Chapter 2
Model of Exoskeleton Suit

2.1 Introduction

To facilitate the analysis of the characteristics of the exoskeleton suit system, assume that every link of the exoskeleton suit is rigid connecting rod, and the connecting rods are connected through joints to form a multi-rigid-body chain structure. The abstract model based on the assumption can be used as the object of our theoretical research and analysis.

The mathematical model of multi-connecting rods' object generally consists of two parts: the kinematics and dynamics [1]. In the control of the exoskeleton suit, the position relationship and the speed relationship among the joint connecting rods of exoskeleton suit can be directly measured, namely the angle and the angular velocity of each joint. How to calculate the pose of the end effector in the operating space according to the position and velocity of each connecting rod (the terminal of support phase is gravity center of the body, and the terminal of the swing phase is ankle) is known as exoskeleton suit's kinematic analysis or forward kinematics problem. On the contrary, according to the position of the end effector in the operating space and speed expectation, calculate the corresponding position and speed of each joint's connecting rod reversely, which is called kinematic synthesis or inverse kinematics problem of the robot. And the relationship between exoskeleton suit' movement and the force and torque that produce this movement becomes the dynamics problem, which also can be divided into two problems: according to the torque or force acting on the joints, calculating the joints' position, velocity and acceleration is the forward dynamics problem; according to the desired joints' position, velocity and acceleration, calculating the required torque or force is called inverse dynamics problem [1–5].

This chapter first describes the book's research object, the exoskeleton suit in detail. Then, it analyzes the forward kinematics and inverse dynamics problems based on the mechanical model of an exoskeleton suit and establishes the kinematics and dynamics models of the exoskeleton suit. At the same time, it describes

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the human body model, human-machine interaction model, and the virtual prototype research method of the exoskeleton suit and provides the human-machine interaction models that can satisfy the requirement of simulation assumptions.

2.2 Basic Description of Exoskeleton Suit

2.2.1 Mechanical Model of Exoskeleton Suit

From the aspects of energy consumption and realized functions, there are two development trends for the exoskeleton suit.

1. The whole body exoskeleton suit

The whole body-type exoskeleton suit refers to the exoskeleton that contains upper and lower limbs and the exoskeleton whose all main movement joints have actuators. This exoskeleton suit's representative is the "WEAR" developed by SARCOS Company in the USA, which mainly drives such joints as the hip, knee and ankle joints of two legs, and shoulder, elbow, and wrist joints of two arms. The WEAR tries to reduce the operator's energy consumption as much as possible and increase more functions, such as protective armor, the body air-conditioning equipment, color clothes, and flexible fabric (instruments), and even realize the function to fly.

The whole body-type exoskeleton suit looks at the future, in order to meet human's dream of becoming superman. But it requires more drive joints: Not only the structure is complicated, but also energy consumption is large. Now, we only can stay in the laboratory dragging long cable under the condition without miniaturization, high density, and portable energy devices.

2. The lower limb-type exoskeleton suit

Lower limb-type exoskeleton suit refers to the exoskeleton that only contains lower limbs. The representative of the exoskeleton suit is Berkeley's HULC of the USA. The exoskeleton suit's desired function is very simple, that is, to help soldiers carry load and reduce the energy consumption. It drives a few joints selectively, for example, HULC and MIT exoskeleton suit only drives the knee joint, trying to transfer the load to the ground as much as possible.

Because the lower limb-type exoskeleton suit's function is single and the number of its driving joints is reduced, its structure design is relatively simple. And because of its lighter weight, smaller volume, reduced power consumption, and increased portability, the function is easy to be realized under current technical level.

This book mainly introduces the lower limb-type exoskeleton suit, providing an instance of exoskeleton suit as shown in Fig. 2.1 [6]. The exoskeleton suit is composed of seven rigid connecting rods, namely the trunk (back frame), two legs, two shanks, and two feet. The trunk of exoskeleton suit carries the load, and the load can be seen as part of the trunk.

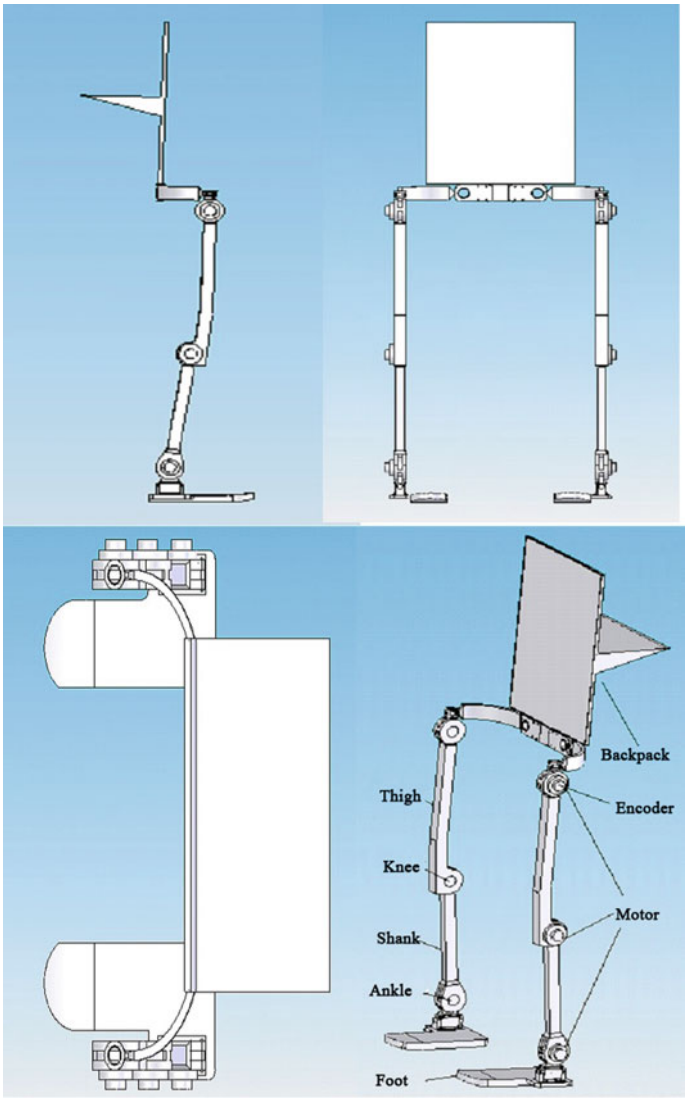


Fig. 2.1 Mechanical model of the exoskeleton suit

Simplify the mechanical model drawings as shown in Fig. 2.1 to plane sketch as shown in Fig. 2.2. The *Y*-axis of the vertical direction and the *X*-axis of the horizontal direction are in a longitudinal plane, as shown in Fig. 2.2a. When analyzing the exoskeleton suit's movement and control theoretically, considering exoskeleton suit's most basic movement form is forward walking in flat ground, and the range of each joint movement in longitudinal plane is bigger than the range in other planes. Therefore, this book will make theoretical analysis and research based on the

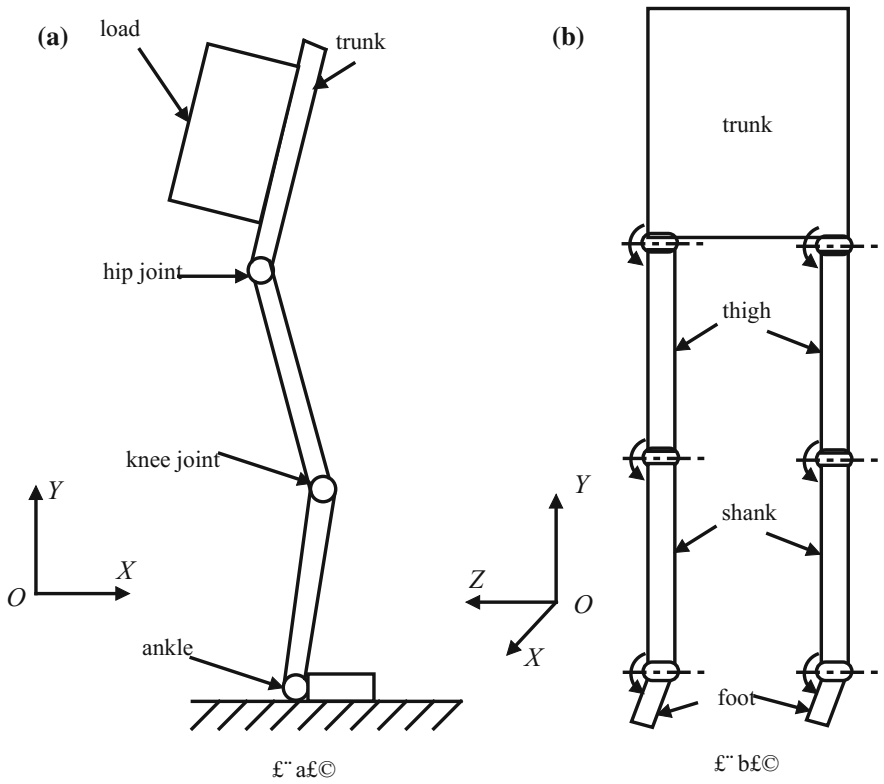


Fig. 2.2 Simplified model of the exoskeleton suit: **a** side view and **b** front view

exoskeleton suit models as shown in Figs. 2.1 and 2.2 and will limit the theoretical analysis and discussion of the exoskeleton suit within the range of the longitudinal plane, only researching each link's flexion and extension movement around coronary axis, imposing drive, and ignoring movement other planes, which decreases the complexity of the model and simplifies the design of the structure. Although there is no drive in transverse plane (refers to the external drive), there is also need to design the corresponding degrees of freedom, and it is driven by the human body's energy, in order to follow the human body's movement; the energy consumption is lesser, so it can be ignored.

