

Chapter 2

Discrete Fracture Model

Abstract This chapter introduces the concept of discrete fracture model. It started by reviewing the background and the state of the art of discrete fracture model. It then explains three numerical methods to solve discrete fracture model including the Galerkin finite element method, the control volume method, and the mimetic finite difference method. In this chapter, detailed process of the establishment of mathematical model and the corresponding solution for the three numerical methods are explained. Then these numerical methods are applied to some examples. By the end of the chapter, the embedded discrete fracture model is introduced. A full discussion of the establishment and solving for the embedded discrete fracture mathematical model is included.

Keywords Discrete fracture model • Numerical simulation • Galerkin finite element method • Control volume method • Mimetic finite difference method • Embedded discrete fracture model

2.1 Background and the State of the Art

Fracture, the smallest geological structure (Van Golf-Racht 1982), is any break or fracture occurring in the rock caused by the deformation or the physical diagenesis in the rock. All rocks in the earth's crust are fractured to some extent. In ground-water dynamics, the rock masses with well-developed fractures are known as fractured porous media. The problems of two-phase flow in fractured porous media widely exist in different engineering practices, such as the oil and gas field development, the prevention and control of groundwater pollution, and the disposal of underground nuclear wastes (Slough et al. 1999; Yuan et al. 2004).

In fracture porous media, two types of media may be distinguished: fractured media of single porosity and fractured media of double porosity. Both media are composed of a network of fractures surrounding rock blocks, but what differentiates the two types of reservoirs are the porosity and permeability of the rock blocks. In the first case, the rock blocks are practically impervious while in the second case the

porosity and permeability are quite significant. Fractures distribute very randomly and display multiscale features (Zhang 2005; Zhou and Wang 2004; Rutqvist et al. 2002) (as shown in Fig. 2.1). Therefore, it is difficult to establish an accurate and effective mathematical flow model and the corresponding numerical simulation method, which is the current research focus of petroleum industry and rock hydraulics (Feng et al. 2009; Huang et al. 2010; Reichenberger et al. 2006; Yao et al. 2010; Zhang and Wu 2010; Zhou 2007).

Since the 1980s, discrete fracture model (DFM) has been a great deal of development. The major characteristic of DFM is explicit expression and reducing dimension. For DEM, fractures are viewed as entity and the relationship between fractures and matrix can be established without interporosity flow function. This kind of model keeps their computational accuracy while the data volume and the cost of computation are reduced. Meanwhile, the model considers the permeability of matrix, namely fluid flow in matrix as well as in fractures. For this reason, DFM can not only accurately describe the flow character within fractures, but also describe the inhomogeneous character and seepage character of fracture media.

Originally, Noorishad and Mehran (1982) put forward DFM to solve single-phase flow problem in 2-D porous media. In this model, fractures are viewed as 1-D entity and finite element method is used to solve transient transport equation.

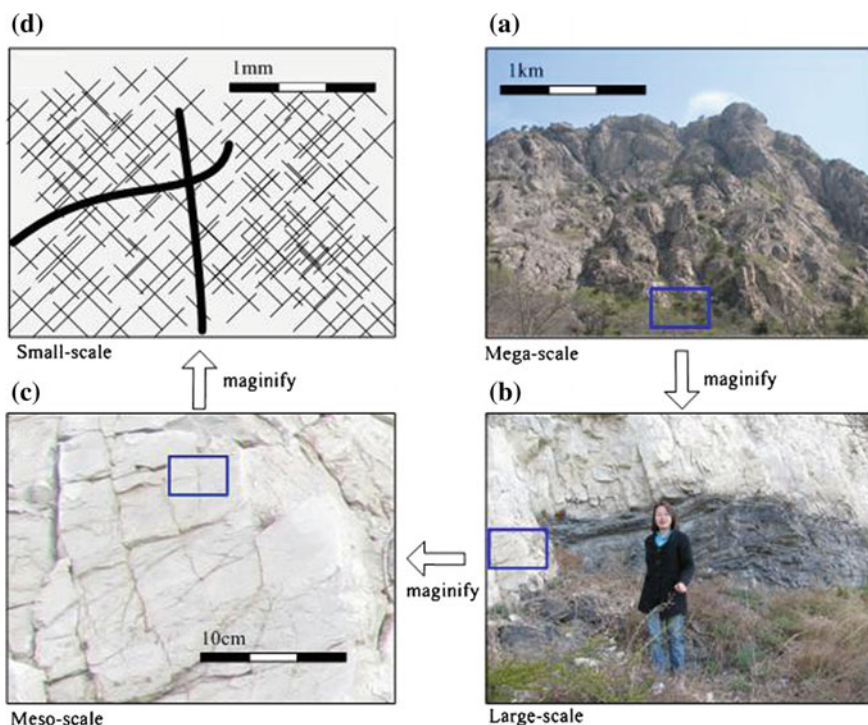


Fig. 2.1 Fractured porous media of different scales

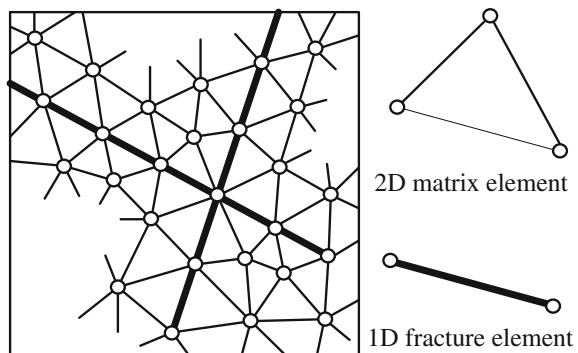
While Kim and Deo (2000) adopt finite element to do discretization to discrete fracture model and combine matrix and fractures according to superposition principle. For nonlinear partial differential equations, pressure and saturation fully implicit scheme and Newton's method can solve it.

In 2003, Karimi-Fard and Firoozabadi (2003) adopt discrete fracture model to solve the two-phase flow problem of fractured media. As shown in Fig. 2.2, the model used line element to do discretization for fractures and used irregular mesh element such as triangle to do discretization for matrix. Furthermore, Galerkin finite element method is used to process numerical simulation based on implicit pressure–explicit saturation equation. The method greatly simplifies the problem so it can be applied to any complex structure in fractured media. There is an excellent match between the result of this method and traditional numerical simulation method that is based on single-porosity model. On this basis, Jun Yao et al. did further research, and the validity of model and algorithm has been verified by computation examples (Yao et al. 2010). By analyzing the impacts of fractures on the water flooding development effect, the discrete fracture model is regarded as the method which has good applicability for reservoir of low development degree of fractures, especially when the reservoir has several large fractures that control the direction of flow.

Lange et al. (2004) put forward a new discretization method of discrete fracture model. Based on the concept of dual media, the model does discretization to complex fractures according to minimum principia of calculation amount based on the geologic model and confirms fracture pressure at real fractures. Fractures are discretized by the model in each horizontal plane of formation, which means confirm compute nodes at every intersection and the end points of fractures and combine rock blocks with every fracture elements by rapid processing algorithms according to minimum principia of distance from fracture mesh, as shown in Fig. 2.3.

Above scholars mostly use finite element method when solving the model while finite element method cannot ensure locally mass conserving, so some scholars apply finite volume method which is based on physical conservation to discrete fracture model. P. Bastian et al. performed two-phase flow numerical simulations of fractured media to discrete fracture model by finite volume method and developed corresponding simulator (Bastian et al. 2000). In 2004, S. Geiger et al. applied

Fig. 2.2 Schematic of mesh generation in discrete fracture model



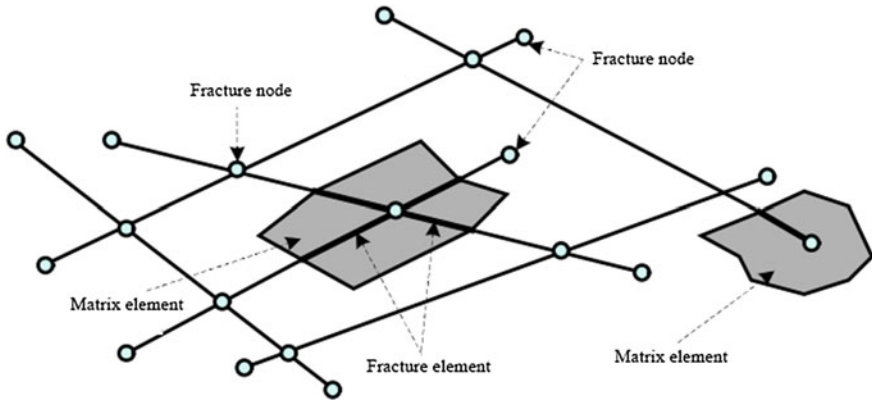


Fig. 2.3 Schematic of discrete fracture model

control volume method to solve flow potential equations while solving saturation equations by finite volume method (Geiger et al. 2004).

In recent years, discrete fracture model is getting more and more attention interiorly and becoming a hot research topic with the development of unconventional resources such as shale gas, tight reservoirs, and fractured reservoir. Further research has been done by Huang et al. (2011) in two-phase flow simulation of fractured reservoirs based on discrete fracture model. Combining the appropriate unstructured mesh generation technique, discrete fracture model can keep the arbitrary of development and distribution for fractures very well, describes the heterogeneity, anisotropy, and discontinuity of fractured media, and depicts the unique flow characteristic in fractures. Lv et al. (2012) did some research of discrete fracture mesh flow simulation based on control volume method. The high efficiency of calculation and the validity of flow simulation theory and algorithm for discrete fracture model based on finite volume method have been verified by examples (Lv 2010; Lv et al. 2012).

In recent 15 years, discrete fracture model has had a great development in fractured reservoir numerical simulation and several numerical discrete forms have sprung up, which include finite difference method, Galerkin finite element method, control volume method, finite volume method, mixed finite element method, mimetic finite difference method, etc.

(1) Finite difference method

Slough et al. applied finite difference method to do some research about multiphase flow problem for discrete fractured media based on discrete fracture model (Slough et al. 1999). Discrete fracture model is discretized into regular structured mesh to adapt to finite difference computation format in this study, while the discrete fracture always has complex geometry in practical problems. So this method has not been widely promoted.

After that, Lee et al. first proposed an embedded discrete fracture model to make full use of the existing mature finite difference reservoir numerical simulator and

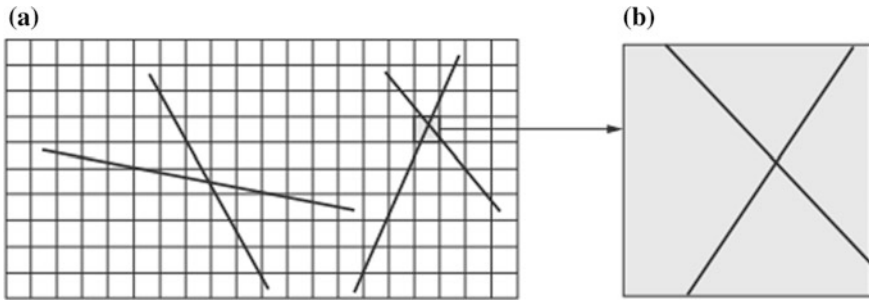


Fig. 2.4 Schematic of embedded discrete fracture model

adapt to the complex geometry of discrete fracture model (Lee et al. 2001). As shown in Fig. 2.4a, this model is typical nonmatching grid. In recent years, Li, Moinfar, Panfili, and Zhou et al. did further promotion and improvement to the model (Li and Lee 2008; Moinfar et al. 2012; Panfili et al. 2013; Zhou et al. 2014). Recently, Xia Yan et al. established a new embedded discrete fracture numerical computation format based on mimetic finite difference method to adapt to the condition of full tensor permeability (Yan et al. 2014).

(2) Galerkin finite element method

Based on the work of Kim and Deo (1999, 2000), Karimi-Fard and Firoozabadi used Galerkin finite element method to study water flooding numerical simulation in discrete fracture model (Karimi-Fard et al. 2003). Considering the influence of different wettability and comparing with the numerical result of single-porosity model (fractures are viewed as narrow high permeable zone), the validity of discrete fracture model can be verified. However, although Galerkin finite element method has whole conservation, the local conservation of elements cannot be guaranteed, especially on the singularities such as injection and production well. The oscillation of solutions is existent even when applied in the upstream format. Toward this, Zhang et al. put forward local conservation Galerkin finite element method (Zhang et al. 2013). In essence, the method meets the quantity of flow continuity condition at boundary of elements by the post-processing of element node to guarantee the local conservation of elements, which is similar to mixed finite element method. The method has not been given strict mathematic proof, so whether the method can extend to discrete fracture model remains to be studied.

(3) Control volume method

Based on control volume method, Monteagudo and Firoozabadi established a discrete fracture numerical computation format that has good local conservation to make up the short of Galerkin finite element method (Monteagudo and Firoozabadi 2007). Furthermore, they did some research about 3-D two-phase immiscible flow problem of fractured media. After that, Matthai et al. did further study about the method. A mixed mesh computation format was established to improve applicability of this method to the discrete fracture model (Matthäi and Belayneh 2004).

Reichenberger et al. developed a fully implicit numerical computation format based on control volume method (Reichenberger et al. 2006). Two sets of mesh are needed in control volume computation to solve the control volume of element node in every mesh: one is the initial mesh based on element nodes; another one is the auxiliary mesh system based on the central point of element. Consequently, the computational amount of this method will be increased compared with Galerkin finite element method.

(4) Finite volume method

Based on finite volume method, Granet et al. established a set of new discrete fracture numerical computation format and studied the 2-D incompressible two-phase flow (Granet et al. 1998). After that, Karimi-Fard et al. extended the method to 3-D multiphase flow problem based on GPRS reservoir numerical simulator of Stanford University (Karimi-Fard et al. 2003, 2004). Above computation format belongs to two-point flux approximation (TPFA). Accordingly, it cannot adapt to the condition of full tensor permeability. Sandve et al. deduced multipoint flux approximation (MPFA) of discrete fracture model to solve the problem (Sandve et al. 2012).

Finite volume method has good local conservation and little computation compared with finite element method and been widely used in reservoir numerical simulation. However, finite volume method is not as convenient as finite element method when it deal with cross fracture in discrete fracture model. To solve this problem, Karimi-Fard et al. put forward Delta-Star method to deal with cross fracture with the experience of resistance analysis method in cross circuit (Karimi-Fard et al. 2004). For single-phase flow, the method has high computation precision; for two-phase flow, Karimi-Fard et al. indicate that the computation error can meet the requirements only when the fracture densities are small, while the applicability and validity have not been verified for large-scale computation in reservoir.

(5) Mixed finite element method

In 1970s, Raviart and Thomas successfully applied the mixed finite method to reservoir numerical simulation and put forward the famous low-order RT_0 mixed finite element computation format (Raviart and Thomas 1977). Mixed finite element method is viewed as the finite volume method in finite element method for the good local conservation. Recently, Hoteit and Firroozabadi studied incompressible two-phase flow problem in discrete fracture model by combining mixed finite element method and discontinuous Galerkin finite element method (Hoteit and Firroozabadi 2006). They put forward an upstream weighted computation format that has high computational accuracy when they deal with cross fracture. For mixed finite element method, the point is the structure of the pressure and velocity basis function. For triangle, quadrangle, and regular hexahedron, the structure of the basis function has matured theory and method. The matured universal method for 3-D unstructured mesh such as tetrahedron and irregular polyhedron element has not been developed, which restricts the development and application of mixed finite element method in discrete fracture reservoir numerical simulation to some extent.

(6) Mimetic finite difference method

Huang et al. deduced a new discrete fracture numerical computation format based on mimetic finite difference method and studied incompressible two-phase flow problem (Huang et al. 2014). The mimetic finite difference method which is put forward by Breezi et al. (2005) has been widely used in research such as computational fluid dynamics (Lie et al. 2012; Lipnikov et al. 2014), electromagnetism, reservoir numerical simulation, etc., for the good local conservation and application to complicate mesh. The method is treated as the mixed finite element method in finite volume method, which means the computation format is similar to mixed finite element method and the difference is structure of element computation format. Mimetic finite difference method can structure computation format only based on single mesh element, so it can adapt to arbitrary complicate mesh system even the concave mesh. Because of reducing requirement to the mesh, mimetic finite difference method is more applicable than mixed finite element method for the flow simulation of complicate discrete fracture model.

2.2 Galerkin Finite Element Numerical Simulation

2.2.1 Discrete Fractured Model

Affected by the generation environment (stress, deposition, erosion, effloresce, etc.), fractures have complicated geometric configuration. It is necessary to simplify the fractures for convenience. Usually fractures are simplified into a parallel plate model inside which flow follows Navier–Stokes equation. For laminar flow conditions, velocity distribution along the fracture aperture can be obtained. Rewriting the quantity of flow in the form of equivalent Darcy’s law gives the fractures’ equivalent permeability. Evidently, the flow parameters and correlative physical quantities keep constant along the direction of the fracture in aperture, so reducing the dimension of the fracture in aperture direction is feasible. Fractures are simplified into 1-D line element for 2-D problem, and 2-D surface area element for 3-D problem (Fig. 2.5). Such simplification is the fundamental concept of the discrete fractured model.

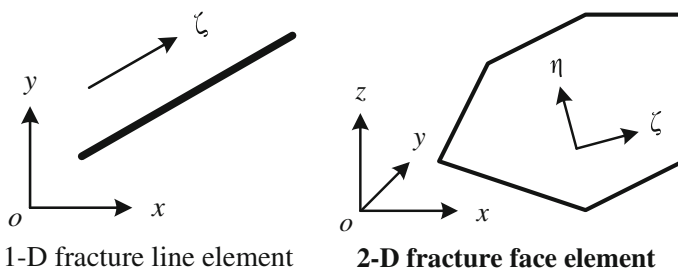


Fig. 2.5 Schematic of simplified fracture

The matrix system comprising microfissure and rock mass are regarded as equivalent porous continuum and the macroscopic fractures are represented manifestly as discrete fractures. As shown in Fig. 2.1, fractures occur at a variety of scales, from microscopic to field scale. Therefore, the division of microfissure and macroscopic fissure should comply with the specific research problem and the required precision of numerical simulation. Generally, a large fracture should be longer than a mesh of numerical simulation.

Therefore, the whole fracture porous media consists of matrix system and fracture system. The research region is $\Omega = \Omega_m + \sum a_i \times (\Omega_f)_i$, where m represents matrix, f represents fracture, and a is the aperture of the i -th fracture. Assuming the representative element volumes of both matrix and fracture system exist, the two-phase flow equations FEQ (Flow Equations) are applicable to the entire research area. Then for the discrete fractured model, the integral form of the flow equation can be expressed as

$$\int_{\Omega} \text{FEQ} \, d\Omega = \int_{\Omega_m} \text{FEQ} \, d\Omega_m + \sum_i a_i \times \int_{(\Omega_f)_i} \text{FEQ} \, d(\Omega_f)_i \quad (2.1)$$

When neglecting the storage and seepage ability of matrix system, the research area only include fracture system and the above model degenerates into DFN (Discrete Fracture Network) model. When we consider the storage and seepage ability of the matrix system, the above model is the discrete fracture model. If the fractures are treated as microfissure, the above model only includes matrix system and changes into classical porous media flow model.

2.2.2 Two-Phase Flow Mathematical Model

For simplicity, we only consider isothermal flow of incompressible fluid, which is similar to the analysis of other flow problem. Flow equations include mass conservation equation, generalized Darcy's law, saturation auxiliary equation, and capillary pressure relationship. Specific equations are as follows:

$$\phi \frac{\partial S_{\alpha}}{\partial t} + \nabla \cdot \mathbf{v}_{\alpha} = q_{\alpha}, \quad \alpha = w, n \quad (2.2)$$

$$\mathbf{v}_{\alpha} = -\mathbf{K} \frac{k_{r\alpha}}{\mu_{\alpha}} (\nabla p_{\alpha} + \rho_{\alpha} g \nabla z), \quad \alpha = w, n \quad (2.3)$$

$$S_w + S_n = 1 \quad (2.4)$$

$$p_c(S_w) = p_n - p_w, \quad (2.5)$$

where ϕ is porosity; S_l is saturation; v_i is seepage velocity, (m/s); D is Hamilton operator; q_l is source term, (1/s); w, n denote wetting phase and non-wetting phase, respectively; $\mathbf{K}$ is permeability tensor, (m^2); k_{rl} is relative permeability; μ_l is fluid viscosity, (Pa s); p_l is fluid pressure, (Pa); ρ_l is fluid density, (kg/m^3); g is acceleration of gravity; z denotes highness, positive on the upward side, (m); p_c is capillary pressure, (Pa). Herein, we define the flow potential Φ_l as follows:

$$\Phi_\alpha = p_\alpha + \rho_\alpha g z \quad (2.6)$$

and the corresponding capillary force potential Φ_c as

$$\Phi_c = \Phi_n - \Phi_w = p_c + (\rho_n - \rho_w) g z \quad (2.7)$$

Based on the above definitions, flow Eqs. (2.2), (2.3), and (2.5) can be written as

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot (-\mathbf{K} \lambda_w \nabla \Phi_w) = q_w \quad (2.8)$$

$$\phi \frac{\partial S_n}{\partial t} + \nabla \cdot (-\mathbf{K} \lambda_n \nabla \Phi_n) = q_n \quad (2.9)$$

$$\Phi_c = \Phi_n - \Phi_w, \quad (2.10)$$

where

$$\lambda_w = \frac{k_{rw}}{\mu_w}, \quad \lambda_n = \frac{k_{rn}}{\mu_n} \quad (2.11)$$

denote the mobility coefficient of wetting phase and non-wetting phase, respectively.

