

Chapter 2

Cable Dynamics Elements

Abstract To study the dynamics and control of space net, the dynamic characteristics of the cable element need to be investigated. In this chapter, the dynamic models of cable element, including continuous model, spring damper model and multi-rigid-body model, are developed. And the process of stress wave propagating in the cable and reflecting at the interface are investigated. Specifically, the fluid force acting on the cable elements is discussed using the spring damper model.

2.1 Geometric Nonlinear Characteristics of Cable

The cable has the special property of “can pull and cannot press.” Even under the assumption of small linear elastic deformation, its dynamics response also has geometrical nonlinear characteristics. Consider a simple cable system as shown in Fig. 2.1. The ends of the cable are fixed at points B and D , which are separated by a distance of $2L$. The stiffness is marked as EA , the length is $2L_0$, and the prestrain and pretension are ε_0 and T_0 , respectively. With the action of external load F , the midpoint C has a vertical displacement v , and the cable elongates by $2\Delta L$. The strain increases to ε , tension increases to T , and midpoint C moves to C' .

With the action of an external load, the strain of the cable is denoted as follows:

$$\begin{aligned}\varepsilon &= \frac{\Delta L}{L_0} = \frac{(1 + \varepsilon_0)L_0 \sec \theta_B - L_0}{L_0} = (1 + \varepsilon_0) \sec \theta_B - 1 \\ &= (1 + \varepsilon_0) \sqrt{1 + \left(\frac{v}{L}\right)^2} - 1\end{aligned}\quad (2.1)$$

With small deformation assumption, the following approximate relationship exists:

$$\sin \theta_B \approx \frac{v}{L} \quad (2.2)$$

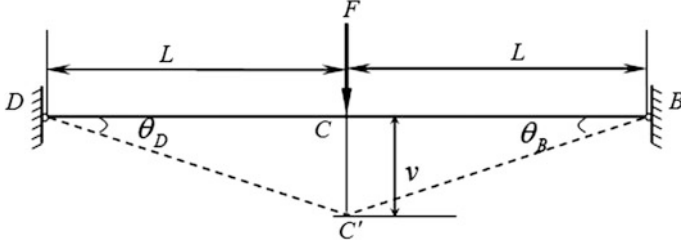


Fig. 2.1 Schematic diagram of simple cable system

$$\varepsilon = (1 + \varepsilon_0) \sqrt{1 + \left(\frac{v}{L}\right)^2} - 1 \stackrel{v/L \rightarrow 0}{=} (1 + \varepsilon_0) \left(\frac{v^2}{2L^2} + 1 \right) - 1 \quad (2.3)$$

According to the conditions for static equilibrium, it can be obtained at the midpoint C' ,

$$2T \sin \theta_B = F \quad (2.4)$$

And by substituting Eq. (2.2) into Eq. (2.4), the following can be obtained:

$$T = \frac{FL}{2v}. \quad (2.5)$$

Under the assumption of linear elastic deformation, the system equations are

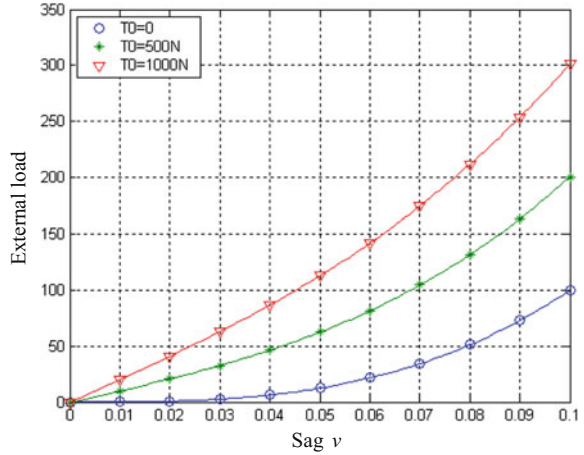
$$T = \varepsilon EA, \quad \varepsilon_0 = \frac{T_0}{EA} \quad (2.6)$$

And by combining Eq. (2.3) with Eqs. (2.4), (2.5) and (2.6), the following system dynamic control equation can be obtained:

$$\left(\frac{EA}{L^3} + \frac{T_0}{L^3} \right) v^3 + \frac{2T_0}{L} v = F \quad (2.7)$$

Equation (2.7) indicates that even under the assumption of a small linear elastic deformation, a cable system's external force and displacement do not have a simple linear relationship. For a simple system, as shown in Fig. 2.1, suppose that the cable's stiffness is $EA = 80,000$ N, and its original length is $2L_0 = 2$ m. Figure 2.2 shows the curve for the midpoint sag and external load for prestress values of 0 N, 500 N, and 1000 N. It can be seen that a larger prestress results in a greater slope for the curve, which indicates that the cable has a greater resistance to external force. The dynamic problem of the cable is characteristic of nonlinear dynamics, and is different from the normal linear system.

Fig. 2.2 Curve of cable sag and external load



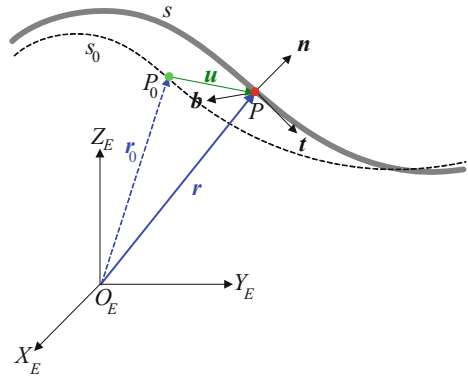
2.2 Continuous Model of Cable

2.2.1 General Dynamics Equation of Space Cable Coordinate System

As shown in Fig. 2.3, any cable in a Euclidean space can be approximated as a curve. Cable coordinates can be specified based on the cable length, and then arbitrary points for the cable can be expressed using the distance between each point and the starting point. Take the local north-vertical-east coordinate system as a reference inertia system. The position of point P at the cable in the cable coordinates is denoted as s , and $\mathbf{r}$ is defined as the position vector in the inertial space. We then obtain

$$\mathbf{r} = xi + yj + zk \quad (2.8)$$

Fig. 2.3 Coordinate system of space cable



where i, j , and k are unit vectors in three directions in the inertial space, and (x, y, z) are the coordinates of point P in the inertial space. Based on the spatial arc length formula, the following relationship exists between position s of point P in the cable coordinates and space coordinates:

$$s = \int_0^P \sqrt{\left(\frac{\partial x}{\partial s}\right)^2 + \left(\frac{\partial y}{\partial s}\right)^2 + \left(\frac{\partial z}{\partial s}\right)^2} ds \quad (2.9)$$

The local Frenet coordinate system Pmb is developed with P as the base point, where axes Pt , Pn , and Pb are directed in the local tangent direction, main normal direction, and binormal direction, respectively, and e_t , e_n , and e_b are the corresponding unit vectors, according to the differential geometry theory,

$$e_t = \frac{\partial r}{\partial s} \quad (2.10)$$

From the Frenet equation of the space curve, the unit vectors in three directions satisfy the following relationship:

$$\frac{d}{ds} \begin{bmatrix} e_t \\ e_n \\ e_b \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} e_t \\ e_n \\ e_b \end{bmatrix} \quad (2.11)$$

where κ and τ are respectively the curvature and torsion of point P .

The inertial and Frenet coordinate systems can be converted into each other using a rotating axis. The conversion order is set to be “3-2-1,” and the corresponding rotating angles are respectively φ , θ , and ϕ . Then, the base vectors of the two coordinate systems satisfy the following relation:

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = B_{Et} \begin{bmatrix} e_t \\ e_n \\ e_b \end{bmatrix} = L_t(\phi) L_n(\theta) L_b(\varphi) \begin{bmatrix} e_t \\ e_n \\ e_b \end{bmatrix} \quad (2.12)$$

where B_{Et} is the conversion matrix from the Frenet to inertial coordinate system, and L_t , L_n and L_b are the rotating axes of the three conversion matrices.

2.2.2 Dynamics of Continuous Space Cable

P_0 is the undeformed location of point P , and the position vector in the inertial coordinate system is r_0 , with the corresponding coordinates (x_0, y_0, z_0) , setting the displacement vector as u . Then,

$$\mathbf{u} = u_x \mathbf{i} + u_y \mathbf{j} + u_z \mathbf{k} \quad (2.13)$$

The location vectors between the deformed and undeformed cables satisfy the following equation,

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{u} \quad (2.14)$$

The coordinate of P_0 in the chord coordinate system is as follows:

$$s_0 = \int_0^{P_0} \sqrt{\left(\frac{\partial x_0}{\partial s_0}\right)^2 + \left(\frac{\partial y_0}{\partial s_0}\right)^2 + \left(\frac{\partial z_0}{\partial s_0}\right)^2} ds_0 \quad (2.15)$$

Equation (2.9) can also be expressed like s_0 ,

$$s_0 = \int_0^P \sqrt{\left(\frac{\partial x}{\partial s_0}\right)^2 + \left(\frac{\partial y}{\partial s_0}\right)^2 + \left(\frac{\partial z}{\partial s_0}\right)^2} ds_0 \quad (2.16)$$

The strain ε of point P is as follows,

$$\varepsilon = \frac{ds}{ds_0} - 1 \quad (2.17)$$

Using Eq. (2.16), strain ε can also be expressed as follows,

$$\varepsilon = \sqrt{\left(\frac{\partial x}{\partial s_0}\right)^2 + \left(\frac{\partial y}{\partial s_0}\right)^2 + \left(\frac{\partial z}{\partial s_0}\right)^2} - 1 \quad (2.18)$$

