. The stress state of an arbitrary point in the plastic deformation zone of the workpiece is shown in Fig. 2.13.

In the forming process, y-axis is rotating with the roller, while y-axis is perpendicular to the contact surface, and σ_y is the average pressure force on the contacting plane.

The frictional model: The contact between the roller addendum and the workpiece is mainly indentation, the contact between the flank and the tooth of the workpiece is rolling and pressing, and the friction forms on different contact positions are different. According to the contact characteristics, when analyzed with the slip-line theory, constant friction model should be used in the contact area between addendum of the roller and the workpiece, and the coulomb friction model should be used in the contact area between flank of the roller and the workpiece.

The stress-equilibrium differential equation, the strain-displacement relation, and the characteristic of plain strain are all suitable for cold rolling of external spline, that is:

Fig. 2.13 Diagram of the stress state



$$\begin{cases} \sigma_z = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_m = \sigma_2 \\ \tau_{zx} = \tau_{zy} = 0 \end{cases} \quad (2.29)$$

The other stress components can be expressed as follows:

$$\begin{cases} \sigma_x = \sigma_m - K \sin 2\omega \\ \sigma_y = \sigma_m + K \sin 2\omega \\ \tau_{xy} = K \cos 2\omega \end{cases} \quad (2.30)$$

where

K is the shear yield strength;
 ω is the direction angle of the slip line; and
 σ_m is the mean stress.

Expressed by the principal stress, the yielding criterion is as follows:

$$\sigma_1 - \sigma_3 = 2K \begin{cases} Tresca : K = \sigma_s/2 \\ Mises : K = \sigma_s/\sqrt{3} \end{cases} \quad (2.31)$$

H. Hencky stress equation that reflects the rule of the mean stress in slip line is as follows:

$$\begin{cases} \sigma_m - 2K\omega = \xi(\beta), & (\text{Along line } \alpha) \\ \sigma_m + 2K\omega = \xi(\alpha), & (\text{Along line } \beta) \end{cases} \quad (2.32)$$

2.6.2 Fundamental Plastic Forming Theory of Stress Analysis

In the cold rolling precision forming process, there is no deformation along the axis, and the displacement and strain are zero. In the theory of plastic forming, if all of the particles deform only in the same coordinate plane, there is no deformation at the other normal directions, and thus, it can be called as plane deformation or plane strain. Obviously, the cold rolling precision forming is a kind of plane deformation. Characteristics of the plane strain state and stress-equilibrium differential equation and geometrical equation (relation of the displacement increment and strain increment) are all suitable for this process.

In the cold rolling process, the formed teeth must be rolled for several times and work hardening will occur on the surface of the workpiece, and the average stress in the contact area of the forming process can be obtained by setting up of the slip-line field of each tooth from entering to detaching the contact zone.

2.6.2.1 Fundamental Equation of the Plane Strain

Because plastic forming is only occurred in the coordinate plane of xOy , there is no deformation in the plane perpendicular to the deforming plane, and there is no relation between the variations of displacement in the deforming object and coordinate z . Thus

$$u = u(x, y), v = v(x, y), w = 0 \quad (2.33)$$

According to the geometrical equation, the strain components are as follows:

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z} = 0 \\ \gamma_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \gamma_{xz} = \frac{1}{2} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) = 0, \gamma_{yz} = \frac{1}{2} \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) = 0 \end{cases} \quad (2.34)$$

From $\varepsilon_z = 0$, it can be deduced that $\dot{\varepsilon}_z = 0$.

According to Saint-Venant's principle, $\dot{\varepsilon}_{ij} = \dot{\lambda} \sigma'_{ij}$, then:

$$\dot{\varepsilon}_z = \dot{\lambda} \sigma'_z = \dot{\lambda} (\sigma_z - \sigma_m) = 0 \quad (2.35)$$

where $\dot{\lambda} = d\lambda/dt$, $d\lambda$ is the positive constant;

σ_m is the average stress, $\sigma_m = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$.