Substituting Eqs. (2.4) and (2.10) into Eqs. (2.8) and (2.9) leads to the flow potential equation and phase saturation equation of wetting phase, written in the form of matrix:

$$\begin{bmatrix} 0 & 0 \\ 0 & \phi \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} \Phi_w \\ S_w \end{bmatrix} + \nabla \cdot \left\{ - \begin{bmatrix} \mathbf{K}(\lambda_w + \lambda_n) & \mathbf{K} \lambda_n p'_c \\ \mathbf{K} \lambda_w & 0 \end{bmatrix} \nabla \begin{bmatrix} \Phi_w \\ S_w \end{bmatrix} \right\} = \begin{bmatrix} q_n + q_w \\ q_w \end{bmatrix}, \quad (2.12)$$

where

$$p'_c \nabla S_w = \nabla \Phi_c = \frac{d\Phi_c}{dS_w} \nabla S_w = \frac{dp_c}{dS_w} \nabla S_w \quad (2.13)$$

The initial condition and boundary conditions are stated below:

(1) Initial conditions

$$\Phi_\alpha(x, 0) = \Phi_\alpha(x), S_\alpha(x, 0) = S_\alpha(x), \quad \text{at } t = 0 \quad (2.14)$$

(2) Dirichlet boundary conditions

$$\Phi_\alpha(x, t) = \Phi_\alpha, S_\alpha(x, t) = S_\alpha, \quad \text{on } \Gamma_D \quad (2.15)$$

(3) Neumann boundary conditions (the outer boundary is impermeable), i.e.,

$$\begin{cases} \mathbf{v}_\alpha \cdot \mathbf{n} = (-K\lambda_\alpha \nabla \Phi_\alpha) \cdot \mathbf{n} = 0 \\ \nabla S_\alpha \cdot \mathbf{n} = 0 \end{cases}, \quad \text{on } \Gamma_N, \quad (2.16)$$

where $\mathbf{n}$ is the outer normal unit vector of outer boundary. N points to the outer normal direction of the interface of fracture line and outer boundary for 2-D problem and of fracture surface and outer boundary for 3-D problem.

(4) Internal impermeable boundary conditions mainly refer to the impermeable internal boundaries such as fault and fracture filled with mud, etc.

$$\mathbf{v}_l \cdot \mathbf{n} = (-K\lambda_\alpha \nabla \Phi_\alpha) \cdot \mathbf{n} = 0, \quad \text{on } \Gamma_F, \quad (2.17)$$

where $\mathbf{n}$ is normal unit vector of internal boundary.

By substituting Eq. (2.12) as two-phase flow equation FEQ of fractured porous media into Eq. (2.1) and with the above initial conditions and boundary conditions, the complete mathematical model of the discrete fractured model can be developed.

2.2.3 Finite Element Numerical Formula

The discrete fractured model usually has complex fracture network structure and fractures distribute randomly. So in the numerical calculations, unstructured meshes are often used to adapt to its complex geometrical configuration. Therefore, the finite element method is applied to solve the numerical problem. The Galerkin weighted residual method is used to deduce the finite element calculation formula of Eq. (2.12). For convenience, the flow potential equation and phase saturation equation of wetting phase in Eq. (2.12) are derived separately, and the corresponding weight functions are variations of flow potential and saturation, respectively. Specific equations are as follows:

(1) Flow potential equation.

$$\begin{aligned} & \int_{\Omega} \nabla \cdot [-\mathbf{K}(\lambda_w + \lambda_n) \nabla \Phi_w] \delta \Phi_w d\Omega + \int_{\Omega} \nabla \cdot (-\mathbf{K} \lambda_n p'_c \nabla S_w) \delta \Phi_w d\Omega \\ &= \int_{\Omega} (q_n + q_w) \delta \Phi_w d\Omega \end{aligned} \quad (2.18)$$

(2) Saturation equation.

$$\int_{\Omega} \phi \frac{\partial S_w}{\partial t} \delta S_w d\Omega + \int_{\Omega} \nabla \cdot (-\mathbf{K} \lambda_w \nabla \Phi_w) \delta S_w d\Omega = \int_{\Omega} q_w \delta S_w d\Omega \quad (2.19)$$

After integration by parts, combining impermeable boundary conditions (2.16), we obtain

$$\begin{aligned} & \int_{\Omega} [\mathbf{K}(\lambda_w + \lambda_n) \nabla \Phi_w] \nabla (\delta \Phi_w) d\Omega + \int_{\Omega} (\mathbf{K} \lambda_n p'_c \nabla S_w) \nabla (\delta \Phi_w) d\Omega \\ &= \int_{\Omega} (q_n + q_w) \delta \Phi_w d\Omega \end{aligned} \quad (2.20)$$

$$\int_{\Omega} \phi \frac{\partial S_w}{\partial t} \delta S_w d\Omega + \int_{\Omega} (\mathbf{K} \lambda_w \nabla \Phi_w) \nabla (\delta S_w) d\Omega = \int_{\Omega} q_w \delta S_w d\Omega \quad (2.21)$$

For 2-D problem, Delaunay triangular mesh is employed to subdivide the whole research region and 1-D line element is employed to represent fracture. For 3-D problem, Delaunay triangular mesh is used to subdivide the fracture surface; the entire research region is subdivided by relevant tetrahedron or hexahedron, as shown in Fig. 2.6.

In each element, finite element approximation of flow potential and saturation is

$$\Phi_w \approx \sum_{i=1}^m N_i(\Phi_w)_i = \mathbf{N}(\mathbf{x}) \Phi_w(t) \quad S_w \approx \sum_{i=1}^m N_i(S_w)_i = \mathbf{N}(\mathbf{x}) S_w(t), \quad (2.22)$$

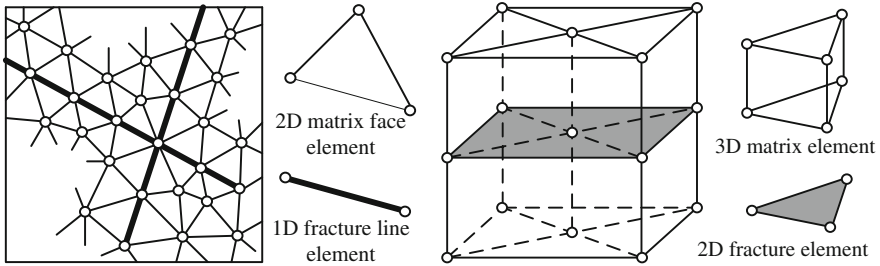


Fig. 2.6 Mesh schematics of discrete fractured model. **a** 2-D problem; **b** 3-D problem

where m is the number of element nodes; $N = [N_1, \dots, N_m]$ is shape function; $\Phi_w = [(\Phi_w)_1, \dots, (\Phi_w)_m]^T$ is flow potential value of wetting phase at element nodes; $S_w = [(S_w)_1, \dots, (S_w)_m]^T$ is saturation value of wetting phase at element nodes.

Substituting Eq. (2.22) into Eqs. (2.20) and (2.21) and considering the arbitrariness of variation results in the following equation:

$$\begin{bmatrix} 0 & 0 \\ 0 & M_S \end{bmatrix} \begin{bmatrix} \dot{\Phi}_w \\ \dot{S}_w \end{bmatrix} + \begin{bmatrix} B_{\Phi 1} & B_{\Phi 2} \\ B_{S1} & B_{S2} \end{bmatrix} \begin{bmatrix} \Phi_w \\ S_w \end{bmatrix} = \begin{bmatrix} Q_\Phi \\ Q_S \end{bmatrix}, \quad (2.23)$$

where

$$\begin{aligned} B_{\Phi 1} &= \sum_e B_{\Phi 1}^e = \sum_e \int_{\Omega^e} \nabla^T N [K(\lambda_w + \lambda_n)] \nabla N d\Omega^e; \\ B_{\Phi 2} &= \sum_e B_{\Phi 2}^e = \sum_e \int_{\Omega^e} \nabla^T N (K \lambda_n p'_c) \nabla N d\Omega^e; \\ Q_\Phi &= \sum_e Q_\Phi^e = \sum_e \int_{\Omega^e} \nabla^T N (q_n + q_w) d\Omega^e; \quad M_S = \sum_e M_S^e \int_{\Omega^e} N^T \phi N d\Omega^e; \\ B_{S1} &= \sum_e B_{S1}^e = \sum_e \int_{\Omega^e} \nabla^T N (K \lambda_w) \nabla N d\Omega^e; \quad B_{S2} = 0; \quad Q_S = \sum_e Q_S^e = \sum_e \int_{\Omega^e} \nabla^T N q_w d\Omega^e, \end{aligned}$$

where e denotes the elements set.

The interface of fractures and the matrix system needs special numerical calculation, for the reducing dimension of fractures in the discrete fractured model. As shown in Fig. 2.6, the nodes of fracture element and the matrix element are coincident at the interface. Assuming the water is wetting phase, the pressure of water phase at interface of fracture and the matrix is continuous. Therefore, the flow potential of water is continuous. After calculating the fracture element and the matrix element, respectively, and using superposition principle, the completely matrix equation can be developed, as shown in Fig. 2.7.

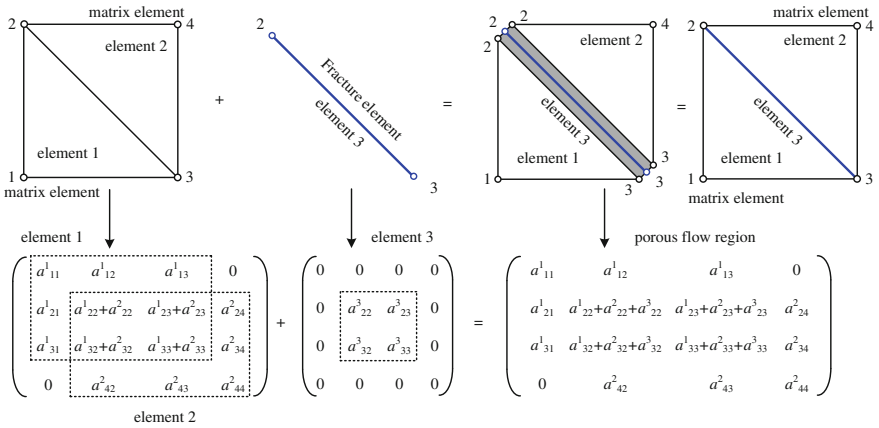
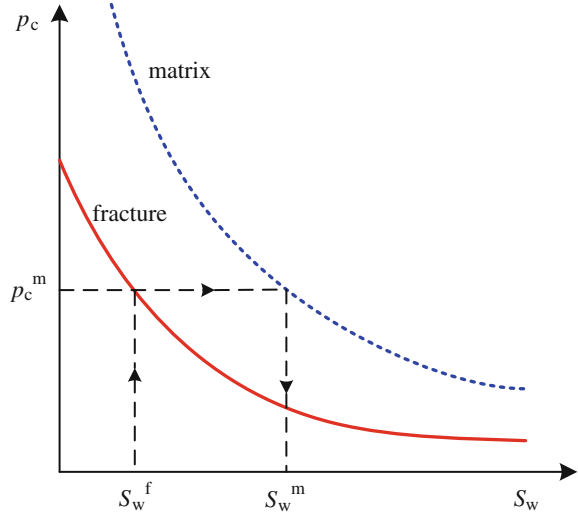


Fig. 2.7 Schematic of processing fracture and matrix element

Fig. 2.8 Capillary pressure curve for fracture and matrix



Generally, the capillary pressure curves of fractures and the matrix system are different. The saturation at interface of fractures and the matrix system is not always continuous, as shown in Fig. 2.8. Herein, the water phase saturation equation of fractures needs special treatment in above superposing process. For the flow potential of water is continuous, the capillary force potential and the capillary pressure are both continuous. Combining Fig. 2.8, the saturation of fractures and the matrix system meet the equation as follows:

$$\lambda^{\text{up}} = \begin{cases} \lambda_i & \text{if } \Phi_i \geq \Phi_j \\ \lambda_j & \text{if } \Phi_i < \Phi_j \end{cases} \quad (2.24)$$

For fractures, Eq. (2.12) can be written as

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} \Phi_w^f \\ S_w^f \end{bmatrix} + \nabla \cdot \left\{ - \begin{bmatrix} \mathbf{K}^f (\lambda_w^f + \lambda_n^f) & \mathbf{K}^f \lambda_n^f (p_c^f)' \\ \mathbf{K}^f \lambda_w^f & 0 \end{bmatrix} \nabla \begin{bmatrix} \Phi_w^f \\ S_w^f \end{bmatrix} \right\} = \begin{bmatrix} q_w^f + q_n^f \\ q_w^f \end{bmatrix} \quad (2.25)$$

Substituting the water phase flow potential continuous conditions and Eq. (2.24) into Eq. (2.25), we obtain

$$\begin{aligned} & \begin{bmatrix} 0 & 0 \\ 0 & \phi^f \frac{ds_w^f}{ds_w^m} \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} \Phi_w^m \\ S_w^m \end{bmatrix} + \nabla \cdot \left\{ - \begin{bmatrix} \mathbf{K}^f (\lambda_w^f + \lambda_n^f) & \mathbf{K}^f \lambda_n^f (p_c^f)' \frac{ds_w^f}{ds_w^m} \\ \mathbf{K}^f \lambda_w^f & 0 \end{bmatrix} \nabla \begin{bmatrix} \Phi_w^m \\ S_w^m \end{bmatrix} \right\} \\ & = \begin{bmatrix} q_w^f + q_n^f \\ q_w^f \end{bmatrix} \end{aligned} \quad (2.26)$$

As shown in Eq. (2.26), only when $\frac{ds_w^f}{ds_w^m} = 1$, $S_w^f = S_w^m$ can be derived, the saturation continuous, i.e.,

obtaining the matrix equations by combining equations of fractures and the matrix system according to Eq. (2.1) and Fig. 2.7. For each node, the corresponding network element at the interface of fractures is not always in the same fracture line or fracture surface. Therefore, the global coordinates change into local coordinates when we get the specific property matrix. After superposing the above specific property matrix, the algebra equations can be derived. For the time term, the backward difference will be used to solve the problem. Then, the flow potential and distribution of matrix saturation of water phase can be developed. The saturation value of water phase in fractures can be deduced by Eq. (2.24). The mass matrix M_s of water phase saturation equation is usually off-diagonal consistent mass matrix. The row-sum lumping method is used to get lumped mass matrix (Reddy 1993), and specific equation in e element as follows:

$$[\hat{M}]_{ii}^e = \sum_{j=1}^m \int_{\Omega} N_i^e \phi N_j^e d\Omega, \quad [\hat{M}]_{ij}^e = 0 \quad (2.27)$$

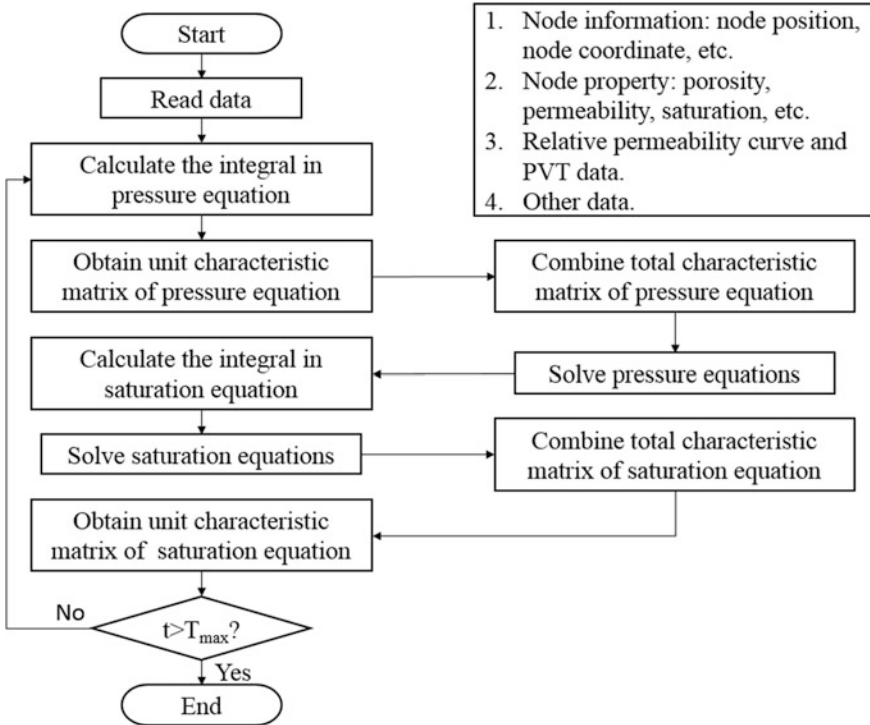


Fig. 2.9 Schematic of discrete fracture finite element numerical calculation process

Based on above numerical calculation formula, the MATLAB programming language is used to program corresponding finite element numerical calculation program of two-phase discrete fracture. The specific process is shown in Fig. 2.9.

It is possible that numerical oscillation occurs if convection dominated, when the standard Galerkin finite method is used to solve two-phase flow problem. Therefore, the upwind Galerkin calculation formula is used to solve the equations, specific equation as follows:

$$\lambda^{\text{up}} = \begin{cases} \lambda_i, & \Phi_i \geq \Phi_j \\ \lambda_j, & \Phi_i < \Phi_j \end{cases}, \quad (2.28)$$

where the fluidity coefficient is defined at every nodes of elements. The upwind Galerkin calculation formula has well stability and convergence. Corresponding analysis refers to references (Dalen 1979; Helmig and Huber 1998).

2.2.4 Numerical Examples and Applications

(1) Single fracture model

First, consider a two-phase flow problem in a single fracture. For search convenience, the wetting phase is always water and the non-wetting phase is always oil in the following context. Assume that fracture is filled with oil initially, water is injected from the left end in a constant speed and the pressure in the right end keeps the initial pressure. Make further assumption as follows: the fracture is partially filled and the length is 100 m, porosity $\phi = 0.25$, aperture is 1 mm, absolute permeability $K = 1 \mu\text{m}^2$, viscosity of water $\mu_w = 1 \text{ mPa s}$, viscosity of oil $\mu_o = 5 \text{ mPa s}$, both irreducible water saturation and residual oil saturation are zero, water phase relative permeability $k_{rw} = S_w^2$, oil phase relative permeability $k_m = (1 - S_w)^2$, initial pressure is 10 MPa, and injection rate $q = 6.0 \times 10^{-6} \text{ m/s}$.

Uniform mesh is applied, and the number of nodes is 251; the elements are quadratic; neglect the gravity and capillary pressure; the fluid is incompressible. This example is a typical Buckley–Leverett problem and its analytical solution is expressed as

$$x = \frac{f'_w(S_w)}{\phi A} \int_0^t q_{\text{in}} dt, \quad (2.29)$$

where A is the cross-sectional area of fracture, (m^2); $f_w = \lambda_w/(\lambda_w + \lambda_o)$ is water cut, $f'_w(S_w) = df_w/dS_w$. Figure 2.10 shows that there is an excellent match between numerical solution and analytical solution.

Fig. 2.10 Comparison between numerical and analytical solution

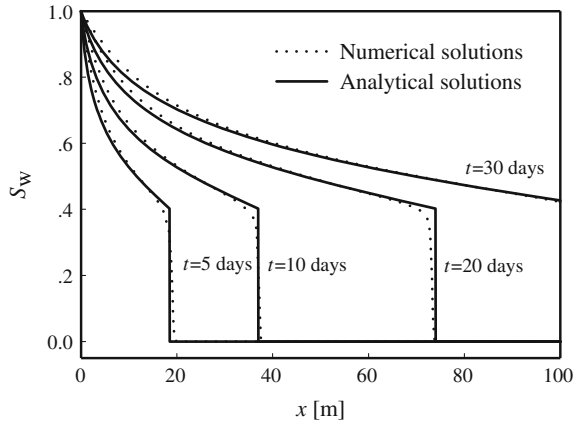
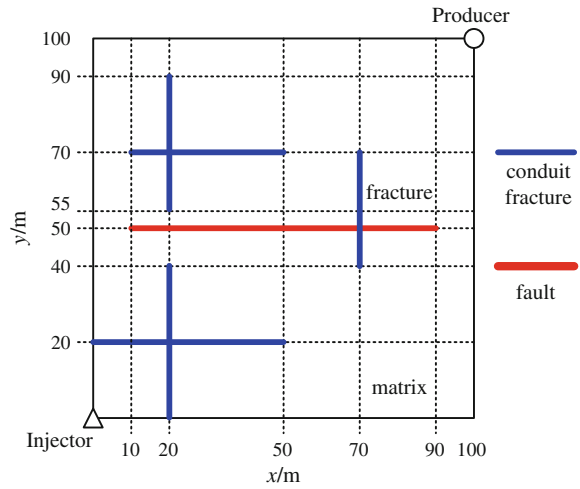


Fig. 2.11 Fractured reservoir with complex fractures

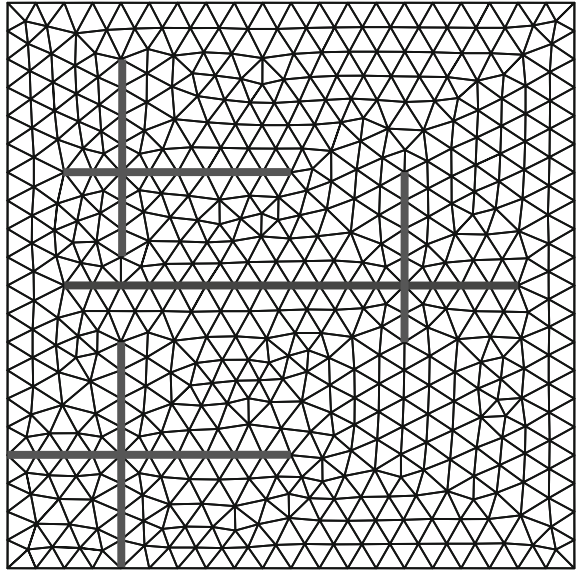


(2) Double porosity example

Consider the complex fracture reservoir model in Fig. 2.11, which includes diversion fracture and fault. The thickness of the reservoir is 10 m. Porosity of homogeneous isotropic matrix $\phi = 0.2$, permeability $K_m = 1000 \mu\text{m}^2$; fracture aperture $a = 1 \text{ mm}$, permeability $K_f = a^2/12 = 8.33 \times 10^4 \mu\text{m}^2$. Viscosity of water $\mu_w = 1 \text{ mPa s}$ viscosity of oil $\mu_o = 5 \text{ mPa s}$, irreducible water saturation is $S_{wr} = 0$, residual oil saturation is $S_{or} = 0.2$.