If the deformation of the cable is small, then strain ε can also be expressed using displacement u ,

$$\begin{aligned} \varepsilon &= \frac{\partial u_x}{\partial s_0} \frac{\partial x_0}{\partial s_0} + \frac{\partial u_y}{\partial s_0} \frac{\partial y_0}{\partial s_0} + \frac{\partial u_z}{\partial s_0} \frac{\partial z_0}{\partial s_0} + \frac{1}{2} \left(\frac{\partial u_x}{\partial s_0}\right)^2 + \frac{1}{2} \left(\frac{\partial u_y}{\partial s_0}\right)^2 \\ &\quad + \frac{1}{2} \left(\frac{\partial u_z}{\partial s_0}\right)^2 = \frac{\partial u}{\partial s_0} \frac{\partial r_0}{\partial s_0} + \frac{1}{2} \left(\frac{\partial u}{\partial s_0} \frac{\partial u}{\partial s_0}\right) \end{aligned} \quad (2.19)$$

Assuming the length of the deformed cable is L , then the kinetic energy V of the deformed cable is as follows,

$$V = \frac{1}{2} \int_0^L \rho A (\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_z^2) ds \quad (2.20)$$

Assuming the cable is a linear isotropic elastic material with elastic modulus E , then the potential energy of the whole cable is as follows,

$$U = \int_0^L \left(\frac{1}{2} EA \varepsilon^2 + \rho A g \right) ds \quad (2.21)$$

where A is the cross-sectional area. According to the continuous equation of the cable, we obtain the following equation,

$$A(1 + \varepsilon) = A_0 \quad (2.22)$$

where A_0 is the cross-sectional area of the undeformed cable.

The forces of the cable unit at point P are respectively f_x , f_y , and f_z in three directions. Then, with the Hamilton principle, we get the following equation,

$$\delta \int_{t_0}^{t_1} (T - U) dt + \int_{t_0}^{t_1} \delta W dt = 0 \quad (2.23)$$

The dynamics of the space continuous cable can be derived as follows,

$$\begin{cases} \rho A_0 \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial}{\partial s_0} \left(\frac{EA \varepsilon}{(1 + \varepsilon)} \frac{\partial x}{\partial s_0} \right) + f_x (1 + \varepsilon) \\ \rho A_0 \frac{\partial^2 u_y}{\partial t^2} = \frac{\partial}{\partial s_0} \left(\frac{EA \varepsilon}{(1 + \varepsilon)} \frac{\partial y}{\partial s_0} \right) + f_y (1 + \varepsilon) - \rho A_0 g \\ \rho A_0 \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial}{\partial s_0} \left(\frac{EA \varepsilon}{(1 + \varepsilon)} \frac{\partial z}{\partial s_0} \right) + f_z (1 + \varepsilon) \end{cases} \quad (2.24)$$

2.2.3 Simplification for Dynamics of Continuous Space Cable

To determine the dynamics of the cable, the cable is normally regarded as a curve moving in a plane, which means the torsion τ equals to zero during motion. With the assumption that the motion plane of the cable is $X_E O_E Y_E$, then the conversion relationship between the inertial and Frenet coordinate systems is as follows,

$$\begin{bmatrix} i \\ j \end{bmatrix} = B_{Et} \begin{bmatrix} e_t \\ e_n \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} e_t \\ e_n \end{bmatrix} \quad (2.25)$$

Then, the dynamic model of the cable only needs the first two items of Eq. (2.24).

When investigating the tension of a linear density cable, if the positive force is smaller than the elastic force, the bending deformation can be neglected, and the

dynamic model of the cable can be simplified into a one-dimensional model. If the cable is parallel to the O_EX_E axis, then,

$$s_0 = x_0, \quad s = x \quad (2.26)$$

Submitting Eq. (2.26) into dynamic model of the cable,

$$\rho \frac{\partial^2 u_x}{\partial t^2} = E \frac{\partial^2 u_x}{\partial x^2} \quad (2.27)$$

Equation (2.27) is the governing equation of the stress wave propagating in a one-dimensional elastic cable. For $u = u_x$ in a one-dimensional situation, Eq. (2.27) can also expressed as follows,

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (2.28)$$

where c is the elastic velocity. The partial differential equation (Eq. 2.28) can be solved under the definite condition. The definite condition normally includes the initial condition and boundary condition, where the initial condition describes the initial state of the cable, which is normally expressed as follows,

$$\begin{cases} u(x, 0) = u(x) \\ v(x, 0) = v(x) \\ \sigma(x, 0) = \sigma(x) \end{cases} \quad (2.29)$$

The boundary condition reflects the constraints at both ends of the cable, which normally includes the velocity boundary and stress boundary.

$$\begin{cases} v(0, t) = v_0(t), & v(L, t) = v_L(t) \\ \sigma(0, t) = \sigma_0(t), & \sigma(L, t) = \sigma_L(t) \end{cases} \quad (2.30)$$

where L is the cable length. When the boundary velocity is always zero, the corresponding boundary condition is called a fixed boundary; when the boundary stress is always zero, the corresponding boundary condition is called a free boundary.

2.2.4 Reflection and Transmission of Stress Wave Between Cable Sections

To research the stress wave propagating in a cable, the cable is normally regarded as a homogeneously continuous elastic or viscoelastic material with isotropy. If the research goals are multiple cable units with different parameters, then when the

stress wave propagates to a section, because of the obvious differences in the materials at the section, only some of the energy crosses the section and propagates forward, while the rest is reflected at the section. This chapter will first derive the reflection and transmission equations of the stress wave at a section, and will then analyze and discuss them concisely.

2.2.4.1 Basic Equation

As Fig. 2.4 shows, there are two homogeneously continuous cables (cable 1 and cable 2), where the density, cross-sectional area, elastic modulus, and propagating velocity are, respectively, ρ_i , A_i , E_i and $c_i (i = 1, 2)$. The two cables are tightly connected at section NN' , and the displacement, velocity, and internal force satisfy the continuous condition. According to the dynamic equation of the space cable, the motion of the cables at the ends satisfies the following governing equations,

$$\begin{cases} \frac{\partial^2 u_i}{\partial t^2} = \frac{E_i}{\rho_i} \frac{\partial^2 u_i}{\partial x^2} = c_i^2 \frac{\partial^2 u_i}{\partial x^2} \\ \frac{\partial v_i}{\partial x} = \frac{\partial \varepsilon_i}{\partial t} \\ \sigma_i = E_i \varepsilon_i \end{cases} \quad (i = 1, 2) \quad (2.31)$$

If the stress wave propagates to the right in cable 1 at a moment, and the wave propagates to section NN' , this section will produce reflection and transmission (Fig. 2.4) based on the differences in its materials and structure. Setting the incident wave, reflected wave, and transmitted wave as I , R and T , respectively, the continuous condition at section NN' gives the following,

$$\begin{cases} u_{N1} = u_{N2} \\ v_{N1} = v_{N2} \\ \sigma_{N1} A_{N1} = \sigma_{N2} A_{N2} \end{cases} \quad (2.32)$$

where u_{Ni} , v_{Ni} and $\sigma_{Ni} (i = 1, 2)$ represent the displacement, velocity, and stress of both ends.

According to superposition, Eq. (2.32) can be written as follows,

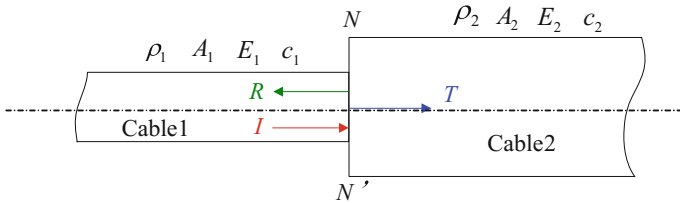


Fig. 2.4 Reflection and transmission of stress wave between cable sections

$$\begin{cases} u_I + u_R = u_T \\ v_I + v_R = v_T \\ (\sigma_I + \sigma_R)A_1 = \sigma_TA_2 \end{cases} \quad (2.33)$$

Because the incident wave propagates to the right, the reflected wave propagates to the left, and the transmitted wave propagates to the right, the corresponding characteristic equations are as follows,

$$\begin{cases} \text{Incident wave: } dx = c_1 dt \\ \text{Reflected wave: } dx = -c_1 dt \\ \text{Transmitted wave: } dx = c_2 dt \end{cases} \quad (2.34)$$

Then, we get the following equations,

$$\begin{cases} \frac{\partial u_I}{\partial t} = c_1 \frac{\partial u_I}{\partial x} \\ \frac{\partial u_R}{\partial t} = -c_1 \frac{\partial u_R}{\partial x} \\ \frac{\partial u_T}{\partial t} = c_2 \frac{\partial u_T}{\partial x} \end{cases} \quad (2.35)$$

For $v = \frac{\partial u}{\partial t}$, $\varepsilon = \frac{\partial u}{\partial x}$, substituting Eq. (2.35) into Eq. (2.33) produces the following equations,

$$\begin{cases} v_I + v_R = v_T \\ \rho_1 c_1 A_1 (v_I - v_R) = \rho_2 c_2 A_2 v_T \end{cases} \quad (2.36)$$

Then, the motion velocities of the cables after the reflected wave, transmitted wave, and incident wave satisfy the following equations,

$$\begin{cases} v_R = \frac{\rho_1 c_1 A_1 - \rho_2 c_2 A_2}{\rho_1 c_1 A_1 + \rho_2 c_2 A_2} v_I = \frac{\alpha - 1}{\alpha + 1} v_I \\ v_T = \frac{2\rho_1 c_1 A_1}{\rho_1 c_1 A_1 + \rho_2 c_2 A_2} v_I = \frac{2\alpha}{\alpha + 1} v_I \end{cases} \quad (2.37)$$

where α is defined as follows,

$$\alpha = \frac{\rho_1 c_1 A_1}{\rho_2 c_2 A_2} \quad (2.38)$$

To describe the problem conveniently, the generalized impedance of the cable is defined as the product of the linear density and wave velocity (ρcA), and α is the generalized impedance ratio of the connected cables.