There is $\sigma_z - \sigma_m = 0$

$$\sigma_z = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_m = \sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3) \quad (2.36)$$

In the plain deformation, the plane of plastic flow is perpendicular to z -axis, there is no relative movement, inclination, and torsion between surfaces, that is:

$$\tau_{zx} = \tau_{zy} = 0 \quad (2.37)$$

The maximum shear stress can be expressed by the first and third main stresses:

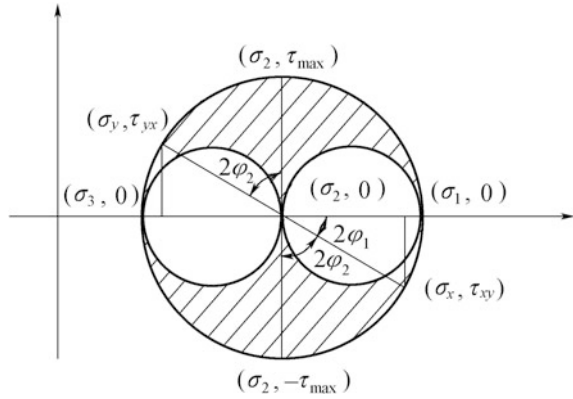
$$\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_3) \quad (2.38)$$

If the stress state of an arbitrary point Q is expressed by the mean stress σ_m and the maximum shear stress $\tau_{\max}$:

$$\sigma_1 = \sigma_m + \tau_{\max}, \sigma_2 = \sigma_m, \sigma_3 = \sigma_m - \tau_{\max} \quad (2.39)$$

At a plane strain state, the stress Mohr's circle of an arbitrary point Q in the deforming object is shown in Fig. 2.14.

Fig. 2.14 Stress Mohr's circle at plane strain state



From the stress Mohr's circle, the direction angle φ_1 of the main stress and the direction angle φ_2 of the maximum shear stress can be calculated as follows:

$$\tan 2\varphi_1 = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \tan 2\varphi_2 = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \quad (2.40)$$

2.6.2.2 Yield Criteria

When the particles of an object are in a multistress state, the stress state of a point can be determined by the six stress components. Under certain deforming conditions (deforming temperature and speed), plastic deforming occur on the particles only when each stress component coincides with a certain relationship.

There are two commonly used yield criteria to determine the plastic deformation state: Tresca criterion and Mises criterion [17]. In this book, Mises criterion is used; that is, under certain deforming conditions, when the elastic potential energy changing the unite volume shape of the material reaches a certain constant, the material will yield. The mathematical description of Mises criterion is as follows:

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = 4K^2 = 4\left(\frac{\sigma_s}{\sqrt{3}}\right)^2 \quad (2.41)$$

where

K is the shear yield strength and

σ_s is the yield stress.

The criterion can be expressed by the main stress as follows:

$$\sigma_1 - \sigma_2 = 2K \quad (2.42)$$

where $K = \frac{\sigma_s}{\sqrt{3}}$.

2.6.2.3 The Strengthening Model

In the cold rolling forming of spine shaft, the final shape of the component is obtained by multipass rolling deformation of the billet applied by rollers. Through the cold plastic deformation and the work hardening of the materials, the mechanical performance of the material can be obtained and thus improved the strength of the spline shafts.

In the modern plastic forming theory, there are many constitutive relation hypothesis and material strengthening models, in which the following three models are most widely utilized in theoretical and engineering analysis: isotropic hardening model, kinematic hardening model, and anisotropic hardening model [18].

- (1) Isotropic hardening model: In the plastic deforming process, the subsequent yield surface and the loading surface are enlarged with a similar shape in the stress space and the center is fixed. In the π plane, the loading surface is a curve, which is similar to the initial curve. In the isotropic hardening model, the anisotropy resulted by plastic forming has been neglected. The result coincides with practice under a medium deformation and small variation of the stress deviation ratio.
- (2) Kinematic hardening model: It is a simplified model in which Bauschinger effect is considered. The material is assumed to be hardened in the plastic deformation direction and equally softened at the opposite direction. Therefore, in the loading process, with the development of the plastic deformation, the size and shape of the yield surface are invariant and just translation integrally in the stress space.
- (3) Anisotropic hardening model: In order to reflect the Bauschinger effect of the materials perfectly, the isotropic hardening model and kinematic hardening model can be combined together. It can be considered that the shape, size, and position of the subsequent yield surface varied with the development of the plastic deformation. Although the result of this model fits well with the experimental result, the application of the model is limited because of its complexity.