Water phase relative permeability $k_{rw} = S_w^2$, oil phase relative permeability $k_m = (1 - S_w)^2$, initial pressure is 10 MPa, both injection and production rates are

Fig. 2.12 Finite element meshes



$q = 30 \text{ m}^3/\text{d}$. Assume that the reservoir model is water wetting, and the capillary pressure curve follows the Brooks–Corey capillary pressure function, and that both the matrix’s capillary pressure and the fracture’s pressure are considered.

$$p_c(S_w) = p_d \left(\frac{S_w - S_{wr}}{1 - S_{wr} - S_{or}} \right)^{-\frac{1}{\lambda}}, \quad 0.2 \leq \lambda \leq 3.0 \quad (2.30)$$

For matrix, threshold value $p_c = 10000 \text{ Pa}$, λ equals to 2.0. For fractures, threshold value $p_d = 1000 \text{ Pa}$, λ equals to 1.0.

As illustrated in Fig. 2.12, the finite element meshes consist of 532 nodes and 982 elements. Figure 2.13 is representing the water saturation distributions at different times. The results indicate that the injected water displaces oil down the matrix and then moves forward rapidly along fractures when the oil/water front encounters conduit fractures, as suggested by Fig. 2.13a. At the same time, fault acts as a flow barrier, which forces the fluid flow along the extension direction of the fault, as shown in Fig. 2.13b; conduit fractures connected with fault can rapidly introduce lower fluid into upper reservoir across the fault, as shown in Fig. 2.13c, d. Evidently, the existence of fractures results in strong heterogeneity and anisotropy, which have a great influence in the water flooding development. Owing to the presence of capillary pressure, the recovery of water flooding development is improved because of the expansion of sweep area; but the entire development effect is still controlled by fractures.

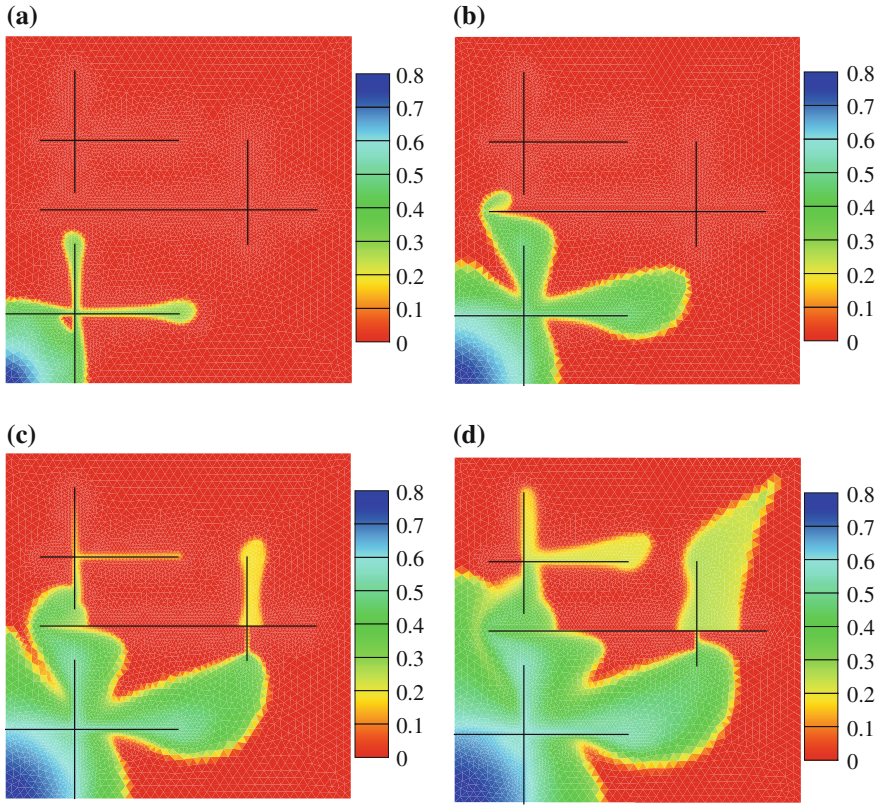


Fig. 2.13 Water saturation distributions at different times. **a** After 20 days; **b** after 50 days; **c** after 80 days; **d** after 120 days

2.3 Control Volume Method Numerical Simulation

First, two-phase flow control equations are developed. Then results in discrete fracture model according to the equivalence principle of quantity of flow for single fracture are presented, accompanied with the saturation relationship at the interface of inhomogeneous media. The fundamental assumptions of the reservoir model are as follows:

- (1) The flow in this reservoir model is isothermal flow;
- (2) Considering the existence of two-phase: water and oil, which cannot dissolve and react with each other, and their flow both follow the Darcy law;
- (3) The fluid in matrix and fractures is slight compressible;
- (4) Neglecting the compressibility of rock mass;
- (5) Neglecting the effect of gravity, considering the effect of capillary pressure.

2.3.1 Two-Phase Flow Control Equations

The two-phase flow control equations comprise mass conservation equation, the Darcy law, state equation, saturation equation, capillary pressure relationship, etc. Considering the gravity and capillary pressure of fluid, the mathematic model which could describe the two-phase flow of slightly compressible fluid in reservoir can be established.

(1) Mass conservation equation.

Based on the principle of mass conservation, the continuity equation of oil phase and water phase can be established, respectively.

For oil phase:

$$-\nabla \cdot (\rho_o \cdot \mathbf{v}_o) + Q_o = \frac{\partial(\phi \rho_o S_o)}{\partial t} \quad (2.31)$$

For water phase:

$$-\nabla \cdot (\rho_w \cdot \mathbf{v}_w) + Q_w = \frac{\partial(\phi \rho_w S_w)}{\partial t}, \quad (2.32)$$

where o represents oil, w represents water; ρ_i is fluid density, kg/m^3 ; $\mathbf{v}_i$ is fluid velocity, m/s ; ϕ is formation porosity; S_i fluid saturation; Q_i is source term which represents mass change in unit time and unit volume, and Q_i equals to positive value for injection well and Q_i equals to negative value for producing well, $\text{kg}/(\text{m}^3 \text{ s})$.

(2) Momentum equation.

When the fluid in reservoir follows the Darcy law, flow velocity can be expressed as follows:

For oil phase:

$$\mathbf{v}_o = -\frac{k_{ro}\mathbf{K}}{\mu_o}(\nabla p_o + \rho_o g \nabla z) \quad (2.33)$$

For water phase:

$$\mathbf{v}_w = -\frac{k_{rw}\mathbf{K}}{\mu_w}(\nabla p_w + \rho_w g \nabla z), \quad (2.34)$$

where μ_i is fluid viscosity, Pa s ; p_i is fluid pressure; g is acceleration of gravity, m/s^2 ; z is highness from a reference plane, positive on the upward side, m ; K_{ri} is relative permeability; $\mathbf{K}$ is permeability tensor, which changes into scalar K in isotropic matrix, m^2 .

For 2-D problem, the permeability tensor $\mathbf{K}$ is defined as

$$\mathbf{K} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}$$

For 3-D problem, the permeability tensor $\mathbf{K}$ is defined as

$$\mathbf{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

If the feature vector of $\mathbf{K}$ coincides with the direction of coordinate axis, $\mathbf{K}$ degenerates into the tensor in diagonal form:

$$\mathbf{K} = \begin{bmatrix} K_{xx} & & \\ & K_{yy} & \\ & & K_{zz} \end{bmatrix}$$

The direction of permeability tensor is always different from coordinate axis, especially for the complex reservoir. Therefore, full permeability tensor is needed for making up the deviation. But most of the simulator cannot simulate this kind of permeability now (Durlofsky 1993).

(3) State equation.

Considering the compressibility of oil and water, we obtain

For oil phase:

$$C_o = \frac{1}{\rho_o} \frac{d\rho_o}{dp_o} \quad (2.35)$$

For water phase:

$$C_w = \frac{1}{\rho_w} \frac{d\rho_w}{dp_w}, \quad (2.36)$$

where ρ_i is fluid density, kg/m^3 ; p_i is fluid pressure, Pa; C_l is elastic compression coefficient of fluid, Pa^{-1} .

(4) Auxiliary equation.

Saturation equation:

$$S_o + S_w = 1 \quad (2.37)$$

Capillary pressure equation:

$$p_o - p_w = p_c(S_w), \quad (2.38)$$

where p_c is capillary pressure, Pa.

Substituting above momentum equations of oil phase and water phase into continuity equation, respectively, we obtain

For oil phase:

$$\nabla \cdot \left(\rho_o \cdot \frac{k_{ro} \mathbf{K}}{\mu_o} (\nabla p_o + \rho_o g \nabla z) \right) + Q_o = \frac{\partial(\phi \rho_o S_o)}{\partial t} \quad (2.39)$$

For water phase:

$$\nabla \cdot \left(\rho_w \cdot \frac{k_{rw} \mathbf{K}}{\mu_w} (\nabla p_w + \rho_w g \nabla z) \right) + Q_w = \frac{\partial(\phi \rho_w S_w)}{\partial t} \quad (2.40)$$

Equations (2.39) and (2.40) are simplified by compound function derivation law, we obtain

For oil phase:

$$\nabla \cdot \left(\rho_o \cdot \frac{k_{ro} \bar{\mathbf{K}}}{\mu_o} (\nabla p_o + \rho_o g \nabla z) \right) + Q_o = \phi \left(\rho_o \frac{\partial S_o}{\partial t} + S_o \frac{\partial \rho_o}{\partial t} \right) \quad (2.41)$$

For water phase:

$$\nabla \cdot \left(\rho_w \cdot \frac{k_{rw} \bar{\mathbf{K}}}{\mu_w} (\nabla p_w + \rho_w g \nabla z) \right) + Q_w = \phi \left(\rho_w \frac{\partial S_w}{\partial t} + S_w \frac{\partial \rho_w}{\partial t} \right) \quad (2.42)$$

Assuming condition as follows:

$$\frac{\partial \rho_i}{\partial t} = \frac{\partial \rho_i}{\partial p_i} \frac{\partial p_i}{\partial t} \quad l = (o, w) \quad (2.43)$$

Then Eqs. (2.41) and (2.42) can be expressed as

For oil phase:

$$\nabla \cdot \left(\rho_o \cdot \frac{k_{ro} \mathbf{K}}{\mu_o} (\nabla p_o + \rho_o g \nabla z) \right) + Q_o = \phi \rho_o \left(\frac{\partial S_o}{\partial t} + S_o \frac{1}{\rho_o} \frac{\partial \rho_o}{\partial p_o} \frac{\partial p_o}{\partial t} \right) \quad (2.44)$$

For water phase:

$$\nabla \cdot \left(\rho_w \cdot \frac{k_{rw} \mathbf{K}}{\mu_w} (\nabla p_w + \rho_w g \nabla z) \right) + Q_w = \phi \rho_w \left(\frac{\partial S_w}{\partial t} + S_w \frac{1}{\rho_w} \frac{\partial \rho_w}{\partial p_w} \frac{\partial p_w}{\partial t} \right) \quad (2.45)$$

Substituting state equation into Eqs. (2.44) and (2.45), we obtain

For oil phase:

$$\nabla \cdot \left(\rho_o \cdot \frac{k_{ro} \mathbf{K}}{\mu_o} (\nabla p_o + \rho_o g \nabla z) \right) + Q_o = \phi \rho_o \frac{\partial S_o}{\partial t} + \phi \rho_o S_o C_o \frac{\partial p_o}{\partial t} \quad (2.46)$$

For water phase:

$$\nabla \cdot \left(\rho_w \cdot \frac{k_{rw} \mathbf{K}}{\mu_w} (\nabla p_w + \rho_w g \nabla z) \right) + Q_w = \phi \rho_w \frac{\partial S_w}{\partial t} + \phi \rho_w S_w C_w \frac{\partial p_w}{\partial t} \quad (2.47)$$

Divide Eq. (2.46) and Eq. (2.47) by fluid density ρ_i ($i = \{o, w\}$), and standard control equation can be developed, which can describe immiscible displacement of two-phase slightly compressible fluid.

For oil phase:

$$\phi \frac{\partial S_o}{\partial t} + \phi S_o C_o \frac{\partial p_o}{\partial t} - \nabla \cdot \left(\frac{k_{ro} \mathbf{K}}{\mu_o} (\nabla p_o + \rho_o g \nabla z) \right) - q_o = 0 \quad (2.48)$$

For water phase:

$$\phi \frac{\partial S_w}{\partial t} + \phi S_w C_w \frac{\partial p_w}{\partial t} - \nabla \cdot \left(\frac{k_{rw} \mathbf{K}}{\mu_w} (\nabla p_w + \rho_w g \nabla z) \right) - q_w = 0, \quad (2.49)$$

where $q_i = Q_i/\rho_i$, which represents quantity of flow into or out in unit volume unit time, S^{-1} .

Assuming condition as follows:

$$\lambda_l = \frac{k_{rl} \mathbf{K}}{\mu_l} \quad (2.50)$$

The flow potential of l phase can be defined as

$$\Phi_l = p_l + \rho_l g z \quad (2.51)$$

Then the capillary pressure flow potential can be defined as

$$\Phi_c = \Phi_o - \Phi_w = P_c + (\rho_o - \rho_w) g z \quad (2.52)$$

Based on the above definition, permeability scalar K is employed to replace permeability tensor in homogeneous isotropic matrix. Neglecting gravity, the immiscible flow control equation of two-phase slightly compressible fluid can be expressed as

For oil phase:

$$\phi \frac{\partial S_o}{\partial t} + \phi S_o C_o \frac{\partial p_o}{\partial t} - \nabla \cdot (\lambda_o \nabla p_o) - q_o = 0 \quad (2.53)$$

For water phase:

$$\phi \frac{\partial S_w}{\partial t} + \phi S_w C_w \frac{\partial p_w}{\partial t} - \nabla \cdot (\lambda_w \nabla p_w) - q_w = 0 \quad (2.54)$$

Add control equations of oil phase and water phase up and keep the water phase control equation. Combine two auxiliary equations and define composite compressibility as $C_t = S_w C_w + S_o C_o$, $\frac{\partial p_c}{\partial t} \approx 0$. Then above mathematical model can be expressed as two partial differential equations:

$$-\phi C_t \frac{\partial p_w}{\partial t} + \nabla \cdot ((\lambda_o + \lambda_w) \nabla p_w) + \nabla \cdot (\lambda_o \nabla p_c) + (q_o + q_w) = 0 \quad (2.55)$$

$$\phi \frac{\partial S_w}{\partial t} + \phi S_w C_w \frac{\partial p_w}{\partial t} - \nabla \cdot (\lambda_w \nabla p_w) - q_w = 0 \quad (2.56)$$

Equation (2.55) is pressure equation, and Eq. (2.56) is saturation equation.

The mathematical model initial conditions are

$$p_i(\mathbf{x}, 0) = p_i(\mathbf{x}), \quad S_i(\mathbf{x}, 0) = S_i(\mathbf{x}), \quad t = 0, \quad i = \{w, o\} \quad (2.57)$$

The boundary conditions can be all forms of Dirichlet boundary condition, Neumann boundary condition, and mixed mode.

Dirichlet condition is

$$p_i(\mathbf{x}, t) = p_i, \quad S_i(\mathbf{x}, t) = S_i, \quad i = \{w, o\}, \quad \text{on } \Gamma_D \quad (2.58)$$

Neumann condition is (assume the boundary is impermeable)

$$v_i \cdot \vec{n} = -(\lambda_i \nabla p_i) \cdot \vec{n} = 0, \quad \nabla S_i \cdot \vec{n} = 0, \quad i = \{w, o\}, \quad \text{on } \Gamma_N \quad (2.59)$$

Then the immiscible flow mathematical model of two-phase slightly compressible fluid is developed.

2.3.2 Discrete Fracture Mathematical Model

Based on the equivalence principle of quantity of flow for single fracture, discrete fracture model can be developed, as shown in Fig. 2.14. The model is built upon a parallel plate of single fracture and is fracture aperture. Assume that flow of fluid in fracture follows N-S equation. The flow in parallel plate is laminar flow when fluid velocity is small. The velocity distribution and quantity of flow along the fracture aperture can be obtained. According to Darcy's law, the quantity of flow gives the fractures' equivalent permeability and the equivalent flow velocity distribution. And the value is kept constant along the direction of fracture in aperture. Evidently, the instability of the flow parameters and correlative physical quantities along the direction of the fracture in aperture reduce the dimension of the fracture in aperture and develop the discrete fracture model.

Fractures are simplified into 1-D line element for 2-D problem, and 2-D surface area element for 3-D problem, as shown in Fig. 2.15.

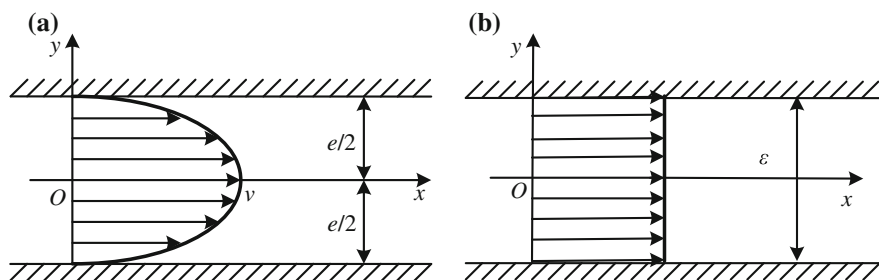


Fig. 2.14 Velocity distribution in single fracture. **a** Realistic velocity, **b** equivalent velocity

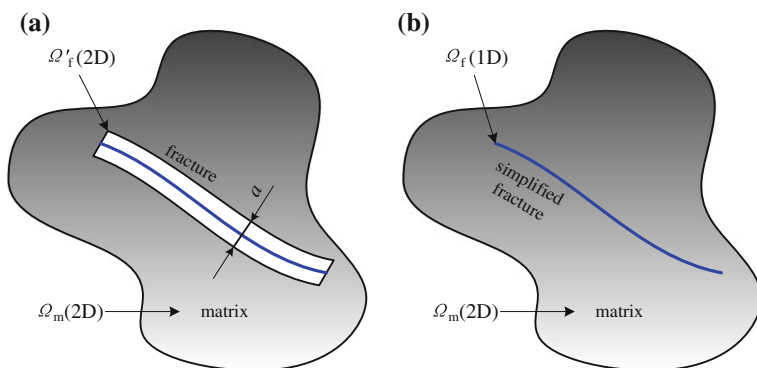


Fig. 2.15 Schematic of discrete fracture model. **a** Single-porosity model, **b** discrete fracture model

Considering the 2-D porous media region in single fracture, the whole area is Ω , and the matrix area is Ω_m , the fracture area is Ω_f in single fracture model and is $a\Omega'_f$ in discrete fracture model. So based on the discrete fracture model, the whole region of reservoir can be expressed as

$$\Omega = \Omega_m + a\Omega'_f \quad (2.60)$$

where a is fracture aperture. In above 2-D discrete fracture model, the 2-D control equation system of the matrix region is

$$-\phi^m C_t \frac{\partial p_w^m}{\partial t} + \nabla \cdot ((\lambda_o^m + \lambda_w^m) \nabla p_w^m) + \nabla \cdot (\lambda_o^m \nabla p_c^m) + (q_o^m + q_w^m) = 0 \quad (2.61)$$

$$\phi^m \frac{\partial S_w^m}{\partial t} + \phi^m S_w^m C_w \frac{\partial p_w^m}{\partial t} - \nabla \cdot (\lambda_w^m (\nabla p_w^m)) - q_w^m = 0 \quad (2.62)$$

The 1-D control equation system of the matrix region is

$$-\phi^f C_t \frac{\partial p_w^f}{\partial t} + \left((\lambda_o^f + \lambda_w^f) \frac{\partial p_w^f}{\partial \xi} \right) + \frac{\partial}{\partial \xi} \left(\lambda_o^f \frac{\partial p_c^f}{\partial \xi} \right) + (q_w^f + q_o^f) = 0 \quad (2.63)$$

$$\phi^f \frac{\partial S_w^f}{\partial t} + \phi^f S_w^f C_w \frac{\partial p_w^f}{\partial t} - \frac{\partial}{\partial \xi} \left(\lambda_w^f \left(\frac{\partial p_w^f}{\partial \xi} \right) \right) - q_w^f = 0, \quad (2.64)$$

where ξ is coordinate system along the direction of fracture in aperture.