Using the same method, the stresses produced by the reflected wave, transmitted wave, and incident wave are as follows,

$$\begin{cases} (\sigma_R A_1) = \frac{\rho_1 c_1 A_1 - \rho_2 c_2 A_2}{\rho_1 c_1 A_1 + \rho_2 c_2 A_2} (\sigma_I A_1) = -\frac{\alpha-1}{\alpha+1} (\sigma_I A_1) \\ (\sigma_T A_2) = \frac{2\rho_2 c_2 A_2}{\rho_1 c_1 A_1 + \rho_2 c_2 A_2} (\sigma_I A_1) = \frac{2}{\alpha+1} (\sigma_I A_1) \end{cases} \quad (2.39)$$

The displacement relationship can be derived by integrating Eq. (2.37),

$$\begin{cases} u_R = \frac{\rho_1 c_1 A_1 - \rho_2 c_2 A_2}{\rho_1 c_1 A_1 + \rho_2 c_2 A_2} u_I = \frac{\alpha-1}{\alpha+1} u_I \\ u_T = \frac{2\rho_1 c_1 A_1}{\rho_1 c_1 A_1 + \rho_2 c_2 A_2} u_I = \frac{2\alpha}{\alpha+1} u_I \end{cases} \quad (2.40)$$

Equations (2.37), (2.39), and (2.40) are the state variable equations of the stress wave reflected and transmitted at the section between two cables.

2.2.4.2 Discussion of Several Cases

As the above equations show, the ratios of the reflected wave and transmitted wave to the incident wave are mainly determined by the generalized impedance ratio α of the connected cables. Analysis of different α values is given in the following.

(1) $\alpha = 1$

The generalized impedance of the connected cables matches each other, and

$$(\rho_1 c_1 A_1) = (\rho_2 c_2 A_2) \quad (2.41)$$

According to Eqs. (2.37), (2.39), and (2.40), the reflected wave satisfies the following equation,

$$v_R = \sigma_R = u_R = 0 \quad (2.42)$$

The transmitted wave satisfies the following equations,

$$\begin{cases} v_T = v_I \\ (\sigma_T A_2) = (\sigma_I A_1) \\ u_T = u_I \end{cases} \quad (2.43)$$

As Eqs. (2.42) and (2.43) show, the stress wave will not be reflected at the conjunction for cables with completely different materials and structures with their generalized impedances matching each other.

(2) $\alpha < 1$

The stress wave can propagate from the cable with a small generalized impedance to that with a large generalized impedance,

$$\begin{cases} \frac{v_R}{v_I} = \frac{\alpha - 1}{\alpha + 1} < 0, & \frac{v_T}{v_I} = \frac{2\alpha}{\alpha + 1} < 1 \\ \frac{\sigma_R}{\sigma_I} = -\frac{\alpha - 1}{\alpha + 1} > 0, & \frac{\sigma_T A_2}{\sigma_I A_1} = \frac{2}{\alpha + 1} > 1 \\ \frac{u_R}{u_I} = \frac{\alpha - 1}{\alpha + 1} < 0, & \frac{u_T}{u_I} = \frac{2\alpha}{\alpha + 1} < 1 \end{cases} \quad (2.44)$$

Here, the reflected and transmitted waves of a tensile wave are still tensile waves. According to the superposition principle, the motion velocity and displacement can be damped, and the force increases in the passing area of the reflected wave, as illustrated in Fig. 2.5a.

$\alpha \rightarrow 0$ can be regarded as an extreme example of $\alpha < 1$ for a stress wave propagating to a fixed boundary. Then based on Eq. (2.44), we derive the following equations,

$$\begin{cases} v_R = -v_I, & v_T = 0 \\ \sigma_R = \sigma_I, & \sigma_T = 2\sigma_I \\ u_R = -u_I, & u_T = 0 \end{cases} \quad (2.45)$$

According to the superposition principle, the stress doubles while the velocity and displacement disappear in the passing area of the reflected wave, and the stress wave reflects completely without producing a transmitted wave at the conjunction.

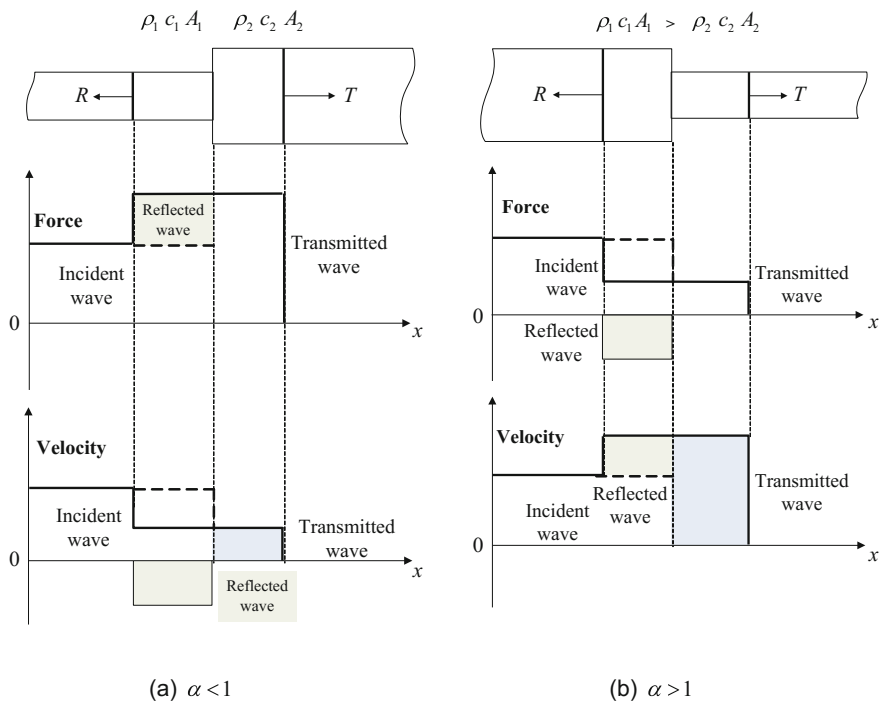


Fig. 2.5 Reflection and transmission of stress wave at section between cables

(3) $\alpha > 1$

The stress wave can propagate from a cable with a large generalized impedance to that with a small generalized impedance. Then,

$$\begin{cases} \frac{v_R}{v_I} = \frac{\alpha-1}{\alpha+1} > 0, & \frac{v_T}{v_I} = \frac{2\alpha}{\alpha+1} > 1 \\ \frac{\sigma_R}{\sigma_I} = -\frac{\alpha-1}{\alpha+1} < 0, & \frac{\sigma_T A_2}{\sigma_I A_1} = \frac{2}{\alpha+1} < 1 \\ \frac{u_R}{u_I} = \frac{\alpha-1}{\alpha+1} > 0, & \frac{u_T}{u_I} = \frac{2\alpha}{\alpha+1} > 1 \end{cases} \quad (2.46)$$

Here, the reflected wave of the tensile wave is the compressional wave. According to the superposition principle, the motion velocity and displacement increase, and the stress decreases in the passing area of the reflected wave, as illustrated in Fig. 2.5b.

$\alpha \rightarrow \infty$ can be regarded as the extreme example of $\alpha > 1$ for the stress wave reflected at the free boundary. Then based on Eq. (2.46), we derive the following equations,

$$\begin{cases} v_R = v_I, & v_T = 2v_I \\ \sigma_R = -\sigma_I, & \sigma_T = 0 \\ u_R = u_I, & u_T = 2u_I \end{cases} \quad (2.47)$$

According to the superposition principle, the velocity doubles while the stress disappears in the passing area of the reflected wave.

2.2.5 Numerical Solution of Stress Waves with Characteristic Line

Stress wave propagation can be solved using a hyperbolic partial differential equation. However, because of the complexity of the partial differential equation, most practical problems cannot be solved analytically, but a theoretical analysis can be used with the specified constitutive relation and boundary condition. During the process of an umbrella hatch dragging a parachute pack, the motion of the hatch provides a boundary for the connection cable and stress wave of the pack. Meanwhile, the stress wave propagating in the connection cable and parachute pack will also affect the motion of the umbrella hatch, and tight coupling exists between the propagating stress wave and motion of the umbrella hatch. The stress wave propagating in the connection cable and pack can only be solved using mathematical methods.

The current wave equation solutions include the characteristic line method, harmonic wave method, Laplace transform method, precise integration method, and finite difference method (Lax-Wendroff, Godnov method), which are normally used in computational fluid mechanics. Among these methods, the characteristic line method is widely used for its precise and stable calculation of the stress wave.

However, the characteristic line method cannot solve the reflection and transmission of the stress wave at adjacent cables. Thus, the node splitting method is adopted at adjacent cables in the present article, which can make the characteristic line method applicable.