2.2.2 Segment Properties of Exoskeleton Suit

In the later parts of this book, we will build the exoskeleton suit's kinematic and dynamic models according to the above exoskeleton suit models. Before modeling, first define the physical properties of the studied exoskeleton suit, as follows: call

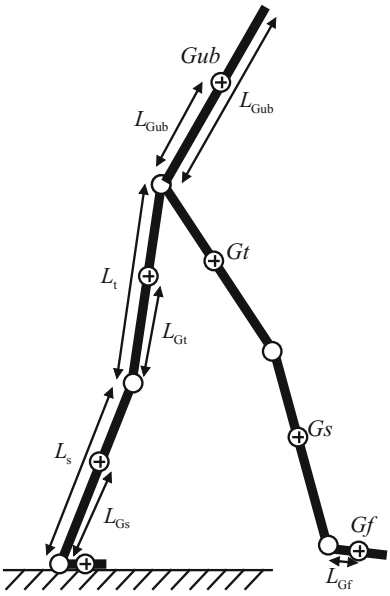


Fig. 2.3 Link properties of the exoskeleton suit

each connecting rod of the exoskeleton suit, namely the thighs, the shanks, feet, and torso, a link. The links' attributes include quality, rotational inertia, and length as shown in Fig. 2.3. These attribute parameters, as the most basic modeling parameter, must be determined in advance.

Define that m represents quality, I represents rotational inertia around the center of mass, G represents the center of mass, L represents the rod length, and L_G represents the components from joint points to the center of gravity in the reference coordinate system. Then the parameters of exoskeleton suit's each part are as follows:

(1) Feet: m_f, I_f, L_f, L_{Gf}
(2) Shanks: m_s, I_s, L_s, L_{Gs}
(3) Thighs: m_t, I_t, L_t, L_{Gt}
(4) Trunk: $m_{ub}, I_{up}, L_{up}, L_{Gup}$.

For the convenience of study, this book ignores the offset of the center of gravity of the exoskeleton suit trunk (including load) in the longitudinal plane. In the theoretical analysis stage of verification, this assumption is reasonable.

2.2.3 *Definition of Coordinate System*

For both kinematics model and dynamics model, the pose of exoskeleton suit's each connecting rod needs to be defined. To define the pose of each connecting rod, we

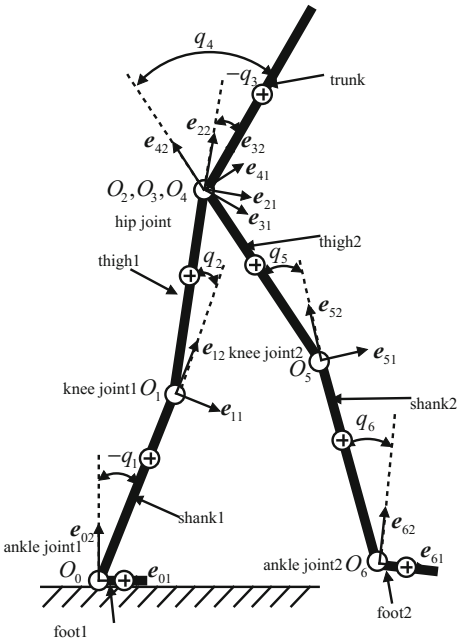


Fig. 2.4 Coordinate system and symbol definition

need first define the local coordinate system of each rod, and using coordinate system describes geometric relationship of all the exoskeleton suit's entities. The local coordinate system defining the exoskeleton suit is shown in Fig. 2.4 [7].

Reference coordinate system (coordinate system 0) is defined in the landing place of the heel, $\boldsymbol{e}_{01}$ is in parallel to the instep, and pointing from ankle to tiptoe, $\boldsymbol{e}_{02}$, is perpendicular to the instep. Other coordinate systems except reference coordinate system are associated with the system status. The other coordinate systems are defined as follows:

(1) Coordinate system 1 is fixed in knee joint 1 of the standing leg 1, and $-\boldsymbol{e}_{12}$ points to the ankle 1 of standing foot.
(2) Coordinate system 2 is fixed in the hip joint of standing leg 1, and $-\boldsymbol{e}_{22}$ points to the knee joint 1 of the standing leg.
(3) Coordinate system 3 is fixed in the hip joint of the trunk, and $\boldsymbol{e}_{32}$ points to the head.
(4) Coordinate system 4 is fixed in the hip joint of swing leg 2, and $-\boldsymbol{e}_{42}$ points to knee joint 2 of the swing leg.
(5) Coordinate system 5 is fixed in knee joint 2 of swing leg 2, and $-\boldsymbol{e}_{52}$ points to ankle 2 of the swing leg.
(6) Coordinate system 6 is fixed in ankle 2 of swing foot 2, and $-\boldsymbol{e}_{61}$ points to the tiptoe of swing foot 2.

In the figure, $O_i(i = 0, \ldots, 6)$ represents the original point of each coordinate system; $\boldsymbol{e}_{ij}$ represents the unit vector expressed by the i in the coordinate system. q_i $(i = 1, \ldots, 6)$ represents each joint angle, and the counterclockwise is positive; q_i is defined as follows, respectively:

(1): q_1 represents the flexion angle of ankle joint 1.
(2): q_2 represents the flexion angle of knee joint 1.
(3): q_3 represents hip joint's stretching angle of thigh 1.
(4): q_4 represents the hip joint's flexion angle of thigh 2.
(5): q_5 represents the flexion angle of knee joint 2.
(6): q_6 represents the flexion angle of ankle joint 2.

2.2.4 Partition of the Model

According to the constraint relationship between the exoskeleton suit and the ground and at the same time considering the movement status of the legs, Racine has divided the exoskeleton suit's walking process into several main models [7], as follows:

(1) Jumping model is the state in which the feet are off the ground and do not touch the ground.
(2) Single support model is the state in which one foot touches the ground and the other foot is completely off the ground.
(3) Double support model is the state in which the two soles fully touch the ground.
(4) Double support and single redundance state is the state in which a sole fully touches the ground, and for the other one, only the toe or heel touches the ground.
(5) Double support and double redundance state is the state in which for either foot, only the toe or heel touches the ground.

When using this classification method, there are too many models, and the dynamic equation is complex. For example, the single support model as shown in Fig. 2.4, due to considering the movement of two legs at the same time, there are seven degrees of freedom (in the figure, the rotational degree of freedom of the tiptoe rotating on the ground caused by tiptoe touching the land and heel being off the land is not given).

In this book, models of each leg are classified alone, which are divided into support model and swing model. This method is more simple, for example, for the left leg, in every moment, no matter what state the right leg is in, there are only two kinds of states for the left leg, one being the support model and the other being the swing model, and there are only three degrees of freedom for each model (only considering the longitudinal plane). For the support model as shown in Fig. 2.5a,

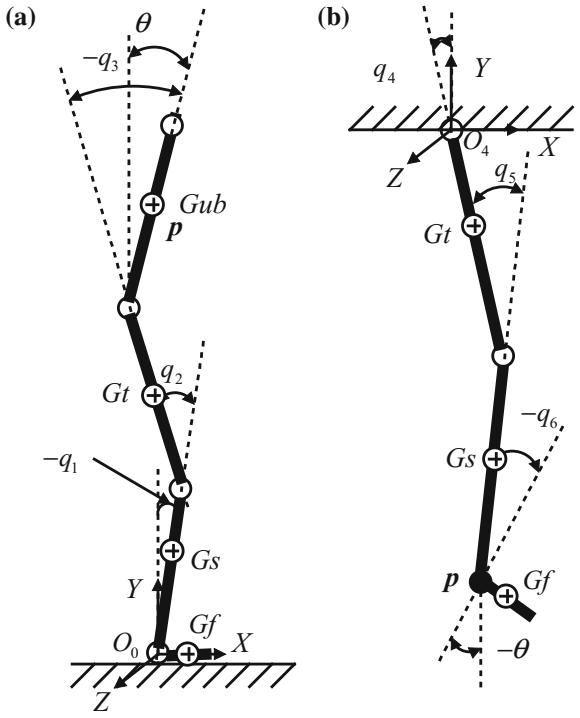


Fig. 2.5 Exoskeleton suit's model classification and the end effector's schematic diagram: **a** the support leg and **b** the swing leg

the left leg is composed of three connecting rods of shanks, thighs, and torso. For the swing model as shown in Fig. 2.5b, the left leg is composed of three connecting rods of thighs, shanks, and feet. The model classification method simplifies the exoskeleton suit's model, and for each leg's models, just switch between these two kinds.