Assume that the material is an isotropic strain hardening material, according to the isotropic hardening model, the shape and central position of the subsequent yield locus are unchanged. The loading surface by Tresca criterion is a series of concentric hexagonal prisms surface; while under Mises criterion, the loading surface is a series of concentric cylindrical surface, as shown in Fig. 2.15.

2.6.2.4 The Slip-Line Stress Field Theory

Slip-line theory is a powerful tool for solving ideal rigid-plastic plane strain problems [19–21]. Slip line is the trajectory of the maximum tangent stress direction on the plastic flow plane (plastic zone). Each point Q satisfying the yield

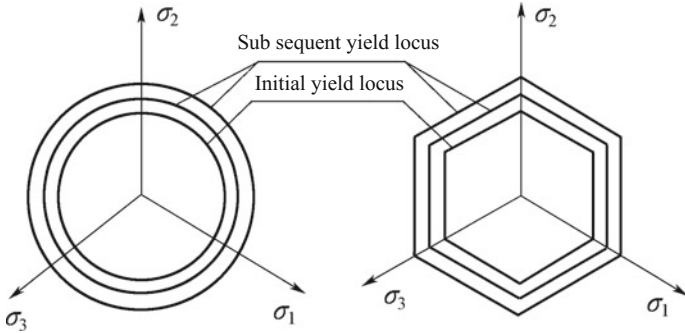
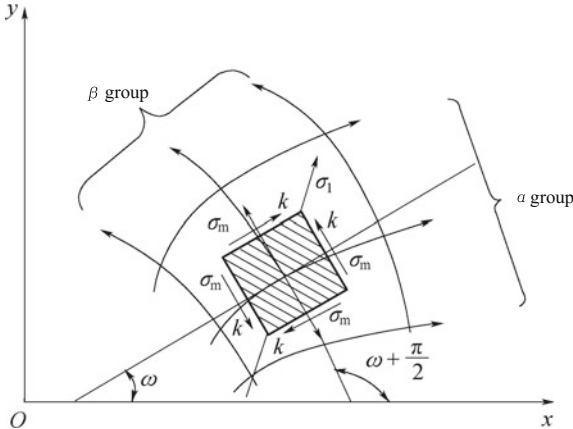


Fig. 2.15 Subsequent yield locus of isotropic strain hardening materials

criteria in the plastic zone has two orthogonal shear directions: the first one and the second one, combining graphic and analysis method, connect each point of the plastic zone along the first and second shear direction, then two group of orthogonal curves α and β can be obtained, which is called as slip line. Mutual orthogonal slip lines consist of the slip-line network and form the slip-line field. Applying the property and stress boundary conditions of the slip line, the distribution of the stress and strain speed, and finally the limited load in the plastic zone can be solved, and thus solve the mechanical problems of plane plastic flow in the cold rolling forming of spline.

The slip line and slip-line field are shown in Fig. 2.16. The direction angle of slip line ω is the angle of the first shear direction, which is the angle formed by the coordinate axis Ox rotating to the direction of the first shear direction clockwise. The differential equation of α and β group of the slip line is as follows [7]:

Fig. 2.16 Slip line and slip-line field



$$\begin{cases} \frac{dy}{dx} = \tan \omega & \text{along } \alpha \\ \frac{dy}{dx} = -\cot \omega & \text{along } \beta \end{cases} \quad (2.43)$$

According to H. Hencky stress equation, the average stress on the slip line can be obtained as follows:

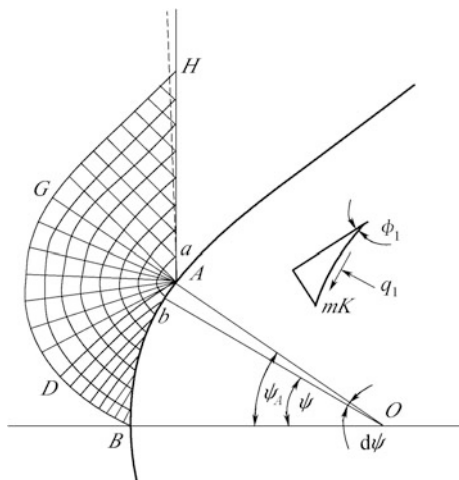
$$\begin{cases} \sigma_m - 2K\omega = \xi(S_\beta) & \text{along } \alpha \\ \sigma_m + 2K\omega = \eta(S_\alpha) & \text{along } \beta \end{cases} \quad (2.44)$$

In the above equation, the value of $\xi(S_\beta)$ (or $\eta(S_\alpha)$) can be determined with the following method: While moving along one of the slip line of group α or β , it is a constant; while moving from one slip line to the other, the constant will change correspondingly. The value of these constants can be obtained by the stress boundary conditions of the slip-line fields.