2.2.5.1 Discretization of Characteristic Line Equation

The characteristic line method is normally used to solve the partial differential equation by transforming the governing equation of the stress wave into an ordinary differential equation along the characteristic line, and then using a given boundary condition. For two cables, according to governing equation (Eq. 2.31) of the propagating stress wave, the corresponding characteristic line equation is as follows:

$$\frac{dx}{dt} = \pm c_i \quad (2.48)$$

Substituting Eq. (2.48) into the governing equation, the compatible equation is as follows:

$$d\sigma_i \mp \rho_i c_i dv_i = 0 \quad (2.49)$$

Integrating both sides, we obtain the following equation:

$$\sigma_i \mp \rho_i c_i v_i = \text{const} \quad (2.50)$$

The stress and velocity maintain a constant relationship along the characteristic line. Based on this, the stress, strain, and velocity in the time domain and spatial domain can be solved along the characteristic line with a given boundary condition and initial condition. With this idea, cable i can be discretized into $\Delta x_i \times \Delta t_i$, and time step Δt_i and spatial step Δx_i satisfy the following equation:

$$\Delta x_i = c_i \Delta t_i \quad (2.51)$$

For the same time step in the spatial domain during the calculation process, the corresponding spatial steps of cable 1 and cable 2 satisfy the following equation:

$$\frac{\Delta x_1}{\Delta x_2} = \frac{c_2}{c_1} \quad (2.52)$$

According to the compatible equation, the stress σ_j^{n+1} and velocity v_j^{n+1} of point j at t^{n+1} can be calculated using points $j - 1$ and $j + 1$ at t^n :

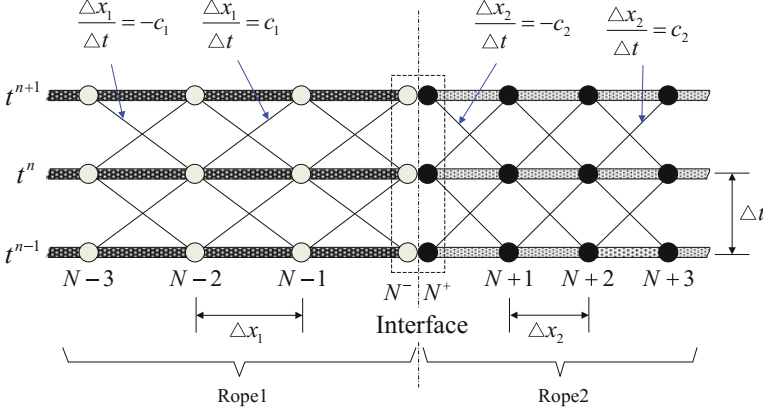


Fig. 2.6 Discretization of connected cables in time domain and spatial domain

$$\begin{cases} \sigma_j^{n+1} - \rho_j c_j v_j^{n+1} = \sigma_{j-1}^n - \rho_{j-1} c_{j-1} v_{j-1}^n \\ \sigma_j^{n+1} + \rho_j c_j v_j^{n+1} = \sigma_{j+1}^n + \rho_{j+1} c_{j+1} v_{j+1}^n \end{cases} \quad (2.53)$$

For the adjacent cables (cable 1 and cable 2), the end point of cable 1 and beginning point of cable 2 are glued tightly together. The two points have different density, generalized wave impedance, stress and strain values, but have the same displacement, velocity, and force. Point N at the adjacent cables is divided into N^- and N^+ , where N^- is the end of cable 1, and N^+ is the beginning of cable 2, as shown in Fig. 2.6. Then, the stress and velocity of the adjacent cables at t^{n+1} can be calculated using point $N-1$ of cable 1 and point $N+1$ of cable 2 at t^n .

$$\begin{cases} \sigma_{N^-}^{n+1} A_1 - \rho_1 c_1 A_1 v_{N^-}^{n+1} = \sigma_{N-1}^{n+1} A_2 - \rho_1 c_1 A_2 v_{N-1}^{n+1} \\ \sigma_{N^+}^{n+1} A_2 + \rho_1 c_1 A_2 v_{N^+}^{n+1} = \sigma_{N+1}^{n+1} A_2 + \rho_1 c_1 A_2 v_{N+1}^{n+1} \\ \sigma_{N^-}^{n+1} A_1 = \sigma_{N^+}^{n+1} A_2 \end{cases} \quad (2.54)$$

2.2.5.2 Numerical Simulation

To test the validity of the model and numerical method, the stress wave propagating in the two connected cables is simulated. As Fig. 2.7 shows, the right end of cable 1 is tightly connected to the left end of cable 2. They are both in a static situation at the initial time of the simulation, with the left end of cable 1 loaded with constant velocity v_0 from $t = 0$, and the right end of cable 2 kept free.

For a clear description of the problem, density ρ_1 , modulus E_1 , cross-sectional area A_1 , length L_1 , wave velocity c_1 and time L_1/c_1 are chosen as the characteristic parameters, and the corresponding parameters are made dimensionless. Setting the

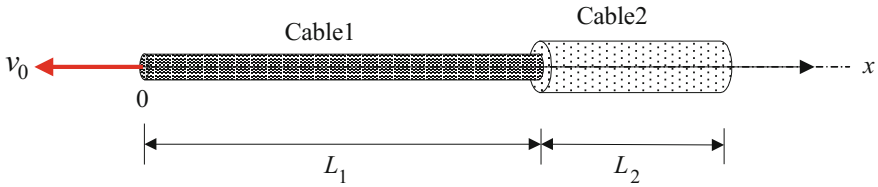


Fig. 2.7 Velocity loaded at connected cables

Table 2.1 Dimensionless parameters of two cables

Example	Cable 1						Cable 2					
	$\bar{\rho}_1$	$\bar{A}_1$	$\bar{E}_1$	$\bar{L}_1$	$\bar{c}_1$	α_1	$\bar{\rho}_2$	$\bar{A}_2$	$\bar{E}_2$	$\bar{L}_2$	$\bar{c}_2$	α_2
Example 1	1	1	1	1	1	1	1	1	1	0.5	1	1
Example 2	1	1	1	1	1	1	1	2	0.25	0.5	0.5	1
Example 3	1	1	1	1	1	0.1	1	10	1	0.5	1	10
Example 4	1	1	1	1	1	10	1	0.1	1	0.5	1	0.1

dimensionless velocity $\bar{v}_0 = -0.01$, the dimensionless parameters of the corresponding cable material under different simulation conditions are listed in Table 2.1.

Among the four simulation examples, the stress wave propagating from a cable with a small generalized impedance to a cable with a large generalized impedance is simulated in Example 3, similar to the relationship of the connected cable and pack during the pack dragging out, and the reflection and transmission of the stress wave between the connected cable and pack can be described.

Here, α_1 is the generalized impedance ratio of cable 1 to cable 2, α_2 is the generalized impedance ratio of cable 2 to cable 1, and α_1 is the reciprocal of α_2 ($\alpha_1 = 1/\alpha_2$).

The velocity and force distribution of the cable in Example 1 at different times are illustrated in Fig. 2.8. As this figure shows, the tensile wave propagating to the right is reflected at the right boundary, which doubles the velocity, causes the force to disappear, and makes the cable slack. The tensile wave becomes a compressional wave at the free boundary. In addition, there is no reflection when the stress wave propagates to the adjacent cable. The cable is completely slack when the reflected wave propagates to the left end at $\bar{t} = 3$.

Although the materials and structures of cable 1 and cable 2 in Example 2 are different, their generalized impedances match each other, and there is no reflection of the stress wave at the adjacent cable, which is similar to the propagation process in Example 1. For $\bar{c}_2 = 0.5$ of cable 2, the propagation time of the stress wave in Example 2 is twice that of Example 1, which indicates that the stress wave reaches the right end of cable 2 at $\bar{t} = 2$, and both cables become slack at $\bar{t} = 4$.

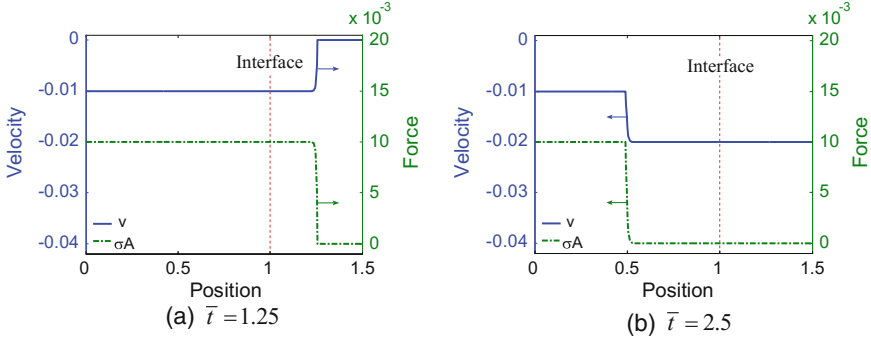


Fig. 2.8 Velocity and force distribution of cable in Example 1 at different time

The situation of a stress wave propagating from a small generalized impedance cable to a large generalized impedance cable is simulated in Example 3. Figure 2.9 shows the velocities and force distributions of the cable at different times, and as illustrated in Fig. 2.9, the simulation results for the incident wave, reflected wave, and transmitted wave match the equations in Sect. 1.2.2, which verifies the validity of the numerical method. Then, the characteristics of the stress wave propagating in Example 3 are as follows.

The loading wave induced at the left end of cable 1 propagates to the interface. A small part will transmit into cable 2, while the largest part will reflect. Then, the force increases, and the velocity decreases in the area after the reflected wave passing.

The transmitted wave reflects into the unloading wave at the right end of cable 2. Then, the velocity doubles and force disappears. When the unloading wave reaches the interface, the largest part transmits into cable 1, the cable velocity increases and the force decreases for the superposition with the loading wave in cable 1.

With the repeated superposition of the loading wave at the left end of cable 1 and the unloading wave at the right end of cable 2, both cables become slack at $\bar{t} = 8.51$. Then, the velocities of cable 1 and cable 2 vary obviously, and the velocity of cable 2 is about twice the loading velocity.

The situation of a stress wave propagating from a large generalized impedance cable to a small generalized impedance cable is simulated in Example 4. Figure 2.10 shows the velocities and force distributions of the cable at different times, and the simulation results for the incident wave, reflected wave, and transmitted wave match the equations in Sect. 2.2.2, which verifies the validity of the numerical method. The characteristics of the stress wave propagating in Example 4 are as follows.