Strictly speaking, there are mutual influence relations between two legs. For example, when the left leg is in the support model and the right leg in the swing model, all the weight of the torso is borne by the left leg in the support model; when the right leg is in the support model, the weight of the torso is borne by the two legs together. For left leg support state model, because of the change of right leg model, the relative weight of the torso changes, causing the change of model parameters, but this change can be predicted by judging the right leg model, so the influence on system model parameter can be remedied. Due to the limitation of the book's length, this book does not research the mutual influence between the two legs for the moment.

In this book, we will establish the models of exoskeleton suit's each leg separately, and we divided each leg's models into support model and swing model for analysis. The method of establishing the kinematics model and dynamics model will be introduced in the next parts.

2.3 Kinematics Model of Exoskeleton Suit

2.3.1 Position and Orientation Description of Rigid Body

Any joint connecting rod of the exoskeleton suit can be regarded as a rigid body, and the pose of the rigid body in space is made up of the rigid body's position ${}^0\boldsymbol{p} = [p_x \quad p_y \quad p_z]^{\mathrm{T}}$ and orientation in reference coordinate system $\{0\}$. The position can be represented with three-dimensional vector $\boldsymbol{p}$, namely

$$ {}^0\boldsymbol{p} = [p_x \quad p_y \quad p_z]^{\mathrm{T}} \tag{2.1} $$

And orientation can be represented with rotation matrix $\boldsymbol{R}$ from the local coordinate system $\{i\}$ fixed on the rigid body to the reference coordinate system $\{0\}$, namely

$$ {}^0_i\boldsymbol{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{2.2} $$

The pose of any point $\boldsymbol{p}$ in the space is different for different coordinate systems, and the expression of $\boldsymbol{p}$ among different coordinate systems can be obtained by translating coordinate transformation and rotating coordinate transformation. If the orientation of coordinate system $\{i\}$ and that of coordinate system $\{j\}$ are same, but their origins are different, then the origin's vector of coordinate system $\{j\}$ in the coordinate system $\{i\}$ is ${}^i\boldsymbol{p}_{Oj}$. Assuming position vector of the point $\boldsymbol{p}$ in coordinate system $\{j\}$ is ${}^j\boldsymbol{p}$, then, its position vector ${}^i\boldsymbol{p}$ in coordinate system $\{i\}$ can be obtained by the formula

$$ {}^i\boldsymbol{p} = {}^j\boldsymbol{p} + {}^i\boldsymbol{p}_{o_j} \tag{2.3} $$

If the origin O_i of coordinate system $\{i\}$ coincides with the origin O_j of coordinate system $\{j\}$, their orientations are different, as shown in Fig. 2.6. Coordinate system $\{j\}$ can be obtained in this way: First, coordinate system $\{i\}$ revolves ψ angle around y_i-axis to obtain coordinate system $\{O_i x' y_i z'\}$, then $\{O_i x' y_i z'\}$ revolves θ angle around z'-axis to obtain coordinate system $\{O_i x_j y' z'\}$, and then $\{O_i x_j y' z'\}$ revolves γ angle around x_j-axis to obtain coordinate system $\{O_j x_j y_j z_j\}$, namely coordinate system $\{j\}$. Use the ${}^i_j\boldsymbol{R}$ representing rotation matrix of the coordinate system $\{j\}$ relative to the coordinate system $\{i\}$; use the ${}^i_{O_i x' y_i z'}\boldsymbol{R}$ representing rotation matrix of the coordinate system $\{O_i x' y_i z'\}$ relative to the coordinate system $\{i\}$; use the ${}^{O_i x' y_i z'}_{O_i x_j y' z'}\boldsymbol{R}$ representing rotation matrix of the coordinate system $\{O_i x_j y' z'\}$ relative to the coordinate system $\{O_i x' y_i z'\}$; use the ${}^{O_i x_j y' z'}_{j}\boldsymbol{R}$ representing rotation

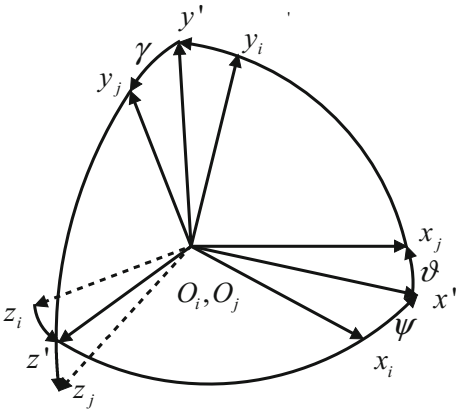


Fig. 2.6 Rotation transformation of the coordinate

matrix of the coordinate system $\{j\}$ relative to the coordinate system $\{O_i x_j y' z'\}$; then,

$$ {}_{O_i x' y_i z'}^{\ \ \ \ \ \ \ \ i}\boldsymbol{R} = \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix} \tag{2.4} $$

$$ {}_{O_i x_j y' z'}^{O_i x' y_i z'}\boldsymbol{R} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.5} $$

$$ {}_{j}^{\ \ \ \ \ \ \ \ O_i x_j y' z'}\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \tag{2.6} $$

$$ {}_{j}^{i}\boldsymbol{R} = {}_{O_i x' y_i z'}^{\ \ \ \ \ \ \ \ i}\boldsymbol{R} \cdot {}_{O_i x_j y' z'}^{O_i x' y_i z'}\boldsymbol{R} \cdot {}_{j}^{\ \ \ \ \ \ \ \ O_i x_j y' z'}\boldsymbol{R} \tag{2.7} $$

Assuming position vector of the point $\boldsymbol{p}$ in coordinate system $\{j\}$ is ${}^{j}\boldsymbol{p}$, then, its position vector ${}^{i}\boldsymbol{p}$ in coordinate system $\{i\}$ can be obtained by the formula

$$ {}^{i}\boldsymbol{p} = {}_{j}^{i}\boldsymbol{R} \cdot {}^{j}\boldsymbol{p} \tag{2.8} $$

Using rotation matrix ${}_{j}^{i}\boldsymbol{R}$, three-dimensional space can be transformed from a given posture to any posture; therefore, the three variables (ψ, θ, γ) can represent any posture, often called yawing–pitching–rolling notation, also known as $X-Y-Z$ Euler angle.

For the most general case, namely, the origin of coordinate system $\{j\}$ does not coincide with that coordinate system $\{j\}$, and their orientations are different. Then, the position vector can be obtained through the composite transformation of

translation transformation and rotation transformation; namely, position vector of the point $\boldsymbol{p}$ in coordinate system $\{j\}$ is ${}^{j}\boldsymbol{p}$

$$ {}^{i}\boldsymbol{p} = {}^{i}_{j}\boldsymbol{R} \cdot {}^{j}\boldsymbol{p} + {}^{i}\boldsymbol{p}_{Oj} \tag{2.9} $$

But the equation (2.9) is not homogeneous for ${}^{j}\boldsymbol{p}$ and so generally transforms the equation (2.9) into equivalent homogeneous transformation form

$$ \begin{bmatrix} {}^{i}\boldsymbol{p} \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{i}_{j}\boldsymbol{R} & {}^{i}\boldsymbol{p}_{Oj} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} {}^{j}\boldsymbol{p} \\ 1 \end{bmatrix} \tag{2.10} $$

Under the condition of without causing confusion, use 4×1 column vector representing point coordinates of three-dimensional coordinate system, called homogeneous coordinates, and still mark them as ${}^{i}\boldsymbol{p}$ and ${}^{i}\boldsymbol{p}$. Then, the equation (2.10) can be written in the following form:

$$ {}^{i}\boldsymbol{p} = {}^{i}_{j}\boldsymbol{T} \cdot {}^{j}\boldsymbol{p} \tag{2.11} $$

In the equation,

$$ {}^{i}_{j}\boldsymbol{T} = \begin{bmatrix} {}^{i}_{j}\boldsymbol{R} & {}^{i}\boldsymbol{p}_{Oj} \\ 0 & 1 \end{bmatrix} \tag{2.12} $$

The equation (2.12) is called the homogeneous transformation matrix. If we have known the position vector of point $\boldsymbol{p}$ in the coordinate system $\{i\}$ or $\{j\}$, then we can easily obtain the position vector in another coordinate system by homogeneous transformation equation (2.11).

2.3.2 Kinematics Model

Kinematics model describes the relationship between the space pose and joint angle. The space pose refers to the space pose of the end effector, and the space in which the space pose is located is generally referred to as the operating space, work space, or task space, and in this book, we will call the space as operating space. The space in which the joint angle is located is called the joint space. So we can say the kinematics model describes the relationship between the joint space and operating space. Generally, the robot's end effector is unique, while the exoskeleton suit's end effector is related to the system's state. If a certain leg of the exoskeleton suit is in the support model, then the foot-supporting point of exoskeleton suit is regarded as the base coordinate, and the trunk's center of gravity of the exoskeleton suit is regarded as the end effector of the supporting leg connecting rod, as shown in

Fig. 2.5a. If a certain leg of the exoskeleton suit is in the swing model, then the hip joint of the leg is regarded as the base coordinate, and the ankle of exoskeleton suit is regarded as the end effector of the swing leg, as shown in Fig. 2.5a. In the next part, we will analyze the kinematics of exoskeleton suit based on the position and speed relationship of operating space and the joint space, respectively.