In single fracture model, if f is used to represent pressure equation and saturation equation system, the integral form of the whole equation can be written as

$$\int_{\Omega} f d\Omega = \int_{\Omega_m} f^m d\Omega_m + \int_{\Omega_f} f^f d\Omega_f = 0 \quad (2.65)$$

According to $\Omega = \Omega_m + \varepsilon\Omega'_f$, the integral form of pressure equation and saturation equation in the discrete fracture model can be expressed as

$$\int_{\Omega} f d\Omega = \int_{\Omega_m} f^m d\Omega_m + \varepsilon \int_{\Omega'_f} f^f d\Omega'_f = 0 \quad (2.66)$$

That way, the two-phase discrete fracture model flow equation is developed. In theory, the discrete fracture model can be applied to any fractured porous media that has complex form. Compared to single fracture model, the integration in fractures of the discrete fracture model can simplify the problem to a great extent. And fracture aperture will appear in front of 1-D integral form as a coefficient in order to keep the integral form.

2.3.3 Saturation Discontinuity Treatment at the Interface

After discretized space and time, and performing linearization of the nonlinear term in Eq. (2.66), we can get the discrete equation of system:

$$\int_{\Omega} \mathbf{f} d\Omega = \mathbf{A}^m \mathbf{x}^m - \mathbf{b}^m + \mathbf{A}^f \mathbf{x}^f - \mathbf{b}^f = 0, \quad (2.67)$$

where

$$\mathbf{x} = [\mathbf{p}_w, \mathbf{S}_w]^T$$

Neglecting the compressibility of fluid, Karimi-Fard, Firoozabadi, Kim, Deo (Karimi-Fard and Firoozabadi 2003; Kim and Deo 2000) have changed Eq. (2.67) into Eq. (2.68).

$$(\mathbf{A}^m + \mathbf{A}^f) \mathbf{x} - \mathbf{b}^m - \mathbf{b}^f = 0 \quad (2.68)$$

There is an implicit relationship $\mathbf{x}^m = \mathbf{x}^f = \mathbf{x}$ in Eq. (2.68) and it is only applied to particular circumstances. Hence, the relationship between the matrix system and fracture as well as the corresponding equation in the interface of the matrix and fracture needs to be developed based on a real physical meaning.

There is no change of fluid mass in the interface Γ_{mf} of the matrix and fracture, so the quantity of flow and direction of normal velocity are continuous, i.e.,

$$q_i^{f*} = q_i^{m*}, \quad v_i^m \cdot \mathbf{n}_{mf} = v_i^m \cdot \mathbf{n}_{mf}, \quad i = \{w, o\}, \quad \text{on } \Gamma_{mf}, \quad (2.69)$$

where $\mathbf{n}_{mf}$ is the normal vector at the matrix and fracture.

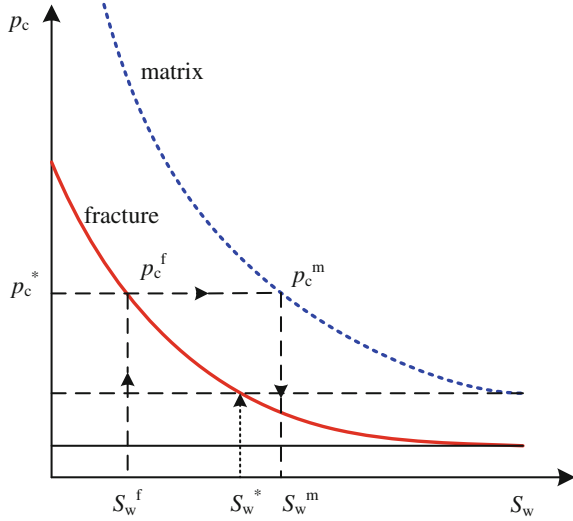
Superposition principle is applied to integrate flow equations in discrete fracture mode and these terms will be eliminated when flow equations are added up. So the quantity of flow at interface of the matrix and fracture can be neglected in flow equations.

The coordinate of arbitrary fixed point z at interface of the matrix and fracture is same and $\Phi_i^m = \Phi_i^f$ can be known from Eq. (2.51). The capillary pressure potential is same too as shown in Eq. (2.52), i.e.,

$$\Phi_c^m(S_w^m) = \Phi_c^f(S_w^f) \quad (2.70)$$

It is equivalent to the continuity of capillary pressure. Figure 2.16 is capillary pressure at the interface of two different phases. Because the capillary pressure at the interface of the matrix and fracture is same, $p_c^f = p_c^m = p_c^*$, and the water saturation deciding capillary pressure is discontinuous at the interface. The physical relationship of S_w^f and S_w^m at the interface can be developed using the continuity condition of capillary pressure.

Fig. 2.16 Capillary pressure at the interface between matrix and fracture



$$S_w^m = \begin{cases} 1, & S_w^f > S_w^* \\ [p_c^m]^{-1} p_c^f(S_w^f), & S_w^f \leq S_w^* \end{cases} \quad (2.71)$$

Making use of Eq. (2.71) and compound function derivation law, the saturation equation of fractures can be applied to the water saturation S_w^m of the matrix and be expressed as follows:

$$\phi^f \frac{dS_w^f}{dS_w^m} \frac{\partial S_w^m}{\partial t} + \phi^f S_w^f C_w \frac{\partial p_w^f}{\partial t} - \frac{\partial}{\partial \xi} \left(\lambda_w^f \left(\frac{\partial p_w^f}{\partial \xi} \right) \right) - q_w^f = 0 \quad (2.72)$$

According to the assumption $\mathbf{x}^m = \mathbf{x}^f = \mathbf{x}$ in Eq. (2.68), it can be applied only when the capillary pressure function of the matrix and fracture is same, namely when $dS_w^f/dS_w^m = 1$. Thus the corresponding dS_w^f/dS_w^m should be calculated for different capillary pressure functions of the matrix and fracture. The fluid exchange term can be neglected, because it will be eliminated when added up in control volume element.

2.3.4 Control Volume Numerical Formulation

The control volume method was originally used in computational fluid dynamics. And it is essentially a finite volume computation format based on Delaunay mesh dual element. Hence, the Delaunay dual mesh is needed first when developing the computation format based on control volume method. Then by integrating the

pressure equation and the saturation equation at each control volume element, the numerical computation format can be established.

(1) Discrete fracture model control volume mesh generation

Unstructured mesh is applied to accomplish geometrical discrete of discrete fracture model for the distribution of fractures in fractured reservoir is random. For 2-D problem, first, Delaunay triangle mesh is generated. Triangle element will be employed to discrete the matrix and 1-D line element represents the fracture area, as shown in Fig. 2.17a. So-called control volume is a polygonal area. Each area controlled by a node is connected by the center of gravity of adjacent triangle element and midpoint of side that links to the node. And volume element will be divided into three areas by lines connected by the center of gravity and midpoint of side.

As shown in Fig. 2.17b, the adjacent nodes of node a are $\{b_1, b_2, \dots, b_6\}$, triangles which take node a as the vertex are $\{T_1, T_2, \dots, T_6\}$, the center of gravity of triangles are $\{G_1, G_2, \dots, G_6\}$, midpoints of sides which take node a as vertex are $\{M_{ab_1}, M_{ab_2}, \dots, M_{ab_6}\}$. The control volume element of node a is polygon $G_1M_{ab_1}G_2M_{ab_2}G_3M_{ab_3}G_4M_{ab_4}G_5M_{ab_5}G_6M_{ab_6}$, which can be obtained by connecting the center of gravity of triangles with corresponding midpoint of sides. Control volume element of the other nodes in research region can be obtained in the same way, where ab_1 represents fractures. In standard control volume element, Delaunay triangle is local homogeneous while control volume element might be nonhomogeneous. The major characteristics of polygon control volume are covering the entire calculation region without overlap and keeping calculation accuracy by the cross-distribution of triangle elements and control volume.

The matrix of fractured porous media is homogeneous. Considering saturation variable (S_w, S_o) is constant in each control volume element while flow pressure variable (p_w, p_o, p_c) can be estimated with linear approximation by the value of Delaunay mesh element (triangle or tetrahedron) which comprise control volume element:

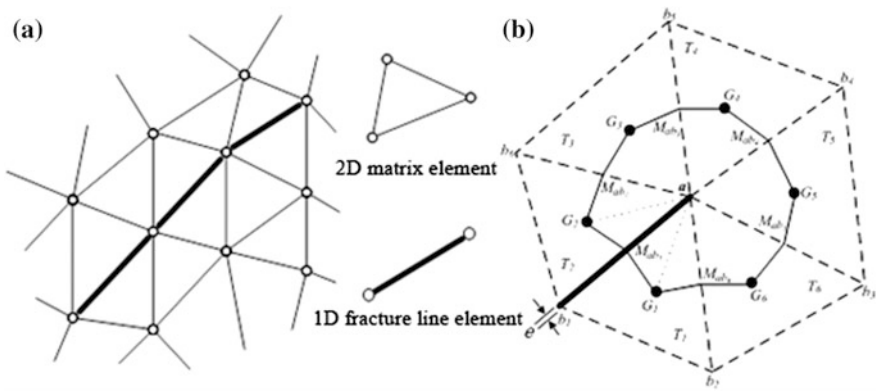


Fig. 2.17 Delaunay triangle mesh and the dual control volume mesh of 2-D discrete fracture model

$$\Psi(\mathbf{x}) = \sum_{i=1}^m S_i(\mathbf{x}) \Psi_i, \quad (2.73)$$

where x is coordinate in the dimension of corresponding control volume element; m is the number of vertex; Ψ_i is arbitrary variable of node i at coordinate $\mathbf{x}_i$; S_i is shape factor and defined as follows:

For triangle:

$$S_i(\mathbf{x}) = \frac{\alpha_i + \beta_i x + \gamma_i y}{2A} \quad (2.74)$$

For tetrahedron:

$$S_i(\mathbf{x}) = \frac{\alpha_i + \beta_i x + \gamma_i y + \delta_i z}{6V}, \quad (2.75)$$

where A is area of triangle element, m^2 ; V is volume of tetrahedron element, m^3 ; $(\alpha_i, \beta_i, \gamma_i, \delta_i)$ are constants about geometric coordinates of element nodes.

Assume that constants are used to coding triangle element nodes and anti-clockwise direction is positive direction. Thus $\alpha_i, \beta_i, \gamma_i$ in Eq. (2.74) can be expressed as follows, respectively (Wang 2003):

$$\begin{aligned} \alpha_i &= \begin{vmatrix} x_j & y_j \\ x_k & y_k \end{vmatrix} = x_j y_k - x_k y_j \\ \beta_i &= - \begin{vmatrix} 1 & x_j \\ 1 & y_k \end{vmatrix} = y_j - y_k \\ \gamma_i &= \begin{vmatrix} 1 & x_j \\ 1 & x_k \end{vmatrix} = -x_j + x_k \end{aligned} \quad (2.76)$$

From Eq. (2.73), the gradient of arbitrary variable in a triangle is

$$\nabla \Psi = \sum_{i=1}^m \Psi_i \nabla S_i(\mathbf{x}) \quad (2.77)$$

For 3-D tetrahedron, it is similar to the above 2-D triangle.

(2) The establishment of control volume computation format

To establish numerical computation format of mathematical model by control volume method, integral should be done first for pressure Eq. (2.55) and saturation Eq. (2.56) in every control volume element, respectively. In 2-D discrete fracture model, the control volume discrete computation format method has to establish saturation equation which is:

Applying integral to Eq. (2.56) in arbitrary control volume element CV_i , we can obtain

$$\iint_{\Omega} \left(\phi \frac{\partial S_w}{\partial t} + \phi S_w C_w \frac{\partial p_w}{\partial t} \right) dA - \iint_{\Omega} \nabla \cdot (\lambda_w \nabla p_w) dA - \iint_{\Omega} q_w dA = 0 \quad (2.78)$$

Assuming that porosity only changes at space and transforms surface integral into line integral for the second term of the left side of Eq. (2.78) with Gauss divergence theorem, we obtain

$$\iint_{\Omega} \left(\phi \frac{\partial S_w}{\partial t} + \phi S_w C_w \frac{\partial p_w}{\partial t} \right) dA - \int_{\Gamma} (\lambda_w \nabla p_w) \cdot \mathbf{n} d\Gamma - \iint_{\Omega} q_w dA = 0, \quad (2.79)$$

where Γ is boundary of control volume element CV_i ; $\mathbf{n}$ is unit outward normal vector on boundary Γ .

The water saturation of the matrix and fractures can be connected with each other by Eq. (2.71), the first term of the left side of Eq. (2.79) can be approximately expressed as

$$\iint_{\Omega} \left(\phi \frac{\partial S_w}{\partial t} + \phi S_w C_w \frac{\partial p_w}{\partial t} \right) dA \approx A_{\phi i} \left(\frac{\partial S_w^m}{\partial t} + S_w C_w \frac{\partial p_w}{\partial t} \right), \quad (2.80)$$

where

$$A_{\phi i} = \sum_{k=1}^t \phi_k A_k \varphi_k^m + \sum_{l=1}^s \frac{dS_w^f}{dS_w^m} e_l |L_l| \varphi_l^f, \quad (2.81)$$

where $A_{\phi i}$ is pore volume of CV_i ; t is the number of Delaunay triangle element which takes node i as vertex; φ_k is triangle k 's area ratio to Delaunay triangle k in control volume element CV_i ; A_k is the area of Delaunay triangle k ; ϕ_k^m is porosity of the matrix system in triangle k ; s is the number of fracture in CV_i ; ϕ_l^f , e_l , and $|L_l|$ are porosity, aperture, and length of fracture l in control volume element CV_i , respectively.

The first term of the right side of Eq. (2.81) represents pore volume of the matrix in CV_i ; the second term represents pore volume of fractures in CV_i . To express the whole equation with the matrix water saturation, the second term will be multiplied by $\frac{dS_w^f}{dS_w^m}$.

The second integral of the left side of Eq. (2.79) can be expressed as

$$\int_{\Gamma} (\lambda_w \nabla \Phi_w) \cdot \mathbf{n} d\Gamma \approx \sum_{k=1}^t |s_k| [\lambda_w^m (S_w^{m,up}) \nabla p_w]_k \cdot \mathbf{n}_k + \sum_{l=1}^s e_l \lambda_w^f (S_w^{f,up}) \frac{\partial p_w^f}{\partial \xi}, \quad (2.82)$$

where $|s_k|$ is internal boundary of CV_i in triangle k and has unit outward normal vector; ∇p_w is water phase flowing pressure gradient at $|s_k|$, which can be estimated by Eq. (2.47); ξ is local coordinate along the direction of fracture; $\partial p_w^f / \partial \xi$ is flowing potential gradient on fracture l .

The first term of the right side of Eq. (2.82) represents quantity of flow through control volume element CV_i 's boundary; the second term represents quantity of flow through every fracture in CV_i . The value of saturation comply with upstream standard, where superscript up represents upstream value. For the flow in fractures can viewed as 1-D, $\partial p_w^f / \partial \xi$ can be estimated by the following equation:

$$\frac{dp_w^f}{d\xi} = \frac{p_j - p_i}{2|L_l|}, \quad (2.83)$$

where p_i, p_j represent pressure of 1-D adjacent element.

The third term of the left side of Eq. (2.79) can be approximately expressed as

$$\iint_{\Omega} q_w dA \approx q_{wi} A_i, \quad (2.84)$$

where A_i is the area of 2-D control volume element CV_i and can be calculated by the following equation:

$$q_{wi} A_i = q_{wi}^m \sum_{k=1}^t \varphi_k A_k + \sum_{l=1}^s e_l |L_l| q_{w,l}^f \quad (2.85)$$

Based on above approximation, the numerical computation format of saturation equation in every control volume element can be written as

$$\begin{aligned} A_{\phi i} \left(\frac{\partial S_w^m}{\partial t} + S_w C_w \frac{\partial p_w}{\partial t} \right) - \left[\sum_{k=1}^t |s_k| [\lambda_w^m (S_w^{m,up}) \nabla p_w]_k \cdot \mathbf{n}_k + \sum_{l=1}^s e_l \lambda_w^f (S_w^{f,up}) \frac{\partial p_w^f}{\partial \xi} \right] \\ - q_{wi} A_i = 0 \end{aligned} \quad (2.86)$$

The assumption that flow pressure of corresponding mesh at interface of the matrix and fractures is similar has been done, above step could be applied to pressure equation too. For 2-D matrix system and 1-D fracture system, the numerical computation format of flowing pressure equation can be written as

$$\begin{aligned} A_{\phi i} C_t \frac{\partial p_w}{\partial t} - \left[\sum_{k=1}^t |s_k| [\lambda^m \nabla p_w + \lambda_o^m \nabla p_c]_k \cdot \mathbf{n}_k + \sum_{l=1}^s \left[\lambda^f \frac{\partial p_w}{\partial \xi} + \lambda_o^f \frac{\partial p_c}{\partial \xi} \right] e_l \right] \\ - (q_{wi} + q_{oi}) A_i = 0 \end{aligned} \quad (2.87)$$

where $\lambda = \lambda_w + \lambda_o$ is the total fluidity. Fluidity in equation should comply with upstream standard and capillary pressure gradient in fractures can be estimated with following equations: Eqs. (2.86) and (2.87) are the numerical computation format of pressure equation and saturation equation that are based on control volume method in 2-D research region. The method can be easily extended to 3-D discrete fracture model.

(3) Inhomogeneous matrix discrete fracture model numerical formulation

Control volume computation format of discrete fracture model for homogeneous matrix has been established. There have been some researchers who studied the inhomogeneous problem with the method of control volume. While they mostly had focus on the inhomogeneity of absolutely permeability and the anisotropy of unidirectional flow (Edwards 2002). For permeability, as it changes rapidly at the interface of inhomogeneous media, there will be imprecise velocity field when we use the standard control volume method at the interface (Durlofsky 1994). As shown in Fig. 2.18a, Delaunay triangle is locally homogeneous and control volume element polygon is inhomogeneous in standard control volume method. Some researchers put forward locally homogeneous control volume element to get precise velocity field, as shown in Fig. 2.18b. The fact that Delaunay triangle is inhomogeneous can be known from Fig. 2.18b.

To pressure equation and saturation equation, control volume method is used to process spatial discretization. First, do the integral to equations in a control volume element, which is the dual mesh connected by the center of gravity and midpoint of 2-D Delaunay triangle or 3-D tetrahedron. Figure 2.19a is fractured porous media control volume element schematic for homogeneous matrix, and the two-phase numerical computation format of slightly compressible fluid in discrete fracture model has been established. Figure 2.19b is fractured porous media control volume element schematic for inhomogeneous matrix, and the two-phase numerical computation format of slightly compressible fluid will be established below.

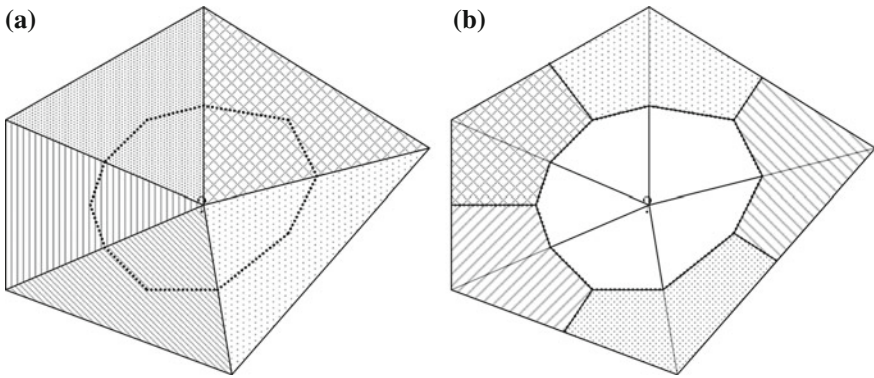


Fig. 2.18 Inhomogeneous media control volume element. **a** Standard control volume element, **b** local homogeneous control volume element

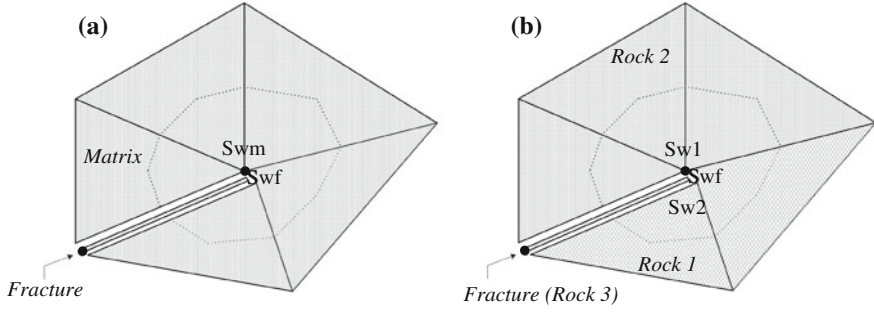


Fig. 2.19 Control volume element with fractures. **a** Homogeneous matrix, **b** Inhomogeneous matrix

Flowing pressure equation and saturation equation both are composed of three terms:

- (1) Time derivative term, it is $\phi C_t \frac{\partial p_w}{\partial t}$ in flowing pressure equation and is $\phi \frac{\partial S_w}{\partial t}$ and $\phi S_w C_w \frac{\partial p_w}{\partial t}$ in saturation equation;
- (2) Source term q_i , where $i = (o, w)$;
- (3) Divergence term $\nabla \cdot \mathbf{F}$, flow vector $\mathbf{F} = \mathbf{F}(S_w)$, which is equal to $(\lambda_o + \lambda_w) \nabla p_w$ and $\lambda_o \nabla p_c$ in flow pressure equation and is equal to $\lambda_w \nabla p_w$ in saturation equation.

As shown in Fig. 2.19b, there exists one fracture and two kinds of matrix rock mass in one control volume element. As same as the interface of matrix and fractures in above section, capillary pressure between different matrix is continuous. Hence, saturations of inhomogeneous matrix rock mass can be connected:

$$\int_A \left(\phi \frac{\partial S_w}{\partial t} + \phi S_w C_w \frac{\partial p_w}{\partial t} \right) dA = \sum_{k=1}^m \left(\frac{\partial}{\partial t} S_w^k \phi^k A^k + S_w^k C_w^k \phi^k A^k \frac{\partial p_w}{\partial t} \right), \quad (2.88)$$

where m represents the number of media in control volume element.