The loading wave induced at the left end of cable 1 propagates to the interface, where the largest part is transmitted into cable 2, and a small part reflects. Then, the force decreases and the velocity increases in the area after the reflected wave passing.

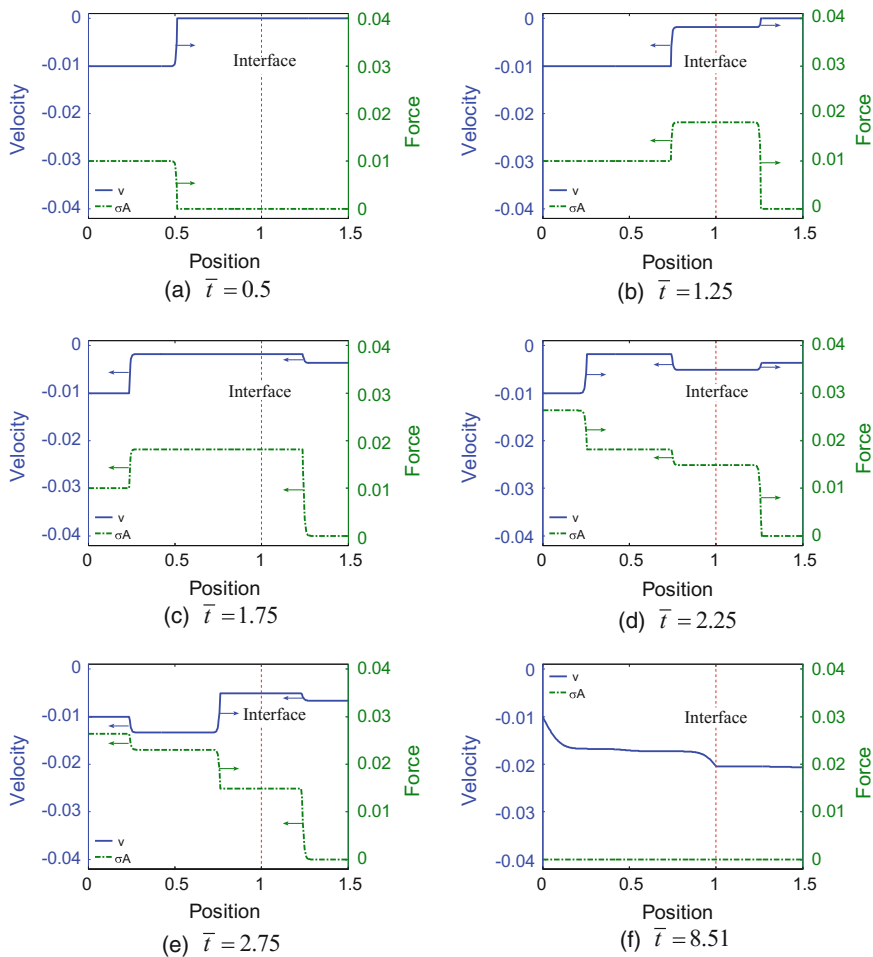


Fig. 2.9 Velocities and force distributions of cable in Example 3 at different times

The transmitted wave reflects into the unloading wave at the right end of cable 2, and the velocity triples. When the unloading wave reaches the interface, only a small part is transmitted into cable 1, the cable velocity is twice that at the left end of cable 1, and force becomes zero.

When the reflected wave reaches the left end of cable 1, the unloading wave still begins from the left end of cable 1 for the cable velocity is larger than that at the left end of cable 1. It then comes across the unloading wave transmitted from cable 2, which induces a velocity gap in cable 1 when the cable is slack.

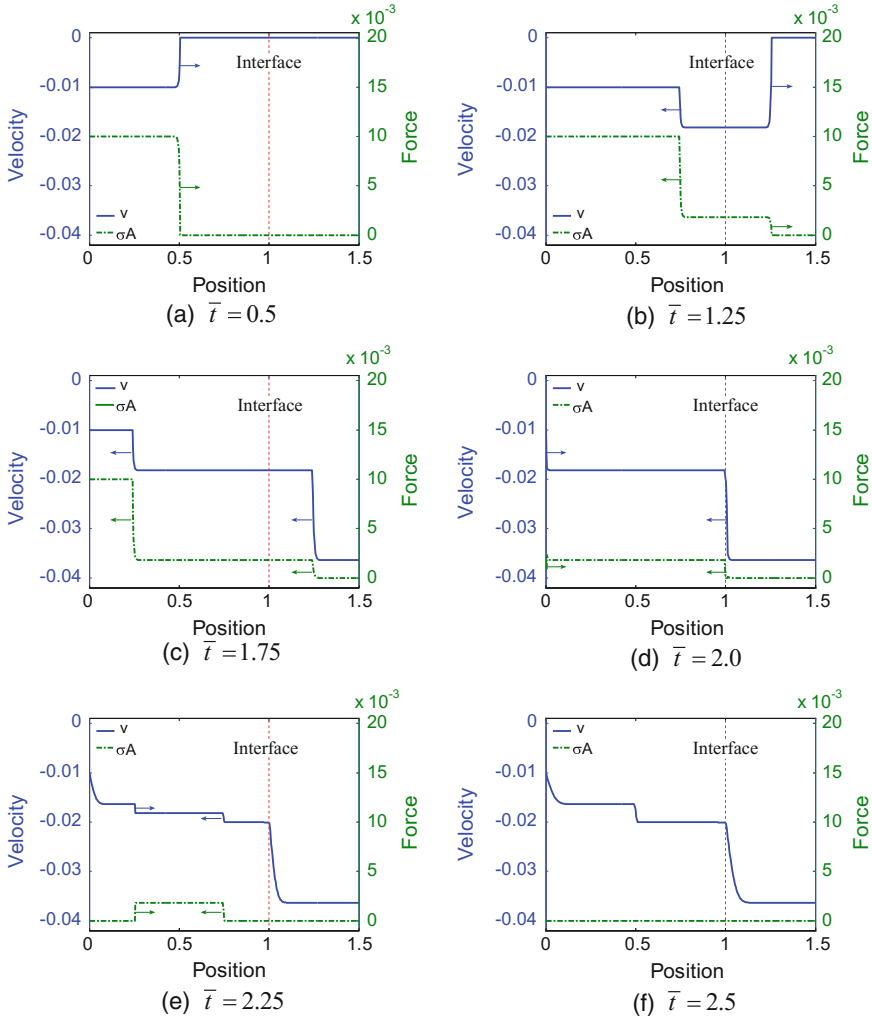


Fig. 2.10 Velocities and force distributions of cable in Example 4 at different times

2.3 Spring Damper Model of Cable

2.3.1 Description of Model

In order to analyze the dynamics of the cable, the cable is normally discretized into several elements, and the mass is distributed at the knot. Because the cable is slack and can only bear tensile force, the knots are connected by a spring, which can only bear tensile force. Considering the damping effects, the cable elements are treated as a lumped mass damper spring, which is called the “lumped mass spring model” or

“semi-linear springs and dampers.” The n th cable element is treated as the “springs and dampers model,” and the rigidity and damping index are respectively k_n and c_n , as shown in Fig. 2.11. The coordinates of the n th cable element are x_n , y_n , and z_n .

Figure 2.12 shows the body-fixed coordinate system of the n th cable element, where the three axes correspond to the main normal vector, hypo-normal vector, and tangent vector. The inertial and body-fixed coordinate system can be transformed through φ_n , θ_n , and ψ_n . The inertial coordinate system first rotates by φ_n around the oz_d axis, with $ox_dy_dz_d$ converted to ONy_1z_d . Then, it rotates by θ_n around ON , with ONy_1z_d converted to ONy_1z_n . Finally, it rotates by ψ_n around oz_n , with oNy_2z_n converted to $oNx_ny_nz_n$. The conversion matrix is as follows,

Fig. 2.11 Diagram of cable element

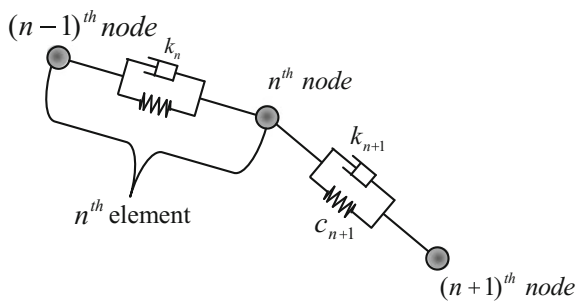
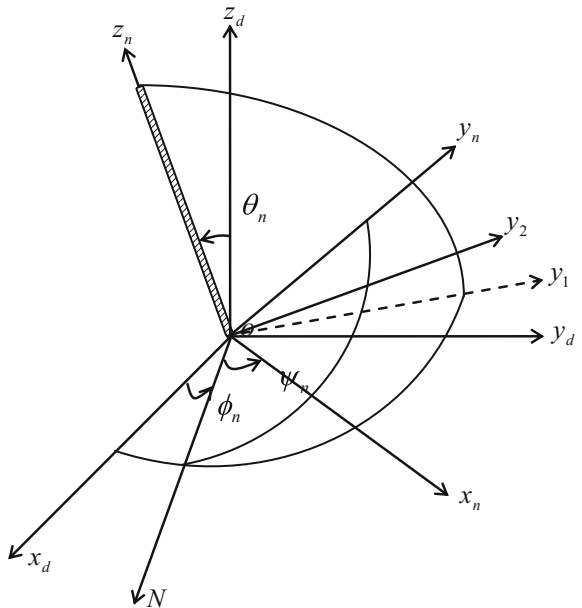