2.6.3 Slip-Line Field and Stress Analysis in the Initial Rolling Stage

In the initial stage, the feed rate is very small; the involute flank of the workpiece has not formed. The plastic deforming area is the contact area between the addendum of roller and workpiece. Taken y-axis as the horizontal direction, the free boundary of the workpiece was an arc, which can be simplified as a straight line. And the arc of the roller addendum circle was divided according to the area of the contact zone. In order to ensure the calculated accuracy, the subsection of arc $\widehat{AB}$ should satisfy $d\psi < 5^\circ$. According to the stress boundary condition, the slip-line

Fig. 2.17 Slip-line field at a small feeding rate



field can be set up, as shown in Fig. 2.17. The boundary shown in dash line is the arc of the external circle on the workpiece before simplified.

In the built slip-line field $ABDGH$, AH is the free surface without applied force on the workpiece, and the slip-line field in zone AGH is a uniform stress field with a free boundary. The surface AB contacted with the addendum of the roller is the non slip-line smooth boundary in the plastic deformation zone. Therefore, in the area ABD , the second class of boundary value problem is suitable. The slip-line field can be obtained with the graphic method. The angle between the slip line that contact with the roller addendum and the contact surface is ϕ_1 , which is related to friction. Area ADG is the central sector field, and area AGH is a free boundary uniform stress field.

According to the slip-line field, the stress state on the contact surface can be obtained. Take a point a on the free surface AH and a point b on the contact surface AB , and points a and b are on the same slip line of ab . The stress states of points a and b are shown in Fig. 2.18a and 2.18b, respectively.

Slip line of ab is the β slip line. Point a is on the free surface, and from the stress boundary types, the slip-line direction angle of point a can be easily obtained as follows:

$$\omega_a = -\frac{3\pi}{4};$$

Additionally, $\sigma_1 = 0$

From the yield criterion, substituting $\sigma_1 = 0$ to Eq. (2.42), it can be obtained as follows:

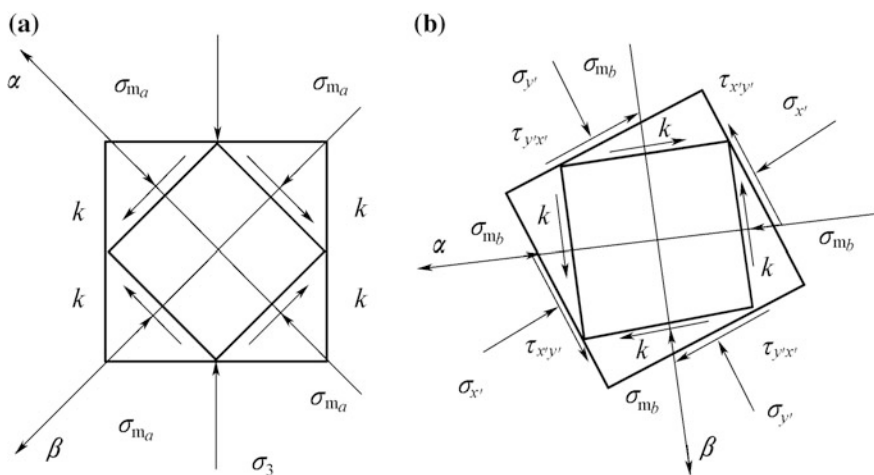


Fig. 2.18 Stress states of points on the free surface and the contact surface **a** stress state of point a ; **b** stress state of point b

$$\sigma_3 = -2K$$

The average stress σ_{m_a} of point a on the slip line is as follows:

$$\sigma_{m_a} = -K$$

The direction angle of point b on the contact surface is related to friction:

$$\omega_b = -(\phi_1 + \psi)$$

where ϕ_1 is the angle between the slip line determined by the friction model and the contact surface;