Equation (2.88) can be expressed on the basis of reference media S_w^+ which comply with the parameter B in capillary pressure model Eq. (2.89).

$$P_c^k = -B^k \ln S_w^k, \quad (2.89)$$

where superscript $k = 1 \cdots n_m$ is the index of media in control volume element.

The relationship between different media's saturations can be established based on the concept of capillary pressure continuation. For example, as far as fracture media:

$$S_w^m / S_w^f = \exp(-B^m / B^f)$$

The maximum of B^k in Eq. (2.89) is chosen as reference media in here. Based on Eq. (2.88) and compound function derivation method, we obtain

$$\int_A \left(\phi \frac{\partial S_w}{\partial t} + \phi S_w C_w \frac{\partial p_w}{\partial t} \right) dA = \left(\sum_{k=1}^m \frac{dS_w^k}{dS_w^+} \phi^k A^k \right) \frac{\partial}{\partial t} S_w^+ + \left(\sum_{k=1}^m S_w^k C_w^k \phi^k A^k \right) \frac{\partial p_w}{\partial t} \quad (2.90)$$

The integral of source term can be written as

$$\int_A q_w dA = \sum_{k=1}^m A^k q_w^k \quad (2.91)$$

The integral of divergence term can be confirmed in the light of equation below:

$$\int_A \nabla \cdot \mathbf{F} dA = \int_{\Gamma_A} \mathbf{F} \cdot \mathbf{n} d\Gamma_A \approx \sum_{j=1}^{n_b} [\mathbf{F} \cdot \mathbf{n}]_j, \quad (2.92)$$

where n_b is the number of boundary element.

Every triangle included in control volume element is local homogeneous. Saturation between different media can be calculated by Eq. (2.71). Combining above three equations, we obtain the numerical computation format of pressure equation and saturation equation.

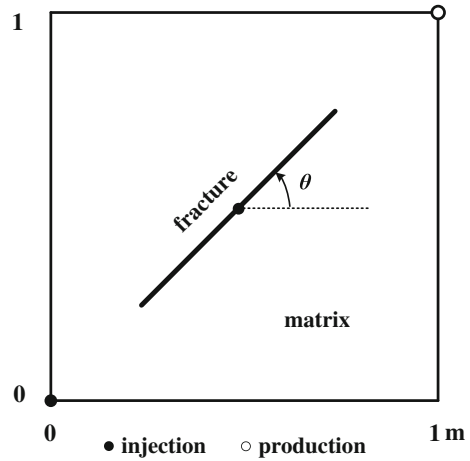
$$\left(\sum_{k=1}^m \frac{dS_w^k}{dS_w^+} \phi^k A^k \right) \frac{\partial}{\partial t} S_w^+ + \left(\sum_{k=1}^m S_w^k C_w^k \phi^k A^k \right) \frac{\partial p_w}{\partial t} - \sum_{j=1}^{n_b} [\mathbf{F} \cdot \mathbf{n}]_j - \sum_{k=1}^m A^k q_w^k = 0 \quad (2.93)$$

With this, based on control volume method, the discrete fracture model numerical computation format that considers inhomogeneous matrix has been established.

2.3.5 Numerical Examples

As shown in Fig. 2.20, there is a simple 1/4 five-point water injection scheme, the size of porous media model is 1 m $\times$ 1 m, initial pressure is $p_i = 10$ MPa, porosity of homogeneous isotropic matrix is $\phi = 0.2$, permeability is $K_m = 1 \times 10^{-3} \mu\text{m}^2$. Considering the existence of fractures in porous media with azimuthal angle of $\theta = 0^\circ$, $\theta = 45^\circ$, $\theta = 90^\circ$, and $\theta = 135^\circ$, and the center of fracture and porous media are overlapping, the length of fractures is $L = 60 \sqrt{2}$ cm, fracture aperture is

Fig. 2.20 Schematic of 2-D reservoir model



$a = 1$ mm, permeability is $K_f = a^2/12 = 8.33 \times 10^4 \mu\text{m}^2$. There is a water injection well in the lower left corner and a production well in the top right corner. The injection rate is $q_{\text{in}} = 0.01$ PV/day (PV is the acronym of Pore Volume which represents multiple of pore volume) and production rate is $q_{\text{out}} = 0.01$ PV/day. Viscosity of water phase is $\mu_w = 1$ mPa s, viscosity of oil phase is $\mu_o = 5$ mPa s, density of water phase is $\rho_w = 1$ kg/m³, density of oil phase is $\rho_o = 0.8$ kg/m³, compressibility of oil phase is $C_o = 10 \times 10^{-4}$ MPa⁻¹, compressibility of water phase is $C_w = 5 \times 10^{-4}$ MPa⁻¹, irreducible water saturation is $S_{wc} = 0$, residual oil saturation is $S_{or} = 0$, normalized saturation is $S_e = (S_w - S_{wc})/(1 - S_{wc} - S_{or})$, relative permeability of water phase for matrix and fractures is $k_{rw} = S_e$, relative permeability of oil phase for matrix and fractures is $k_{ro} = 1 - S_e$, initial water saturation is 0. Ignore the impact of capillary pressure and gravity.

As shown in Fig. 2.21, porous media models which have one fracture with angle of $\theta = 0^\circ$, $\theta = 45^\circ$, $\theta = 90^\circ$, or $\theta = 135^\circ$ (in degrees from horizontal) are described with Delaunay triangle mesh based on three models. There are 874, 878, 875, and 878 control volume elements and 1646, 1654, 1648, and 1654 triangle mesh elements after mesh generation, respectively. For single-porosity model, the fracture aperture that is 1 mm and the scale of research region can differ by three magnitudes, so it is essential to do mesh refinement for real fractures. In single-porosity model I, the number of control volume element node is 9466 and the number of elements is 18,854 for horizontal fracture model whose azimuthal angle is $\theta = 0^\circ$; the number of control volume element node is 9343 and the number of triangle elements is 18,620 for the fracture whose azimuthal angle is $\theta = 45^\circ$; the number of control volume element node is 9294 and the number of triangle elements is 18,510 for vertical fracture whose azimuthal angle is $\theta = 90^\circ$; the number of control volume element node is 9236 and the number of triangle elements is 18,406 for the fracture whose azimuthal angle is $\theta = 135^\circ$. The single-porosity model II that have not been done mesh refinement around fracture are meshed to verify the high

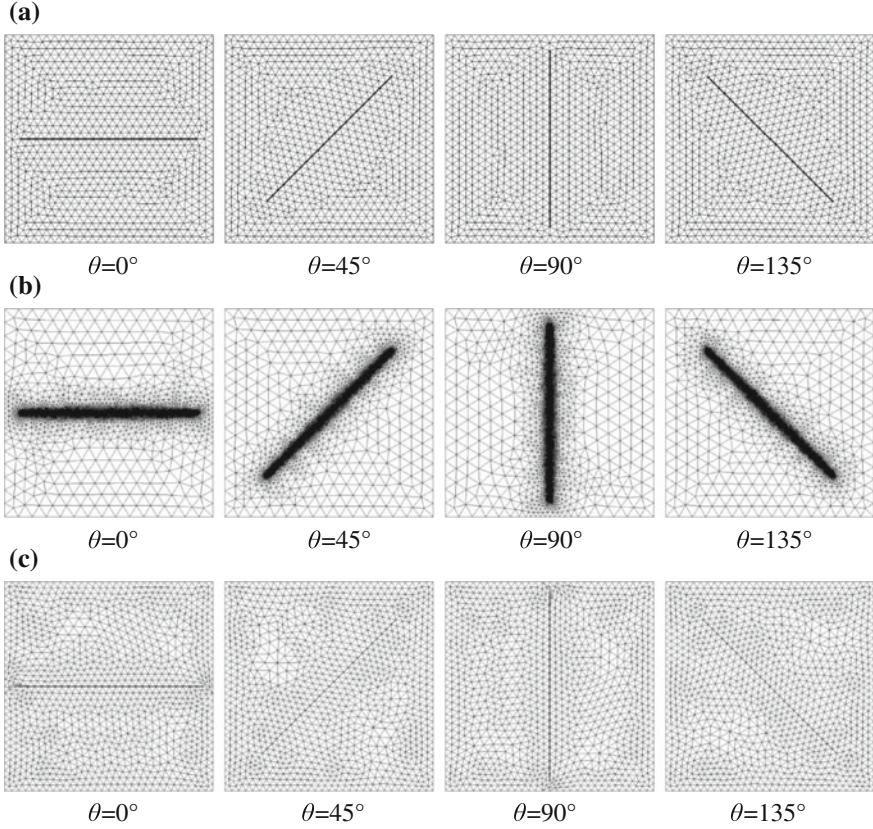


Fig. 2.21 Mesh distribution for porous media that have single fracture with different azimuthal angle. **a** Discrete fracture model, **b** single-porosity model I, **c** single-porosity model II

efficiency of discrete fracture model method. 1094, 1088, 1096, and 1092 control volume element nodes and 2068, 2054, 2070, and 2062. Delaunay triangle elements can be obtained after mesh generation. It is observed that the number of control volume element for discrete fracture model and single-porosity model tends to be similar, and both of them are much less than the control volume element number for single-porosity model I.

Water saturation section for the injection volume of 0.5 PV can be obtained based on control volume method, as shown in Fig. 2.22. As can be seen from the figure, fractures have a significant impact on fluid flow, and the computed result of discrete fracture model and single-porosity model is almost same.

Figure 2.23 is schematic of recovery degree for single-porosity media that have different azimuthal angles based on discrete fracture model and single-porosity

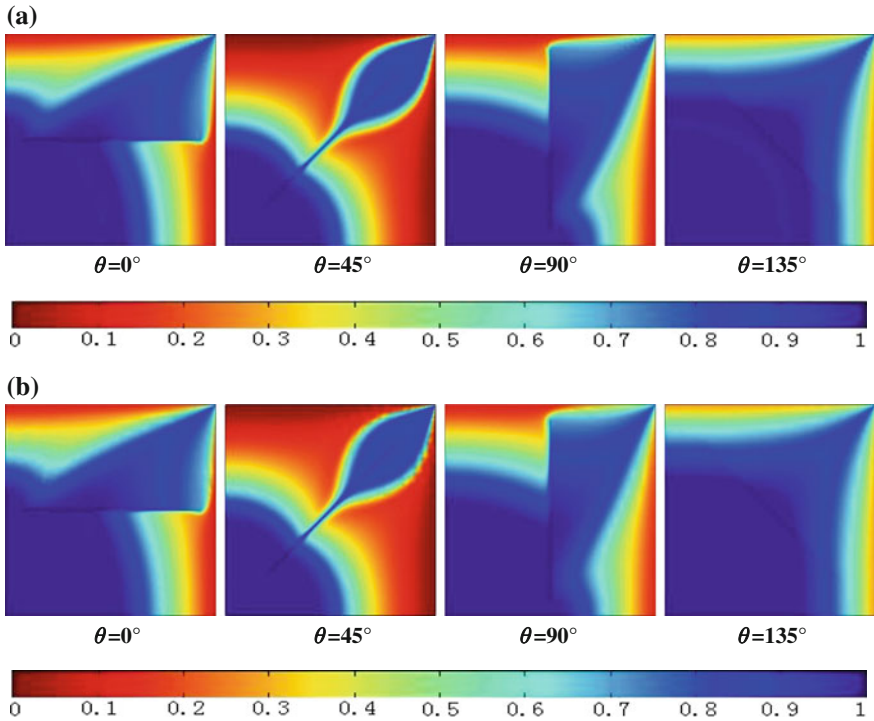


Fig. 2.22 Schematic of water saturation distribution for two models when the injection volume is 0.5 PV. **a** Discrete fracture model, **b** single-porosity model I

model I when the well has been produced for 300 days. As can be seen from the figure, the computed result of discrete fracture model and single-porosity model I showed a great consistency.

As shown in Figs. 2.22 and 2.23, regarding the single-porosity model I which has local mesh refinement as reference solution, the validity of numerical method can be verified for the computed result of discrete fracture model correlated well with the reference solution.

To verify the efficiency of discrete fracture model, we consider geometric models with different dips and computing times based on discrete fracture model or single-porosity model which has local mesh refinement or not, where CPU clock speed is 2.93 GHz. The corresponding computing time is shown in Table 2.1.

As can be seen from Table 2.1, discrete fracture model has the same calculation accuracy with single-porosity model while the former has less computing time than the latter, which could explain that discrete fracture model have high efficiency. In addition, the latter has a poorer convergence than discrete fracture model for the wide difference between meshes around fracture.

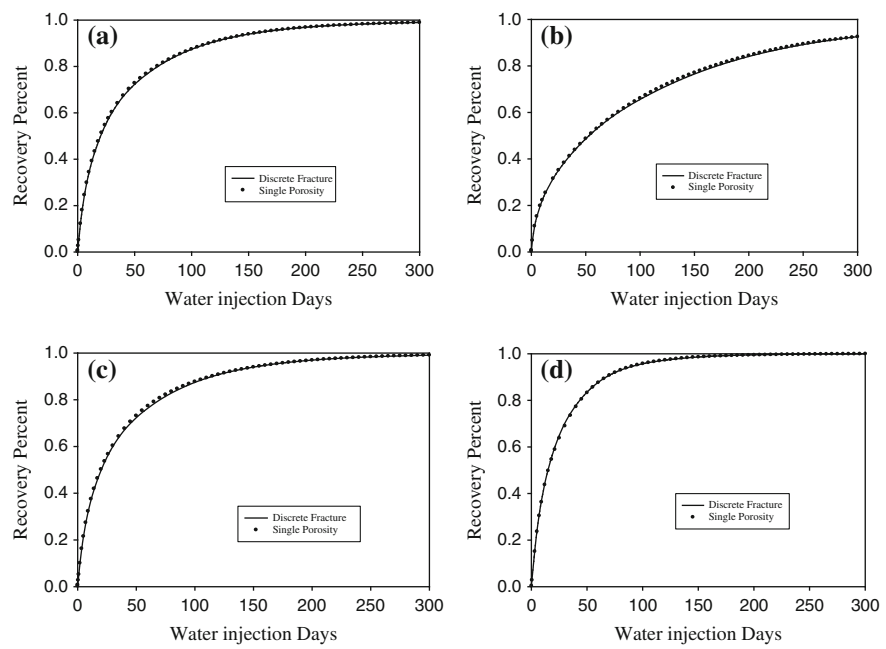


Fig. 2.23 Comparison diagram of recovery degree for single-porosity media based on two models. **a** $\theta = 0^\circ$, **b** $\theta = 45^\circ$, **c** $\theta = 90^\circ$, **d** $\theta = 135^\circ$

Table 2.1 Computing time of different porous media based on different models

Computation times/s	$\theta = 0^\circ$	$\theta = 45^\circ$	$\theta = 90^\circ$	$\theta = 135^\circ$
Discrete fracture model	47.42	48.81	52.33	54.63
Single-porosity model I	402.83	397.00	382.06	390.59
Single-porosity model II	144.93	132.07	138.74	141.22

2.4 Mimetic Finite Difference Numerical Simulation

The existing numerical calculation methods of discrete fracture flow mainly include two categories as below: finite volume method and finite element method. The former needs to simplify and equivalent the processes, which leads to reducing calculation accuracy when processing the mass calculation; the latter has some defects in conservation-type calculation format and computational stability. As a new numerical calculation method, MFD (Mimetic Finite Difference) gets a successful application in the numerical simulation calculation of fluid mechanics, electromagnetic field, and oil reservoir, because of its good local conservation and the applicability of the complex grid. In this section, we have further put this

method into use to flow numerical simulation research of discrete fracture model, elaborated the basic principle of Mimetic Finite Difference, set up a corresponding discrete fracture numerical format, and we have solved the two-phase flow problem by the method of IMPES (Implicit Pressure and Explicit Saturation Scheme). In the end, we have proved the validity of this method by an example.

2.4.1 Two-Phase Fluid Flow Mathematical Model

For brevity, we only consider incompressible oil–water two-phase flow problems, and other problems' methods remain the same. Here, we use the classical fractional flow mathematical model, of which the pressure equation is

$$\mathbf{v} = -\mathbf{K}\lambda \cdot \nabla p + \mathbf{K} \cdot (\lambda_w \cdot \rho_w + \lambda_o \rho_o) \mathbf{G}, \quad \nabla \cdot \mathbf{v} = q, \quad (2.94)$$

where $\mathbf{v} = \mathbf{v}_w + \mathbf{v}_o$ stands for total seepage velocity; $\mathbf{K}$ is permeability tensor; $\lambda = \lambda_w + \lambda_o$ stands for overall coefficient of fluidity, of which we impose $\lambda = k_{rl}/\mu_l$ ($l = w, o$), and define shunt function $f_l = \lambda_l/\lambda$; k_{rl} is relative permeability of l -phase fluid; μ_l is viscosity of l -phase fluid; ρ_l is the density of two-phase fluid; $\mathbf{G} = -g\nabla z$ is gravity item, of which g is gravitational acceleration; z is reservoir depth (positive upward); $q = q_w + q_o$ is source or sink term; and the global pressure p is defined as below:

$$p = p_o - \int_1^{S_w} f_w(\xi) \frac{\partial p_c}{\partial S_w}(\xi) d\xi \quad (2.95)$$

where p_c is capillary force, and S_w is water phase saturation.

The corresponding water phase saturation equation is

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot \mathbf{v}_w = q_w \quad (2.96)$$

$$\mathbf{v}_w = f_w[\mathbf{v} + \mathbf{K}\lambda_o \cdot \nabla p_c + \mathbf{K}\lambda_o \cdot (\rho_w - \rho_o) \mathbf{G}] \quad (2.97)$$

where ϕ denotes porosity.

Assume that the flow in the matrix and fracture meet the Darcy's law, therefore, the above equations are applicable to the entire area of the fractured media. In this text, we use IMPES solution to solve Eqs. (2.94) and (2.96) in turns: where we employ IMPES to solve pressure Eq. (2.94), and use finite volume method to get an explicit solution of formula (2.96).

In order to adapt to complex geometry of A discrete fracture model, we adopt unstructured mesh generation technology to discretize the research area, as shown in Fig. 2.24. Due to the small fracture aperture, for this fracture, we employ

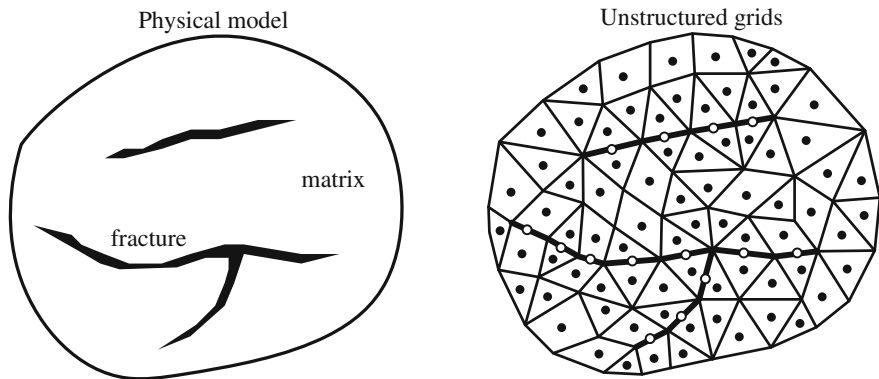


Fig. 2.24 The discrete fracture model and unstructured mesh generation diagram

dimensionality reduction, that is, fracture is simplified as fracture line element in the 2-D problems, and simplified as fracture plane unit in the 3-D problems. By dimension reduction process, the number of grids can be cut down so that computational efficiency is enhanced; however, fracture aperture is only considered in specific numerical calculation.

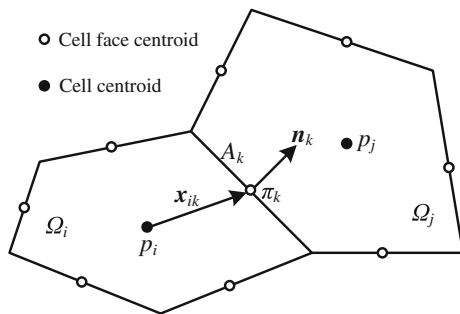
2.4.2 Solution Strategies for the Pressure Equation

(1) Matrix section

Assume that the research area $\Omega \in \mathbf{R}^d$ is subdivided by a set of nonoverlapping polygon ($d = 2$) or polyhedron ($d = 3$) grids $\Omega_h = \{\Omega_i\}$. As shown in Fig. 2.21, we can take any unit Ω_i to analyze, and Ω_j is adjacent unit. $A_k = \Omega_i \cap \Omega_j$ is interface, $\mathbf{n}_k = |A_k| \hat{\mathbf{n}}_k$ is the area-weighted normal vector of interface area A_k . $\hat{\mathbf{n}}_k$ is the unit outward normal vector (Fig. 2.25).

First of all, on the unit center $\mathbf{x}_i$ and boundary surface center $\mathbf{x}_k$, we can, respectively, define unit pressure p_i and boundary surface pressure π_k as follows:

Fig. 2.25 Analysis schematic diagram of finite difference grid cell simulation



$$p_i = \frac{1}{|\Omega_i|} \int_{\Omega_i} p d\Omega, \quad \pi_k = \int_{A_k} p dA \quad (2.98)$$

Notice that if the gravity is taken into account, the pressure equations described above should be regarded as flow potential. By Darcy's law, it is easy to know that the normal seepage velocity v_i on boundary surface can be written as the following formula:

$$v_i = \mathbf{T}_i \cdot (e_i p_i - \pi_i) \quad (2.99)$$

In this formula, $\mathbf{T}_i$ is transmission matrix, $v_i = [v_1, \dots, v_m]^T$, m is the number of boundary surface of the unit Ω_i , $e_i = [1, \dots, 1]^T$. The structure of the matrix $e_i = [1, \dots, 1]^T$ is the key of MFD simulation.