Fig. 2.12 Coordinate system of cable element



$$B_n = \begin{bmatrix} \cos \varphi_n \cos \psi_n - \sin \varphi_n \cos \theta_n \sin \psi_n & \sin \varphi_n \cos \psi_n + \cos \varphi_n \cos \theta_n \sin \psi_n & \sin \theta_n \sin \psi_n \\ -\cos \varphi_n \sin \psi_n - \sin \varphi_n \cos \theta_n \cos \psi_n & -\sin \varphi_n \sin \psi_n + \cos \varphi_n \cos \theta_n \cos \psi_n & \sin \theta_n \cos \psi_n \\ \sin \varphi_n \sin \theta_n & -\cos \varphi_n \sin \theta_n & \cos \theta_n \end{bmatrix} \quad (2.55)$$

Because the cable is very slack, it cannot bear the normal moment and shear force. Then, ψ_n can be neglected or considered to be zero, and the conversion matrix can be simplified as follows,

$$B_n = \begin{bmatrix} \cos \varphi_n & \sin \varphi_n & 0 \\ -\sin \varphi_n \cos \theta_n & \cos \varphi_n \cos \theta_n & \sin \theta_n \\ \sin \varphi_n \sin \theta_n & -\cos \varphi_n \sin \theta_n & \cos \theta_n \end{bmatrix} \quad (2.56)$$

2.3.2 Computation of Internal Force

Because the precise dynamic stress-strain curve of the cable is difficult to obtain, the static stress-strain curve is normally adopted in engineering, which deals with piecewise linearization. The constitutive relationship of the cable is simplified as linear elastic and damping, and the stress of the n th cable unit is as follows,

$$T_n = \begin{cases} 0 & \varepsilon_n \leq 0 \\ p_n(\varepsilon_n) + B_n \dot{\varepsilon}_n & \varepsilon_n > 0 \end{cases} \quad (2.57)$$

where $p_n(\varepsilon_n)$ is the linear stress function of the n th cable element, B_n is the stress damping parameter, and strain ε_n is as follows,

$$\varepsilon_n = \frac{L_n - L_{n,0}}{L_{n,0}} \quad (2.58)$$

where $L_{n,0}$ is the unstretched length, and L_n is the stretched length.

$$L_n = \sqrt{(x_{n-1} - x_{n-1})^2 + (y_n - y_{n-1})^2 + (z_n - z_{n-1})^2} \quad (2.59)$$

The above equation is considered using the differential method, and the strain differential is as follows,

$$\dot{\varepsilon}_n = [(x_n - x_{n-1})(\dot{x}_n - \dot{x}_{n-1}) + (y_n - y_{n-1})(\dot{y}_n - \dot{y}_{n-1}) + (z_n - z_{n-1})(\dot{z}_n - \dot{z}_{n-1})] / (L_{n,0} L_n) \quad (2.60)$$

2.3.3 Computation of External Force

The external forces include gravity, fluid forces and so on. This section mainly discusses the fluid forces, which include the added mass force, normal drag, and tangential drag.

As Fig. 2.13 shows, assuming the cable element is a cylinder, G is the mass center, P is an arbitrary point of the cable element, ζ is the length of GP , $\mathbf{t}$ is the axial vector of the cable element, and the fluid resistance of the arbitrary cable element can be expressed as follows,

$$\mathbf{f} = C_M \frac{\rho \pi d^2}{4} \mathbf{a}_N + C_N \frac{\rho d}{2} |\mathbf{V}_N| \mathbf{V}_N + C_T \frac{\rho d}{2} |\mathbf{V}_T| \mathbf{V}_T \quad (2.61)$$

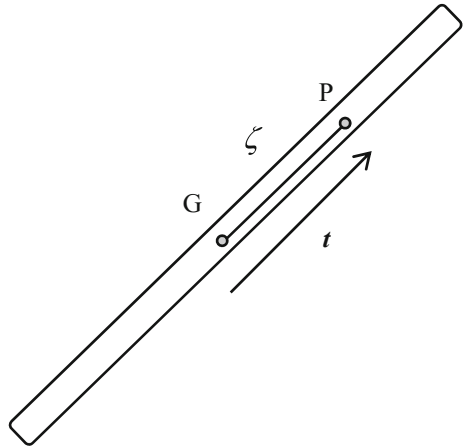
where ρ is the density of the fluid, d is the diameter of the cable section, $\mathbf{a}_N$ is the normal acceleration, C_M is the additional mass force index, $\mathbf{V}_N$ and $\mathbf{V}_T$ are respectively the normal velocity and tangible velocity of the fluid with respect to the cable element, and C_N and C_T are respectively the normal force index and tangible force index.

According to the research in Sect. 2.3.2, the additional mass force is induced by the relative acceleration between the cable and fluid. For a cylindrical cable element, the additional mass index is normally

$$C_M = 1 \quad (2.62)$$

The fluid resistance is mainly induced by the viscosity of the fluid and is relevant to the Reynolds number. If the theoretical analysis is difficult, the normal force index C_N and tangible force index C_T are determined using an experimental equation, and Webster et al. provided the following equation,

Fig. 2.13 Cable element moving in fluid



$$C_N = \begin{cases} 1.455 + 8.55(R_e)^{-0.4} & 1 < R_e < 30 \\ 1 + 4(R_e)^{-0.5} & 30 \leq R_e < 100 \\ 2.35 - 0.45 \ln(R_e) & 100 \leq R_e < 1000 \\ 0.9 & 1000 \leq R_e < 4000 \\ 1.05 + 0.54 \ln(R_e) & 4000 \leq R_e < 15000 \\ 1.21 & 15000 \leq R_e < 150000 \\ 0.3 & 150000 \leq R_e \end{cases} \quad (2.63)$$

$$C_T = \begin{cases} 1.88/(R_{eT})^{0.74} & 0.1 < R_{eT} \leq 100.55 \\ 0.062 & R_{eT} > 100.55 \end{cases} \quad (2.64)$$

Assuming that μ is the dynamic viscosity of the fluid, the Reynolds numbers R_e and R_{eT} can be determined as follows,

$$R_e = \frac{d|V_N|}{\mu} \quad (2.65)$$

$$R_{eT} = \frac{d|V_T|}{\mu} \quad (2.66)$$

The velocity of the fluid around the cable element is V_W , the velocity of point P is V_P , the velocity of the cable element center is V_G , the angular velocity of the cable element is ω , and the velocity of arbitrary point P is founded as follows,

$$V_{W/P} = V_W - V_P = V_W - V_G - (\omega \times t)\zeta \quad (2.67)$$

The normal velocity of arbitrary point P is

$$\begin{aligned} V_N &= V_{W/P} - (V_{W/P} \cdot t)t \\ &= (V_W - V_G) - [(V_W - V_G) \cdot t]t - (\omega \times t)\zeta \end{aligned} \quad (2.68)$$

The normal velocity of the fluid at the mass center is

$$V_{GN} = (V_W - V_G) - [(V_W - V_G) \cdot t]t \quad (2.69)$$

Then, the normal velocity of point P is

$$V_N = V_{GN} - (\omega \times t)\zeta \quad (2.70)$$

If the length of the cable element is L , then the normal fluid resistance $\mathbf{F}_{b.N}$ of the whole cable element is

$$\mathbf{F}_{b.N} = \frac{C_N \rho d}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} |\mathbf{V}_N| \mathbf{V}_N d\zeta \quad (2.71)$$

For a convenient calculation, the normal velocity $|\mathbf{V}_N|$ is expressed as

$$|\mathbf{V}_N| = |a + b\zeta + c\zeta^2| \quad (2.72)$$

where

$$\begin{aligned} a &= \mathbf{V}_{GN} \cdot \mathbf{V}_{GN} \\ b &= -2\mathbf{V}_{GN} \cdot (\boldsymbol{\omega} \times \mathbf{t}) \\ c &= (\boldsymbol{\omega} \times \mathbf{t}) \cdot (\boldsymbol{\omega} \times \mathbf{t}) \end{aligned} \quad (2.73)$$

and

$$\begin{aligned} H_1 &= \left(a + \frac{bL}{2} + \frac{cL^2}{4} \right)^{\frac{1}{2}} \\ H_2 &= \left(a - \frac{bL}{2} + \frac{cL^2}{4} \right)^{\frac{1}{2}} \\ K_1 &= \left(a + \frac{bL}{2} + \frac{cL^2}{4} \right)^{\frac{3}{2}} \\ K_2 &= \left(a - \frac{bL}{2} + \frac{cL^2}{4} \right)^{\frac{3}{2}} \end{aligned} \quad (2.74)$$

Then, the normal fluid resistance of the whole cable element is

$$\mathbf{F}_{b.N} = C_N \frac{d\rho}{2} [A_1 \mathbf{V}_{GN} - A_2 (\boldsymbol{\omega} \times \mathbf{t})] \quad (2.75)$$

where

$$\begin{aligned}
A_1 &= \int_{-\frac{l}{2}}^{\frac{l}{2}} (a + bx + cx^2) dx = \left(\frac{Lc + b}{4c} \right) H_1 \\
&\quad + \left(\frac{Lc - b}{4c} \right) H_2 + \frac{4ac - b^2}{8c^{\frac{3}{2}}} \ln \frac{H_1 + \frac{b+Lc}{2c^{\frac{1}{2}}}}{H_2 + \frac{b-L}{2c^{\frac{1}{2}}}} \\
A_2 &= \int_{-\frac{l}{2}}^{\frac{l}{2}} x(a + bx + cx^2)^{\frac{1}{2}} dx = \frac{c}{3} [K_1 - K_2] \\
&\quad + \frac{b}{8c^2} [(b + cL)H_1 - (b - cL)H_2] \\
&\quad + \frac{b^3 - 4abc}{16c^{\frac{5}{2}}} \ln \frac{H_1 + \frac{b+Lc}{2c^{\frac{1}{2}}}}{H_2 + \frac{b-L}{2c^{\frac{1}{2}}}}
\end{aligned} \tag{2.76}$$