The position relationship

We take the left supporting leg as the example to make the kinematics analysis. Refer to coordinate system definition in Fig. 2.4: The origin of reference coordinate system $\{0\}$ is defined as the point O_0, and then, the position vector of the left knee joint point O_1 in the reference coordinate system $\{0\}$ is

$$ {}^{0}\boldsymbol{p}_{O1} = [\,-L_s \sin q_1 \quad L_s \cos q_1 \quad 0\,]^{\mathrm{T}} \tag{2.13} $$

The rotation transformation matrix from left knee joint coordinate system $\{1\}$ to the reference coordinate system $\{0\}$ is

$$ {}^{0}_{1}\boldsymbol{R} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.14} $$

Then, the homogeneous transformation matrix from coordinate system $\{1\}$ to the reference coordinate system $\{0\}$ is as follows:

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} {}^{0}_{1}\boldsymbol{R} & {}^{0}\boldsymbol{p}_{O1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & -L_s \sin q_1 \\ \sin q_1 & \cos q_1 & 0 & L_s \cos q_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.15} $$

In the same way, the homogeneous transformation matrix from left hip joint coordinate system $\{2\}$ to the left knee joint coordinate system $\{1\}$ is as follows:

$$ {}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} {}^{1}_{2}\boldsymbol{R} & {}^{1}\boldsymbol{p}_{O2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & -L_t \sin q_2 \\ \sin q_2 & \cos q_2 & 0 & L_t \cos q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.16} $$

The origin O_3 of the trunk coordinate system $\{3\}$ coincides with the origin O_2 of the left hip joint coordinate system $\{2\}$, so

$$ {}^{2}_{3}\boldsymbol{T} = \begin{bmatrix} {}^{2}_{3}\boldsymbol{R} & {}^{2}\boldsymbol{p}_{O3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_3 & -\sin q_3 & 0 & 0 \\ \sin q_3 & \cos q_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.17} $$

The vector of trunk's center of gravity in trunk coordinate system $\{3\}$ can be expressed as

$$^{2}\boldsymbol{p}_{\text{Gub}} = [\,-L_{\text{Gub}}\sin q_3 \quad L_{\text{Gub}}\cos q_3 \quad 0\,]^{\text{T}} \tag{2.18}$$

The homogeneous transformation matrix from trunk's center of gravity to the left hip joint coordinate system is as follows:

$$^{2}_{3}\boldsymbol{T}_{\text{Gub}} = \begin{bmatrix} ^{2}_{3}\boldsymbol{R} & ^{2}\boldsymbol{p}_{\text{Gub}} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_3 & -\sin q_3 & 0 & -L_{\text{Gub}}\sin q_3 \\ \sin q_3 & \cos q_3 & 0 & L_{\text{Gub}}\cos q_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.19}$$

So, according to the chain rule of the homogeneous transformation matrix, the homogeneous transformation matrix from trunk's center of gravity to reference coordinate system is as follows:

$$^{0}_{3}\boldsymbol{T}_{\text{Gub}} = {}^{0}_{1}\boldsymbol{T}\cdot{}^{1}_{2}\boldsymbol{T}\cdot{}^{2}_{3}\boldsymbol{T}_{\text{Gub}} \tag{2.20}$$

Making use of symbolic operation toolbox of MATLAB, we can calculate to obtain the $^{0}_{3}\boldsymbol{T}_{\text{Gub}}$ quickly:

$$^{0}_{3}\boldsymbol{T}_{\text{Gub}} = \begin{bmatrix} ^{0}_{3}\boldsymbol{R} & ^{0}\boldsymbol{p}_{\text{Gub}} \\ 0 & 1 \end{bmatrix} \tag{2.21}$$

In the equation,

$$^{0}_{3}\boldsymbol{R} = \begin{bmatrix} \cos(q_1+q_2+q_3) & -\sin(q_1+q_2+q_3) & 0 \\ \sin(q_1+q_2+q_3) & \cos(q_1+q_2+q_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.22}$$

$$^{0}\boldsymbol{p}_{\text{Gub}} = \begin{bmatrix} -L_s\sin q_1 - L_t\sin(q_2+q_1) - L_{\text{Gub}}\sin(q_3+q_2+q_1) \\ L_s\cos q_1 + L_t\cos(q_2+q_1) + L_{\text{Gub}}\cos(q_3+q_2+q_1) \\ 0 \end{bmatrix} \tag{2.23}$$

Making use of the same method, we can obtain the homogeneous transformation matrix from the swing leg's ankle joint to hip joint coordinate system.

2.3.2.1 Speed Relationship

If we define the $\dot{\boldsymbol{q}}$ to represent the exoskeleton suit's joint velocity, define the $\dot{\boldsymbol{p}}$ to represent 3×1 translational velocity vector of trunk's center of gravity of

exoskeleton suit relative to reference coordinate system, and define $\boldsymbol{\omega}$ to represent the 3×1 rotation angular velocity vector of trunk's center of gravity of exoskeleton suit relative to reference coordinate system, then the velocity of trunk's center of gravity in reference coordinate system can be expressed as

$$\boldsymbol{v} = \begin{bmatrix} \dot{\boldsymbol{p}} \\ \boldsymbol{\omega} \end{bmatrix} \tag{2.24}$$

And there is such relationship between $\boldsymbol{v}$ and $\dot{\boldsymbol{q}}$:

$$\boldsymbol{v} = \boldsymbol{J}(\boldsymbol{q}) \cdot \dot{\boldsymbol{q}} \tag{2.25}$$

In the equation, $\boldsymbol{J}$ is a 6×3 matrix, representing the geometric Jacobian matrix of the trunk's center of gravity of exoskeleton suit, and $\boldsymbol{J}$ can be divided into two parts, namely

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{J}_{\mathrm{p}} \\ \boldsymbol{J}_{\omega} \end{bmatrix} \tag{2.26}$$

The two parts correspond to the translational velocity and angular velocity In equation (2.24) respectively.
We take the derivative of the equation (2.24), and then, we can obtain

$$\boldsymbol{J}_{\mathrm{p}} = [\boldsymbol{J}_{\mathrm{p1}} \quad \boldsymbol{J}_{\mathrm{p2}} \quad \boldsymbol{J}_{\mathrm{p3}}]^{\mathrm{T}} \tag{2.27}$$

In the equation,

$$\boldsymbol{J}_{\mathrm{p1}} = \begin{bmatrix} -L_{\mathrm{s}} \cos q_1 - L_{\mathrm{t}} \cos(q_2 + q_1) - L_{\mathrm{Gub}} \cos(q_3 + q_2 + q_1) \\ -L_{\mathrm{t}} \cos(q_2 + q_1) - L_{\mathrm{Gub}} \cos(q_3 + q_2 + q_1) \\ -L_{\mathrm{Gub}} \cos(q_3 + q_2 + q_1) \end{bmatrix}^{\mathrm{T}} \tag{2.28}$$

$$\boldsymbol{J}_{\mathrm{p2}} = \begin{bmatrix} -L_{\mathrm{s}} \sin q_1 - L_{\mathrm{t}} \sin(q_2 + q_1) - L_{\mathrm{Gub}} \sin(q_3 + q_2 + q_1) \\ -L_{\mathrm{t}} \sin(q_2 + q_1) - L_{\mathrm{Gub}} \sin(q_3 + q_2 + q_1) \\ -L_{\mathrm{Gub}} \sin(q_3 + q_2 + q_1) \end{bmatrix}^{\mathrm{T}} \tag{2.29}$$

$$\boldsymbol{J}_{\mathrm{p3}} = [0 \quad 0 \quad 0] \tag{2.30}$$

There are such relationships among the angular velocity $\boldsymbol{\omega}$, the rotation matrix, and its differential:

$$\dot{{}^{0}_{3}\boldsymbol{R}} = \boldsymbol{S}(\boldsymbol{\omega}) \cdot {}^{0}_{3}\boldsymbol{R} \tag{2.31}$$

If the $\boldsymbol{\omega}$ can be expressed as follows:

$$\boldsymbol{\omega} = [\,\omega_{\mathrm{x}} \quad \omega_{\mathrm{y}} \quad \omega_{\mathrm{z}}\,]^{\mathrm{T}} \tag{2.32}$$

Then, $\boldsymbol{S}(\boldsymbol{\omega})$ can be defined as follows:

$$\boldsymbol{S} = \begin{bmatrix} 0 & -\omega_{\mathrm{z}} & \omega_{\mathrm{y}} \\ \omega_{\mathrm{z}} & 0 & -\omega_{\mathrm{x}} \\ -\omega_{\mathrm{y}} & \omega_{\mathrm{x}} & 0 \end{bmatrix} \tag{2.33}$$

And $\boldsymbol{S}(\boldsymbol{\omega})$ is skew symmetric matrix, namely

$$\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{\omega}) = -\boldsymbol{S}(\boldsymbol{\omega}) \tag{2.34}$$

By the equation (2.31), we can obtain

$$\boldsymbol{S}(\boldsymbol{\omega}) = {}^{0}_{3}\dot{\boldsymbol{R}} \cdot {}^{0}_{3}\boldsymbol{R}^{\mathrm{T}} \tag{2.35}$$

Equation (2.22) differentiates the time, and then, the following can be obtained:

$${}^{0}_{3}\dot{\boldsymbol{R}} = (\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \cdot \begin{bmatrix} -\sin(q_1 + q_2 + q_3) & -\cos(q_1 + q_2 + q_3) & 0 \\ \cos(q_1 + q_2 + q_3) & -\sin(q_1 + q_2 + q_3) & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.36}$$