ψ is the angle between Ob and y -axis. According to the characteristic that the variation of the average stress along the slip line in the slip-line field is in proportion to the variation of the direction angle of the slip line, the average stress σ_{m_b} on point b on the slip line and the average stress σ_{m_a} of point a has the following relationship:

$$\sigma_{m_b} - \sigma_{m_a} = -2K(\omega_b - \omega_a)$$

The average stress of point b on the contact surface can be obtained as follows:

$$\sigma_{m_b} = -2K\left(\frac{1}{2} + \frac{3\pi}{4} - \phi_1 - \psi\right) \quad (2.45)$$

2.6.4 Slip-Line Field and Stress Analysis in the Stable Rolling Stage

After the preliminary indexing, tooth profile has been formed on the workpiece surface. From the analysis of the contact area, it can be seen that when the workpiece enters the contact zone, the tooth surface contacts with the addendum and the flank of the roller. The contact position of the spline tooth is different at different rotation angles, and only the involute spline on the tooth flank is in contacting. Therefore, the contact condition can be simplified as the contact of two cylinders on the contact point, and the stress can be analyzed with the slip-line field method in simple cross rolling. Here, the slip-line field and stress of the contact zone are mainly analyzed and discussed under the conditions the addendum and the flank contact with the workpiece simultaneously.

y -axis is still chosen as the horizontal direction. The boundary of the contact zone can be simplified by substituting the involute on the roller flank with the tangent line on the intersection point of the pitch circle and the involute of the flank on conditions of rolling meshing. The non-contact free boundary on the workpiece

substituted by cord lines, and the slip-line field can be obtained with the graphic method.

In order to obtain the stress state of the points on the contact line, point a is taken from the free surface CH and point c is taken from the contact surface AC . Points a and c are in the same group of slip line, and point d is taken from the straight-line slip-line field of area $ADEF$. Points c and d are in the same group slip line. And points d and b are in the same group slip line. The stress states of points a and b are shown in Fig. 2.18, and the stress state of point d is similar to the stress state of point b . According to the stress states, it can be confirmed that lines ac and db are the β slip line, and line cd is the α slip line.

By analyzing of the free surface, it is easy to obtain from the stress boundary type, that, at point a :

$$\omega_a = -\frac{3\pi}{4}, \sigma_{m_a} = -K$$

The direction angle of the slip line at point c is related with friction:

$$\omega_c = -\left(\frac{\pi}{2} - \alpha + \phi_2\right)$$

where ϕ_2 is the angle between the slip line decided by the friction model and the contact surface.

According to the clan property of the slip line, there is:

$$\sigma_{m_c} = -2K\left(\frac{1}{2} + \frac{\pi}{4} + \alpha - \phi_2\right) \quad (2.46)$$

The slip line cd is a straight line, therefore

$$\omega_d = \omega_c, \sigma_{m_d} = \sigma_{m_c}$$

The same as the initial rolling contact, at point b on the contact arc of the roller addendum:

$$\omega_b = -(\phi_1 + \psi)$$

From Hencky stress equation (2.44), obtain:

$$\sigma_{m_b} - \sigma_{m_d} = -2K(\omega_b - \omega_d)$$

It can be obtained as follows:

$$\sigma_{m_b} = -2K\left(\frac{1}{2} + \frac{3\pi}{4} - \phi_1 - \psi\right) \quad (2.47)$$

Equations (2.46) and (2.47) are the formulas of the average stress on contact surface, while the roller addendum and the flank are both contacted with the workpiece. In which Eq. (2.47) has the same format with the average stress expressed in Eq. (2.45) at the initial rolling stage, only the addendum of the roller is contacted with the roller.

2.7 Average Pressure on the Contact Surface During the Forming Process

The theoretical calculation method of the plastic forming force mainly includes main stress method, slip-line field method, upper bound method and finite element method [22].

The unit average pressure on the contact surface of simple cross rolling has been solved by A. Д. ТОМЛЕНОВ with slip-line method. But friction and work hardening were not considered in the solving process, and it is difficult to be applied in the cold rolling process of the spline. On the basis of this method, modification has been carried out to meet the requirement of the average stress calculation of the spline cold rolling precision forming process.