Through a similar derivation, the tangible velocity of point P is

$$\mathbf{V}_T = (\mathbf{V}_{W/P} \cdot \mathbf{t}) \mathbf{t} \tag{2.77}$$

The relative tangible velocity of the fluid at the mass center is V_{GT} , and the relative tangible velocity of point P can be further expressed as

$$\mathbf{V}_T = (\mathbf{V}_{W/G} \cdot \mathbf{t}) \mathbf{t} = V_{GT} \tag{2.78}$$

Then, the tangible fluid resistance of the whole cable element is

$$\begin{aligned}
\mathbf{F}_{b,T} &= \frac{C_T \rho d}{2} \int_{-\frac{l}{2}}^{\frac{l}{2}} |\mathbf{V}_T| \mathbf{V}_T d\zeta \\
&= \frac{C_T \rho d L |\mathbf{V}_{GT}| \mathbf{V}_{GT}}{2}
\end{aligned} \tag{2.79}$$

If the acceleration of the fluid is $\mathbf{a}_W$, and the acceleration of the cable element at mass center G is $\mathbf{a}_G$, then the relative fluid acceleration at mass center G is

$$\mathbf{a}_{W/G} = \mathbf{a}_W - \mathbf{a}_G \tag{2.80}$$

The normal velocity of the fluid at mass center G is

$$\mathbf{a}_{GN} = \mathbf{a}_{W/G} - (\mathbf{a}_{W/G} \cdot \mathbf{t}) \mathbf{t} \tag{2.81}$$

If the angular velocity and angular acceleration of the cable elements are set as ω and ε , respectively, the relative normal acceleration of the fluid at arbitrary point P can be expressed as follows,

$$\mathbf{a}_N(\zeta) = \mathbf{a}_{GN} - \varepsilon \times \mathbf{t}\zeta - \omega \times \omega \times \mathbf{t}\zeta - \zeta(\omega \cdot \mathbf{t})^2 \mathbf{t} - \zeta|\omega|^2 \mathbf{t} \quad (2.82)$$

The inertial force induced by the additional mass can be expressed as

$$\mathbf{F}_{b,A} = C_M \frac{\pi d^2}{4} \int_{-L/2}^{L/2} \mathbf{a}_N d\zeta = C_M \frac{L\pi d^2}{4} \mathbf{a}_{GN} \quad (2.83)$$

The additional mass of the cable element can be expressed as

$$m_{a,n} = C_M \frac{L\pi d^2}{4} \quad (2.84)$$

Based on Eqs. (2.75) and (2.79), the fluid force of the cable elements can be expressed as

$$\mathbf{F}_b = \mathbf{F}_{b,N} + \mathbf{F}_{b,T} \quad (2.85)$$

2.3.4 System Dynamics

The mass of a cable element is set as m_n . Then, the mass matrix of the n th cable element in the body-fixed coordinate system can be expressed as

$$\mathbf{M}_{b,n} = \text{diag}[m_n + m_{a,n} \quad m_n + m_{a,n} \quad m_n] \quad (2.86)$$

If half the n th mass and half the $n + 1$ th mass are concentrated on the n th cable element with an approximate processing method, then the mass matrix of the n th cable element in the inertial coordinate system can be expressed as

$$\mathbf{M}_n = \frac{1}{2} [\mathbf{B}_n \mathbf{M}_{b,n} \mathbf{B}_n^T + \mathbf{B}_{n+1} \mathbf{M}_{b,n+1} \mathbf{B}_{n+1}^T] \quad (2.87)$$

If the array of fluid force $\mathbf{F}_{b,n}$ in the fixed body coordinate system is $\mathbf{F}_{b,n}$, and half the n th external force $\mathbf{F}_{b,n}$ and half the $n + 1$ th external force $\mathbf{F}_{b,n+1}$ are concentrated on the n th cable element with an approximate processing method, then the fluid force of the n th cable element in the inertial coordinate system can be expressed as

$$F_n = \frac{1}{2}(B_n F_{b,n} + B_{n+1} F_{b,n+1}) \quad (2.88)$$

If the array of internal force T_n in the body coordinate system $ox_n y_n z_n$ is T_n , then based on Newton's second law, the dynamic equation of the n th cable element can be expressed as

$$M_n [\ddot{x}_n \quad \ddot{y}_n \quad \ddot{z}_n]^T = B_{n+1} T_{n+1} - B_n T_n + F_n + M_n g \quad (2.89)$$

If the accelerations of all the nodes are solved using the above equations, then the velocity of the whole cable can be obtained.

2.4 Multi-rigid-body Model of Cable

2.4.1 Description of Model

As shown in Fig. 2.14, bodies A and B are connected by a long cable. Then, the cable is dispersed into N elements, and each cable element is assumed to be a slim rigid rod with the mass concentrated at the end, with a spherical hinge connecting the adjacent rods, the cable system is processed into a multi-rigid-body system. Bodies A and B are set as mass points, and the additional masses of the cable elements are neglected for simplification.

The bodies at the ends of the cable are marked as the 0th and N th points. The length of the n th cable unit is L_n , and the concentrated masses at the ends are marked as m_{n-1} and m_n , which can be used to refer to the ends of the n th cable unit for simplification. The coordinate vectors of the n th point in inertial coordinate system $ox_d y_d z_d$ are x_n , y_n , and z_n . As shown in Fig. 2.15, the location of each cable

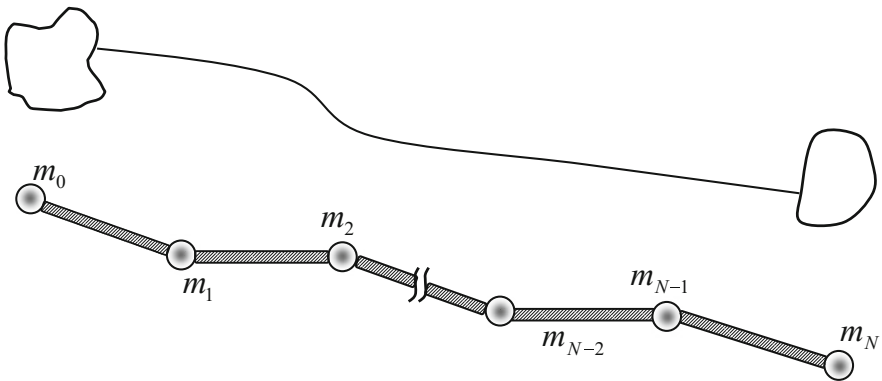
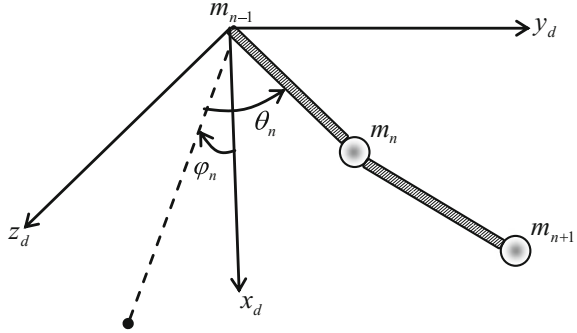


Fig. 2.14 Diagram of cable system

Fig. 2.15 Geometry description of cable element



element in the inertial coordinate system can be defined by Euler angles φ_n and θ_n , where θ_n is the angle between the n th cable element and coordinate plane ox_dz_d , and φ_n is the angle between the ejection of the n th cable unit in coordinate plane ox_dz_d and axis ox_d . The recurrence relation of the n th cable element and $n - 1$ th cable element is as follows,

$$\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = \begin{bmatrix} x_{n-1} \\ y_{n-1} \\ z_{n-1} \end{bmatrix} + L_n \begin{bmatrix} \cos \theta_n \cos \varphi_n \\ \sin \theta_n \\ \cos \theta_n \sin \varphi_n \end{bmatrix}, \quad n = 1, 2, \dots, N \quad (2.90)$$

With the derivation of the above equation, the recurrence relation of the cable element velocity ($V_{n,x}$, $V_{n,y}$, $V_{n,z}$) is as follows,

$$\begin{bmatrix} V_{n,x} \\ V_{n,y} \\ V_{n,z} \end{bmatrix} = \begin{bmatrix} V_{n-1,x} \\ V_{n-1,y} \\ V_{n-1,z} \end{bmatrix} + L_n \begin{bmatrix} -\dot{\theta}_n \sin \theta_n \cos \varphi_n - \dot{\varphi}_n \cos \theta_n \sin \varphi_n \\ \dot{\theta}_n \cos \theta_n \\ -\dot{\theta}_n \sin \theta_n \sin \varphi_n + \dot{\varphi}_n \cos \theta_n \cos \varphi_n \end{bmatrix}, \quad (2.91)$$

($n = 1, 2, \dots, N$)

Then, with the derivation of Eq. (2.91), the recurrence relation of the cable element acceleration ($a_{n,x}$, $a_{n,y}$, $a_{n,z}$) is as follows,

$$\begin{bmatrix} a_{n,x} \\ a_{n,y} \\ a_{n,z} \end{bmatrix} = \begin{bmatrix} a_{n-1,x} \\ a_{n-1,y} \\ a_{n-1,z} \end{bmatrix} + L_n \begin{bmatrix} -[(\dot{\varphi}_n)^2 + (\dot{\theta}_n)^2] \cos \theta_n \cos \varphi_n + 2\dot{\theta}_n \dot{\varphi}_n \sin \theta_n \sin \varphi_n \\ -(\dot{\theta}_n)^2 \sin \theta_n \\ -[(\dot{\varphi}_n)^2 + (\dot{\theta}_n)^2] \cos \theta_n \sin \varphi_n - 2\dot{\theta}_n \dot{\varphi}_n \sin \theta_n \cos \varphi_n \end{bmatrix} \\ + L_n \begin{bmatrix} -\ddot{\theta}_n \sin \theta_n \cos \varphi_n - \ddot{\varphi}_n \cos \theta_n \sin \varphi_n \\ \ddot{\theta}_n \cos \theta_n \\ -\ddot{\theta}_n \sin \theta_n \sin \varphi_n + \ddot{\varphi}_n \cos \theta_n \cos \varphi_n \end{bmatrix} \quad N = 1, 2, \dots, N \quad (2.92)$$

Equations (2.90) and (2.91) are obviously the geometric constraint and motion constraint of the cable system, respectively.