So, by the equations (2.22), (2.35) and (2.36), we can obtain

$$\boldsymbol{S}(\boldsymbol{\omega}) = \begin{bmatrix} 0 & -(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) & 0 \\ \dot{q}_1 + \dot{q}_2 + \dot{q}_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.37}$$

Contrasting Eq. (2.33) with Eq. (2.37), we can obtain

$$\begin{aligned} \omega_{\mathrm{x}} &= 0 \\ \omega_{\mathrm{y}} &= 0 \\ \omega_{\mathrm{z}} &= \dot{q}_1 + \dot{q}_2 + \dot{q}_3 \end{aligned} \tag{2.38}$$

Namely,

$$\boldsymbol{\omega} = [\,0 \quad 0 \quad \dot{q}_1 + \dot{q}_2 + \dot{q}_3\,]^{\mathrm{T}} \tag{2.39}$$

So,

$$\boldsymbol{J}_{\omega} = \begin{bmatrix} \boldsymbol{J}_{\omega 1} \\ \boldsymbol{J}_{\omega 2} \\ \boldsymbol{J}_{\omega 3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \tag{2.40}$$

After observing Eqs. (2.23), (2.39), we find that because of only considering the longitudinal plane, the position of trunk's center of gravity in the z-axis direction in the reference coordinate system is always zero. And the rotation angular velocity of the trunk revolving around x-axis and y-axis is also zero; therefore, we can use the way of dimension reduction to show the generalized position of exoskeleton suit's trunk in the operating space, namely defining the generalized coordinates of the exoskeleton suit's trunk:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{p} \\ \theta \end{bmatrix} \tag{2.41}$$

In the equation, $\boldsymbol{p} = [p_{\mathrm{x}} \quad p_{\mathrm{y}}]^{\mathrm{T}} \in \mathbb{R}^2$ is the plane position vector of the trunk's center of gravity of exoskeleton suit in the operating space, and by Eq. (2.23), we can obtain

$$p_{\mathrm{x}} = -L_{\mathrm{s}} \sin q_1 - L_{\mathrm{t}} \sin(q_2 + q_1) - L_{\mathrm{Gub}} \sin(q_3 + q_2 + q_1) \tag{2.42}$$

$$p_{\mathrm{y}} = L_{\mathrm{s}} \cos q_1 + L_{\mathrm{t}} \cos(q_2 + q_1) + L_{\mathrm{Gub}} \cos(q_3 + q_2 + q_1) \tag{2.43}$$

And the $\theta = q_1 + q_2 + q_3$ represents the orientation of the exoskeleton suit's trunk, and then, $\dot{\theta} = \omega_{\mathrm{z}}$. According to Eq. (2.41), the generalized velocity of exoskeleton suit's trunk is as follows:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{p}} \\ \dot{\theta} \end{bmatrix} \tag{2.44}$$

According to Eqs. (2.25), (2.28), and (2.29), we obtain the equation:

$$\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{2.45}$$

In the equation, $\boldsymbol{J}(\boldsymbol{q})$ is the Jacobian matrix of 3×3, and

$$\boldsymbol{J}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{J}_{\mathrm{p1}} \\ \boldsymbol{J}_{\mathrm{p2}} \\ \boldsymbol{J}_{\omega 3} \end{bmatrix} \tag{2.46}$$

Similarly, for the swing leg, as shown in Fig. 2.5b, also we can obtain

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{p} \\ \theta \end{bmatrix} = \begin{bmatrix} p_{\mathrm{x}} \\ p_{\mathrm{y}} \\ \theta \end{bmatrix} = \begin{bmatrix} L_{\mathrm{t}} \sin q_1 + L_{\mathrm{s}} \sin(q_1 + q_2) \\ L_{\mathrm{t}} \cos q_1 + L_{\mathrm{s}} \cos(q_1 + q_2) \\ q_1 + q_2 + q_3 \end{bmatrix} \tag{2.47}$$

Taking the (2.47) in the form of partial differential equation, we can obtain the Jacobian matrix

$$\boldsymbol{J}(\boldsymbol{q}) = \begin{bmatrix} L_{\mathrm{t}} \cos q_1 + L_{\mathrm{s}} \cos(q_1 + q_2) & L_{\mathrm{s}} \cos(q_1 + q_2) & 0 \\ -L_{\mathrm{t}} \sin q_1 - L_{\mathrm{s}} \sin(q_1 + q_2) & -L_{\mathrm{s}} \sin(q_1 + q_2) & 0 \\ 1 & 1 & 1 \end{bmatrix} \tag{2.48}$$

In the actual movement, in some certain condition, the exoskeleton suit's pose can lead to Jacobian matrix's module $|\boldsymbol{J}| = 0$, and namely, singular value exists. In the singular value, the Jacobian matrix fails, therefore, in the control of exoskeleton suit, and the occurrence of singular pose should be tried to avoid.

Equations (2.41) and (2.45) are the expression of exoskeleton suit's kinematics model. The kinematics model describes the relationship between the joint space and the operation space. In this book, Chap. 3 discusses the questions in joint space, and from Chaps. 4–7, questions are discussed in joint space and operation space at the same time.

2.4 Dynamics of Exoskeleton Suit

Dynamics modeling is conducted on the basis of kinematics modeling, which mainly researches the force/torque's acting relationship among all the system's parts in the process of walking, to determine the driving torque of each joint on the basis of the drive and control. The exoskeleton suit corresponds to different models under the different movement models, so in the dynamic modeling, we need to make modeling analysis, respectively, according to different models.

2.4.1 The Method of Dynamics Modeling

For the multi-rigid-body system, there are a lot of dynamic modeling methods. Euler–Lagrange method is more mature classical method of multi-rigid-body dynamic modeling [8]. Euler–Lagrange method is the method that differentiates the system variables and time based on the energy term. For simple cases, using this method is more complicated than using Newton's mechanics method; however, with the increase of system's complexity, using the Euler–Lagrange method becomes relatively easier. The Euler–Lagrange method is based on the following

two basic equations: one for rectilinear motion and the other for rotary motion. First of all, define the Lagrange functions as

$$L = KE - V \tag{2.49}$$

In the equation, KE is system dynamics; V is potential energy of the system. So

$$\boldsymbol{T} = \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\boldsymbol{q}}} - \frac{\partial L}{\partial \boldsymbol{q}} \tag{2.50}$$

In the equation, $\boldsymbol{T}$ is all the resultant external moment vector in the movement; $\boldsymbol{q}$ is the system's joint angle motion vector. The other method widely used in robot dynamics is KANE method [9, 10], and it can also be applied to establish dynamic equation of a multi-body system. As same as the Lagrange method, KANE method is also a standard method to write dynamic equation. The concrete form is as follows:

$$\boldsymbol{K}_{\mathrm{r}} + \boldsymbol{K}_{\mathrm{r}}^{*} = 0 \tag{2.51}$$

In the equation, $\boldsymbol{K}_{\mathrm{r}}$ and $\boldsymbol{K}_{\mathrm{r}}^{*}$ are generalized active force and generalized inertia force, respectively.

Comparing the above two methods, their main differences are as follows: For the mechanical system of n degrees of freedom, Lagrange equation gives n second-order differential equations, and KANE method gives the $2n$ first-order differential equation. The Lagrange equation only need write the system's kinetic energy and potential energy shown in generalized coordinates, and the concept is clear, while the KANE method must construct appropriate generalized velocity and partial velocity. In fact, when adopting the generalized coordinates adopted by Lagrange, the models derived by the two methods are equivalent [11].

Another method is the R/W method proposed by Roberson of University of San Diego in the USA and Wittenburg of University of Karlsruhekars in Germany. They have introduced the graph theory in mathematics into dynamics to study the safety problem of car crash [12]. Because this method is more complex, it is used less.

In this book, we use Euler–Lagrange method to make a dynamic modeling analysis of the exoskeleton suit.

2.4.2 Modeling Procedures

The modeling method of support state model and swing state model is the same. That is, firstly calculate the kinetic energy and potential energy to obtain the Lagrange function shown in Eq. (2.49), and then, calculate to obtain the corresponding dynamic model by differential operation shown in Eq. (2.50). The following takes the example of the steps of modeling the support state model to explain [7].