Suppose that pressure on the unit has linear variation, that is, $p = \mathbf{a} \cdot \mathbf{x} + b$, so by the Darcy law, we can get binding equations:

$$v_k = -\mu^{-1} |A_k| \hat{\mathbf{n}}_k \cdot \mathbf{K} \cdot \nabla p = \mu^{-1} |A_k| \hat{\mathbf{n}}_k \cdot \mathbf{K} \cdot \mathbf{a} \quad (2.100)$$

Combining Eqs. (2.99) and (2.100), and considering $p_i - \pi_k = \mathbf{a} \cdot (\mathbf{x}_i - \mathbf{x}_k)$, we can get the following equation:

$$\mathbf{v}_i = \mathbf{T}_i \cdot \begin{bmatrix} x_1 - x_i \\ \vdots \\ x_k - x_i \\ \vdots \\ x_m - x_i \end{bmatrix} \cdot \mathbf{a} = \mu^{-1} \begin{bmatrix} |A_1| \hat{\mathbf{n}}_1 \\ \vdots \\ |A_k| \hat{\mathbf{n}}_k \\ \vdots \\ |A_m| \hat{\mathbf{n}}_m \end{bmatrix} \cdot \mathbf{K} \cdot \mathbf{a} \Rightarrow \mathbf{T}_i \mathbf{X} = \mu^{-1} \mathbf{N} \mathbf{K} \quad (2.101)$$

In this formula, $\mathbf{X} = [\mathbf{X}_1], \dots, [\mathbf{X}_d]$, $\mathbf{N} = [\mathbf{N}_1], \dots, [\mathbf{N}_d]$, and $\mathbf{N}^T \mathbf{X} = [Z_{ij}]_{d \times d}$. In this definition, $\mathbf{x}^{(i)}$ denotes the i th dimension Cartesian coordinates of $\mathbf{x}$, so we get

$$Z_{ij} = \mathbf{N}_i^T \mathbf{X}_j = \sum_{k=1}^m |A_k| \hat{\mathbf{n}}_k^{(i)} (\mathbf{x}_k - \mathbf{x}_i)^{(j)} \quad (2.102)$$

Notice, $\mathbf{x}_k - \mathbf{x}_i = \frac{1}{|A_k|} \int_{A_k} (\mathbf{x} - \mathbf{x}_i) dA$, and combine divergence theorem, then we can obtain format as follows:

$$\begin{aligned} Z_{ij} &= \sum_{k=1}^m |A_k| \hat{\mathbf{e}}_i \cdot \hat{\mathbf{n}}_k \frac{1}{|A_k|} \int_{A_k} (\mathbf{x} - \mathbf{x}_i)^{(j)} dA = \sum_{k=1}^m \hat{\mathbf{e}}_i \cdot \int_{A_k} (\mathbf{x} - \mathbf{x}_i)^{(j)} \cdot \hat{\mathbf{n}}_k dA \\ &= \hat{\mathbf{e}}_i \cdot \int_{\Omega_i} \nabla \cdot (\mathbf{x} - \mathbf{x}_i)^{(j)} d\Omega = \hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_j |\Omega_i| = \delta_{ij} |\Omega_i| \end{aligned} \quad (2.103)$$

$$\delta_{ij} \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

Namely, $N^T X = |\Omega_i| E_d$, where E_d is d order unit matrix, therefore. Through the equations, we can obtain conductance matrix T_i as follows:

$$T_i = \frac{1}{\mu|\Omega_i|} N K N^T + T_2 \quad (2.104)$$

where $T_2 Z = 0$. In order to ensure the existence of inverse matrix, we apply Brezzi–Lipnikov–Simoncini theorem (Thomas et al. 1983) to construct matrix. In this paper, we employ the following form:

$$T_i = \frac{1}{\mu|\Omega_i|} \left[N K N^T + \frac{6}{d} \text{trace}(K) A (E_m - Q Q^T) A \right] \quad (2.105)$$

in which, $A = \text{diag}(|A_k|)$, $Q = \text{orth}(AX)$. For the continuity equation in this equation, we can directly integral divergence theorem in the unit Ω_i , and get

$$\sum_{k=1}^m v_k^f = \int_{\Omega_i} q_i d\Omega \quad (2.106)$$

Consider that speed continuity conditions on the surface of the cell boundaries, combine Eqs. (2.102) and (2.106), so MFD numerical calculation format can be obtained as follows:

$$\begin{bmatrix} B & -C & D \\ C & 0 & 0 \\ D^T & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ \pi \end{bmatrix} = \begin{bmatrix} g \\ q \\ f \end{bmatrix} \quad (2.107)$$

where $v = [v_k]$ is seepage velocity array of unit boundary surface; $p = [p_i]$ is unit center pressure array; $\pi = [\pi_k]$ is pressure array on unit boundary surface center. $g = [g_k]$ is gravity item. $q = [q_i]$ is source sink term of unit Ω_i . $f = [f_i]$ is flow boundary conditions. $f = 0$ represents impermeable barrier. The first line of equation corresponds to the Darcy's law. The second line corresponds to the continuity equation. The third line is continuity conditions for normal speed on the surface of the cell boundaries. The coefficient matrix of above equation is specific as follows:

$$B = \begin{bmatrix} T_1^{-1} & & \\ & \ddots & \\ & & T_{N_e}^{-1} \end{bmatrix}, C = \begin{bmatrix} e_1 & & \\ & \ddots & \\ & & e_{N_e} \end{bmatrix}, D = \begin{bmatrix} I_1 & & \\ & \ddots & \\ & & I_{N_e} \end{bmatrix} \quad (2.108)$$

where N_e is the sum of grid cells: $I_i = E_m$.

Conclusion can be drawn from the above derivation process: MFD method is only based on a single grid cell to construct the numerical format, which is suitable for any complicated grid system. Besides, it has good local conservation property which is similar to the hybrid finite element. However, for complex grid system, the structure of the mixed finite element numerical calculation format has a big difficulty.

(2) Numerical solution of the discrete fracture model

As mentioned earlier, flow of fractures and matrix all meet the Darcy's law. If we consider closed outer boundary, the corresponding equations are as follows:

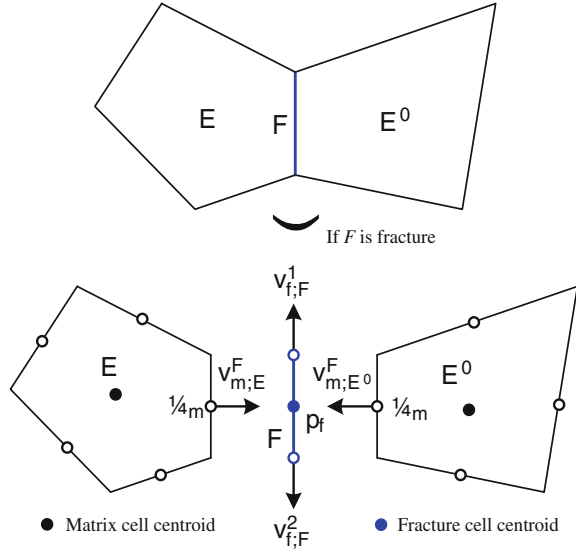
$$\begin{bmatrix} B_m & -C_m & D_m \\ C_m^T & 0 & 0 \\ D_m^T & 0 & 0 \end{bmatrix} \begin{bmatrix} v_m \\ p_m \\ \pi_m \end{bmatrix} = \begin{bmatrix} g_m \\ q_m \\ 0 \end{bmatrix} \quad (2.109)$$

$$\begin{bmatrix} B_f & -C_f & D_f \\ C_f^T & 0 & 0 \\ D_f^T & 0 & 0 \end{bmatrix} \begin{bmatrix} v_f \\ p_f \\ \pi_f \end{bmatrix} = \begin{bmatrix} g_f \\ q_f \\ 0 \end{bmatrix} \quad (2.110)$$

In these equations, the subscripts m and f , respectively, denote matrix and fracture. Notice that, in this paper, the fracture has been reducing dimensions. Therefore, the space dimension of Eq. (2.110) is low 1-D than Eq. (2.109).

The key of MFD discrete fracture numerical format's structure consists in the coupling of pressure equation of matrix and fracture (Fig. 2.26).

Fig. 2.26 Fracture–matrix coupled flow analysis diagram



Withal, consider fracture–matrix coupled hybrid flow grid analysis diagram as shown in Fig. 2.22. Fracture grid cell can be treated as the boundary of the matrix grid cell surface, thus fracture unit pressure p_f and boundary surface pressure π_m of adjacent matrix unit are equal. Therefore, we can just reserve π_m in the numerical format. Seepage velocity term is coupling on the fracturing unit in Eqs. (2.109) and (2.110) in accordance with the following conditions.

(1) If F is diversion fractures, the total flow exchange between adjacent rock element and fractures element can be denoted as Q_f^F . For fracture unit, this flow can be used as a source/sink term. So, the equation is as follows:

$$\begin{cases} v_{m,E}^F + v_{m,E'}^F = Q_f^F \\ \sum_i v_{f,F}^i = Q_f^F + q_f^F \end{cases} \quad (2.111)$$

Where $v_{m,E}^F$, $v_{m,E'}^F$, respectively, are exchange to fracture from matrix elements E and E' ; q_f^F represents sources/sinks; $\sum_i v_{f,F}^i$ in the second line of above equations corresponds to equation of continuity of fracture element.

(2) If F is flow barrier, it will be processed in accordance with impermeable barrier.

At the moment, Eqs. (2.109) and (2.110) can be coupled together to form the corresponding discrete fracture numerical formats as follows:

$$\begin{bmatrix} \mathbf{B}_m & -\mathbf{C}_m & \mathbf{D} & 0 & 0 \\ \mathbf{C}_m^T & 0 & 0 & 0 & 0 \\ \mathbf{D}_m^T & 0 & 0 & -\mathbf{C}_f^T & 0 \\ 0 & 0 & -\mathbf{C}_f & \mathbf{B}_f & \mathbf{D}_f \\ 0 & 0 & 0 & \mathbf{D}_f^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_m \\ \mathbf{p}_m \\ \pi_m \\ \mathbf{v}_f \\ \pi_f \end{bmatrix} = \begin{bmatrix} \mathbf{g}_m \\ \mathbf{q}_m \\ -\mathbf{q}_f \\ \mathbf{g}_f \\ 0 \end{bmatrix} \quad (2.112)$$

2.4.3 The Solution of the Saturation Equation

(1) The calculation format of finite volume method

In this work, we use the IMPES (implicit pressure, explicit saturation) method, which used to be quite popular in the industry. In the IMPES method, the fluid pressure equations (flow equations) are solved implicitly while the saturation field is fixed, yielding the velocities of the fluid phases. These velocities are used to calculate the mass balance of the fluid phases in the transport equations while the pressure field remains fixed. For saturation equation, we apply the finite volume method for solving. We can directly integral the formula (2.96) on element and it can be written as

$$\int_{\Omega_i} \phi \frac{\partial S}{\partial t} d\Omega + \int_{\partial\Omega_i} \{f_w[\mathbf{v} + \mathbf{K}\lambda_o \cdot \nabla p_c + \mathbf{K}\lambda_o \cdot (\rho_w - \rho_o)\mathbf{G}]\} \cdot \mathbf{n}_i d\Gamma = \int_{\Omega_i} q_w d\Omega \quad (2.113)$$

For convenience of writing, we have removed the subscript w of the water saturation S_w . For the time dimension, if we apply θ -rules, the following finite volume numerical discrete format can be obtained:

$$\frac{\phi_i}{\Delta t} (S_i^{n+1} - S_i^n) + \frac{1}{|\Omega_i|} \sum_{k=1}^m [\theta F_k(S^{n+1}) + (1 - \theta) F_k(S^n)] = q_w(S_i^n) \quad (2.114)$$

where

$$F_k(S) = \int_{A_k} [f_w(S)]_k (\mathbf{v} \cdot \hat{\mathbf{n}}_k + \mathbf{K} \lambda_o \cdot \nabla p_c \cdot \hat{\mathbf{n}}_k + \mathbf{K} \lambda_o \cdot (\rho_w - \rho_o) \mathbf{G} \cdot \hat{\mathbf{n}}_k) dA \quad (2.115)$$

where superscript n stands for time step.

On boundary surface A_k , we have applied following format $[f_w(S)]_k$, which is the upstream windward format

$$[f_w(S)]_k = \begin{cases} f_w(S_i) & \text{if } \mathbf{v} \cdot \hat{\mathbf{n}}_k \geq 0 \\ f_w(S_j) & \text{if } \mathbf{v} \cdot \hat{\mathbf{n}}_k < 0 \end{cases} \quad (2.116)$$

We can solve explicit solution for the saturation equation, namely $\theta = 0$. In order to calculate stability, the time step applies the CFL condition as follows:

$$\Delta t \leq \frac{\phi_i |\Omega_i|}{v_i^{\text{in}} \max \{f'_w(S)\}_{0 \leq S \leq 1}} \quad (2.117)$$

where

$$v_i^{\text{in}} = \max(q_i, 0) - \sum_{A_k} \min(v_k, 0) \quad , \quad \frac{\partial f_w}{\partial S} = \frac{\partial f_w}{\partial S^*} \frac{\partial S^*}{\partial S} = \frac{1}{1 - S_{wc} - S_{or}} \frac{\partial f_w}{\partial S^*}$$

In these equations, S^* is the water phase saturation after the normalization; S_{wo} is irreducible water saturation; S_{ro} is residual oil saturation.

(2) The saturation calculation at fractures' intersections

When two or more fractures intersect, the key in the discrete fracture flow simulation is saturation calculation. At present there are mainly two kinds of processing methods: one is the upstream windward format of conductivity calculation based on Delta-Star (Karimi-Fard et al. 2004), which simplifies and equivalently deals with the crossed fracture. Another is upstream wind weighted format (Hoteit and Firoozabadi 2006), which is of high calculation precision, but needs to get real seepage of velocity of every fracture unit at intersections. In this paper, we apply the latter one. As shown in Fig. 2.27, assume that there are N_I fracture elements e_i intersect at I ; each fracture element corresponds to distribution function f_{w,e_i} . v_{f,e_i} is the seepage velocity at the intersection. We can define the inflows and outflows on the intersection I as follows:

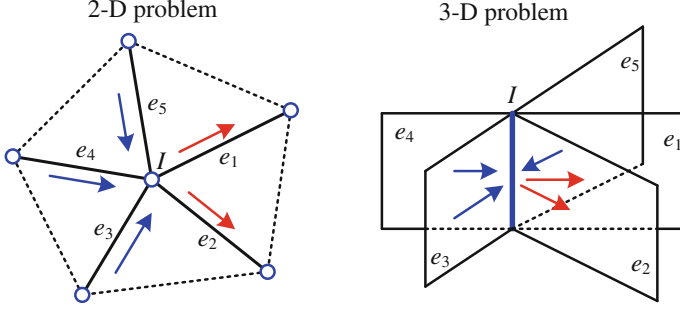


Fig. 2.27 The fracture's intersections saturation calculation diagram

$$\begin{cases} v_{f,e_i} \leq 0, & 0 < i \leq N \quad (\text{efflux}) \\ v_{f,e_i} \geq 0, & N < i < N_I \quad (\text{influx}) \end{cases} \quad (2.118)$$

By the law of conservation of mass, the equation is written as

$$\sum_{i=N+1}^{N_I} v_{f,e_i} = - \sum_{i=1}^N v_{f,e_i} \quad (2.119)$$

Further, by the definition of upstream windward format, we can get

$$\sum_{i=N+1}^{N_I} f_{w,i} v_{f,e_i} = - \sum_{i=1}^N f_{w,I} v_{f,e_i} = -f_{w,I} \sum_{i=1}^N v_{f,e_i} \quad (2.120)$$

Thus, upstream windward weighted distribution function in fracture's intersections I is as follows:

$$f_{w,I} = - \frac{\sum_{i=N+1}^{N_I} f_{w,i} v_{f,e_i}}{\sum_{i=1}^N v_{f,e_i}}. \quad (2.121)$$

2.4.4 Numerical Example

First, this section presents two simple numerical examples of discrete fracture model. And through the comparison of the experimental results, the validity of these methods and procedures have been verified. Then, the calculation examples of complex discrete fracture model have further verified the correctness of the method and the robustness of the program.

(1) The simple calculation example of discrete fracture model

Consider the one-well injection and one-well production physical model, as shown in Figs. 2.24 and 2.25, whose size is $1 \text{ m} \times 1 \text{ m} \times 0.025 \text{ m}$ and can be treated as

planar flow problem. Figure 2.24 is for a single fracture model and Fig. 2.25 is for two intersecting fractures model. They all are produced by glass (160–180 mesh) sand combined with epoxy resin by compaction and cementation, and then encapsulated by transparent organic glass. Matrix can be regarded as homogeneous isotropic medium, whose porosity is $\phi \approx 0.4$, and permeability $K_m = 10 \mu\text{m}^2$. Fractures are replaced by ultrathin sheet steel when modeling. It will be dissociated after model's cementation, its aperture is about 1 mm, and its permeability is $K_f = a^2/12 = 8.33 \times 10^4 \mu\text{m}^2$. The flow of water injection well is $q_{\text{in}} = 0.01$ PV/min. And production well connects to the barometric pressure. The viscosity of water is $\mu_w = 1$ mPa s, the oil viscosity is $\mu_o = 5$ mPa s, the density of water is $\rho_w = 1000 \text{ kg/m}^3$, and the density of oil is $\rho_o = 800 \text{ kg/m}^3$.

The initial value of oil saturation, irreducible water saturation, and residual oil saturation of model is all zero. The water phase relative permeability of matrix and fractures is $k_{rw} = S_w$, and the oil phase relative permeability is $k_{ro} = 1 - S_w$. When we use the method mentioned in this paper to do numerical modeling, we can ignore the influence of capillary force and gravity in the calculation. The corresponding Delaunay triangle mesh subdivision and the results of numerical simulation are shown in Figs. 2.24 and 2.25. By comparison with true flow process in experiment we can see that the results of numerical calculation and experimental results are basically identical. Thus we have verified the correctness of the method and procedure in this paper. It is worth noting that, the rapid flow phenomenon appears on the left border in Fig. 2.28(a), which is due to the poor sealing of the experimental model (Fig. 2.29).