2.4.2 Recurrence of Constraint Force

For the multi-rigid-body model of the cable system, each cable end is loaded with the gravity force, hinge constraint force, and other external forces (aerodynamic resistance, etc.), and the forces loaded on the n th cable end are marked as $\mathbf{F}_n(F_{n.x}, F_{n.y}, F_{n.z})$. According to Newton's second law, the dynamic equation of the 0th cable end is derived as follows,

$$m_0 \begin{bmatrix} a_{0.x} \\ a_{0.y} \\ a_{0.z} \end{bmatrix} = \begin{bmatrix} T_1 \cos \theta_1 \cos \varphi_1 \\ T_1 \sin \theta_1 \\ T_1 \cos \theta_1 \sin \varphi_1 \end{bmatrix} + \begin{bmatrix} m_0 g \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} F_{0.x} \\ F_{0.y} \\ F_{0.z} \end{bmatrix} \quad (2.93)$$

where T_1 is the internal force in the 1st cable. The dynamic equations of the other cable ends can be derived using a similar method.

$$m_n \begin{bmatrix} a_{n.x} \\ a_{n.y} \\ a_{n.z} \end{bmatrix} = - \begin{bmatrix} T_n \cos \theta_n \cos \varphi_n \\ T_n \sin \theta_n \\ T_n \cos \theta_n \sin \varphi_n \end{bmatrix} + \begin{bmatrix} T_{n+1} \cos \theta_{n+1} \cos \varphi_{n+1} \\ T_{n+1} \sin \theta_{n+1} \\ T_{n+1} \cos \theta_{n+1} \sin \varphi_{n+1} \end{bmatrix} + \begin{bmatrix} m_n g \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} F_{n.x} \\ F_{n.y} \\ F_{n.z} \end{bmatrix} \quad (2.94)$$

$$m_N \begin{bmatrix} a_{N.x} \\ a_{N.y} \\ a_{N.z} \end{bmatrix} = - \begin{bmatrix} T_N \cos \theta_N \cos \varphi_N \\ T_N \sin \theta_N \\ T_N \cos \theta_N \sin \varphi_N \end{bmatrix} + m_N \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} F_{N.x} \\ F_{N.y} \\ F_{N.z} \end{bmatrix} \quad (2.95)$$

Substituting Eq. (2.92) into Eq. (2.94), we get

$$\begin{aligned} & \frac{m_n}{m_{n-1}} \left\{ \begin{bmatrix} T_n \cos \theta_n \cos \varphi_n \\ T_n \sin \theta_n \\ T_n \cos \theta_n \sin \varphi_n \end{bmatrix} - \begin{bmatrix} T_{n-1} \cos \theta_{n-1} \cos \varphi_{n-1} \\ T_{n-1} \sin \theta_{n-1} \\ T_{n-1} \cos \theta_{n-1} \sin \varphi_{n-1} \end{bmatrix} + m_{n-1} \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} F_{n-1.x} \\ F_{n-1.y} \\ F_{n-1.z} \end{bmatrix} \right\} \\ & + L_n m_n \begin{bmatrix} -\ddot{\theta}_n \sin \theta_n \cos \varphi_n - \ddot{\varphi}_n \cos \theta_n \sin \varphi_n \\ \ddot{\theta}_n \cos \theta_n \\ \ddot{\theta}_n \sin \theta_n \sin \varphi_n + \ddot{\varphi}_n \cos \theta_n \cos \varphi_n \end{bmatrix} L_n m_n \begin{bmatrix} -[(\dot{\varphi}_n)^2 + (\dot{\theta}_n)^2] \sin \theta_n \cos \varphi_n + 2\dot{\theta}_n \dot{\varphi}_n \sin \theta_n \sin \varphi_n \\ -(\dot{\theta}_n)^2 \sin \theta_n \\ -[(\dot{\varphi}_n)^2 + (\dot{\theta}_n)^2] \cos \theta_n \sin \varphi_n - 2\dot{\theta}_n \dot{\varphi}_n \sin \theta_n \cos \varphi_n \end{bmatrix} \\ & = - \begin{bmatrix} T_n \cos \theta_n \cos \varphi_n \\ T_n \sin \theta_n \\ T_n \cos \theta_n \sin \varphi_n \end{bmatrix} + \begin{bmatrix} T_{n+1} \cos \theta_{n+1} \cos \varphi_{n+1} \\ T_{n+1} \sin \theta_{n+1} \\ T_{n+1} \cos \theta_{n+1} \sin \varphi_{n+1} \end{bmatrix} + m_n \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} F_{n.x} \\ F_{n.y} \\ F_{n.z} \end{bmatrix} \quad (2.96) \end{aligned}$$

After simplification, the above equation transforms into

$$\begin{bmatrix} T_{n+1} \\ \dot{\theta}_n \\ \ddot{\theta}_n \end{bmatrix} = G_n^{-1}(\theta_n, \varphi_n, \theta_{n+1}, \varphi_{n+1}) K_n \quad (2.97)$$

where

$$G_n(\theta_n, \varphi_n, \theta_{n+1}, \varphi_{n+1}) = \begin{bmatrix} \cos \theta_{n+1} \cos \varphi_{n+1} & L_n m_n \sin \theta_n \cos \varphi_n & L_n m_n \cos \theta_n \sin \varphi_n \\ \sin \theta_{n+1} & -L_n m_n \cos \theta_n & 0 \\ \cos \theta_{n+1} \sin \varphi_{n+1} & L_n m_n \sin \theta_n \sin \varphi_n & -L_n m_n \cos \theta_n \cos \varphi_n \end{bmatrix} \quad (2.98)$$

$$K_n = -\frac{m_n}{m_{n-1}} \left\{ \begin{bmatrix} T_{n-1} \cos \theta_{n-1} \cos \varphi_{n-1} \\ T_{n-1} \sin \theta_{n-1} \\ T_{n-1} \cos \theta_{n-1} \sin \varphi_{n-1} \end{bmatrix} + \begin{bmatrix} F_{n-1,x} \\ F_{n-1,y} \\ F_{n-1,z} \end{bmatrix} \right\} + \frac{m_n + m_{n-1}}{m_{n-1}} \begin{bmatrix} T_n \cos \theta_n \cos \varphi_n \\ T_n \sin \theta_n \\ T_n \cos \theta_n \sin \varphi_n \end{bmatrix} - \begin{bmatrix} F_{n,x} \\ F_{n,y} \\ F_{n,z} \end{bmatrix} \\ + L_n \begin{bmatrix} -[(\dot{\varphi}_n)^2 + (\dot{\theta}_n)^2] \sin \theta_n \cos \varphi_n + 2\dot{\theta}_n \dot{\varphi}_n \sin \theta_n \sin \varphi_n \\ -(\dot{\theta}_n)^2 \sin \theta_n \\ -[(\dot{\varphi}_n)^2 + (\dot{\theta}_n)^2] \sin \theta_n \sin \varphi_n - 2\dot{\theta}_n \dot{\varphi}_n \sin \theta_n \cos \varphi_n \end{bmatrix} \quad (2.99)$$

The recurrence relation of the hinge constraint force can be derived through Eq. (2.97):

$$T_{n+1} = A_{n+1} T_n + B_{n+1} T_{n-1} + C_{n+1}, \quad n = 2, 3, \dots, N-1 \quad (2.100)$$

Because matrices G_n and K_n are only relevant to the displacement and velocity, indexes A_{n+1} , B_{n+1} and C_{n+1} are only relevant to the displacement and velocity. The recurrence relation of the constraint force between the beginning point and end point can be obtained using a similar method,

$$T_2 = A_2 T_1 + C_2 \quad (2.101)$$

$$A_{N+1} T_N + B_{N+1} T_{N-1} + C_{N+1} = 0 \quad (2.102)$$

Equations (2.100)–(2.102) include

$$\alpha T_1 + \beta = 0 \quad (2.103)$$

where α and β are only relevant to the velocity and displacement. Based on the multi-rigid-body model, the constraint force T_n is solved using the CFA method, and the acceleration of each cable end can be calculated using Eqs. (2.93)–(2.95).

2.5 Conclusion

This chapter mainly discussed the dynamics of a slim cable. First, the nonlinear characteristics of the cable are shown through a simple numeric example. Then, the spring damper model, multi-rigid-body model, and continuous model are developed. Specifically, the fluid force loaded on the cable elements is discussed using the spring damper model. The dynamic models of a cable proposed in this chapter can be applied in the dynamics and control of a flexible net.

Dynamics and Design of Space Nets for Orbital Capture

Yang, L.; Zhang, Q.; Zhen, M.; Liu, H.

2017, XV, 174 p. 148 illus., Hardcover

ISBN: 978-3-662-54062-6