2.4.2.1 The Unit Vector

If we define $\boldsymbol{e}_{ij/k}$ to represent the unit vector $\boldsymbol{e}_{ij}$ expressed in the coordinate system k, as shown in Fig. 2.4, then in the inertial coordinate system, the unit vector can be expressed as

$$\begin{aligned} \boldsymbol{e}_{01/0} &= [1 \quad 0 \quad 0]^{\mathrm{T}} \\ \boldsymbol{e}_{02/0} &= [0 \quad 1 \quad 0]^{\mathrm{T}} \\ \boldsymbol{e}_{03/0} &= [0 \quad 0 \quad 1]^{\mathrm{T}} \end{aligned} \tag{2.52}$$

By Eq. (2.8), we can know that the vector $\boldsymbol{e}_{/i}$ expressed in the coordinate system i can be expressed as the vector in the coordinate system j by transforming $\boldsymbol{e}_{/j} = {}^{j}_{i}\boldsymbol{R} \cdot \boldsymbol{e}_{/i}$. Therefore, $\forall i \in [1, 2, 3]$ and $\forall j \in [1, 2, 3]$, and then, the unit vector of each coordinate system in the inertial coordinate system can be expressed as

$$\boldsymbol{e}_{ij/0} = {}^{0}_{i}\boldsymbol{R} \cdot \boldsymbol{e}_{ij/i} \tag{2.53}$$

2.4.2.2 The Position Vector

Each key position vector is transformed into the position vector in the reference coordinate system, and let the $\boldsymbol{r}_{O1O2/0}$ express the expression of the vector from point $O1$ to point $O2$ in the coordinate system $\{0\}$ (the reference coordinate system), and the rest can be done in the same manner; so there are the following:

The vector from the ankle to the shank's center of gravity is

$$\boldsymbol{r}_{O0\mathrm{Gs}/0} = L_{\mathrm{Gs}}\boldsymbol{e}_{12/0} \tag{2.54}$$

The vector from the ankle to the knee joint is

$$\boldsymbol{r}_{O0O1/0} = L_{\mathrm{s}}\boldsymbol{e}_{12/0} \tag{2.55}$$

The vector from the knee joint to thigh's center of gravity is

$$\boldsymbol{r}_{O1\mathrm{Gt}/0} = L_{\mathrm{Gt}}\boldsymbol{e}_{22/0} \tag{2.56}$$

The vector from the knee joint to the hip joint is

$$\boldsymbol{r}_{O1O2/0} = L_{\mathrm{t}}\boldsymbol{e}_{22/0} \tag{2.57}$$

The vector from the hip joint to the vector of trunk's center of gravity is

$$\boldsymbol{r}_{O3\mathrm{Gub}/0} = L_{\mathrm{Gub}}\boldsymbol{e}_{32/0} \tag{2.58}$$

2.4.2.3 Link Angular Velocity

$\boldsymbol{\omega}_{ij}$ expresses the rotating angular velocity vector of link i relative to link j, and link 0 expresses the reference coordinate system. Then, for given $i \in [1, 2, 3]$, the angular velocity of two adjacent links can be expressed as

$$\boldsymbol{\omega}_{i(i-1)} = \dot{q}_i \boldsymbol{e}_{03/0} \tag{2.59}$$

For given $i \in [2, 3]$, the angular velocity of each link relative to the reference coordinate system can be expressed as

$$\boldsymbol{\omega}_{i0} = \boldsymbol{\omega}_{i(i-1)} + \boldsymbol{\omega}_{(i-1)0} \tag{2.60}$$

2.4.2.4 The Velocity About the Point

Let $\boldsymbol{v}_{O/0}$ express the linear velocity of the point O in the reference coordinate system, and the rest can be done in the same manner; so there are the following:

The linear velocity of shank's center of gravity is

$$\boldsymbol{v}_{\mathrm{Gs}/0} = \boldsymbol{\omega}_{20} \times \boldsymbol{r}_{O0\mathrm{Gs}/0} \tag{2.61}$$

The linear velocity of the knee joint is

$$\boldsymbol{v}_{O1/0} = \boldsymbol{\omega}_{10} \times \boldsymbol{r}_{O0O1/0} \tag{2.62}$$

The linear velocity of thigh's center of gravity is

$$\boldsymbol{v}_{\mathrm{Gt}/0} = \boldsymbol{v}_{O1/0} + \boldsymbol{\omega}_{20} \times \boldsymbol{r}_{O1\mathrm{Gt}/0} \tag{2.63}$$

The linear velocity of hip joint is

$$\boldsymbol{v}_{O2/0} = \boldsymbol{v}_{O1/0} + \boldsymbol{\omega}_{20} \times \boldsymbol{r}_{O1O2/0} \tag{2.64}$$

The linear velocity of trunk's gravity is

$$\boldsymbol{v}_{\mathrm{Gub}/0} = \boldsymbol{v}_{O2/0} + \boldsymbol{\omega}_{30} \times \boldsymbol{r}_{O3\mathrm{Gub}/0} \tag{2.65}$$

2.4.2.5 The System's Kinetic Energy

The shank's kinetic energy is

$$\mathrm{KE_s} = \frac{1}{2} m_\mathrm{s} \boldsymbol{v}_{\mathrm{Gs}/0} \boldsymbol{v}_{\mathrm{Gs}/0} + \frac{1}{2} I_\mathrm{s} \boldsymbol{\omega}_{10} \boldsymbol{\omega}_{10} \tag{2.66}$$

The thigh's kinetic energy is

$$\mathrm{KE_t} = \frac{1}{2} m_\mathrm{t} \boldsymbol{v}_{\mathrm{Gt}/0} \boldsymbol{v}_{\mathrm{Gt}/0} + \frac{1}{2} I_\mathrm{t} \boldsymbol{\omega}_{20} \boldsymbol{\omega}_{20} \tag{2.67}$$

The trunk's kinetic energy is

$$\mathrm{KE_{ub}} = \frac{1}{2} m_\mathrm{ub} \boldsymbol{v}_{\mathrm{Gub}/0} \boldsymbol{v}_{\mathrm{Gub}/0} + \frac{1}{2} I_\mathrm{ub} \boldsymbol{\omega}_{40} \boldsymbol{\omega}_{40} \tag{2.68}$$

If the other leg is also in the support state, then the trunk's weight and rotary inertia are half of that of the current. The total kinetic energy of the system is

$$\mathrm{KE} = \mathrm{KE_s} + \mathrm{KE_t} + \mathrm{KE_{ub}} \tag{2.69}$$

2.4.2.6 The System's Potential Energy

The shank's potential energy is

$$V_\mathrm{s} = m_\mathrm{s} g \boldsymbol{r}_{O0\mathrm{Gs}/0} \boldsymbol{e}_{02/0} \tag{2.70}$$

The thigh's potential energy is

$$V_\mathrm{t} = m_\mathrm{t} g (\boldsymbol{r}_{O0O1/0} + \boldsymbol{r}_{O1\mathrm{Gt}/0}) \cdot \boldsymbol{e}_{02/0} \tag{2.71}$$

The trunk's potential energy is

$$V_\mathrm{ub} = m_\mathrm{ub} g (\boldsymbol{r}_{O0O1/0} + \boldsymbol{r}_{O1O2/0} + \boldsymbol{r}_{O3\mathrm{Gub}/0}) \cdot \boldsymbol{e}_{02/0} \tag{2.72}$$

The total potential energy is

$$V = V_\mathrm{s} + V_\mathrm{t} + V_\mathrm{ub} \tag{2.73}$$

2.4.3 *Dynamics Model*

By Eqs. (2.69) and (2.73), we can obtain the Lagrange function L expressed by Eq. (2.49). So, according to Eq. (2.50), the system's dynamic model can be determined as

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{T} \tag{2.74}$$

In the equation, $\boldsymbol{q} = [\, q_1 \quad q_2 \quad q_3 \,]^\mathrm{T}$; $\boldsymbol{H}(\boldsymbol{q})$ is the inertia matrix; $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the Coriolis term; $\boldsymbol{G}(\boldsymbol{q})$ is the gravity term; $\boldsymbol{T} = [\, T_1 \quad T_2 \quad T_3 \,]$ expresses the resultant

external moment acting on exoskeleton suit, T_1 expresses ankle joint torque, T_2 expresses the knee joint torque, and T_3 expresses the hip joint torque. The concrete forms of $\boldsymbol{H}(\boldsymbol{q})$, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$, and $\boldsymbol{G}(\boldsymbol{q})$ are as follows:

$$\boldsymbol{H}(\boldsymbol{q}) = \begin{bmatrix} H_{11}(\boldsymbol{q}) & H_{12}(\boldsymbol{q}) & H_{13}(\boldsymbol{q}) \\ H_{21}(\boldsymbol{q}) & H_{22}(\boldsymbol{q}) & H_{23}(\boldsymbol{q}) \\ H_{31}(\boldsymbol{q}) & H_{32}(\boldsymbol{q}) & H_{33}(\boldsymbol{q}) \end{bmatrix} \tag{2.75}$$

$$\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{bmatrix} C_{11}(\boldsymbol{q},\dot{\boldsymbol{q}}) & C_{12}(\boldsymbol{q},\dot{\boldsymbol{q}}) & C_{13}(\boldsymbol{q},\dot{\boldsymbol{q}}) \\ C_{21}(\boldsymbol{q},\dot{\boldsymbol{q}}) & C_{22}(\boldsymbol{q},\dot{\boldsymbol{q}}) & C_{23}(\boldsymbol{q},\dot{\boldsymbol{q}}) \\ C_{31}(\boldsymbol{q},\dot{\boldsymbol{q}}) & C_{32}(\boldsymbol{q},\dot{\boldsymbol{q}}) & C_{33}(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix} \tag{2.76}$$

$$G(\boldsymbol{q}) = \begin{bmatrix} G_1(\boldsymbol{q}) \\ G_2(\boldsymbol{q}) \\ G_3(\boldsymbol{q}) \end{bmatrix} \tag{2.77}$$