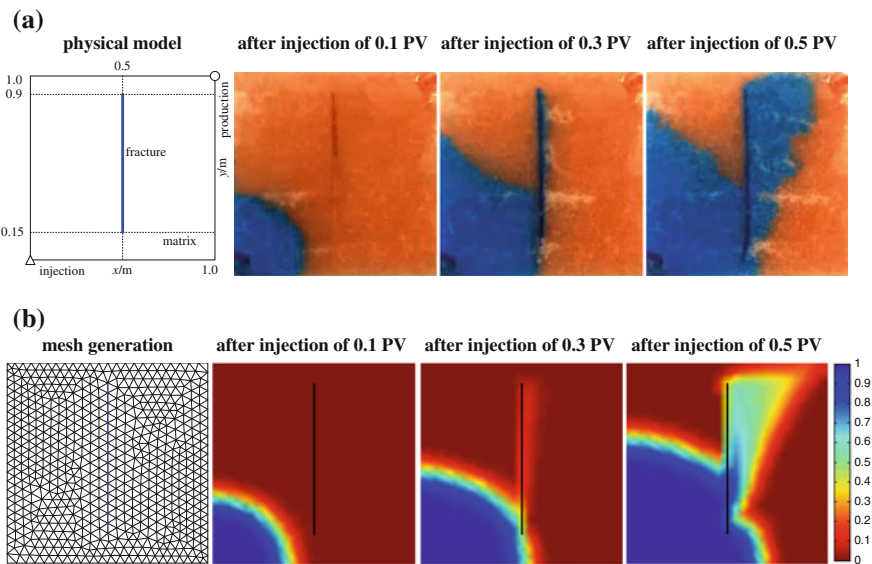


Fig. 2.28 The comparison between results of water saturation and single fracture model. **a** Experimental result, **b** numerical result

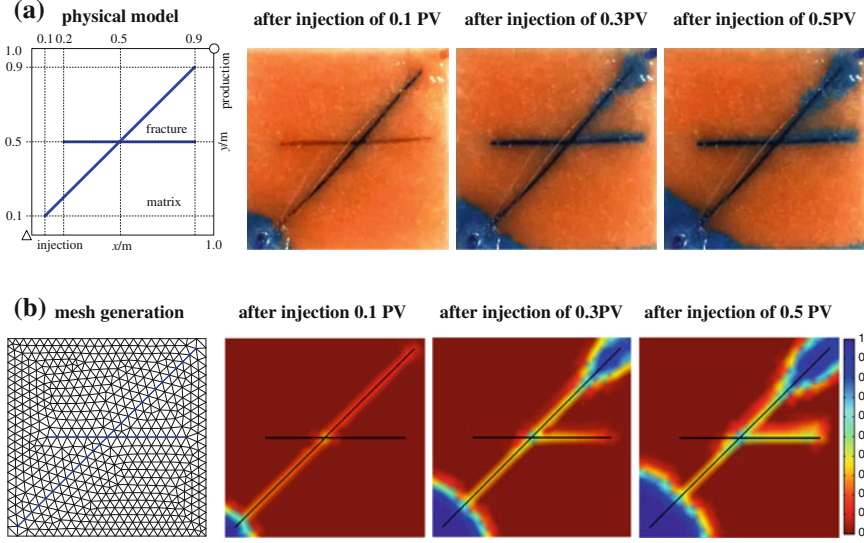


Fig. 2.29 The comparison between results of water saturation and double fracture model. **a** Experimental result, **b** numerical result

(2) Complex discrete fracture model

In this section, we consider a complex fractured model, dimension of $100 \text{ m} \times 50 \text{ m}$ ($x \times y$), as shown in Fig. 2.30, the blue lines represent the fractures which generated random based on the geological statistics. The Delaunay triangular gridding is used to discrete the geometrical model (Fig. 2.26 right). Homogeneous isotropic matrix's porosity $\phi = 0.2$, permeability $K_m = 10 \text{ mD}$ ($1 \text{ mD} = 10^{-3} \mu\text{m}^2$), fracture aperture $a = 1 \text{ mm}$, permeability $K_f = a^2/12 = 8.33 \times 10^7 \text{ mD}$, and physical property parameters of oil and water is in accordance with 5.1 calculation example. The initial reservoir pressure is 10 MPa , initial water saturation is zero, and the speeds of injection wells and production wells are 0.01 PV/day . Water phase relative permeability of matrix and fracture is $K_{rw} = S_w^2$, oil phase relative permeability is $K_{ro} = (1 - S_w)^2$. Assume that model is water-wet reservoir. If we consider the influence of the capillary force in rock and fracture, assume that both types of the capillary force accord with Brooks-Corey capillary force function as shown in formula (2.122). For the matrix, threshold pressure value is $p_d = 1000 \text{ Pa}$, and λ is 2.0 . For fractures, threshold pressure value is $p_d = 1000$, and λ is 1.0 .

$$p_c(S_w) = p_d \left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}} \right)^{-\frac{1}{\lambda}}, \quad 0.2 < \lambda < 3.0. \quad (2.122)$$

Figure 2.31 shows the water saturation distribution at different times. The analogous calculation results indicate: the induced water flows into the fracture quickly; and the existence of fractures results in the strong heterogeneity of

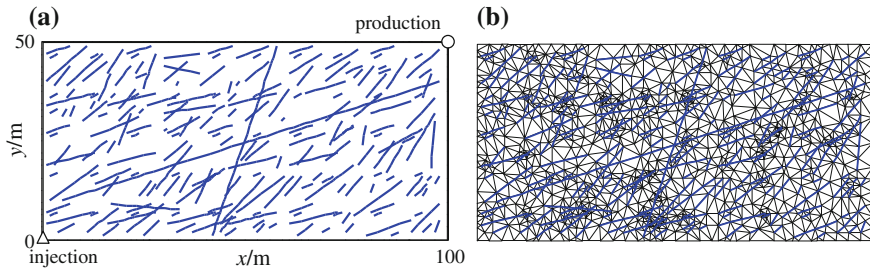


Fig. 2.30 The mode of complex discrete fracture and unstructured grid subdivision. **a** The mode of complex discrete fracture, **b** unstructured grid subdivision

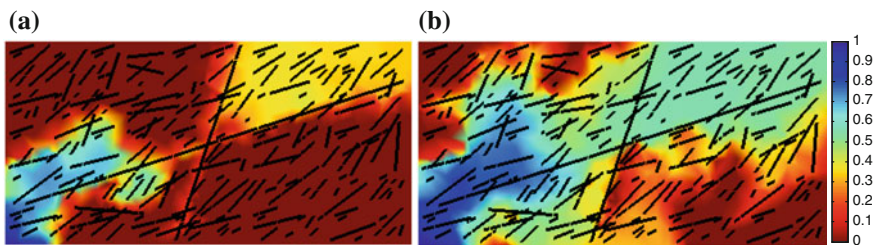


Fig. 2.31 The water saturation distribution at different times. **a** 10 days later, **b** 40 days later

medium; the existence of the capillary force makes the water-flood swept area increased, but the overall effect is still controlled by the macroscopic fracture (macro fracture). Through this calculation example, we further verified the correctness of this suggested method, at the same time we can see that this method still has good applicability for extremely complicated grid system.

2.5 The Embedded Discrete Fracture Numerical Simulation

At present, fractured oil reservoir numerical simulation is mostly based on the double medium model, but this model is only applicable to highly matured fracture in the reservoir. When there are several large fractures that control the direction and the scale of fluid flow, the error in calculation results is bigger. To solve this problem, the discrete fracture model was set up, and with the wide use of artificial fracturing technology in unconventional reservoirs, its corresponding flow simulation technique has a rapid development. However, the existing numerical discrete fracture models are all based on matched grid, that is, we treat the fractures as internal boundary and constrain face for grid subdivision. Due to the complexity of the fracture's geometrical morphology, we need to adopt the unstructured grid

technique, whose subdivision process is very complicated and tedious. Especially when the distance or the angle between the fractures is very small, the mesh generation often is of poor quality, which leads to deviation calculation, as shown in Fig. 2.32a. However, the embedded discrete fracture model does not need to consider the internal fracture morphology when partitioning grid, where matrix system separately generates grids, fracture part generates grids according to the intersection of fracture and matrix grids, as shown in Fig. 2.32b, that greatly reduces the complexity of meshing, so that it can improve the calculation efficiency.

To this end, Lee and Moinfar et al. (2012) put forward embedded discrete fracture model. This model will directly embed fracture network into the matrix structured grid system, which has avoided the complex unstructured grid subdivision process. Although we need to calculate geometry information between the

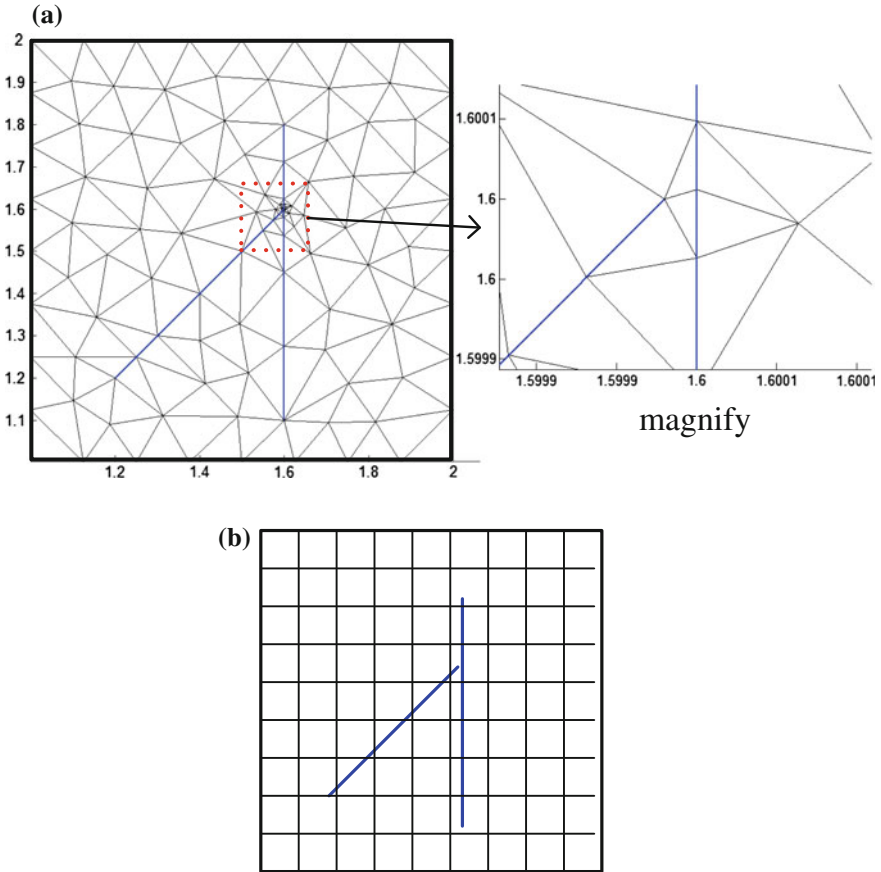


Fig. 2.32 The discrete fracture model and the embedded discrete fracture model mesh generation contrast. **a** Matched unstructured grid, **b** Non-matched structured grid

fracture and grid, computation complexity is significantly reduced to improve computational efficiency, relative to the complex unstructured grid subdivision process.

However, the existing embedded discrete fracture models all adopt finite difference method to solve, so it cannot process accurately the permeability situation of full tensor, and applies only to structured grid. To this, it is necessary to set up a new embedded discrete fracture numerical format, based on the simulation of finite difference method, to apply to numerical simulation of complex fractured reservoir.

Discrete fractures numerical model regards fractures as internal constraint face to generate mesh. Due to the complexity of the fracture's geometrical morphology, we need to adopt the unstructured grid technique, whose subdivision process is very complicated and tedious. Especially when the distance or the angle between the fractures is very small, the mesh generation often is of poor quality, which leads to deviation calculation, as shown in Fig. 2.32a. However, the embedded discrete fracture model do not need to consider the internal fracture morphology when partitioning grid, where matrix system separately generates grids, fracture part generates grids according to the intersection of fracture and matrix grids, as shown in Fig. 2.32b, that greatly reduces the complexity of meshing, so that it can improve the calculation efficiency.

2.5.1 The Mathematical Model of Embedded Discrete Fracture Model

For the convenience of study, illustrate the basic ideas and principals of embedded numerical simulation, based on the 2-D single-phase flow. Assume that the fluid flow process is of constant temperature, regardless of the matrix and the fluid compressibility; matrix system and fluid flow in fracture system meet Darcy's law; ignore the influence of gravity and capillary pressure.

Matrix system mathematical model:

$$\mathbf{v}_m = -\frac{\mathbf{K}_m}{\mu} \cdot \nabla p_m \quad (2.123)$$

$$\nabla \cdot \mathbf{v}_m = q_m + \frac{q_{mf}}{V_m} \delta_{mf} \quad (2.124)$$

Fracture system mathematical model:

$$\frac{K_f}{\mu} \frac{\partial^2 p_f}{\partial \xi^2} = q_f + \frac{q_{mf} + q_{ff} \delta_{ff}}{V_f} \quad (2.125)$$

where

$$\delta_{mf} = \begin{cases} 1 & \text{if there are fractures embedding on the bedrock grid} \\ 0 & \text{if there is no fracture embedding on the bedrock grid} \end{cases}$$

$$\delta_{ff} = \begin{cases} 1, & \text{if one fracture element intersect with another} \\ 0, & \text{if one fracture element doesn't intersect with any other} \end{cases}$$

where v_m is rock seepage velocity; K_m is rock permeability tensor; K_f is fracture permeability (scalar); μ is fluid viscosity; p_m and p_f , respectively, are basement and fracture of the pressure (or streaming potential); V_f and V_m , respectively, are fracture element and the volume of rock unit; q_m and q_f , respectively, represent basement and fracture source sink term; ξ is local coordinate system along the fracture direction; q_{mf} denotes quantity flow between basement and fracture; q_{ff} denotes quantity flow between intersecting fracture elements;

(1) Flow between matrix and fracture element

The fracture aperture is very small compared with mesh scale, and the fracture permeability is greater than the matrix permeability, so we can think that the pressure is of succession on both sides of the fractures. The expression of quantity flow calculation between matrix and fracture element can be written as

$$q_{mf} = -T_{mf}(p_m - p_f) \quad (2.126)$$

where

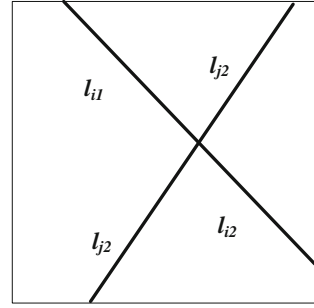
$$T_{mf} = \frac{k_{mf}A_{mf}}{\mu\hat{d}}, \quad \hat{d} = \frac{\int x_{mf}dS}{S}, \quad K_{mf} = \frac{1}{1/K_f + 1/K_m}$$

where $\hat{d}$ represents equivalent distance between the matrix grid and fracture section, that is, the average of vertical distance between all the points in the matrix grids and fracture section; S is the volume of matrix element; K_m denotes permeability (scalar), in the direction perpendicular to fracture; A_{mf} denotes contact area of fracture section and bed rock; x_{mf} is perpendicular distance from all points in the matrix grids to fracture.

(2) Flow between fracture elements

Calculate quantity flow between fractures referring to Karimi-Fard's using transfer coefficient method in calculating intersecting fracture section:

$$q_{ff} = T_{ff}(p_{fi} - p_{fj}) \quad (2.127)$$

Fig. 2.33 Fracture section–intersection diagram

where

$$T_{ff} = \frac{T_{fi}T_{fj}}{T_{fi} + T_{fj}}, \quad T_{fi} = \frac{k_{fi}a_{fi}}{\mu \widehat{d}_i}, \quad T_{fj} = \frac{K_{fj}a_{fj}}{\mu \widehat{d}_j}$$

$$\widehat{d}_i = \frac{l_{i1}}{l_{i1} + l_{i2}} \cdot \frac{1}{2}l_{i1} + \frac{l_{i2}}{l_{i1} + l_{i2}} \cdot \frac{1}{2}l_{i2}$$

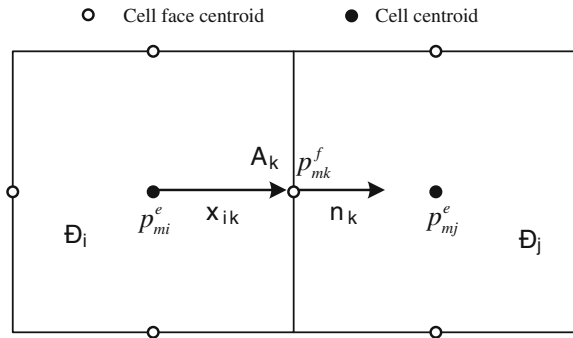
$$\widehat{d}_j = \frac{l_{j1}}{l_{j1} + l_{j2}} \cdot \frac{1}{2}l_{j1} + \frac{l_{j2}}{l_{j1} + l_{j2}} \cdot \frac{1}{2}l_{j2}$$

where a_f denotes fracture aperture, and l denotes the length of fracture (Fig. 2.33).

2.5.2 Numerical Solution of Mathematical Model

(1) Finite difference solution for matrix

Matrix is subdivided by a set of nonoverlapping polygon mesh shown as in Fig. 2.34, we analyze any unit Ω_i , Ω_j is the adjacent cell, the interface $A_k = \Omega_i \cap \Omega_j$, $\mathbf{n}_k = |A_k|\hat{\mathbf{n}}_k$ is vector by the area-weighted method of the interface A_k , $\hat{\mathbf{n}}_k$ is the unit

Fig. 2.34 Simulation of finite difference grid cell analysis diagram

outward normal vector. In the central point $\mathbf{x}_i$ of unit Ω_i and central point $\mathbf{x}_k$ of boundary surface, it is defined, respectively, unit pressure p_{mi}^e and the pressure p_{mk}^f on the boundary surface, as follows:

$$p_{mi}^e = \frac{1}{|\Omega_i|} \int_{\Omega_i} p_m d\Omega, \quad p_{mk}^f = \frac{1}{|A_k|} \int_{A_k} p_m dA \quad (2.128)$$

It is easy to know by Eq. (2.123) that on the surface of the matrix boundary, the normal seepage velocity and pressure gradient have the relationship as follows:

$$\mathbf{v}_m^f = \mathbf{T}_{mi} \cdot (\mathbf{e}_i p_{mi}^e - \mathbf{p}_m^f) \quad (2.129)$$

where $\mathbf{T}_{mi}$ is conductance matrix of matrix grid, $\mathbf{v}_m^f = [v_{m1}, v_{m2}, \dots, v_{mn}]^T$, n is number of boundary surface of the unit Ω_i , $\mathbf{e}_i = [1, \dots, 1]^T$. Therefore, the key of the finite difference simulation is gaining the matrix $\mathbf{T}_{mi}$, hereon, assume that the pressure is linearly varying, $p_m = \mathbf{a}_m \cdot \mathbf{x} + b_m$. Evidently, based on Eq. (2.123), we can get equations as follows:

$$\mathbf{v}_{mk}^f = -\mu^{-1} |A_k| \hat{\mathbf{n}}_k \cdot \mathbf{K}_m \cdot \nabla p_m = -\mu^{-1} |A_k| \hat{\mathbf{n}}_k \cdot \mathbf{K}_m \cdot \mathbf{a}_m \quad (2.130)$$

Meanwhile, $p_{mi}^e - p_{mk}^f = \mathbf{a}_m \cdot (\mathbf{x}_i - \mathbf{x}_k)$, and combine Eqs. (2.129) and (2.130), then we can obtain formula as follows:

$$\mathbf{v}_m^f = \mathbf{T}_{mi} \cdot \begin{bmatrix} \mathbf{x}_1 - \mathbf{x}_i \\ \vdots \\ \mathbf{x}_k - \mathbf{x}_i \\ \vdots \\ \mathbf{x}_n - \mathbf{x}_i \end{bmatrix} \cdot \mathbf{a}_m = \mu^{-1} \begin{bmatrix} |A_1| \hat{\mathbf{n}}_1 \\ \vdots \\ |A_k| \hat{\mathbf{n}}_k \\ \vdots \\ |A_n| \hat{\mathbf{n}}_n \end{bmatrix} \cdot \mathbf{K}_m \cdot \mathbf{a}_m \Rightarrow \mathbf{T}_{mi} \mathbf{X} = \mu^{-1} \mathbf{N} \mathbf{K}_m \quad (2.131)$$

where $\mathbf{X} = [X_1, \dots, X_n]$, $\mathbf{N} = [N_1, \dots, N_n]$, and $\mathbf{N}^T \mathbf{X} = [Z_{ij}]_{n \times n} = |\Omega_i| \mathbf{E}_d$, where $\mathbf{E}_d$ is d order unit matrix.

Therefore, conduction matrix can be obtained by Eq. (2.131), as shown in the following formula:

$$\mathbf{T}_{mi} = \frac{1}{\mu |\Omega_i|} \mathbf{N} \mathbf{K}_m \mathbf{N}^T + \mathbf{T}_2 \quad (2.132)$$

$$\mathbf{T}_{mi} = \frac{1}{\mu |\Omega_i|} \left[\mathbf{N} \mathbf{K}_m \mathbf{N}^T + \frac{6}{d} \text{trace}(\mathbf{K}_m) \mathbf{A} (\mathbf{E}_m - \mathbf{Q} \mathbf{Q}^T) \mathbf{A} \right] \quad (2.133)$$

where

$$\mathbf{A} = \begin{bmatrix} |A_1| & & & \\ & \ddots & & \\ & & |A_k| & \\ & & & \ddots \\ & & & & |A_n| \end{bmatrix}, \quad \mathbf{Q} = \text{orth}(\mathbf{A}\mathbf{X}) \quad (2.134)$$

For this equation, integral directly and use the divergence theorem on the matrix grid cell:

$$\sum_{k=1}^n \mathbf{v}_{mk}^f = \int_{\Omega_i} q_{mi} d\Omega + q_{mf} \delta_{mf} \quad (2.135)$$

Consider the velocity continuity conditions on the cell boundaries surface, combine equation, and obtain simulation of the finite difference numerical formats of matrix section:

$$\begin{bmatrix} \mathbf{B}_m & -\mathbf{C}_m & \mathbf{D}_m \\ \mathbf{C}_m^T & 0 & 0 \\ \mathbf{D}_m^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_m \\ \mathbf{p}_m \\ \boldsymbol{\pi}_m \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{f}_m + \mathbf{Q}_{mf} \\ 0 \end{bmatrix} \quad (2.136)$$

where $\mathbf{v}_m = [v_{mk}^f]$; $\mathbf{p}_m = [p_{mi}^e]$; $\boldsymbol{\pi}_m = [p_{mk}^f]$; $\mathbf{f}_m = [f_{mi}]$, of which $f_{mi} = \int_{\Omega_i} q_{mi} d\Omega$; $\mathbf{Q}_{mf} = [q_{mfi} \delta_{mfi}]$, in order to convenient writing, this item should be on the right side of the equation, and shift it to the left in the final calculation format.

Obviously in this equation, the first line corresponds to the Darcy's law, the second line corresponds to the law of conservation of mass, and the third line represents the normal velocity continuity conditions on the cell boundaries surface. The specific expression of coefficient matrix of above equation is as follows:

$$\mathbf{B}_m = \begin{pmatrix} T_{m1}^{-1} & & \\ & \ddots & \\ & & T_{mN_e}^{-1} \end{pmatrix}, \quad \mathbf{C}_m = \begin{pmatrix} e_1 & & \\ & \ddots & \\ & & e_{N_e} \end{pmatrix}, \quad \mathbf{D}_m = \begin{pmatrix} \mathbf{I}_1 & & \\ & \ddots & \\ & & \mathbf{I}_{N_e} \end{pmatrix} \quad (2.137)$$

where the subscript N_e is the total number of grid cells; $\mathbf{I}_i = \mathbf{E}_n$.

For this Eq. (2.137), the coefficient matrix of Eq. (2.136) only is related to the geometry information and reservoir parameters of grid cell, however, has no requirement for the grid geometry, so it is easy to solve, and applicable to any complex grid in principle.