In the setting up of the slip-line field, different friction conditions are adopted in different contact zones, and in the analysis of the stress, the angle between the slip line and the contact surface should be calculated, respectively, according to the different selected friction models.

Taking stable rolling, for example, in the Fig. 2.19, AB zone is the contact zone between the addendum of the roller and the workpiece, where the constant friction condition is more suitable for the contact deformation; therefore, angle ϕ_1 between the slip-line direction on the contact surface and the tangent direction of the contact surface is as follows:

$$\phi_1 = \frac{1}{2} \arccos m \quad (2.48)$$

where m is the constant friction factor, $0 \leq m \leq 1$.

In Fig. 2.19, zone AC is the contact zone of the roller flank and the workpiece, where Coulomb friction condition is more suitable for the contact deforming friction model. Taking the direction perpendicular with the contact surface as axis y' , according to the average stress and the maximum shear stress, the normal pressure stress and the tangent stress is as follows:

$$\begin{cases} \sigma_{y'} = \sigma_{mc} + K \sin 2(-\phi_2) = -K \left(1 + \frac{\pi}{2} + 2\alpha - 2\phi_2 + \sin 2\phi_2 \right) \\ \tau_{x'y'} = K \cos 2\phi_2 \end{cases} \quad (2.49)$$

According to Coulomb friction condition, Coulomb friction coefficient μ is as follows:

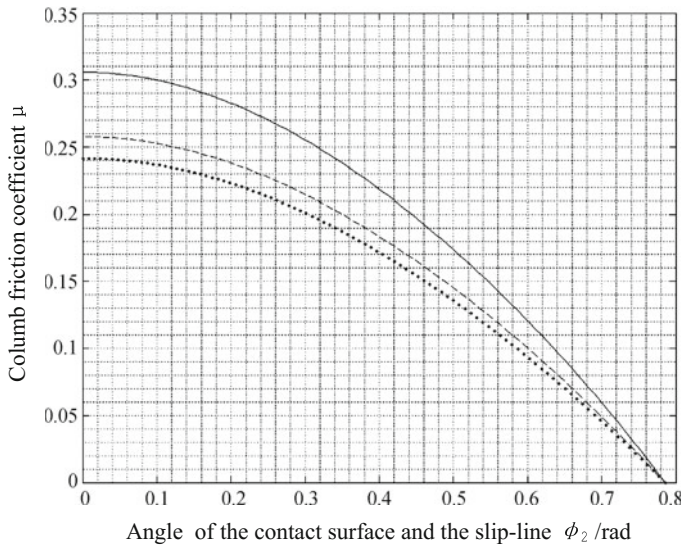


Fig. 2.20 Relationship between Coulomb friction coefficient μ and angle ϕ_2

$$\mu = \frac{|\tau_{x'y'}|}{|\sigma_{y'}|} = \frac{\cos 2\phi_2}{1 + \frac{\pi}{2} + 2\alpha - 2\phi_2 + \sin 2\phi_2} \quad (2.50)$$

It is difficult to solve the angle ϕ_2 from Eq. (2.50) by numerical calculation. Putting a series of ϕ_2 into (2.50), then a series of μ can be obtained. So the diagram can be plotted by the series of ϕ_2 and μ , as shown in Fig. 2.20, and ϕ_2 can thus be obtained from the diagram. In Fig. 2.20, the real line, the broken line, and the point line correspond to $\alpha = 20^\circ$, $\alpha = 37.5^\circ$, and $\alpha = 45^\circ$, respectively. The abscissa represents the change of angle ϕ_2 , and the unit of the angle is radian. For example, when $\alpha = 45^\circ$, it can be obtained from Fig. 2.20: When $\mu = 0.1$, $\phi_2 = 0.5855$; and when $\mu = 0.2$, $\phi_2 = 0.3038$.