$$\begin{aligned} H_{11}(\boldsymbol{q}) = {} & I_{\mathrm{t}} + I_{\mathrm{ub}} + m_{\mathrm{s}}L_{\mathrm{Gs}}^2 + m_{\mathrm{t}}L_{\mathrm{s}}^2 + m_{\mathrm{t}}L_{\mathrm{Gt}}^2 + m_{\mathrm{ub}}L_{\mathrm{s}}^2 + m_{\mathrm{ub}}L_{\mathrm{t}}^2 \\ & + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 + 2m_{\mathrm{t}}L_{\mathrm{Gt}}L_{\mathrm{s}}\cos(q_2) + 2m_{\mathrm{ub}}L_{\mathrm{t}}L_{\mathrm{s}}\cos(q_2) \\ & + 2m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\cos(q_3) + 2m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{s}}\cos(q_2+q_3) \end{aligned} \tag{2.78}$$

$$\begin{aligned} H_{12}(\boldsymbol{q}) = {} & I_{\mathrm{t}} + I_{\mathrm{ub}} + m_{\mathrm{t}}L_{\mathrm{Gt}}^2 + m_{\mathrm{ub}}L_{\mathrm{t}}^2 + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 + 2m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\cos(q_3) \\ & + m_{\mathrm{ub}}L_{\mathrm{t}}\cos(q_2) + m_{\mathrm{t}}L_{\mathrm{Gt}}L_{\mathrm{s}}\cos(q_2) + m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{s}}\cos(q_2+q_3) \end{aligned} \tag{2.79}$$

$$H_{13}(\boldsymbol{q}) = I_{\mathrm{u}} + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 + m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\cos(q_3) + m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{s}}\cos(q_2+q_3) \tag{2.80}$$

$$\begin{aligned} H_{21}(\boldsymbol{q}) = {} & I_{\mathrm{t}} + I_{\mathrm{ub}} + m_{\mathrm{t}}L_{\mathrm{Gt}}^2 + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 + m_{\mathrm{ub}}L_{\mathrm{t}}^2 + m_{\mathrm{t}}L_{\mathrm{Gt}}L_{\mathrm{s}}\cos(q_2) \\ & + m_{\mathrm{ub}}L_{\mathrm{s}}L_{\mathrm{t}}\cos(q_2) + 2m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\cos(q_3) \\ & + m_{\mathrm{ub}}L_{\mathrm{s}}L_{\mathrm{Gub}}\cos(q_2+q_3) \end{aligned} \tag{2.81}$$

$$\begin{aligned} H_{22}(\boldsymbol{q}) = {} & I_{\mathrm{t}} + I_{\mathrm{ub}} + m_{\mathrm{ub}}L_{\mathrm{t}}^2 + m_{\mathrm{t}}L_{\mathrm{Gt}}^2 + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 \\ & + 2m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\cos(q_3) \end{aligned} \tag{2.82}$$

$$H_{23}(\boldsymbol{q}) = I_{\mathrm{ub}} + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 + m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\cos(q_3) \tag{2.83}$$

$$\begin{aligned} H_{31}(\boldsymbol{q}) = {} & I_{\mathrm{ub}} + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 + m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\cos(q_3) \\ & + m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{s}}\cos(q_2+q_3) \end{aligned} \tag{2.84}$$

$$H_{32}(\boldsymbol{q}) = I_{\mathrm{ub}} + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 + m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\cos(q_3) \tag{2.85}$$

$$H_{33}(\boldsymbol{q}) = I_{\mathrm{ub}} + m_{\mathrm{ub}}L_{\mathrm{Gub}}^2 \tag{2.86}$$

$$C_{11}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -2m_{\mathrm{ub}}L_{\mathrm{t}}L_{\mathrm{Gub}}\dot{q}_3\sin(q_3) - 2m_{\mathrm{ub}}L_{\mathrm{s}}L_{\mathrm{t}}\dot{q}_2\sin(q_2) - 2m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_2L_{\mathrm{s}}\sin(q_2+q_3) - 2m_{\mathrm{t}}L_{\mathrm{Gt}}\dot{q}_2L_{\mathrm{s}}\sin(q_2) \tag{2.87}$$

$$C_{12}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_2L_{\mathrm{s}}\sin(q_2+q_3) - m_{\mathrm{ub}}L_{\mathrm{t}}\dot{q}_2L_{\mathrm{s}}\sin(q_2) - m_{\mathrm{t}}L_{\mathrm{Gt}}\dot{q}_2L_{\mathrm{s}}\sin(q_2) - 2m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_3L_{\mathrm{s}}\sin(q_2+q_3) \tag{2.88}$$

$$C_{13}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_3L_{\mathrm{s}}\sin(q_2+q_3) - m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_3L_{\mathrm{t}}\sin(q_3) - 2m_{\mathrm{ub}}L_{\mathrm{t}}\dot{q}_2L_{\mathrm{Gub}}\sin(q_3) - 2m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{s}}\dot{q}_1\sin(q_2+q_3) \tag{2.89}$$

$$C_{21}(\boldsymbol{q},\dot{\boldsymbol{q}}) = m_{\mathrm{ub}}L_{\mathrm{s}}\dot{q}_1L_{\mathrm{t}}\sin(q_2) + m_{\mathrm{ub}}L_{\mathrm{s}}\dot{q}_1L_{\mathrm{Gub}}\sin(q_2+q_3) + m_{\mathrm{t}}L_{\mathrm{Gt}}\dot{q}_1L_{\mathrm{s}}\sin(q_2) \tag{2.90}$$

$$C_{22}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -2m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_3L_{\mathrm{t}}\sin(q_3) \tag{2.91}$$

$$C_{23}(\boldsymbol{q},\dot{\boldsymbol{q}}) = -m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_3L_{\mathrm{t}}\sin(q_3) - 2m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_1L_{\mathrm{t}}\sin(q_3) \tag{2.92}$$

$$C_{31}(\boldsymbol{q},\dot{\boldsymbol{q}}) = m_{\mathrm{ub}}L_{\mathrm{Gub}}\dot{q}_1L_{\mathrm{s}}\sin(q_2+q_3) + m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\dot{q}_1\sin(q_3) \tag{2.93}$$

$$C_{32}(\boldsymbol{q},\dot{\boldsymbol{q}}) = m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\dot{q}_2\sin(q_3) + 2m_{\mathrm{ub}}L_{\mathrm{Gub}}L_{\mathrm{t}}\dot{q}_1\sin(q_3) \tag{2.94}$$

$$C_{33}(\boldsymbol{q},\dot{\boldsymbol{q}}) = 0 \tag{2.95}$$

$$G_1(\boldsymbol{q}) = -m_{\mathrm{ub}}gL_{\mathrm{s}}\sin(q_1) - m_{\mathrm{ub}}gL_{\mathrm{t}}\sin(q_1+q_2) - m_{\mathrm{ub}}gL_{\mathrm{Gub}}\sin(q_1+q_2+q_3) - m_{\mathrm{t}}gL_{\mathrm{s}}\sin(q_1) - m_{\mathrm{t}}gL_{\mathrm{Gt}}\sin(q_1+q_2) - m_{\mathrm{s}}gL_{\mathrm{Gs}}\sin(q_1) \tag{2.96}$$

$$G_2(\boldsymbol{q}) = -m_{\mathrm{ub}}gL_{\mathrm{Gub}}\sin(q_1+q_2+q_3) - m_{\mathrm{t}}gL_{\mathrm{Gt}}\sin(q_1+q_2) - m_{\mathrm{ub}}gL_{\mathrm{t}}\sin(q_1+q_2) \tag{2.97}$$

$$G_3(\boldsymbol{q}) = -m_{\mathrm{ub}}gL_{\mathrm{Gub}}\sin(q_1+q_2+q_3) \tag{2.98}$$

Using the same method, we can obtain the dynamic model of swing leg. Because the book's length is limited, we will not show here, and in the next chapters, these two models will be the research object.

2.4.4 *Properties of the Model*

For dynamic equation as shown by (2.74), we can prove that it meets the following properties [1]:

1. Positive definiteness

For arbitrary $\boldsymbol{q}$, matrix $\boldsymbol{H}(\boldsymbol{q})$ is positive definite.

2. The boundedness

For all the $\boldsymbol{q}, \dot{\boldsymbol{q}}$, matrix function $\boldsymbol{H}(\boldsymbol{q})$ and $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is uniformly bounded, that is, there is positive number λ_m, λ_n and positive definite function. Makes

$$0 \leq \lambda_m \boldsymbol{I} \leq \boldsymbol{H}(\boldsymbol{q}) \leq \lambda_n \boldsymbol{I} \tag{2.99}$$

$$\boldsymbol{C}^{\mathrm{T}}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \leq \boldsymbol{\eta}(\dot{\boldsymbol{q}}) \boldsymbol{I} \tag{2.100}$$

3. The skew symmetry

For arbitrary $\boldsymbol{q}, \dot{\boldsymbol{q}}$, matrix function $\dot{\boldsymbol{H}}(\boldsymbol{q}) - 2 \cdot \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is skew symmetric. Namely, for an arbitrary vector quantity $\boldsymbol{\xi}$, there is

$$\boldsymbol{\xi}^{\mathrm{T}}[\dot{\boldsymbol{H}}(\boldsymbol{q}) - 2 \cdot \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})]\boldsymbol{\xi} = 0 \tag{2.101}$$

4. Linear characteristics

The exoskeleton suit's mathematical model is linear for physical parameter.