(2) Finite Difference Solution for the fracture parts

For 1-D fracture system, we employ implicit difference, and equation multiplies grid cell volume V_f at both ends, so its difference equation is shown as (2.138)

$$T_{\xi_{i+\frac{1}{2}}}^z(p_{fi+1} - p_{fi}) - T_{\xi_{i-\frac{1}{2}}}^z(p_{fi} - p_{fi-1}) = f_{fi} + q_{mfi} + q_{ffi}\delta_{ffi} \quad (2.138)$$

where

$$T_{\xi_{i+\frac{1}{2}}}^z = \frac{K_f}{\mu} \frac{d_{fi}}{0.5(\Delta \xi_{i+1}^z + \Delta \xi_i^z)}; \quad f_{fi} = V_{fi}q_{fi}.$$

(3) The embedded discrete fracture model format

Note that we take the situation with two fractures for example in this section, the calculation format for other situation with more fractures is analogous.

$$\begin{bmatrix} \mathbf{B}_m & -\mathbf{C}_m & \mathbf{D}_m & 0 & 0 \\ \mathbf{C}_m^T & \mathbf{T}_{mf1} + \mathbf{T}_{mf2} & 0 & -\mathbf{T}_{mf1} & -\mathbf{T}_{mf2} \\ \mathbf{D}_m^T & 0 & 0 & 0 & 0 \\ 0 & \mathbf{T}_{mf1} & 0 & \mathbf{T}_{f1} - \mathbf{T}_{mf1} - \mathbf{T}_{ff} & \mathbf{T}_{ff} \\ 0 & \mathbf{T}_{mf2} & 0 & \mathbf{T}_{ff} & \mathbf{T}_{f2} - \mathbf{T}_{mf2} - \mathbf{T}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{v}_m \\ \mathbf{p}_m \\ \boldsymbol{\pi}_m \\ \mathbf{p}_{f1} \\ \mathbf{p}_{f2} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{f}_m \\ 0 \\ \mathbf{f}_{f1} \\ \mathbf{f}_{f2} \end{bmatrix} \quad (2.139)$$

where $\mathbf{T}_{mfi} = [\mathbf{T}_{mfi}]$ represents transmissibility matrix between the i -th fracture and matrix; $\mathbf{T}_{ff} = [\mathbf{T}_{ff}]$ represents transmissibility matrix between fractures; and, respectively, represent finite difference conductivity coefficient matrix of i -th fracture.

2.5.3 Numerical Examples

(1) Fractured medium single-phase flow experimental verification

Consider one-injection and one-production physical model as shown in Fig. 2.35, whose size is 7 cm × 17 cm × 1 cm, and can be treated as situation of plane flow. This model is made up of quartz sand (80–100 mesh) and epoxy resin through the cementation and compaction, then it is encapsulated by the transparent organic glass. Matrix can be regarded as homogeneous isotropic media, its porosity is $\phi = 0.3$, and its permeability is $K_m = 10 \mu\text{m}^2$ while the model is manufactured, fractures are replaced by stalloys. When the model is cemented, we will dissociate the stalloys. In the model, the opening is about 2 mm, fracture permeability is $K_f = 6.67 \times 10^2 \mu\text{m}^2$. Viscosity of water is $\mu_w = 1 \text{ mPa}\cdot\text{s}$, and density of water is $\rho_o = 1000 \text{ kg/m}^3$. The model injects and produces stable discharge with constant pressure difference.

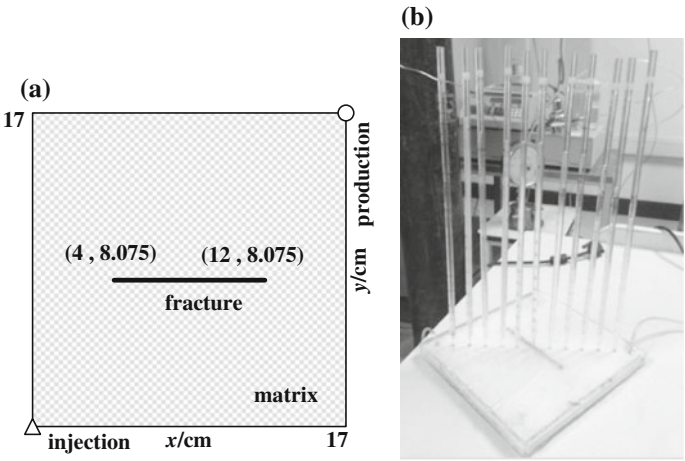


Fig. 2.35 Physical model and experimental model diagrams. **a** Physical model, **b** experimental model

To saturated water of this model, at initial moment, we have calculated its fluid pressure by measuring liquid column height in the glass tube at each point under steady flow state. In this section, we apply the discrete fracture model and the embedded discrete fracture model to simulate the above physical experiment, while gravity influence is ignored. The corresponding numerical simulation results are shown in Fig. 2.36. Figure 2.37 shows pressure curve measured by these two methods in the straight line between the injection–production two points, and it shows comparison results of pressure value measured experiments. From Fig. 2.36, the results of numerical calculation and results of experiment are basically identical, thus the correctness of the method and procedure in this paper is verified. It is worth noting that in this model, the profiles of organic glass and quartz sand glue joint so

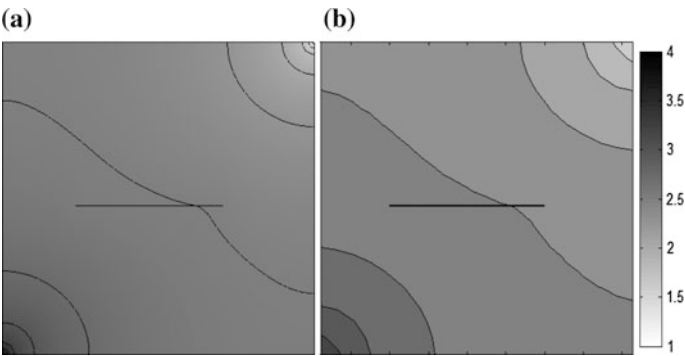


Fig. 2.36 Pressure field distribution by these two methods (KPa). **a** Discrete fracture model, **b** the embedded discrete fracture model (MFD)

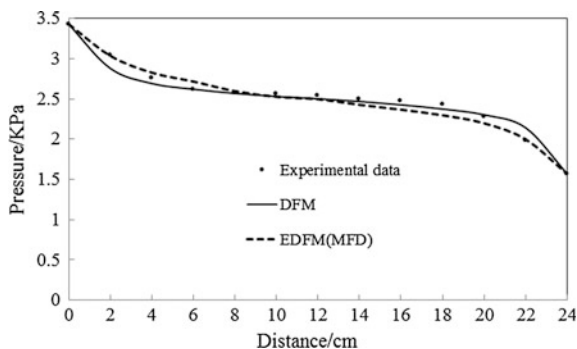


Fig. 2.37 Comparison of pressure distribution on the injection–production diagonal

that model is difficult to achieve completely sealed and quartz sand is hard to be completely homogeneous filling in the model. Therefore, the results will have certain error.

(2) Irregular quadrilateral fractured reservoir

As shown in Fig. 2.38a, it is an irregular quadrilateral fractured reservoir geometry model, matrix permeability is full tensor format. Figure 2.38b, c, respectively, are the embedded triangular mesh which treats the fracture as inner boundary and matched triangular mesh which do not consider fracture subdivision. The basic parameters of the model are as follows: the fracture permeability is $K_f = 1 \times 10 \mu\text{m}^2$, fracture aperture is $a = 1 \text{ mm}$, fluid viscosity is 1 mPa s , and rock permeability is $K_m = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \times 10^{-3} \mu\text{m}^2$.

Based on the above two kinds of grid system, we, respectively, use the discrete fracture model (Fig. 2.39a) and the embedded discrete fracture model combined with simulation of finite difference (Fig. 2.39b) to do single-phase flow numerical simulation of the fractured reservoir. Figure 2.13 indicates two methods on the straight line of two-point source and sink to obtain pressure curve. In the figure, we can see that the method calculated results and the discrete fracture model reference solution are basically identical. The error norm is 2.0 %, and the local maximum

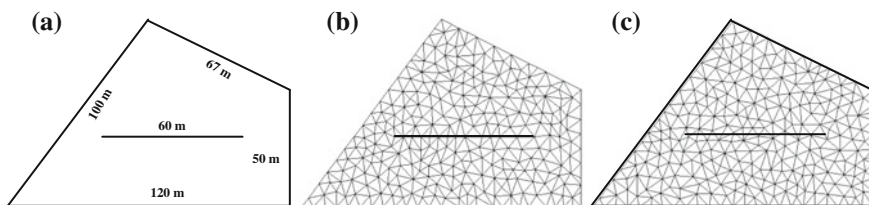


Fig. 2.38 Geometry model of fractured reservoir and result of meshing. **a** Geometry model, **b** matching triangular mesh system, **c** embedded triangular mesh system

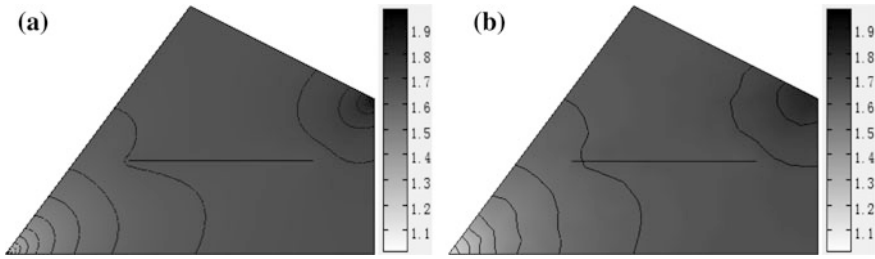


Fig. 2.39 Pressure field distributions calculated by these two methods (MPa). **a** The discrete fracture model, **b** the embedded discrete fracture model (MFD)

error norm is 5.0 %. It can be seen that the method mentioned by this paper also applies to the triangle grid system. Therefore, this method will be applicable for all kinds of complicated boundary shape fractured reservoir flow simulation when combining with a triangular mesh or hybrid mesh (Fig. 2.40).

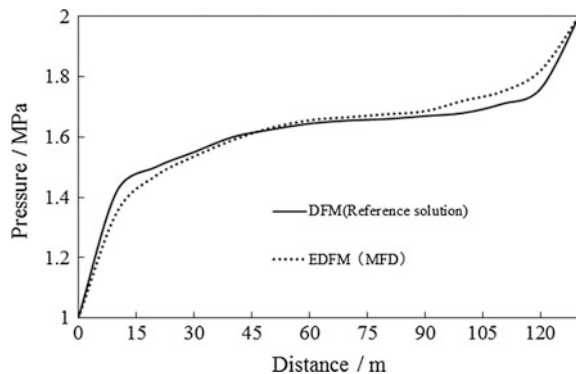
(3) Complex fractured reservoir

According to an actual fractured reservoir, fracture statistical information data include the fracture density, length, aperture, and direction; we have to generate the corresponding actual complex fractured reservoir model, as shown in Fig. 2.14, where fracture penetrates the ground in the vertical direction. Some parameters in the model are shown as follows: the fracture permeability is $K_f = 1 \times 10 \mu\text{m}^2$, fracture aperture is $a = 1 \text{ mm}$, fluid viscosity is 1 mPa s , and rock permeability is

$$K_m = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times 10^{-3} \mu\text{m}^2 \text{ (Fig. 2.41)}$$

We, respectively, use the discrete fracture model (Fig. 2.42a) and the embedded discrete fracture model (Fig. 2.42b), combined with closed constant pressure boundary condition to carry out single-phase flow numerical simulation to the complex fractured reservoir. Figures 2.43 and 2.44, respectively, show pressure

Fig. 2.40 Comparison of pressure distribution on the injection–production diagonal



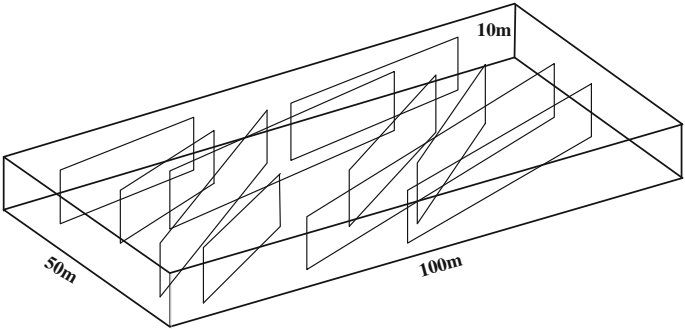


Fig. 2.41 Complex fractured reservoir model

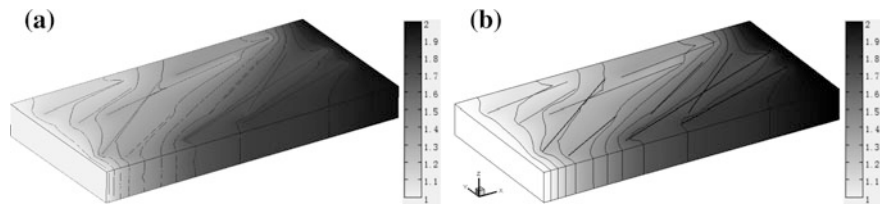


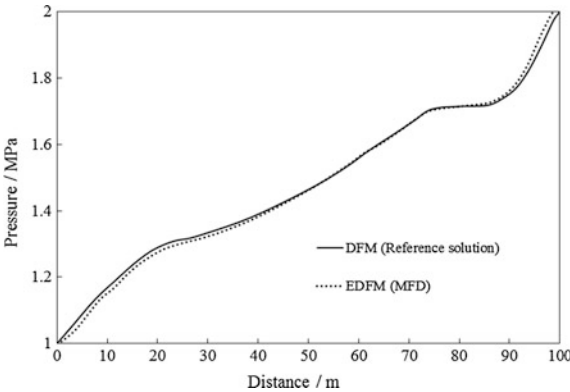
Fig. 2.42 Pressure field distributions by the discrete fracture model and MFD (MPa). **a** The discrete fracture model, **b** the embedded discrete fracture model (MFD)

curve on the two straight lines $y = 26.25\text{ m}$ and $x = 48.75\text{ m}$, which are measured by these two methods. And both results are basically identical.

(4) Calculation example of two-phase flow fractured reservoirs

Based on the embedded discrete fracture single-phase flow model and its solution, and combining the saturation equation by limited volume method in Sect. 2.4.3, the embedded discrete fracture model can be extended to two-phase flow simulation. It is worth noting that this model adapts upstream weighting method to calculate

Fig. 2.43 Comparison of pressure field distribution on the line $y = 26.2\text{ m}$



coefficient of fluidity of two-phase flow channeling item in between fracture and matrix.

Three-dimensional fractured reservoir geometry model is shown in Fig. 2.45, whose size is $40\text{ m} \times 100\text{ m} \times 100\text{ m}$ ($x \times y \times z$). In the reservoir, there are six large fractures, and to the matrix permeability, we should consider scalar form and full tensor form. Parameters of the model are shown in Table 2.2.

We used the embedded discrete fracture model for Water flooding Displacement numerical simulation of the fractured reservoir, and we have, respectively, considered two forms of matrix permeability, scalar form and full tensor form. Figure 2.46 shows the water saturation distribution in the two forms of matrix permeability when exchange of injection water is 0.5 PV. Figure 2.47 shows the injection–production relation curve under different conditions.

Fig. 2.44 Comparison of pressure field distribution on the line $x = 48.75\text{ m}$

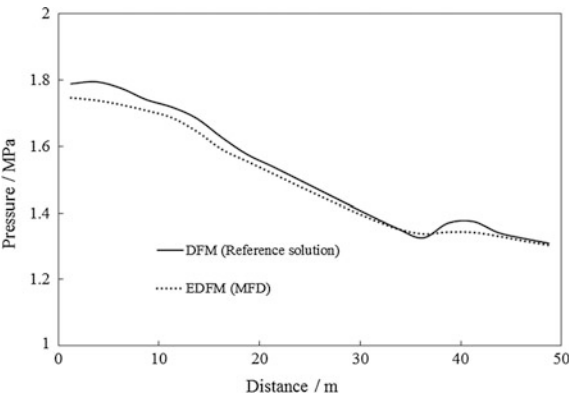


Fig. 2.45 Sketch of geometrical of the 3-D fractured porous medium

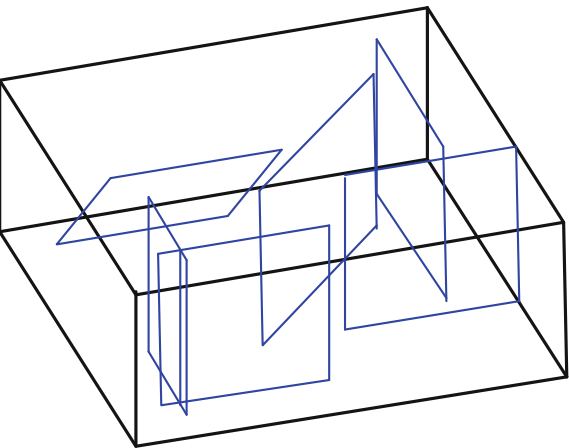


Table 2.2 The parameters of physics models

Name of properties	Value
Matrix properties	$\phi_m = 0.4, K_m = 1 \times 10^{-15} \text{ m}^2, \mathbf{K}_m = \begin{bmatrix} 1 & 0.5 & 0.8 \\ 0.8 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix} \times 10^{-15} \text{ m}^2$
Fracture properties	$\phi_f = 1.0, K_f = 8.33 \times 10^{-8} \text{ m}^2, a_f = 1 \times 10^{-3} \text{ m}$
Fluid properties	$\mu_w = \mu_o = 1.0 \text{ mPa s}, \rho_w = \rho_o = 1000 \text{ kg/m}^3$
Residual saturations in matrix and fractures	$S_{wc} = 0.0, S_{or} = 0.0$
Relative permeabilities in matrix and fractures	$k_{rw} = S_e, k_{ro} = 1 - S_e, S_e = (1 - S_w)/(1 - S_{wc} - S_{or})$
Capillary pressure	Neglected
Water injection and oil production rates	0.01 PV/d

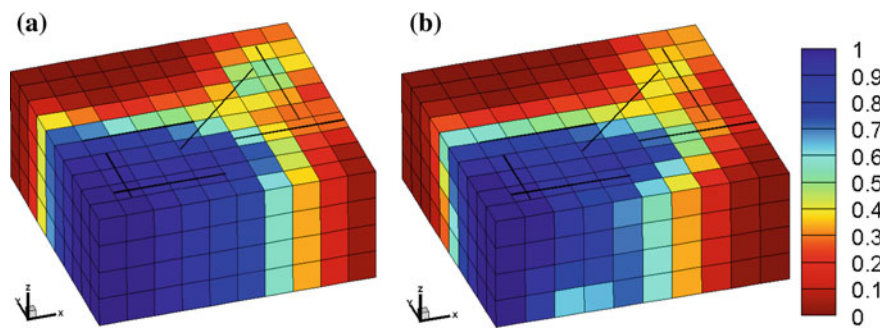
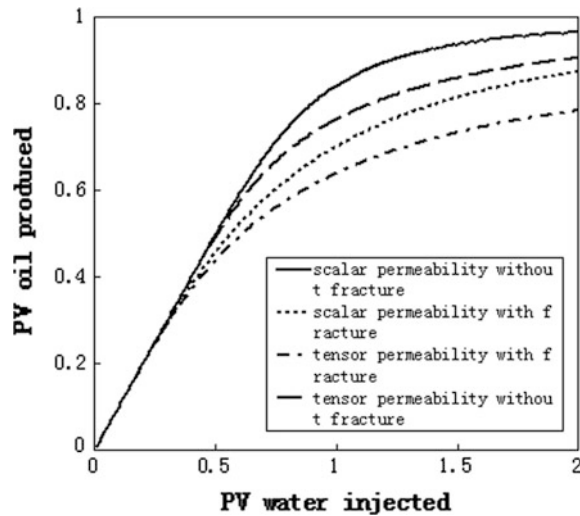


Fig. 2.46 Water saturation profiles after 0.5 PV water injection with different rock permeability. **a** Scalar permeability field, **b** tensor permeability field

Fig. 2.47 Cumulative oil production curve



2.6 Summary and Remarks

(1) Discrete fracture model gives an explicit representation to each fracture on the medium, and has good advantages of high calculation precision and good accuracy. But the large amount of calculation is its disadvantage. With the rapid development of computer technology, based on this model, detailed flow simulation will be possible. At the same time, this model, as a tool, can obtain related parameters of double medium and the equivalent medium model. So, it has broad application prospects. Based on the concept of equivalent single fracture, we have set up the discrete fracture model in this section, and elaborated the basic principle of the model. And a variety of numerical solution of the model is given, including the finite element method, finite volume method, and simulation finite difference method. The correctness of the model and algorithm is verified by calculation examples.

(2) The finite volume method needs to be simplified and equivalent process in fracture's intersections makes calculation accuracy to reduce during large-scale computation; finite element method has certain defects, in the aspects of conservation-type format structure and calculation stability; simulation finite difference method is only based on a single grid node and surface information when structuring numerical computational formulation, and it is applicable to any complex grid system in theory. It also has a good local conservation, and can be applied to the discrete fracture flow simulation research, which indicates that it has a broad application prospect.

(3) The embedded discrete fracture model do not need to consider the reservoir fracture morphology when partitioning grid. It only needs to do simple grid subdivision on the matrix system. Therefore, it can effectively avoid the situation of poor quality of the grid which is caused by too much small distance or angle between the fractures. This model needs to compute fluid channeling information between the fracture element and the matrix grid, however, it treats fracture as the inner boundary and the constraint to do unstructured grid subdivision relative to the discrete fracture model, so that its complexity of grid is reduced greatly, which improves efficiently the computational efficiency. From simulation finite difference method to the embedded discrete fracture model, the latter overcomes the limitations of the former. The limitations of the former are as follows: based on the finite difference method, it cannot effectively deal with the permeability of full tensor, and it is not applicable to complicated boundary shape fractured reservoir.

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