In the cold rolling precision forming of spline shafts, taking the direction perpendicular to the contact surface as axis y' , from the stress analysis, it is known that σ_y is the unit average stress of the contact surface. Homogenizing the unit average stress on the contact zone of the roller addendum, then the normal pressure applied to point b and perpendicular to the contact surface can be expressed as q_1 . From the average stress and maximum shear stress of point b , the unit pressure on the contact surface of the addendum contact zone is as follows:

$$q_1 = -\sigma_{y'} = -[\sigma_{m_b} + K \sin 2(-\phi_1)] = 2K \left(\frac{1}{2} + \frac{3\pi}{4} - \phi_1 - \psi + \frac{\sin 2\phi_1}{2} \right) \quad (2.51)$$

Similarly, while contacting on the flank, the unit average pressure is as follows:

$$q_2 = 2K \left(\frac{1}{2} + \frac{\pi}{4} + \alpha - \phi_2 + \frac{\sin 2\phi_2}{2} \right) \quad (2.52)$$

where

ϕ_1 is the function of the constant friction coefficient m ;

ϕ_2 is the function of Coulomb friction coefficient μ .

Considering the action of work hardening, put the real stress (subsequent yield stress) at every contacting into the function determined by the yield criterion, for the average pressure of the contact surface with determined spline parameters in cold rolling forming, Eqs. (2.51) and (2.52) can be summed up to:

$$\begin{cases} q_1 = f(\sigma_{zs})g_1(m, \psi) \\ q_2 = f(\sigma_{zs})g_2(\mu) \end{cases} \quad (2.53)$$

Function f is determined by the yield criterion. When Mises yield criterion and isotropic hardening model are adopted, the shape of the subsequent yield locus is the same with the initial locus. The size of the yield locus is determined by the properties of materials; during the forming process, the subsequent yield stress s_{zs} will change with the variation of the plastic deformation degree.

Compared with the method of A. Д. Томленов, Eq. (2.53) has considered the position of the contact, different friction models have been used in different contact zones, and the influence of work hardening has been considered. Therefore, it is more close to the real average pressure on the contact surface in the spline shaft cold rolling forming.

Function σ_s is determined by the yield criterion, and once the yield criterion is selected:

$$f(\sigma_s) = C\sigma_s \quad (2.54)$$

where C is a constant.

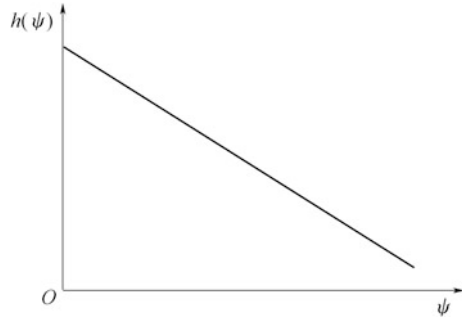
In the plastic forming, the constant m and Coulomb friction coefficient μ are different under different deforming temperatures, different deforming degrees, and different strain rates. But in practice cold rolling forming, the exact value of the parameters is difficult to measure, and the friction coefficient under different conditions can only be measured with matured methods for the theoretical calculation and numerical simulation.

With a determined constant friction factor m and Coulomb friction coefficient μ :

$$\begin{cases} g_1(m, \psi) = C_1 h(\psi) \\ g_2(\mu) = C_2 \end{cases} \quad (2.55)$$

where C_1 and C_2 are constants.

Fig. 2.21 Variation tendency of function $h(\psi)$



Function $h(\psi)$ is the decreasing function of ψ , and the variation trend of it is shown in Fig. 2.21. The maximum and minimum values of g_1 are less than 30%. For the convenience of the calculation, take q_1 on $h(0)$ as the average unit pressure on contact zone (AB) of the contact surface between the roller addendum and the workpiece. Then, Eq. (2.53) can be expressed as follows:

$$\begin{cases} q_{1av} = f(\sigma_s)g_1(m, 0) \\ q_{2av} = f(\sigma_s)g_2(\mu) \end{cases} \quad (2.56)$$

The above deduction is based on the hypothesis of ideal rigid-plastic object, and work hardening of the materials is not considered, what is existed in actual material models. In production, cold plastic forming is always used to improve the mechanical properties of the materials. Therefore, the yield stress before and after plastic forming is different.

For isotropy strain hardening materials, the position of the subsequent yield locus center and the shape after strain hardening is invariable. The shape of the subsequent yield locus is the same with that of the initial one, and the size of the locus is determined by the property of the materials. Therefore, the function f is invariable under determined workpiece materials. However, the stress σ_s should be substituted by the instant flow stress Y (also called as subsequent yield stress), that is:

$$f = f(Y) \quad (2.57)$$

where the real stress Y changes with the variation in the plastic deformation degree.

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