Namely, if we express the constant coefficient of the matrix function $\boldsymbol{H}, \boldsymbol{C}, \boldsymbol{G}$ as a vector quantity $\boldsymbol{\theta}$, we can define the proper matrix $\boldsymbol{\Phi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{v}, \boldsymbol{a})$ and makes

$$\boldsymbol{H}(\boldsymbol{q}) \cdot \boldsymbol{a} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \cdot \boldsymbol{v} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\Phi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{v}, \boldsymbol{a})\boldsymbol{\theta} \tag{2.102}$$

hold.

In the equation, $\boldsymbol{v}$ is the velocity vector and $\boldsymbol{a}$ is the acceleration vector.

2.5 Human-Machine Interaction Model

In fact, the exoskeleton suit separated from the human is meaningless, and exoskeleton suit is a human-machine system. In the human-machine system, the interaction between human and the machine has been the important part of human-machine relationship. One of the important research directions of human-machine system is to realize the convenient communication between human and the machine. Taken together, the human-machine interactive way can be roughly divided into the following kinds: [13–15]

(1) In the traditional sense, operating handle, console, hand (foot) switch, and some others all are the important communication bridge between people and machines.
(2) With the advent of the computer, the interaction between human and the machine has obtained unprecedented development. To realize the interaction

between human and the machine by computer is considered to be the most ideal human-machine interactive way, and the interaction between human and the computer is realized through a user interface.

(3) For the multi-model interaction, the channels cover all kinds of communication methods of users expressing their intents, performing action, or perceiving feedback information, such as language, eye contact, facial expressions, lip movement, hand movement, gestures, head movement, body posture, touch, smell, or taste, and the computer user interface adopting this way is called the "multi-model user interface."

However, after the analysis, we have found that to realize the human-machine interaction of the exoskeleton suit system through the user interface is not appropriate. It is not possible for operators (the human) to wear the exoskeleton suit to hold the keyboard, mouse, and monitor to operate and send control instructions to exoskeleton suit constantly. The commonly used visual interaction needs several fixed cameras to capture the human body's moving posture [14], which is also not appropriate for the exoskeleton suit, because there is no fixed space to install these cameras. Others such as voice, handwriting, posture, sight, and expression are more inappropriate.

The most appropriate interactive way for exoskeleton suit should be touch, which is the most direct and natural interactive way between human and the machine. The touch has four basic functions: touch sense, sliding sense, force sense, and pressure sense. Among them, the force sense is three-dimensional force and three-dimensional torque and is touch's most complex, most comprehensive, and most widely used perception form [16, 17]. If we make the exoskeleton suit have force sense perception ability, then the exoskeleton suit can perceive the human body's movement intention by contacting with the human body naturally to generate the corresponding control command and control the exoskeleton suit to track human movement. The exoskeleton suit's sensitivity amplification control method and the force control method that will be introduced from Chap. 3–7 both require the natural contact of the human and exoskeleton suit to control exoskeleton suit's movement. In the sensitivity amplification control, relying on the force/torque the human body acting on the exoskeleton suit when they are contacting to change the motion state of the exoskeleton suit, and after the sensor installed in the exoskeleton suit have received the change of its motion state, the control signal will be formed to control the joint rotation of the exoskeleton suit, making the exoskeleton suit track human movement. When their movement is consistent, the contact force/torque the human acting on the exoskeleton suit becomes zero, and if keeping the cycle continuously, even the force and torque acted by the human is much lesser, they can drive the exoskeleton suit to move. In the exoskeleton suit's control method, using the multi-axis force/torque sensors to measure the contact force exerted by the human at the end of the exoskeleton suit, and using this information to generate the control signal of the exoskeleton suit to control the exoskeleton suit tracking human movement, then the same effect as that of the sensitivity amplification control method will be realized.

Then, how is the acting force between human and the machine generated? In fact, human and the exoskeleton suit are coupled together by certain links, such as waistcoat, waistband, thigh straps, shank straps, and shoes. When there is error in motion trajectory between human and the exoskeleton suit, the mutual acting force between them will be produced. This force is naturally produced in the process of movement, and there is no need to know the specific mathematical relationship between the error and the force. But if we want to simulate the exoskeleton suit system in computer, the mathematical relationship between them must be determined, namely establishing the mathematical model of human-machine acting force. In general, if we consider the person as an object or environment, then the property of interaction between the exoskeleton suit and human is the same as that of interaction between the robot and environment. The interactive force between the robot and environment can be thought as rigidity, the linear combination of spring and damping [18, 19], as shown in Eq. (2.103)

$$\boldsymbol{f}_{\mathrm{e}} = \boldsymbol{K}_{\mathrm{Pf}}(\boldsymbol{x} - \boldsymbol{x}_{\mathrm{e}}) + \boldsymbol{K}_{\mathrm{Df}}\dot{\boldsymbol{x}} + \boldsymbol{K}_{\mathrm{Mf}}\ddot{\boldsymbol{x}} \tag{2.103}$$

In the equation, $\boldsymbol{f}_{\mathrm{e}}$ expresses the force vector of robots acting on the environment; $\boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}$ expresses position vector, velocity vector, and acceleration vector of the end of robots in the operating space; $\boldsymbol{x}_{\mathrm{e}}$ expresses the position vector of environment; $\boldsymbol{K}_{\mathrm{Pf}}$, $\boldsymbol{K}_{\mathrm{Df}}$, and $\boldsymbol{K}_{\mathrm{Mf}}$ express the spring coefficient matrix (the rigidity coefficient matrix), damping coefficient matrix, and inertia coefficient matrix, respectively. And some use nonlinear model to describe the environment's dynamic condition [18], as shown in Eqs. (2.104) and (2.105)

$$\boldsymbol{f}_{\mathrm{e}} = \boldsymbol{K}_{\mathrm{Pf}}(\boldsymbol{x} - \boldsymbol{x}_{\mathrm{e}})^{a} + \boldsymbol{K}_{\mathrm{Df}}\dot{\boldsymbol{x}}^{b} \tag{2.104}$$

$$\boldsymbol{f}_{\mathrm{e}} = \boldsymbol{K}_{\mathrm{Pf}}(\boldsymbol{x} - \boldsymbol{x}_{\mathrm{e}})^{c} + \boldsymbol{K}_{\mathrm{Df}}(\boldsymbol{x} - \boldsymbol{x}_{\mathrm{e}})^{c}\dot{\boldsymbol{x}} \tag{2.105}$$

For the sake of simplicity, this book does not use the nonlinear model, but is based on the linear model to research. And the environment described by Eq. (2.103) is static, while in the exoskeleton suit system, the human as the environment is kinetic. Then, assume the value of force vector of human acting on exoskeleton suit is $\boldsymbol{f}$ (in contrast to the $\boldsymbol{f}_{\mathrm{e}}$ sign); $\boldsymbol{x}_{\mathrm{h}}, \dot{\boldsymbol{x}}_{\mathrm{h}}, \ddot{\boldsymbol{x}}_{\mathrm{h}}$ express the information of human movement; $\boldsymbol{x}_{\mathrm{e}}, \dot{\boldsymbol{x}}_{\mathrm{e}}, \ddot{\boldsymbol{x}}_{\mathrm{e}}$ express the information of the exoskeleton suit movement; then, the $\boldsymbol{f}$ can be expressed as

$$\boldsymbol{f} = \boldsymbol{K}_{\mathrm{Pf}}(\boldsymbol{x}_{\mathrm{h}} - \boldsymbol{x}_{\mathrm{e}}) + \boldsymbol{K}_{\mathrm{Df}}(\dot{\boldsymbol{x}}_{\mathrm{h}} - \dot{\boldsymbol{x}}_{\mathrm{e}}) + \boldsymbol{K}_{\mathrm{Mf}}(\ddot{\boldsymbol{x}}_{\mathrm{h}} - \ddot{\boldsymbol{x}}_{\mathrm{e}}) \tag{2.106}$$

When taking no account of inertia coefficient, Eq. (2.106) can be simplified to a spring damping model

$$\boldsymbol{f} = \boldsymbol{K}_{\mathrm{Pf}}(\boldsymbol{x}_{\mathrm{h}} - \boldsymbol{x}_{\mathrm{e}}) + \boldsymbol{K}_{\mathrm{Df}}(\dot{\boldsymbol{x}}_{\mathrm{h}} - \dot{\boldsymbol{x}}_{\mathrm{e}}) \tag{2.107}$$

If we do not consider inertia coefficient and damping coefficient, Eq. (2.106) can be simplified as spring model [20]

$$\boldsymbol{f} = \boldsymbol{K}_{\mathrm{Pf}}(\boldsymbol{x}_{\mathrm{h}} - \boldsymbol{x}_{\mathrm{e}}) \tag{2.108}$$

The force human acting on the exoskeleton suit and the force exoskeleton suit acting on human are the mutual reaction force, which can be called human-machine acting force. The human-machine acting force in this book refers to the force human made on the exoskeleton suit. In the later chapters, according to the human-machine interaction model (environment model) described by Eqs. (2.106)–(2.108), we will simulate the human-machine interaction in the computer to complete the simulation of various control methods.

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